7

Mathematics

Teacher's Guide

This instructional material was collaboratively developed and reviewed by educators from public and private schools, colleges, and/or universities. We encourage teachers and other education stakeholders to email their feedback, comments, and recommendations to the Department of Education at action@deped.gov.ph.

We value your feedback and recommendations.

Department of Education
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Teacher's Guide
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First Quarter

GRADE 7 MATH TEACHING GUIDE

Lesson I: SETS: AN INTRODUCTION
Pre-requisite Concepts: Whole numbers
Objectives:

In this lesson, you are expected to:

1. Describe and illustrate
   a. well-defined sets;
   b. subsets;
   c. universal set, and;
   d. the null set.
2. Use Venn Diagrams to represent sets and subsets.

> **NOTE TO THE TEACHER:**
> **This lesson looks easy and fast to teach but don't be deceived. The introductory concepts are always crucial. What differentiates a set from any group is that a set is well defined. Emphasize this to the students.**
>
> **You may vary the activity by giving them a different set of objects to group. You may make this into a class activity by showing a poster of objects in front of the class or even make it into a game. The idea is for them to create their own well-defined groups according to what they see as common characteristics of elements in a group.**

Lesson Proper:
A.

I. Activity

Below are some objects. Group them as you see fit and label each group.

Answer the following questions:

a. How many groups are there?
b. Does each object belong to a group?
c. Is there an object that belongs to more than one group? Which one?

NOTE TO THE TEACHER:
You need to follow up on the opening activity hence, the problem below is important. Ultimately, you want students to apply the concepts of sets to the set of real numbers.

The groups are called sets for as long as the objects in the group share a characteristic and are thus, well defined.

Problem: Consider the set consisting of whole numbers from 1 to 200. Let this be set U. Form smaller sets consisting of elements of U that share a different characteristic. For example, let E be the set of all even numbers from 1 to 200.

Can you form three more such sets? How many elements are there in each of these sets? Do any of these sets have any elements in common?

Did you think of a set with no element?

NOTE TO THE TEACHER:
Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

Important Terms to Remember

The following are terms that you must remember from this point on.

1. A **set** is a well-defined group of objects, called **elements** that share a common characteristic. For example, 3 of the objects above belong to the set of head covering or simply hats (ladies hat, baseball cap, hard hat).
2. The set *F* is a **subset** of set *A* if all elements of *F* are also elements of *A*. For example, the even numbers 2, 4 and 12 all belong to the set of whole numbers. Therefore, the even numbers 2, 4, and 12 form a subset of the set of whole numbers. *F* is a **proper subset** of *A* if *F* does not contain all elements of *A*.
3. The **universal set** *U* is the set that contains all objects under consideration.
4. The **null set** $\varnothing$ is an empty set. The null set is a subset of any set.
5. The **cardinality of a set *A*** is the number of elements contained in *A*.

Notations and Symbols

In this section, you will learn some of the notations and symbols pertaining to sets.

1. Uppercase letters will be used to name sets and lowercase letters will be used to refer to any element of a set. For example, let H be the set of all objects on page 1 that cover or protect the head. We write

   H = {ladies hat, baseball cap, hard hat}

This is the listing or roster method of naming the elements of a set.

Another way of writing the elements of a set is with the use of a descriptor. This is the rule method. For example, *H* = {x| x covers and protects the head}. This is read as “the set H contains the element x such that x covers and protects the head.”

2. The symbol $\varnothing$ or { } will be used to refer to an empty set.
3. If *F* is a subset of *A*, then we write $F \subseteq A$. We also say that *A* contains the set *F* and write it as $A \supseteq F$. If *F* is a proper subset of *A*, then we write $F \subset A$.
4. The cardinality of a set *A* is written as n(*A*).

II. Questions to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER: **It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered.**

Let us answer the questions posed in the opening activity.

1. How many sets are there?

There is the set of head covers (hats), the set of trees, the set of even numbers, and the set of polyhedra. But, there is also a set of round objects and a set of pointy objects. There are 6 well-defined sets.

2. Does each object belong to a set? *Yes.*

3. Is there an object that belongs to more than one set? Which ones?

All the hats belong to the set of round objects. The pine trees and two of the polyhedra belong to the set of pointy objects.

III. Exercises

Do the following exercises. Write your answers on the spaces provided:

1. Give 3 examples of well-defined sets.

Possible answers: The set of all factors of 24, The set of all first year students in this school, The set of all girls in this class.

2. Name two subsets of the set of whole numbers using both the listing or roster method and the rule method.

Example:
Listing or Roster Method:
E = {0, 2, 4, 6, 8, ….}
O = {1, 3, 5, 7, …}

Rule Method:
E = {2x | x is a whole number}
O = {2x+1 | x is a whole number}

3. Let *B* = [1, 3, 5, 7, 9}. List <u>all</u> the possible subsets of *B*.

{ }, {1}, {3}, {5}, {7}, {9}, {1, 3}, {1, 5}, {1, 7}, {1, 9}, {3, 5}, {3, 7}, {3, 9}, {5, 7}, {5, 9}, {7, 9}, {1, 3, 5}, {1, 3, 7}, {1, 3, 9}, {3, 5, 7}, {3, 5, 9}, {5, 7, 9}, {1, 5, 7}, {1, 5, 9}, {1, 7, 9}, {3, 7, 9}, {1, 3, 5, 7}, {1, 3, 5, 9}, {1, 5, 7, 9}, {3, 5, 7, 9}, {1, 3, 7, 9}, {1, 3, 5, 7, 9} – 32 subsets in all.

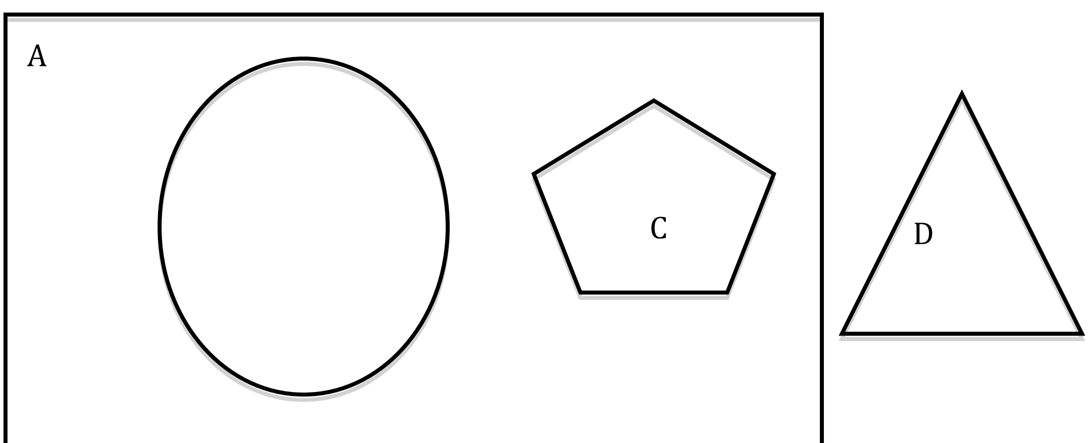
4. Answer this question: How many subsets does a set of n elements have?**There are2^n subsets in all.**

B. Venn Diagrams

NOTE TO THE TEACHER: **A lesson on sets will not be complete without using Venn Diagrams. Note that in this lesson, you are merely introducing the use of these diagrams to show sets and subsets. The extensive use of the Venn Diagrams will be introduced in the next lesson, which is on set operations. The key is for students to be able to verbalize what they see depicted in the Venn Diagrams.**

Sets and subsets may be represented using <u>Venn Diagrams</u>. These are diagrams that make use of geometric shapes to show relationships between sets.

Consider the Venn diagram below. Let the universal set U be all the elements in sets A, B, C and D.



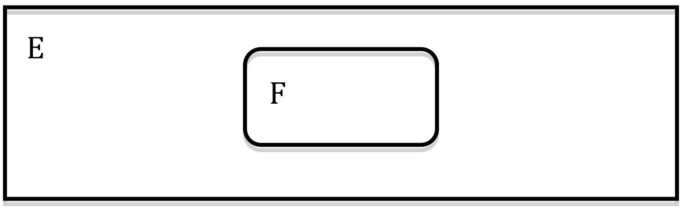
Each shape represents a set. Note that although there are no elements shown inside each shape, we can surmise how the sets are related to each other.Notice that set B is inside set A. This indicates that all elements in B are contained in A. The same with set C. Set D, however, is separate from A, B, C. What does it mean?

Exercise

Draw a Venn diagram to show the relationships between the following pairs or groups of sets:

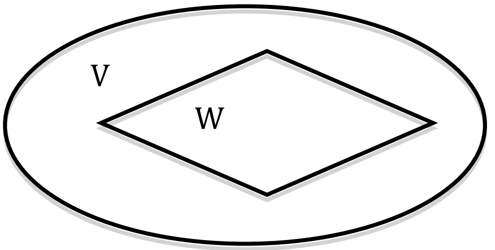
1. E = {2, 4, 8, 16, 32}
   F = {2, 32}

   Sample Answer

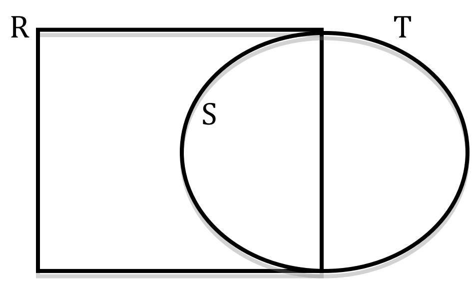
2. V is the set of all odd numbers
   W = {5, 15, 25, 35, 45, 55,….}

   Sample Answer

3. R = {x| x is a factor of 24}
   S = { }
   T = {7, 9, 11}

   Sample Answer:

NOTE TO THE TEACHER: **End the lesson with a good summary.**

Summary

In this lesson, you learned about sets, subsets, the universal set, the null set and the cardinality of the set. You also learned to use the Venn diagram to show relationships between sets.

Lesson 2.1: Union and Intersection of Sets **Time: 1.5 hours**

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

Objectives:

In this lesson, you are expected to:

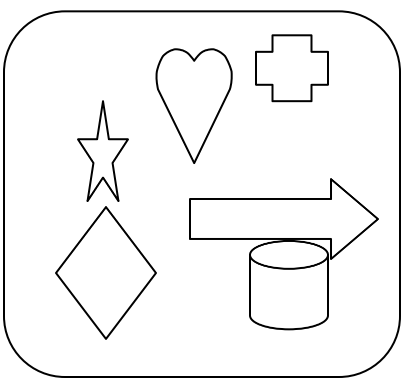
1. Describe and define
    a. union of sets;
    b. intersection of sets.
2. Perform the set operations
    a. union of sets;
    b. intersection of sets.
3. Use Venn diagrams to represent the union and intersection of sets.

Note to the Teacher:

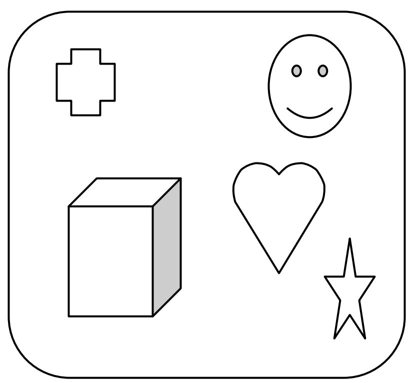
Below are the opening activities for students. Emphasize that just like with the whole number, operations are also used on sets. You may combine two sets or form subsets. Emphasize to students that in counting the elements of a union of two sets, elements that are common to both sets are counted only one.

Lesson Proper:

I. Activities



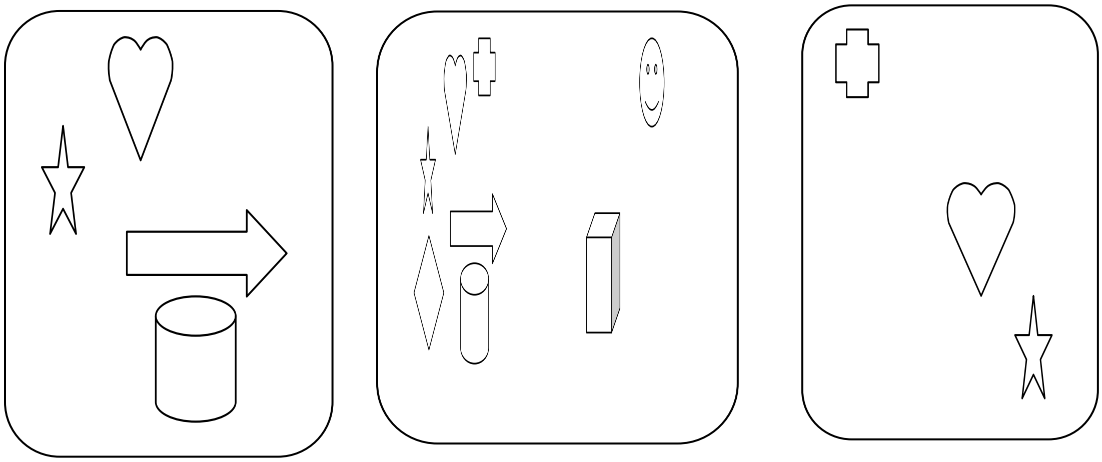
A



B

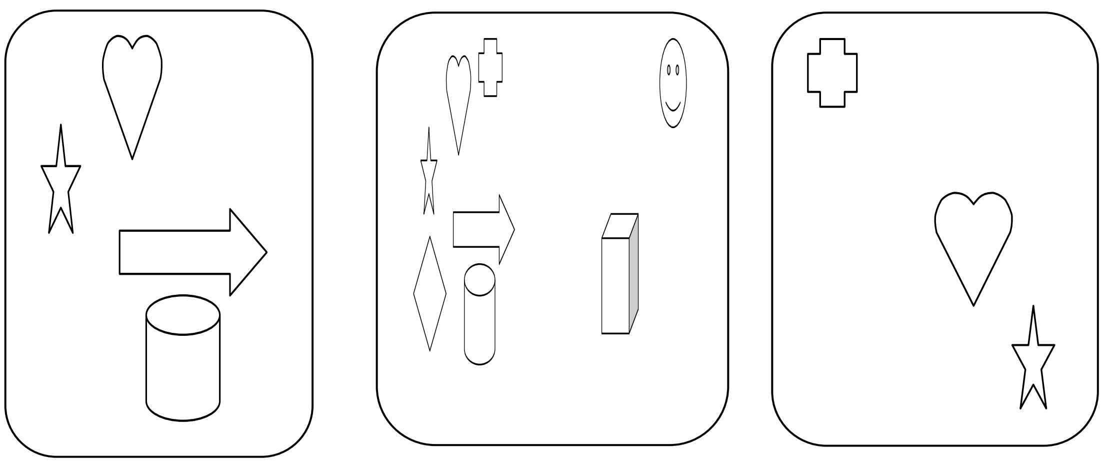
Answer the following questions:

1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?



1 **2** **3**

2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A
and B?



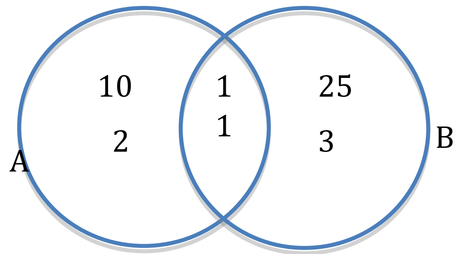
1 **2** **3**

Here's another activity:
Let

$V = \{ 2x \mid x \in I, 1 \leq x \leq 4\}$
$W = \{x^2 \mid x \in I, -2 \leq x \leq 2\}$

What elements may be found in the intersection of V and W? How many are there?
What elements may be found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?



NOTE TO THE TEACHER:
Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

Important Terms/Symbols to Remember

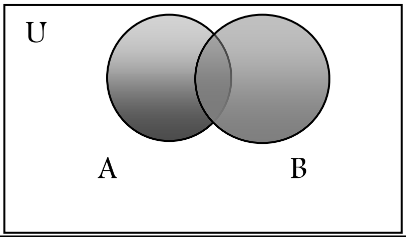
The following are terms that you must remember from this point on.

1. Let A and B be sets. The *union* of the sets A and B, denoted by A ∪ B, is the set that contains those elements that are either in A or in B, or in both.

An element x belongs to the union of the sets A and B if and only if x belongs to A or x belongs to B. This tells us that

A ∪ B = {x I x is in A or x is in B}

Venn diagram:



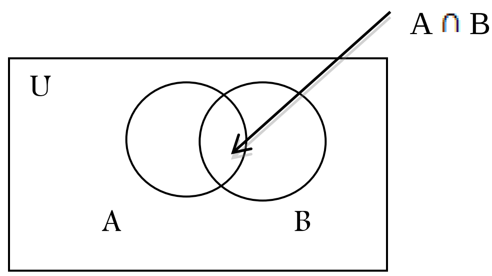
Note to the Teacher:
Explain to the students that in general, the <u>inclusive OR</u> is used in mathematics. Thus, when we say, "elements belonging to A or B", that includes the possibility that the elements belong to both. In some instances, "belonging to both" is explicitly stated when referring to the intersection of two sets. Advise students that from here onwards, OR is used inclusively.

2. Let A and B be sets. The *intersection* of the sets A and B, denoted by A ∩ B, is the set containing those elements in both A and B.

An element x belongs to the intersection of the sets A and B if and only if x belongs to A and x belongs to B. This tells us that

$A \cap B = \{x \mid x \text{ is in A and x is in B}\}$

Venn diagram:



Sets whose intersection is an empty set are called *disjoint sets*.

3. The cardinality of the union of two sets is given by the following equation:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

II. Questions to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER
It is important for you to go over the answers of your students posed in the opening activities in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered. Below are the correct answers to the questions posed in the activities.

Let us answer the questions posed in the opening activity.

1. Which of the following shows the union of set A and set B? **Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.**
2. Which of the following shows the intersection of set A and set B? **Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.**

In the second activity:
V = { 2, 4, 6, 8 }
W = { 0, 1, 4}

Therefore, $V \cap W = \{4\}$ has 1 element and $V \cup W = \{0, 1, 2, 4, 6, 8\}$ has 6 elements. Note that the element { 4 } is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements {1, 12 }; (2) The set that contains elements that belong to A or B or both consists of 6 elements {1, 10, 12, 20, 25, 36 }.

NOTE TO THE TEACHER:
Always ask for the cardinality of the sets if it is possible to obtain such number, if only to emphasize that

$$\mathbf{n(A \cup B) \neq n(A) + n(B)}$$

because of the possible intersection of the two sets. In the exercises below, use every opportunity to emphasize this. Discuss the answers and make sure students understand the "why" of each answer.

III. Exercises

1. Given sets A and B,

Set A **Students who play the guitar**	**Set B** **Students who play the piano**
Ethan Molina	Mayumi Torres
Chris Clemente	Janis Reyes
Angela Dominguez	Chris Clemente
Mayumi Torres	Ethan Molina
Joanna Cruz	Nathan Santos

determine which of the following shows (a) union of sets A and B; and (b) intersection of sets A and B?

Set 1	**Set 2**	**Set 3**	**Set 4**
Ethan Molina Chris Clemente Angela Dominguez Mayumi Torres Joanna Cruz	Mayumi Torres Ethan Molina Chris Clemente	Mayumi Torres Janis Reyes Chris Clemente Ethan Molina Nathan Santos	Ethan Molina Chris Clemente Angela Dominguez Mayumi Torres Joanna Cruz Janis Reyes Nathan Santos

Answers: (a) Set 4. There are 7 elements in this set. (b) Set 2. There are 3 elements in this set.

2. Do the following exercises. Write your answers on the spaces provided:

$A = \{0, 1, 2, 3, 4\}$ $B = \{0, 2, 4, 6, 8\}$ $C = \{1, 3, 5, 7, 9\}$

Answers:

Given the sets above, determine the elements and cardinality of:

a. $A \cup B =$ **$\{0, 1, 2, 3, 4, 6, 8\}$;** $n(A \cup B) =$ **7**
b. $A \cup C =$ **$\{0, 1, 2, 3, 4, 5, 7, 9\}$;** $n(A \cup C) =$ **8**
c. $A \cup B \cup C =$ **$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$;** $n(A \cup B \cup C) =$ **10**
d. $A \cap B =$ **$\{0, 2, 4\}$;** $n(A \cap B) =$ **3**
e. $B \cap C =$ **$\emptyset$;** $n(B \cap C) =$ **0**
f. $A \cap B \cap C =$ **$\emptyset$;** $n(A \cap B \cap C) =$ **0**
g. $(A \cap B) \cup C =$ **$\{0, 1, 2, 3, 4, 5, 7, 9\}$;** $n((A \cap B) \cup C) =$ **8**

NOTE TO THE TEACHER:

In Exercise 2, you may introduce the formula for finding the cardinality of the union of 3 sets. But, it is also instructive to give students the chance to discover this on their own. The formula for finding the cardinality of the union of 3 sets is:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

3. Let $W = \{ x \mid 0 < x < 3 \}$, $Y = \{ x \mid x > 2\}$, and $Z = \{x \mid 0 \le x \le 4 \}$.

Determine (a) $(W \cup Y) \cap Z$; (b) $W \cap Y \cap Z$.

Answers:

Since at this point students are more familiar with whole numbers and fractions greater than or equal to 0, use a partial real numberline to show the elements of these sets.

(a) $(W \cup Y) \cap Z =$ $\mathbf{\{x \mid 0 < x \le 4\}}$

(b) $W \cap Y \cap Z =$ $\mathbf{\{x \mid 2 < x < 3\}}$

NOTE TO THE TEACHER:

End with a good summary. Provide more exercises on finding the union and intersection of sets of numbers.

Summary

In this lesson, you learned about the definition of union and intersection of sets. You learned also how to use Venn diagrams to represent the unions and the intersection of sets.

Lesson 2.2: Complement of a Set **Time: 1.5 hours**

Pre-requisite Concepts: sets, universal set, empty set, union and intersection of sets, cardinality of sets, Venn diagrams

About the Lesson:

The complement of a set is an important concept. There will be times when one needs to consider the elements not found in a particular set A. You must know that this is when you need the complement of a set.

Objectives:

In this lesson, you are expected to:

1. Describe and define the complement of a set;
2. Find the complement of a given set;
3. Use Venn diagrams to represent the complement of a set.

NOTE TO THE TEACHER
Review the concept of universal set before introducing this lesson. Emphasize to the students that there are situations when it is more helpful to consider the elements found in the universal set that are not part of set A.

Lesson Proper:

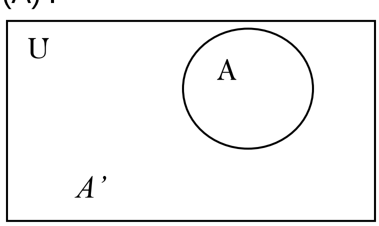
I. Problem

In a population of 8000 students, 2100 are Freshmen, 2000 are Sophomores, 2050 are Juniors and the remaining 1850 are either in their fourth or fifth year in university. A student is selected from the 8000 students and it is not a Sophomore, how many possible choices are there?

Discussion

Definition: The complement of a set A, written as *A'*, is the set of all elements found in the universal set, U, that are not found in set A. The cardinality n (*A'*) is given by

$$n(A') = n(U) - n(A).$$



Examples:

1. Let U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, and A = {0, 2, 4, 6, 8}.

Then the elements of A' are the elements from U that are not found in A.
Therefore, $A' = \{1, 3, 5, 7, 9\}$

2. Let $U = \{1, 2, 3, 4, 5\}$, $A = \{2, 4\}$ and $B = \{1, 5\}$. Then,
   $A' = \{1, 3, 5\}$
   $B' = \{2, 3, 4\}$
   $A' \cup B' = \{1, 2, 3, 4, 5\} = U$

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 7, 8\}$. Then,
   $A' = \{5, 6, 7, 8\}$
   $B' = \{1, 2, 5, 6\}$
   $A' \cap B' = \{5, 6\}$

4. Let $U = \{1, 3, 5, 7, 9\}$, $A = \{5, 7, 9\}$ and $B = \{1, 5, 7, 9\}$. Then,
   $A \cap B = \{5, 7, 9\}$
   $(A \cap B)' = \{1, 3\}$

5. Let U be the set of whole numbers. If $A = \{x \mid x$ is a whole number and $x > 10\}$, then
   $A' = \{x \mid x$ is a whole number and $0 \leq x \leq 10\}$.

The opening problem asks for how many possible choices there are for a student that was selected and known to be a non-Sophomore. Let U be the set of all students and n (U) = 8000. Let A be the set of all Sophomores then n (A) = 2000. The set A' consists of all students in U that are not Sophomores and n (A') = n (U) – n (A) = 6000. Therefore, there are 6000 possible choices for that selected student.

NOTE TO THE TEACHER:

Pay attention to how students identify the elements of the complement of a set. Teach them that a way to check is to take the union of a set and its complement. The union is the universal set U. That is, $A \cup A' = U$. Recall to them as well that $n(A \cup A') = n(A) + n(A') - n(A \cap A') = n(A) + n(A') = n(U)$ since $A \cap A' = \varnothing$ and therefore, $n(A \cap A') = 0$.

In the activity below, use Venn diagrams to show how the different sets relate to each other so that it is easier to identify unions and intersections of sets and complements of sets or complements or unions and intersections of sets. Watch as well the language that you use. In particular, *(A ∪ B)'* is read as "the complement of the union of A and B"

whereas *A'* ∪ *B'* is read as the union of the complement of A and the complement of B."

II. Activity

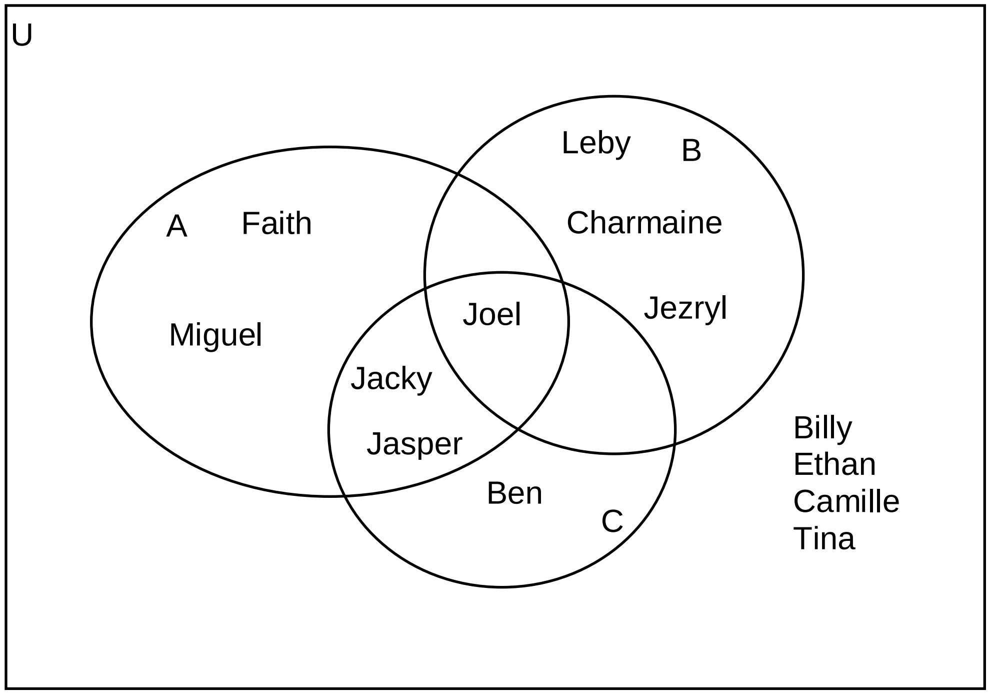
Shown in the table are names of students of a high school class by sets according to the definition of each set.

A **Like Singing**	**B** **Like Dancing**	**C** **Like Acting**	**D** **Don't Like Any**
Jasper Faith Jacky Miguel Joel	Charmaine Leby Joel Jezryl	Jacky Jasper Ben Joel	Billy Ethan Camille Tina

After the survey has been completed, find the following sets.

a. U =
b. A ∪ B' =
c. A' ∪ C =
d. (B ∪ D)' =
e. A' ∩ B =
f. A' ∩ D' =
g. (B ∩ C)' =

The easier way to find the elements of the indicated sets is to use a Venn diagram showing the relationships of U, sets A, B, C, and D. Set D does not share any members with A, B, and C. However, these three sets share some members. The Venn diagram below is the correct picture:



Now, it is easier to identify the elements of the required sets.

a. U = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Joel, Leby, Miguel, Tina}
b. A ∪ *B'* = {Faith, Miguel, Joel, Jacky, Jasper, Ben, Billy, Ethan, Camille, Tina}
c. *A'* ∪ C = {Jasper, Jacky, Joel, Ben, Leby, Charmaine, Jezryl, Billy, Ethan, Camille, Tina}
d. *(B* ∪ *D)'* = {Faith, Miguel, Jacky, Jasper, Ben}
e. *A'* ∩ B = {Leby, Charmaine, Jezryl}
f. *A'* ∩ *D'* = {Leby, Charmaine, Jezryl, Ben}
g. *(B* ∩ *C)'* = {Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Leby, Miguel, Tina}

NOTE TO THE TEACHER

Below are the answers to the exercises. Encourage discussions among students. Take note of the language they use. It is important that students say the words or phrases correctly. Whenever appropriate, use Venn diagrams.

III. Exercises

1. True or False. If your answer is false, give the correct answer.

Let U = the set of the months of the year

X = {March, May, June, July, October}

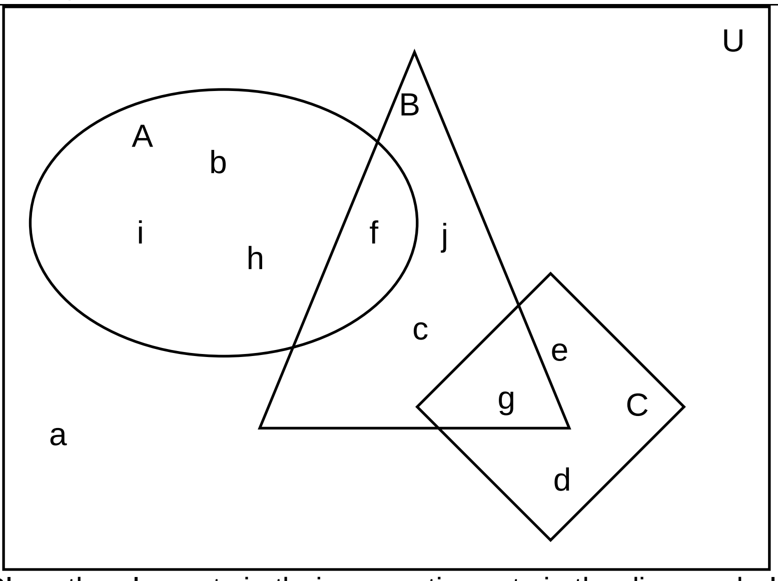
Y = {January, June, July}

Z = {September, October, November, December}

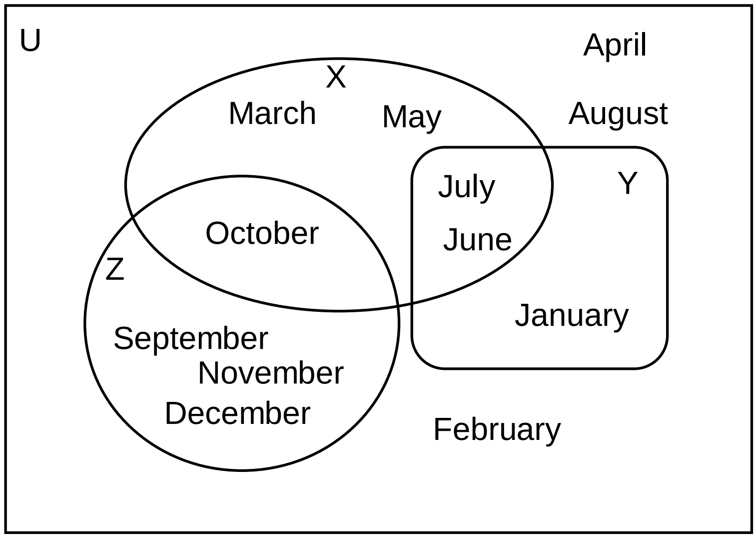
a. Z' = {January, February, March, April, May, June, July, August} **<u>True</u>**
b. X' ∩ Y' = {June, July} **<u>False. X' ∩ Y' = {February, April, August, September, November, December}</u>**
c. X' ∪ Z' = {January, February, March, April, May, June, July, August, September, November, December} **<u>True</u>**
d. (Y ∪ Z)' = {February, March, April, May} **<u>False. (Y ∪ Z)' = {February, March, April, May, August}.</u>**

NOTE TO THE TEACHER

The next exercise is a great opportunity for you to develop students' reasoning skills. If the complement of A, the complement of B and the complement of C all contain the element <u>a</u> then <u>a</u> is outside all three sets but within U. If *B'* and *C'* both contain b but *A'* does not, then A contains b. This kind of reasoning must be clear to students.



2. Place the elements in their respective sets in the diagram below based on the following elements assigned to each set:

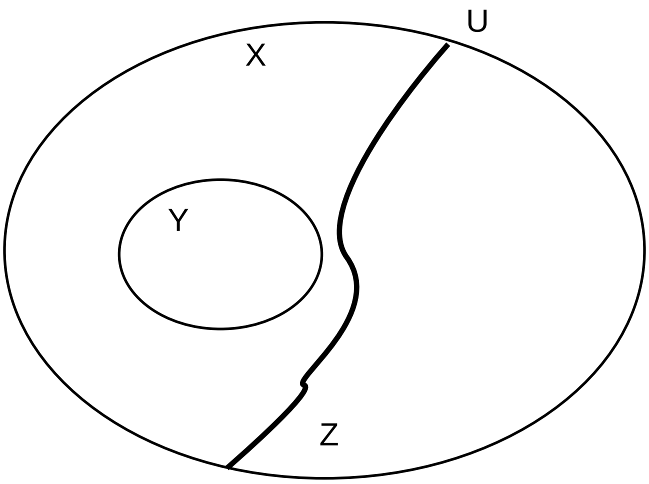


U = {a, b, c, d, e, f, g, h, i, j}
A’ = {a, c, d, e, g, j}
B’ = {a, b, d, e, h, i}
C’ = {a, b, c, f, h, i, j}

NOTE TO THE TEACHER:
In Exercise 3, there are many possible answers. Ask students to show all their work. This is a good opportunity for them to argue and justify their answers. Engage them in meaningful discussions. Encourage them to explain their work. Help them decide which diagrams are correct.

3. Draw a Venn diagram to show the relationships between sets U, X, Y, and Z, given the following information.

- U, the universal set contains set X, set Y, and set Z.
- $X \cup Y \cup Z = U$
- Z is the complement of X.
- Y’ includes some elements of X and the set Z



NOTE TO THE TEACHER
End with a good summary.

Summary

In this lesson, you learned about the complement of a given set. You learned how to describe and define the complement of a set, and how it relates to the universal set, U and the given set.

Lesson 3: **Problems Involving Sets** **Time: 1 hour**

Prerequisite Concepts: Operations on Sets and Venn Diagrams

Objectives:

In this lesson, you are expected to:

1. Solve word problems involving sets with the use of Venn diagrams
2. Apply set operations to solve a variety of word problems.

NOTE TO THE TEACHER
This is an important lesson. Do not skip it. This lesson reinforces what students learned about sets, set operations and the Venn diagram in solving problems.

Lesson Proper:

I. Activity

Try solving the following problem:

In a class of 40 students, 17 have ridden an airplane, 28 have ridden a boat. 10 have ridden a train, 12 have ridden both an airplane and a boat. 3 have ridden a train only and 4 have ridden an airplane only. Some students in the class have not ridden any of the three modes of transportation and an equal number have taken all three.

a. How many students have used all three modes of transportation?
b. How many students have taken only the boat?

NOTE TO THE TEACHER
Allow students to write their own solutions. Allow them to discuss and argue. In the end, you have to know how to steer them to the correct solution.

II. Questions/Points to Ponder (Post-Activity Discussion)

Venn diagrams can be used to solve word problems involving union and intersection of sets. Here are some worked out examples:

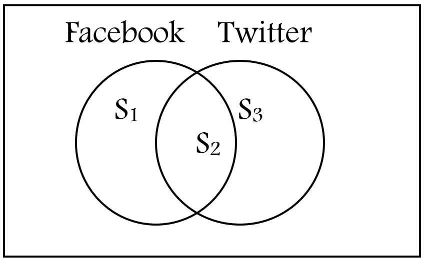
1. A group of 25 high school students were asked whether they use either Facebook or Twitter or both. Fifteen of these students use Facebook and twelve use Twitter.

a. How many use Facebook only?
b. How many use Twitter only?
c. How many use both social networking sites?

<u>Solution:</u>

Let S_1 = set of students who use Facebook only
S_2 = set of students who use both social networking sites
S_3 = set of students who use Twitter only

The Venn diagram is shown below



Finding the elements in each region:

$n(S_1) + n(S_2) + n(S_3) = 25$
$n(S_1) + n(S_2) = 15$

$n(S_3) = 10$

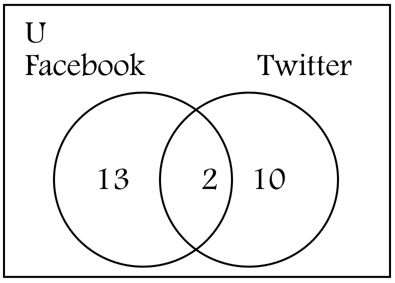
But $n(S_2) + n(S_3) = 12$

$n(S_2) = 2$

$n(S_1) + n(S_2) + n(S_3) = 25$
$n(S_2) + n(S_3) = 12$

$n(S_1) = 13$

The number of elements in each region is shown below



2. A group of 50 students went in a tour in Palawan province. Out of the 50 students, 24 joined the trip to Coron; 18 went to Tubbataha Reef; 20 visited El Nido; 12 made a trip to Coron and Tubbataha Reef; 15 saw Tubbataha Reef and El Nido; 11 made a trip to Coron and El Nido and 10 saw the three tourist spots.
    a. How many of the students went to Coron only?
    b. How many of the students went to Tubbataha Reef only?
    c. How many joined the El Nido trip only?
    d. How many did not go to any of the tourist spots?

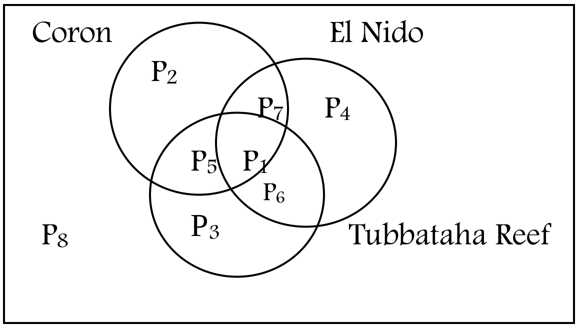
<u>Solution:</u>

To solve this problem, let

P_1 = students who saw the three tourist spots
P_2 = those who visited Coron only
P_3 = those who saw Tubbataha Reef only

P_4 = those who joined the El Nido trip only
P_5 = those who visited Coron and Tubbataha Reef only
P_6 = those who joined the Tubbataha Reef and El Nido trip only
P_7 = those who saw Coron and El Nido only
P_8 = those who did not see any of the three tourist spots

Draw the Venn diagram as shown below and identify the region where the students went.



Determine the elements in each region starting from P_1.

P_1 consists of students who went to all 3 tourist spots. Thus, $n(P_1) = 10$.

$P_1 \cup P_5$ consists of students who visited Coron and Tubbataha Reef but this set includes those who also went to El Nido. Therefore, $n(P_5) = 12 – 10 = 2$ students visited Coron and Tubbatha Reef only.

$P_1 \cup P_6$ consists of students who went to El Nido and Tubbataha Reef but this set includes those who also went to Coron. Therefore, $n(P_6) = 15 – 10 = 5$ students visited El Nido and Tubbataha Reef only.

$P_1 \cup P_7$ consists of students who went to Coron and El Nido but this set includes those who also went to Tubbataha Reef. Therefore, $n(P_7) = 11 – 10 = 1$ student visited Coron and El Nido only.

From here, it follows that

$n(P_2) = 24 – n(P_1) – n(P_5) – n(P_7) = 24 – 10 – 2 – 1 =$ *11 students visited Coron only.*

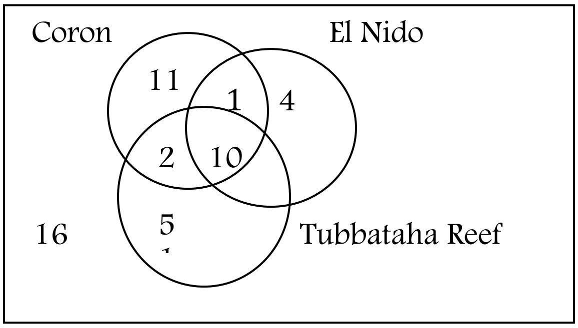
$n(P_3) = 18 – n(P_1) – n(P_5) – n(P_6) = 18 – 10 – 2 – 5 =$ *1 student visited Tubbataha Reef only*

$n(P_4) = 20 – n(P_1) – n(P_6) – n(P_7) = 20 – 10 – 5 – 1 =$ *4 students visited Coron and El Nido only.*

Therefore

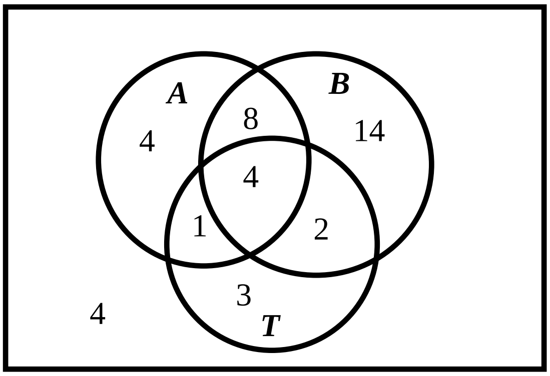
$n(P_8) = 50 – n(P_1) – n(P_2) – n(P_3) – n(P_4) – n(P_5) – n(P_6) – n(P_7) =$ *16 students did not visit any of the three spots.*

The number of elements is shown below.



Now, what about the opening problem? Solution to the Opening Problem (Activity):

Can you explain the numbers?



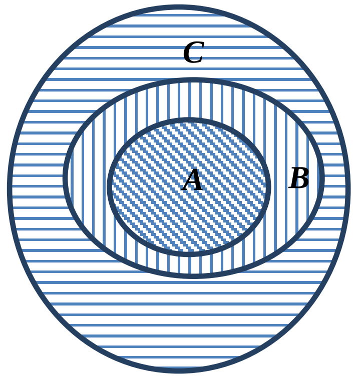
NOTE TO THE TEACHER

Discuss the solution thoroughly and clarify all questions your students might have. Emphasize the notation for the cardinality of a set.

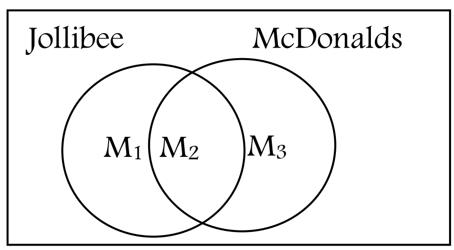
III. Exercises

Do the following exercises. Represent the sets and draw a Venn diagram when needed.

1. If **A** is a set, give two subsets of **A**. **Answer: ∅ and A**
2. **(a)** If A and B are finite sets and $A \subset B$, what can you say about the cardinalities of the two sets?
   (b) If the cardinality of A is less than the cardinality of B, does it follow that $A \subset B$?
   Answer: (a) $n(A) < n(B)$; **(b) No. Example:** $\boldsymbol{A = \{1, 2\}}$, $\boldsymbol{B = \{2, 4, 6\}}$
3. If **A** and **B** have the same cardinality, does it follow that **A = B?** Explain.
   Answer: Not necessarily. Example, A = {1, 2, 3} and B = {4, 8, 9}.
4. If $A \subset B$ and $B \subset C$. Does it follow that $A \subset C$? Illustrate your reasoning using a Venn diagram. **Answer: Yes.**



5. Among the 70 kids in Barangay Magana, 53 like eating in Jollibee while 42 like eating in McDonalds. How many like eating both in Jollibee and in McDonalds? In Jollibee only? In McDonalds only?



<u>Solution:</u>

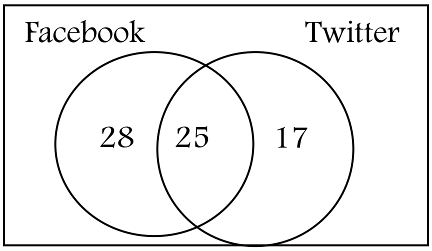
Let $n(M_1)$ = kids who like Jollibee only
$n(M_2)$ = kids who like both Jollibee and McDonalds
$n(M_3)$ = kids who like McDonalds only

Draw the Venn diagram

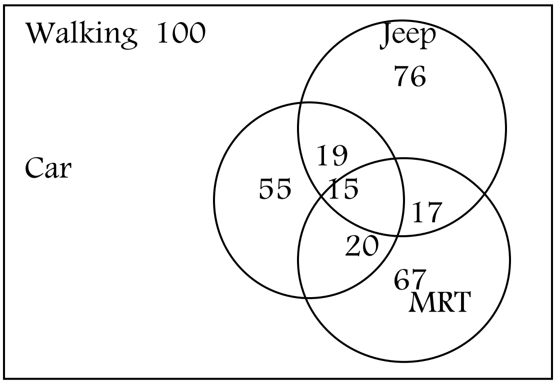
Find the elements in each region

$$\begin{array}{lr} n(M_1) + n(M_2) + n(M_3) & = 70 \\ n(M_1) + n(M_2) & = 53 \\ \hline n(M_3) & = 17 \\ \text{But} \quad n(M_2) + n(M_3) & = 42 \\ \hline n(M_2) & = 25 \end{array} \qquad \begin{array}{lr} n(M_1) + n(M_2) + n(M_3) & = 70 \\ n(M_2) + n(M_3) & = 42 \\ \hline n(M_1) & = 28 \end{array}$$

Check using Venn diagram



6. The following diagram shows how all the First Year students of Maningning High School go to school.

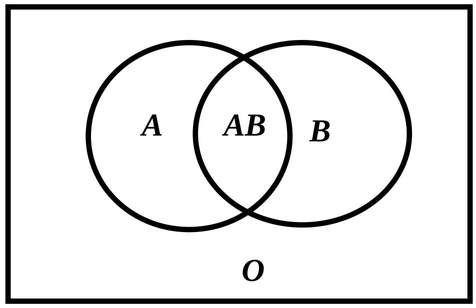


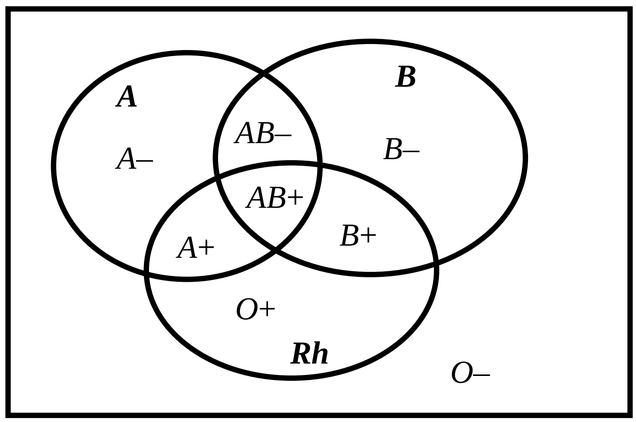
a. How many students ride in a car, jeep and the MRT going to their school? **15**
b. How many students ride in both a car and a jeep? **34**
c. How many students ride in both a car and the MRT? **35**
d. How many students ride in both a jeep and the MRT? **32**
e. How many students go to school
   in a car only **55** a jeep only **76**
   in the MRT only **67** walking **100**
f. How many students First Year students of Maningning High School are there? **269**

7. The blood-typing system is based on the presence of proteins called antigens in the blood. A person with antigen A has blood type A. A person with antigen B has blood type B, and a person with both antigens A and B has blood type AB. If no antigen is present, the blood type is O. Draw a Venn diagram representing the ABO System of blood typing.

   A protein that coats the red blood cells of some persons was discovered in 1940. A person with the protein is classified as Rh positive (Rh+), and a person whose blood cells lack the protein is Rh negative (Rh–). Draw a

Venn diagram illustrating all the blood types in the ABO System with the corresponding Rh classifications.





NOTE TO THE TEACHER

The second problem is quite complex. Adding the 3rd set Rh captures the system without altering the original diagram in the first problem.

Summary

In this lesson, you were able to apply what you have learned about sets, the use of a Venn diagram and set operations in solving word problems.

Lesson 4.1: Fundamental Operations on Integers: Addition of Integers
Time: 1 hour

Pre-requisite Concepts: Whole numbers, Exponents, Concept of Integers

Objectives:

In this lesson, you are expected to:

1. Add integers using different approaches;
2. Solve word problems involving addition of integers.

NOTE TO THE TEACHER

This lesson is a review and deepening of the concept of addition of integers. Keep in mind that the definitions for the operations on integers must retain the properties of the same operations on whole numbers or fractions. In this sense, the operations are merely extended to cover a bigger set of numbers. We present here two models for addition that have been used to represent addition of whole numbers.

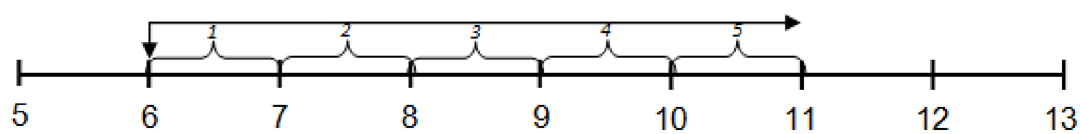
Lesson Proper:

I. Activity

Study the following examples:

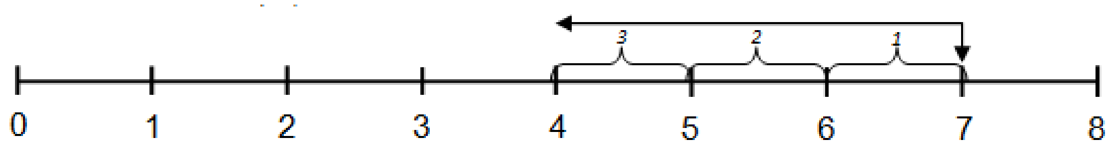
A. Addition Using Number Line

1. Use the number line to find the sum of 6 & 5.



On the number line, start with point 6 and count 5 units to the right. At what point on the number line does it stop ?
It stops at point 11; hence, 6 + 5 = 11.

2. Find the sum of 7 and (-3) .



On the number line, start from 7 and count 3 units going to the left since the sign of 3 is negative.
At which point does it stop?
It stops at point 4; hence, (-3) + (7) = 4.

After the 2 examples, can you now try the next two problems?

a. (-5) + (-4) b. (-8) + (5)

NOTE TO THE TEACHER

More examples may be given if needed to emphasize an interpretation of the negative sign as a direction to the left of the number line.

We now have the following generalization:
Adding a positive integer n to m means moving along the real line a distance of n units to the right from m. Adding a negative integer $-n$ to m means moving along the real line a distance of n units to the left from m.

NOTE TO THE TEACHER

Other objects might be used in this next activity. Signed tiles could be algebra tiles or counters with different colors on each side. Bottle caps are easily obtained and will be very good visual and hands-on materials.

B. Addition Using Signed Tiles

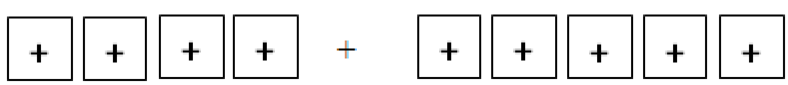
This is another device that can be used to represent integers.The tile $\boxed{+}$ represents integer 1, the tile $\boxed{-}$ represents -1, and the flexible $\boxed{+ \quad -}$ represents 0.

Recall that a number and its negative cancel each other under the operation of addition. This means
$4 + (-4) = 0$
$15 + (-15) = 0$
$(-29) + 29 = 0$
In general, $n + (-n) = (-n) + n = 0$.

NOTE TO THE TEACHER

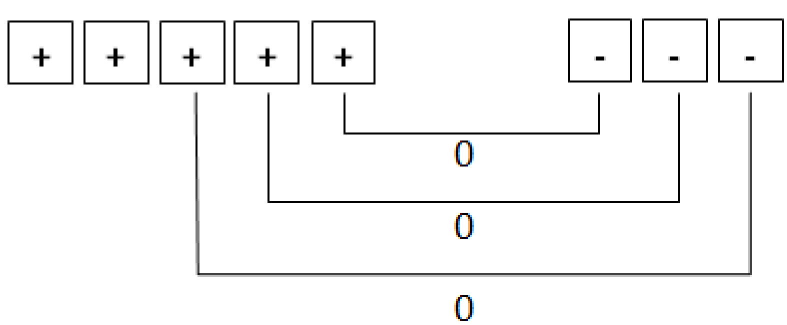
Get the students to model the above equations using signed tiles or colored counters.

Examples:

1. 4 + 5 ------→ 

four (+1) + five (+1)

hence, 4 + 5 = 9

2. 5 + (-3) -----→ 

hence, $5 + (-3) = 2 + 3 + (-3) = 2 + 0 = 2$

3. $(-7) + (-6)$

$$\boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ + \ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}$$

hence $(-7) + (-6) = -13$

Now, try these:

1. (-5) + (-11)
2. (6) + (-9)

Solution:

1. (–5) + (–11)

$$\boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ + \ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}$$

hence, (–5) + (–11) = –16.

2. (6) + (–9)

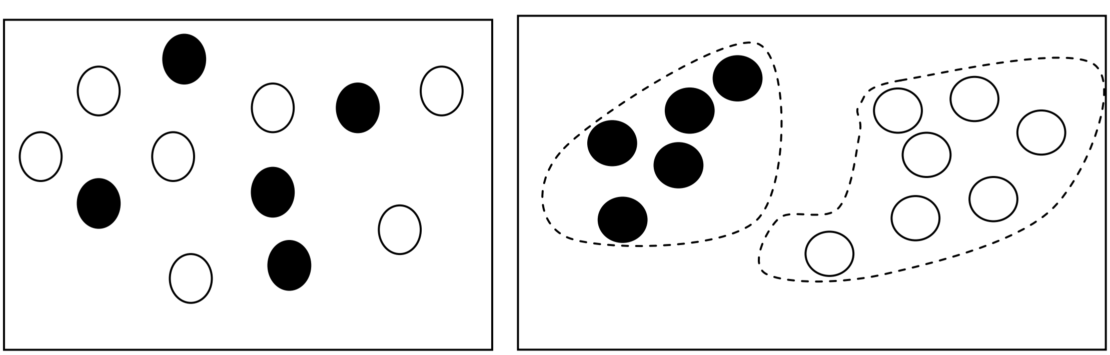
$$\underbrace{\boxed{+}\ \boxed{+}\ \boxed{+}\ \boxed{+}\ \boxed{+}\ \boxed{+}\ + \ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}}_{0}\ \boxed{-}\ \boxed{-}\ \boxed{-}$$

hence, (6) + (–9) = –3.

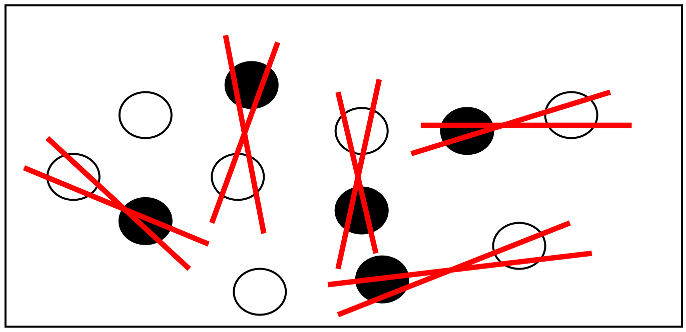
If colored counters (disks) or bottle caps are used, one side of the counter denotes "positive" while the other side denotes "negative". For example, with counters having black and red sides, black denotes "positive", while red denotes "negative". For this module, we will use white instead of red to denote negative.

Examples:

1. The configurations below represent $5 + (-7)$

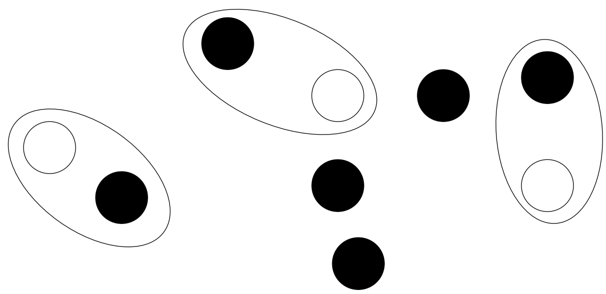


Keeping in mind that a black disk and a white disk cancel each other, take out pairs consisting of a black and a white disk until there are no more pairs left.



This tells us that $5 + (-7) = -2$

2. Give a colored-counter representation of $(-3) + 6$



Therefore, $(-3) + 6 = 3$

The signed tiles model gives us a very useful procedure for adding large integers having different signs.

Examples:

1. $-63 + 25$

   Since 63 is bigger than 25, break up 63 into 25 and 38.

   Hence $-63 + 25 = -38 + (-25) + 25 = -38 + 0 = -38$

2. $724 + (-302) = 422 + 302 + (-302) = 422 + 0 = 422$

II. Questions/ Points to Ponder

Using the above model, we summarize the procedure for adding integers as follows:

1. If the integers have the same sign, just add the positive equivalents of the integers and attach the common sign to the result.

   a. 27 + 30 = + (/27/ + /30/)
      = + (/57/)
      = + 57

   b. (-20) + (-15) = - (/20/ + /15/)
      = - (20 + 15)
      = - (35)
      = - 35

2. If the integers have different signs, get the difference of the positive equivalents of the integers and attach the sign of the larger number to the result.

   a. (38) + (-20)
      Get the difference between 38 and 20: 18
      Since 38 is greater than 20, the sign of the sum is positive.
      Hence $38 + (-20) = 18$

   b. $(-42) + 16$
      Get the difference between 42 and 16: 26
      Since 42 is greater than 16, the sum will have a negative sign.
      Hence $(-42) + 16 = -26$

NOTE TO THE TEACHER **Provide more examples as needed.**

If there are more than two addends in the problem the first step to do is to combine addends with same signs and then get the difference of their sums.
Examples:

1. $(-14) + (22) + (8) + (-16) = -(14 + 16) + (22 + 8)$
   $= -30 + 30 = 0$
2. $31 + 70 + 9 + (-155) = (31 + 70 + 9) + (-155)$
   $= 110 + (-155) = -45$

III. Exercises

A. Who was the first English mathematician who first used the modern symbol of equality in 1557?

(To get the answer, compute the sums of the given exercises below. Write the letter of the problem corresponding to the answer found in each box at the bottom).

A 25 + 95	C. (30) + (-20)	R 65 + 75
B 38 + (-15)	D. (110) + (-75)	O (-120) + (-35)
O 45 + (-20)	T. (16) + (-38)	R (165) + (-85)
R (-65) + (-20)	R (-65) + (-40)	E 47 + 98
E (78) + (-15)	E (-75) + (20)	

-105	25	63	23	-85	-22

140	-55	10	-155	80	35	145

Answer: ROBERT RECORDE

B. Addthe following:

1. (18) + (-11) + (3)
2. (-9) + (-19) + (-6)
3. (-4) + (25) + (-15)
4. (50) + (-13) + (-12)
5. (-100) + (48) + (49)

Answers:
1. 10 2. –34 3. 6 4. 25 5. –3

C. Solve the following problems:

1. Mrs. Reyes charged P3,752.00 worth of groceries on her credit card. Find her balance after she made a payment of P2,530.00.

   Answer: PhP1,222.00
2. In a game, Team Azcals lost 5 yards in one play but gained 7 yards in the next play. What was the actual yardage gain of the team?**Answer: (-5)+7=2 yards**

3. A vendor gained P50.00 on the first day; lost P28.00 on the second day, and gained P49.00 on the third day. How much profit did the vendor gain in 3 days?**Answer: 50+(-28)+49=71. Profit is PhP71.00**

4. Ronnie had PhP2280 in his checking account at the beginning of the month. He wrote checks for PhP450, P1200, and PhP900. He then made a deposit of PhP1000. If at any time during the month the account is overdrawn, a PhP300 service charge is deducted. What was Ronnie's balance at the end of the month?

   Answer: 2280+(-450)+(-1200)+(-900)=-270
   (-270)+(-300)+1000=430
   Balance is PhP430.00

NOTE TO THE TEACHER
Summarize the two models used in this lesson. It is always good to keep these models in mind but make sure that students learn to let go of these models and should be able to add integers eventually even without these models.

Summary
In this lesson, you learned how to add integers using two different methods. The number line model is practical for small integers. For larger integers, the signed tiles model provides a more useful tool.

Lesson 4.2: Fundamental Operation on Integers:Subtraction of Integers
Time: 1 hour

Prerequisite Concepts: Whole numbers, Exponents, Concept of Integers, Addition of Integers

About the Lesson: This lesson focuses on the subtraction of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:

In this lesson, you are expected to:

1. Subtract integers using
   a. Number line
   b. Signed tiles
2. Solve problems involving subtraction of integers.

> **NOTE TO THE TEACHER**
> **This lesson is a continuation of lesson 4.1 in a sense that mastery of the law of signs in addition of integers makes subtraction easy for the learners. Emphasis must be given to how the law of signs in addition is connected to that of subtraction.**

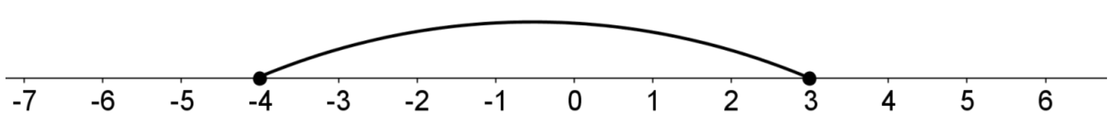
Lesson Proper:
I. Activity

Study the material below.

1. *Subtraction as the reverse operation of addition.*

   Recall how subtraction is defined. We have previously defined subtraction as the reverse operation of addition. This means that when we ask "what is 5 minus 2?", we are also asking "what number do we add to 2 in order to get 5?" Using this definition of subtraction, we can deduce how subtraction is done using the number line.

   a. Suppose you want to compute $(-4) - 3$. You ask "What number must be added to 3 to get -4?

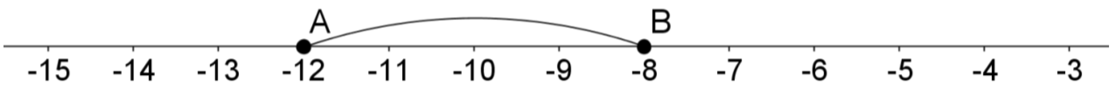


To get from 3 to -4, you need to move 7 units to the left. This is equivalent to adding -7 to 3. Hence in order to get -4, -7 must be added to 3. Therefore,

$(-4) - 3 = -7$

   b. Compute $(-8) - (-12)$

What number must be added to -12 to get -8?



To go from -12 to -8, move 4 units to the right, or equivalently, add 4. Therefore,
$(-8) - (-12) = 4$

2. *Subtraction as the addition of the negative*

   Subtraction is also defined as the addition of the negative of the number. For example, $5 - 3 = 5 + (-3)$. Keeping in mind that n and $-n$ are negatives of each other, we can also have $5 - (-3) = 5 + 3$. Hence the examples above can be solved as follows:

$(-4) - 3 = (-4) + (-3) = -7$

$(-8) - (-12) = (-8) + 12 = 4$

This definition of subtraction allows the conversion of a subtraction problem to an addition problem.

NOTE TO THE TEACHER

You need to follow up on the opening activity, hence the problem below is important to reinforce what was discussed.

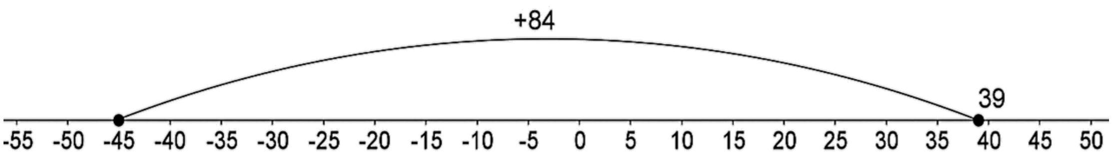
Problem:

Subtract (-45) from 39 using the two definitions of subtraction.

Can you draw your number line?Where do you start numbering it to make the line shorter?

Solution:

1. $39 - (-45)$



What number must be added to -45 in order to obtain 39?

$39 - (-45) = 84$

2. $39 - (-45) = 39 + 45 = 84$

II. Questions/Points to Ponder

Rule in Subtracting Integers

In subtracting integers, add the negative of the subtrahend to the minuend,

$a - b = a + (-b)$

$a - (-b) = a + b$

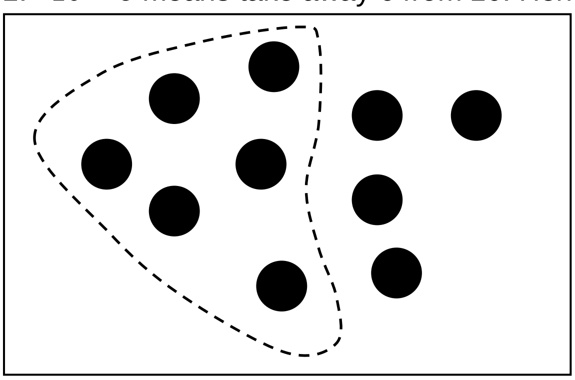
> **NOTE TO THE TEACHER**
> **Give more examples as needed. The next section relies on the use of colored counters or signed tiles. You, the teacher, should study the material so that you may be able to guide yoiur students in understanding the use of these tiles correctly.**

Using signed tiles or colored counters

Signed tiles or colored counters can also be used to model subtraction of integers. In this model, the concept of subtraction as "taking away" is utilized.

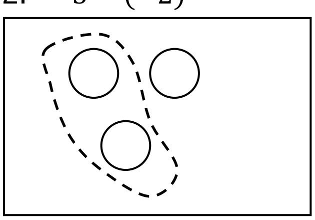
Examples:

1. $10 - 6$ means take away 6 from 10. Hence



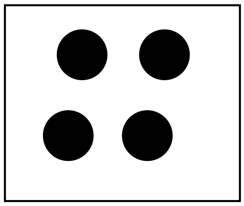
$10 - 6 = 4$

2. $-3 - (-2)$

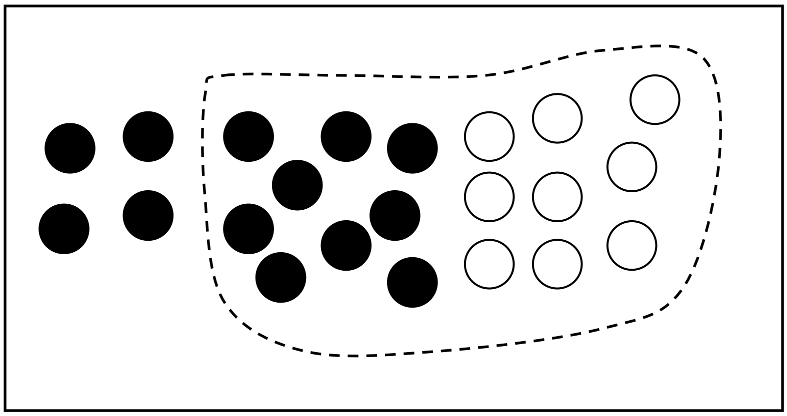


$-3 - (-2) = -1$

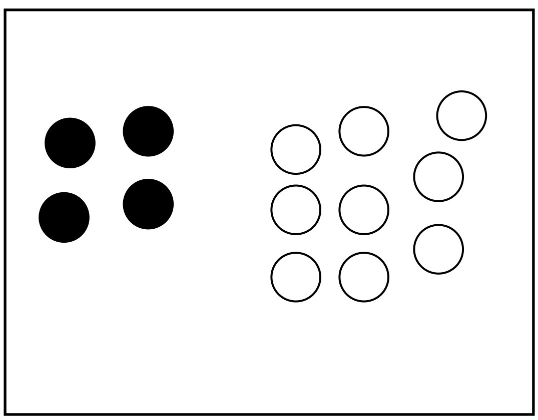
3. $4 - 9$



Since there are not enough counters from which to take away 9, we add 9 black counters and 9 white counters. Remember that these added counters are equivalent to zero.

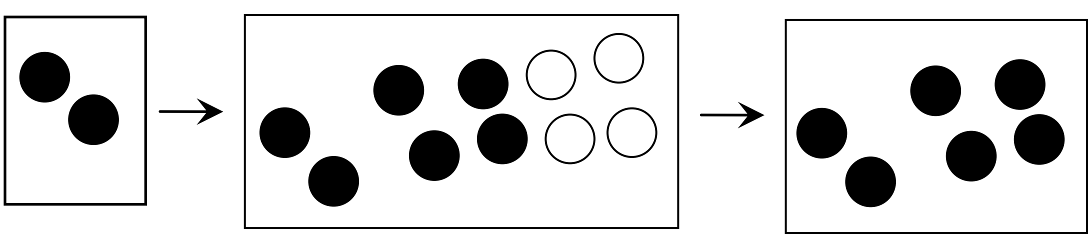


We now take away 9 black counters.



Notice that this configuration is the same configuration for$4 + (-9)$. We proceed with the addition and obtain the answer -5

4. $2 - (-4)$



Hence $2 - (-4) = 6$

The last two examples above illustrate the definition of subtraction as the addition of the negative.
$m - n = m - n + [n + (-n)] = [m - n + n] + (-n) = m + (-n)$

III. Exercices

A. What is the name of the 4th highest mountain in the world?

(Decode the answer by finding the difference of the following subtraction problems. Write the letter to the answer corresponding to the item in the box provided below:

O Subtract (-33) from 99

L Subtract (-30) from 49

H 18 less than (-77)

E Subtract (-99) from 0

T How much is 0 decreased by (-11)?

S (-42) – (-34) – (-9) - 18

79	-95	132	11	-17	99

Answer: LHOTSE

B. Mental Math

Give the difference:

1. 53 -25
2. (-6) - 123
3. (-4) - (-9)
4. 6 - 15
5. 16 - (-20)
6. 25 - 43
7. (-30) - (-20)
8. (-19) - 2
9. 30 –(-9)
10. (-19) - (-15)

Answers:

1. 28 2. –129 3. 5 4. –9 5. 36

6. –18 7. –10 8. –21 9. 39 10. –4

C. Solve the ff. Problems:

1. Maan deposited P53,400.00 in her account and withdrew P19,650.00 after a week. How much of her money was left in the bank?

   Answer: PhP33,750.00

2. Two trains start at the same station at the same time. Train A travels 92km/h, while train B travels 82km/h. If the two trains travel in opposite directions, how far apart will they be after an hour?If the two trains travel in the same direction, how far apart will they be in two hours?

   Answer: 92-(-82)=174 km apart

   2×92-2×82=20 km apart

3. During the Christmas season. The student gov't association was able to solicit 2,356 grocery items and was able to distribute 2,198 to one barangay. If this group decided to distribute 1,201 grocery items to the next barangay, how many more grocery items did they need to solicit?

   Answer: 2356-2198=158 left after the first barangay

   1201-158=1043 needed for the second barangay

NOTE TO THE TEACHER

To end, emphasize the new ideas that this lesson discussed, particularly the new concepts of subtraction and how these concepts allow the conversion of subtraction problems to addition problems.

Summary

In this lesson, you learned how to subtract integers by reversing the process of addition, and by converting subtraction to addition using the negative of the subtrahend.

Lesson 4.3: Fundamental Operations on Integers: Multiplication of Integers
Time: 1 hour

Prerequisite Concepts: Operations on whole numbers, addition and subtraction of integers

About the Lesson: This is the third lesson on operations on integers. The intent of the lesson is to deepen what students have learned in Grade 6, by expounding on the meaning of multiplication of integers.

Objective:

In this lesson; you are expected to:

1. Multiply integers.
2. Apply multiplication of integers in solving problems

NOTE TO THE TEACHER
The repeated addition model for multiplication can be extended to multiplication of two integers in which one of the factors is positive. However, for products in which both factors are negative, repeated addition does not have any meaning. Hence multiplication of integers will be discussed in two parts: the first part looks into products with at least one positive factor, while the second studies the product of two negative integers.

Lesson Proper:

I. Activity

Answer the following question.

How do we define multiplication?

We learned that with whole numbers, *multiplication is repeated addition*. For example, 4×3 means three groups of 4. Or, putting it into a real context, 3 cars with 4 passengers each, how many passenger in all? Thus
$4 \times 3 = 4 + 4 + 4 = 12$.

But, if there are 4 cars with 3 passengers each, in counting the total number of passengers, the equation is $3 \times 4 = 3 + 3 + 3 + 3 = 12$. We can say then that $4 \times 3 = 3 \times 4$and

$4 \times 3 = 3 \times 4 = 3 + 3 + 3 + 3 = 12$.

We extend this definition to multiplication of a negative integer by a positive integer. Consider the situation when a boy loses P6 for 3 consecutive days.His total loss for three days is
$(-6) \times 3$. Hence, we could have
$(-6) \times 3 = (-6) + (-6) + (-6) = -18$.

II. Questions/Points to Ponder

The following examples illustrate further how integers are multiplied.

Example 1. Multiply : 5 ×(-2)

However,

5 × (-2) = (-2) × (5)

Therefore:

(-2) × (5)= (-2) + (-2) + (-2) + (-2) + (-2) = -10

The result shows that the product of a negative multiplier and a positive multiplicand is a negative integer.

Generalization:Multiplying unlike signs

We know that adding negative numbers means adding their positive equivalents and attaching the negative sign to the result, then

$$a \times (-b) = (-b) \times a = \underbrace{(-b) + (-b) + \cdots + (-b)}_{a addends} = -\underbrace{(b + b + \cdots + b)}_{a addends} = -ab$$

for any positive integers a and b.

We know that any whole number multiplied by 0 gives 0. Is this true for any integer as well? The answer is YES. In fact, any number multiplied by 0 gives 0. This is known as the ***Zero Property***.

FOR THE TEACHER: PROOF OF THE ZERO PROPERTY

Since 1 is the identity for multiplication, for any integer a, a×1=a.
The identity for addition is 0, so a×1=a×(1+0)=a .
By the distributive law, a×(1+0)=a×1+a×0=a.
Hence a+a×0=a.
Now 0 is the only number which does not change a on addition. Therefore a×0=0.

What do we get when we multiply two negative integers?

Example 2. Multiply: (-8) × (-3)

We know that $(-8) \times 3 = -24$.

Therefore,

$$\begin{aligned} -24 + (-8) \times (-3) &= (-8) \times 3 + (-8) \times (-3) \\ &= (-8) \times [3 + (-3)] \textit{(Distributive Law)} \\ &= (-8) \times 0 \quad \textit{(3and } -3 \textit{ are additive inverses)} \\ &= 0 \quad \textit{(Zero Property)} \end{aligned}$$

The only number which when added to -24 gives 0 is the additive inverse of -24. Therefore, $(-8) \times (-3)$ is the additive inverse of 24, or

$$(-8) \times (-3) = 24$$

The result shows that the product of two negative integers is a positive integer.

NOTE TO THE TEACHER

The above argument can be generalized to obtain the product (-a)×(-b). The proof may be presented to more advanced students.It is important to note that the definition of the product of two negative integers is not based on the same model as the product of whole numbers (i.e., repeated addition). The basis for the definition of the product of two negative numbers is the preservation of the properties or axioms of whole number operations (distributive law, identity and inverse property).

Generalization:Multiplying Two Negative Integers
If a and b are positive integers, then $(-a) \times (-b) = ab$.

Rules in Multiplying Integers:

In multiplying integers, find the product of their positive equivalents.

1. If the integers have the same signs, their product is positive.
2. If the integers have different signs their product is negative.

III. Exercises

A. Find the product of the following:

1. (5)(12)
2. (-8)(4)
3. (-5)(3)(2)
4. (-7)(4)(-2)
5. (3)(8)(-2)
6. (9)(-8)(-9)
7. (-9)(-4)(-6)

Answers:

1. 60	**2. –32**	**3. –30**	**4. 56**
5. –48	**6. 648**	**7. –216**	

MATH DILEMMA

B. How can a person fairly divide 10 apples among 8 children so that each child has the same share.

To solve the dilemma, match the letter in column II with the number that corresponds to the numbers in column I.

Column I	Column II	
1. (6)(-12)	C	270
2. (-13)(-13)	P	-72
3. (19)(-17)	E	300
4. (-15)(29)	K	-323
5. (165)(0)	A	-435
6. (-18)(-15)	M	0
7. (-15)(-20)	L	16
8. (-5)(-5)(-5)	J	-125
9. (-2)(-2)(-2)(-2)	U	169
10. (4)(6)(8)	I	192

____ ____ ____ ____
5 4 3 7

____ ____ ____ ____ ____
4 1 1 9 7

____ ____ ____ ____ ____
8 2 10 6 7

Answer: MAKE APPLE JUICE

C. Problem Solving

1. Jof has twenty P5 coins in her coin purse. If her niece took 5 of the coins, how much has been taken away?
   Answer: PhP25 $(\mathbf{5 \times 5 = 25})$
2. Mark can type 45 words per minute, how many words can Mark type in 30 minutes?
   Answer: 1350 words $(\mathbf{45 \times 30 = 1350})$
3. Give an arithmetic equation which will solve the following
   a. The messenger came and delivered 6 checks worth PhP50 each. Are you richer or poorer? By how much?
   b. The messenger came and took away 3 checks worth PhP120 each. Are you richer or poorer? By how much?
   c. The messenger came and delivered 12 bills for PhP86 each. Are you richer or poorer? By how much?
   d. The messenger came and took away 15 bills for PhP72 each. Are you richer or poorer? By how much?

Answers:

a. $\mathbf{6 \times 50 = 300}$	**Richer by PhP300**
b. $\mathbf{-3 \times 120 = -360}$	**Poorer by PhP360**
c. $\mathbf{12 \times (-86) = -1032}$	**Poorer by PhP1032**
d. $\mathbf{(-15) \times (-72) = 1080}$	**Richer by PhP1080**

NOTE TO THE TEACHER
Give additional problems and drills, if only to reinforce the rules for multiplying integers. Summarize by emphasizing as well the different types of problems given in this lesson.

Summary

This lesson emphasized the meaning of multiplication to set the rules for multiplying integers. To multiply integers, first find the product of their positive equivalents. If the integers have the same signs, their product is positive. If the integers have different signs their product is negative.

Lesson 4.4: Fundamental Operations on Integers: Division of Integers
Time: 1 hour

Prerequisite Concepts: Addition and subtraction of Integers, Multiplication of Integers

Objective:

In this lesson you are expected to:

1. Find the quotient of two integers.
2. Solve problems involving division of integers.

> **NOTE TO THE TEACHER**
> **This is a short lesson because the sign rules for division of integers are the same as with the multiplication of integers. Division is to be understood as the reverse operation of multiplication, hence making the rules the same with respect to the sign of the quotient.**

Lesson Proper:

I. Activity

Answer the following questions:

What is (-51) ÷ (-3)?
What is (-51) ÷ 3?
What is 51 ÷ (-3)?
What are the rules in dividing integers?

> **NOTE TO THE TEACHER**
> **This exercise emphasizes the need to remember the sign rules for dividing integers.**

II. Questions/Points to Ponder

We have learned that Subtraction is the inverse operation of Addition,
In the same manner, Division is the inverse operation of Multiplication.

Example 1. Find the quotient of (-51) and (-3)

Solution:

Since division is the inverse of multiplication, determine whatnumber multiplied by (-3) produces (-51).

If we ignore the signs for the meantime, we know that

$$3 \times 17 = 51$$

We also know that in order to get a negative product, the factors must have different signs. Hence$(-3) \times 17 = -51$

Therefore

$$(-51) \div (-3) = 17$$

Example 2. What is $(-57) \div 19$?

Solution: $19 \times 3 = 57$

Hence

$$19 \times (-3) = -57$$

Therefore

$$(-57) \div 19 = -3$$

Example 3. Show why $273 \div (-21) = -13$.
Solution: $(-13) \times (-21) = 57$
Therefore, $273 \div (-21) = -13$

NOTE TO THE TEACHER
It is important to give more examples to students. Always, ask students to explain or justify their answers.

Generalization

The quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.However, division by zero is not possible.

NOTE TO THE TEACHER
Since we introduced division as the reverse operation of multiplication, it is now easy to show why division by 0 is not possible.
What is (-10) ÷ 0? Because division is the reverse of multiplication, we must find a number such that when multiplied by 0 gives -10. But, there is no such number. In fact, no number can be divided by 0 for the same reason.

When several operations have to be performed, the GEMDAS rule applies.

Example 4. Perform the indicated operations
1. $2 - 3 \times (-4)$
2. $4 \times 5 + 72 \div (-6)$
3. $9 + 6 - (-3) \times 12 \div (-9)$

Solution:
1. $2 - 3 \times (-4) = 2 - (-12) = 14$
2. $4 \times 5 + 72 \div (-6) = 20 + (-12) = 8$
3. $9 + 6 - (-3) \times 12 \div (-9) = 9 + 6 - (-36) \div (-9) = 9 + 6 - 4 = 11$

III. Exercises:

A. Compute the following

1. $(10 + 15) - 4 \times 3 + 7 \times (-2)$
2. $22 \times 9 \div (-6) - 5 \times 8$
3. $36 \div 12 + 53 + (-30)$
4. $(30 + 26) \div [(-2) \times 7]$
5. $(124 - 5 \times 12) \div 8$

Answers:
1. –1 2. –73 3. 26 4. –4 5. 8

B. What was the original name for the butterfly?

To find the answer find the quotient of each of the following and write the letter of the problems in the box corresponding to the quotient.

R (-352) ÷

U (-120) ÷ 8

B (108) ÷ 9

T (128) ÷ -

L (-444) ÷ (-12)

Y (144) ÷ -3

E (168) ÷ 6

T (-147) ÷ 7

F (-315) ÷ (-

9	37	-15	-8	-8	28	-16	12	-48

Answer: Flutterby

C. Solvethe following problems:

1. Vergara's store earned P8750 a week, How much is her average earning in a day?**Answer: PhP1250.00 (8750÷7=1250)**
2. Russ worked in a factory and earned P7875.00 for 15 days. How much is his earning in a day?**Answer: PhP525.00 (7875÷15=525)**
3. There are 336 oranges in 12 baskets. How many oranges are there in 3 baskets?**Answer: 84 oranges (336÷12×3=84)**
4. A teacher has to divide 280 pieces of graphing paper equally among his 35 students. How many pieces of graphing paper will each student recieve?**Answer: 8 (280÷35=8)**
5. A father has 976 sq. meters lot, he has to divide it among his 4 children. What is the share of each child?**Answer: 244 sq. meters (976÷4=244)**

D. Complete the three-by-three magic square (that is, the sums of the numbers in each row, in each column and in each of the diagonals are the same) using the numbers -10, -7, -4, -3, 0, 3, 4, 7, 10. What is the sum for each row, column and diagonal?

Answer: The sum of all the numbers is 0. Hence each column/row/diagonal will have a sum of $0 \div 3 = 0$. Put 0 in the middle square. Put each number and its negative on either side of 0. A possible solution is

7	10	3
–4	0	4
–3	–10	–7

Summary

Division is the reverse operation of multiplication. Using this definition, it is easy to see that the quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.

Lesson 5: Properties of the Operations on Integers **Time: 1.5 hours**

Prerequisite Concepts: Addition, Subtraction, Multiplication and Division of Integers

Objectives

In this lesson, you are expected to:

1. State and illustrate the different properties of the operations on integers
    a. closure
    b. commutative
    c. associative
    d. distributive
    e. identitiy
    f. inverse
2. Rewrite given expressions according to the given property.

> **NOTE TO THE TEACHER:**
>
> **Operations on integers are some of the difficult topics in elementary algebra and one of the least mastered skills of students based on researches. The different activities presented in this lesson will hopefully give the students a tool for creating their own procedures in solving equations involving operations on integers. These are the basic rules of our system of algebra and they will be used in all succeeding mathematics. It is very important that students understand how to apply each property when solving math problems.**
>
> **In activities 1 and 2, the teacher will try to test the students' ability to give corresponding meaning to the different words exhibited and later on relate said terms to the lesson. In addition, students can show some creativity in activity 2.**

Lesson Proper:

I. A. Activity 1: Try to reflect on these . . .

1. Give at least 5 words synonymous to the word "property".

Activity 2: PICTIONARY GAME: DRAW AND TELL!

The following questions will be answered as you go along to the next activity.

- What properties of real numbers were shown in the Pictionary Game?

  Give one example and explain.
- How are said properties seen in real life?

NOTE TO THE TEACHER **Activity 3 gives a visual presentation of the properties.**

Activity 3: SHOW AND TELL!
Determine what kind of property of real numbers is being illustrated in the following images:
A. Fill in the blanks with the correct numerical values of the motorbike and bicycle riders.

________ ________

+

If ***a*** represents the number of motorbike riders and ***b*** represents the number of bicycle riders, show the mathematical statement for the diagram below.

________ + ________ = ________ + ________

Expected Answer: a + b = b + a

Guide Questions:

- What operation is used in illustrating the diagram? **Addition**
- What happened to the terms in both sides of the equation? **The terms were interchanged.**
- Based on the previous activity, what property is being applied?

Commutative Property of Addition: For integers a, b, a + b = b + a

- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.

  $2 \cdot 3 = 3 \cdot 2$

  $6 = 6$

  Commutative Property of Multiplication: For integers a, b, ab = ba

- Define the property.

  Commutative Property

  Changing the order of two numbers that are either being added or multiplied does not change the result.

- Give a real life situation in which the commutative property can be applied.

  An example is preparing fruit juices - even if you put the powder first before the water or vice versa, the product will still be the same. It's still the same fruit juice.

- Test the property on subtraction and division operations by using simple examples. What did you discover?

  Commutative property is not applicable to subtraction and division as shown in the following examples:

$$6 - 2 = 2 - 6 \qquad\qquad 6 \div 2 = 2 \div 6$$
$$4 \neq -4 \qquad\qquad 3 \neq$$

B. Fill in the blanks with the correct numerical values of the set of cellphones, ipods and laptops.

________ ________ ________

( + ) + 

equals

 + ( + )

If ***a*** represents the number of cellphones, ***b*** represents the ipods and **c** represents the laptops, show the mathematical statement for the diagram below.

(_______ + _______) +_______ = _______ + (_______ + _______)

Expected Answer: (a + b) + c = a + (b + c)

Guide Questions:

What operation is used in illustrating the diagram? **Addition**

- What happened to the groupings of the given sets that correspond to both sides of the equation? **The groupings were changed.**
- Based on the previous activity, what property is being applied?

  Associative Property of Addition

  For integers a, b and c, (a + b) + c = a + (b + c)
- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.

  $$(2 \cdot 3) \cdot 5 = 3 \cdot (2 \cdot 5)$$
  $$6 \cdot 5 = 3 \cdot 10$$
  $$30 = 30$$

  Associative Property of Multiplication

  For integers a, b and c, (a• b)c = a(b• c)
- Define the property.

  Associative Property

  Changing the grouping of numbers that are either being added or multiplied does not change its value.
- Give a real life situation wherein associative property can be applied.

  An example is preparing instant coffee – even if you combine coffee and creamer then sugar or coffee and sugar then creamer the result will be the same – 3-in-1coffee.
- Test the property on subtraction and division operations by using simple examples. What did you discover?

  Associative property is not applicable to subtraction and division as shown in the following examples:

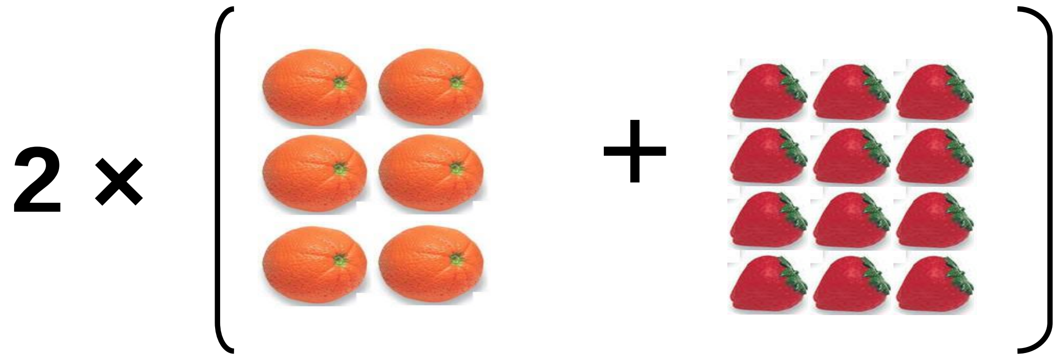
$$(6 - 2) - 1 = 6 - (2 - 1) \qquad (12 \div 2) \div 2 = 12 \div (2 \div 2)$$

$$4 - 1 = 6 - 1 \qquad 6 \div 2 = 12 \div 1$$

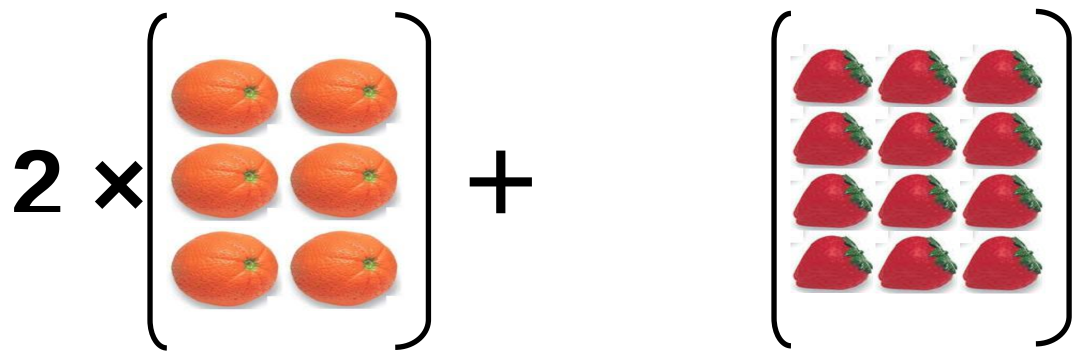
$$3 \neq 5 \qquad 3 \neq 12$$

C. Fill in the blanks with the correct numerical values of the set of oranges and set of strawberries.

_______ _______



equals



_______ _______

If ***a*** represents the multiplier in front, ***b*** represents the set of oranges and **c** represents the set of strawberries, show the mathematical statement for the diagram below.

_______ (_______+_______) = ______ • _______ + _______• ______

Answer: a(b + c) = ab + ac

Guide Questions:

- Based on the previous activity, what property is being applied in the images presented?

  Distributive Property

  For any integers a, b, c, *a(b + c) = ab + ac*

  For any integers a, b, c, *a(b - c) = ab - ac*

- Define the property.

  Distributive Property

  When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same

when each number is multiplied by the factor and the products are then added / subtracted.

- In the said property can we add/subtract the numbers inside the parentheses and then multiply or perform multiplication first and then addition/subtraction? Give an example to prove your answer.

  In the example, we can either add or subtract the numbers inside the parentheses first and then multiply the result; or, we can multiply with each term separately and then add/ subtract the two products together. The answer is the same in both cases as shown below.

-2(4 + 3) = (-2 • 4) + (-2 • 3) ***-2(7) = (-8) + (-6)*** ***-14 = -14*** ***or*** ***-2(4 + 3) = -2(7)*** ***-2(7) = -14*** ***-14 = -14***	

- Give a real life situation wherein distributive property can be applied.
  Your mother gave you four 5-peso coins and your grandmother gave you four 20-peso bills. You now have PhP20 worth of 5-peso coins and PhP80 worth of 20-peso bill. You also have four sets of PhP25 each consisting of a 5-peso coin and a 20-peso bill.

D. Fill in the blanks with the correct numerical representation of the given illustration.

_______ _______ _______

Answer: a + 0 = a

Guide Questions:

- Based on the previous activity, what property is being applied in the images presented?

  Identity Property for Addition
  a + 0 = a

- What will be the result if you add something represented by any number to nothing represented by zero? **The result is the non-zero number.**
- What do you call zero "0" in this case? **Zero, "0" is the additive identity.**
- Define the property.

  Identity Property for Addition *states that 0 is the additive identity, that is, the sum of any number and 0 is the given number.*
- Is there a number multiplied to any number that will result to that same number? Give examples.

  Yes, the number is 1.

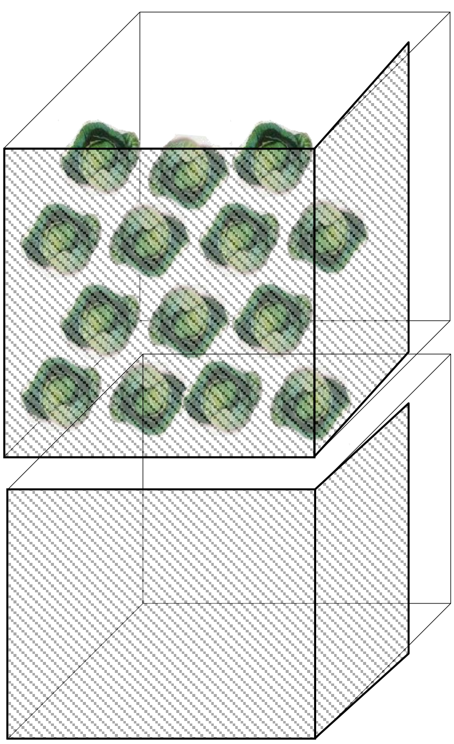
  Examples: 1•2=2 1•3=2 1•4=2
- What property is being illustrated? Define.

  Identity Property for Multiplication says that 1 is the Multiplicative Identity

  - **the product of any number and 1 is the given number, a • 1 = a.**
- What do you call one "1" in this case?

  One, "1" is the multiplicative identity

E. Give the correct mathematical statement of the given illustrations. To do this, refer to the guide questions below.



Guide Questions:

- How many cabbages are there in the crate? **14 cabbages**

- Using integers, represent “put in 14 cabbages” and “remove 14 cabbages”? What will be the result if you add these representations? (+14) + (-14) = 0
- Based on the previous activity, what property is being applied in the images presented?

  Inverse Property for Addition

  $a + (-a) = 0$
- What will be the result if you add something to its negative? **The result is always zero.**
- What do you call the opposite of a number in terms of sign? What is the opposite of a number represented by a?

  Additive Inverse. The additive inverse of the number a is –a.
- Define the property.

  Inverse Property for Addition

  - **states that the sum of any number and its additive inverse or its negative, is zero.**
- What do you mean by reciprocal and what is the other term used for it?

  The reciprocal is *1 divided by that number* or the fraction $\frac{1}{a}$, where *a* represents the number.

  The reciprocal of a number is also known as its multiplicative inverse.
- What if you multiply a number say 5 by its multiplicative inverse $\frac{1}{5}$, what will be the result? ***5*** **$\bullet \frac{1}{5} = 1$**
- What property is being illustrated? Define.

  Inverse Property for Multiplication

  - **states that the product of any number and its multiplicative inverse or reciprocal, is 1.**

  For any number a, the multiplicative inverse is $\frac{1}{a}$.

Important Terms to Remember

The following are terms that you must remember from this point on.

1. Closure Property

   Two integers that are added and multiplied remain as integers. The set of integers is closed under addition and multiplication.
2. Commutative Property

   Changing the order of two numbers that are either being added or multiplied does not change the value.
3. Associative Property

   Changing the grouping of numbers that are either being added or multiplied does not change its value.
4. Distributive Property

When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same when each number is multiplied by the factor and the products are then added / subtracted.

5. Identity Property

   Additive Identity

   - states that the sum of any number and 0 is the given number. Zero, “0” is the additive identity.

   Multiplicative Identity

   - states that the product of any number and 1 is the given number, a • 1 = a. One, “1” is the multiplicative identity.

6. Inverse Property

   In Addition

   - states that the sum of any number and its additive inverse, is zero. The additive inverse of the number a is –a.

   In Multiplication

   - states that the product of any number and its multiplicative inverse or reciprocal, is 1.The multiplicative inverse of the number a is $\frac{1}{a}$.

Notations and Symbols

In this segment, you will learn some of the notations and symbols pertaining to properties of real number applied in the operations of integers.

Closure Property under addition and multiplication	a, b $\in$ *I*, then a+b $\in$ *I*, a•b $\in$ *I*
Commutative property of addition	a + b = b + a
Commutative property of multiplication	ab = ba
Associative property of addition	(a + b) + c = a + (b + c)
Associative property of multiplication	(ab) c = a (bc)
Distributive property	a(b + c) = ab + ac
Additive identity property	a + 0 = a
Multiplicative identity property	a • 1 = a
Multiplicative inverse property	$\frac{1}{a} \bullet a = 1$
Additive inverse property	a + (-a) = 0

NOTE TO THE TEACHER:

It is important for you to examine and discuss the responses by your students to the questions posed in every activity and exercise in order to practice what they have learned for themselves. Remember application as part of the learning process is essential to find out whether the learner gained knowledge of the concept or not. It is also appropriate to encourage brainstorming, dialogues and arguments in the class. After the exchanges, see to it that all questions are resolved.

III. Exercises

A. Complete the Table: Which property of real number justifies each statement?

Given	Property
1. 0 + (-3) = -3	**Additive Identity Property**
2. 2(3 - 5) = 2(3) - 2(5)	**Distributive Property**
3. (- 6) + (-7) = (-7) + (-6)	**Commutative Property**
4. 1 x (-9) = -9	**Multiplicative Identity Property**
5. -4 x $-\frac{1}{4}$ = 1	**Multiplicative Inverse Property**
6. 2 x (3 x 7) = (2 x 3) x 7	**Associative Property**
7. 10 + (-10) = 0	**Additive Inverse Property**
8. 2(5) = 5(2)	**Commutative Property**
9. 1 x ($-\frac{1}{4}$) = $-\frac{1}{4}$	**Multiplicative Identity Property**
10. (-3)(4 + 9) = (-3)(4) + (-3)(9)	**Distributive Property**

B. Rewrite the following expressions using the given property.

1. 12a – 5a → Distributive Property **(12-5)a**
2. (7a)b → Associative Property **7(ab)**
3. 8 + 5 → Commutative Property **5+8**
4. -4(1) → Identity Property **-4**
5. 25 + (-25) → Inverse Property **0**

C. Fill in the blanks and determine what properties were used to solve the equations.

1. 5 x (**-2** + 2) = 0 **Additive Inverse, Zero Property**
2. -4 + 4 = **0** **Additive Inverse**
3. -6 + 0 = **-6** **Additive Identity**
4. (-14 + 14) + 7 = **7** **Additive Inverse, Additive Identity**
5. 7 x (0 + 7) = 49 **Additive Identity**

NOTE TO THE TEACHER

Try to give more of the type of exercises in Exercise C. Combine properties so that you can test how well your students have understood the lesson.

Summary

The lesson on the properties or real numbers explains how numbers or values are arranged or related in an equation. It further clarifies that no matter how these numbers are arranged and what processes are used, the composition of the equation and the final answer will still be the same. Our society is much like these equations - composed of different numbers and operations, different people with varied personalities, perspectives and experiences. We can choose to look at the differences and forever highlight one's advantage or superiority over the others. Or we can focus on the commonality among people and altogether, work for the common good. A peaceful society and harmonious relationship starts with recognizing, appreciating and fully maximizing the positive traits that we, as a people, have in common.

Lesson 6: Rational Numbers in the Number Line
Time: 1 hour

Prerequisite Concepts: Subsets of Real Numbers, Integers

Objective:

In this lesson, you, the students, are expected to

1. Define rational numbers;
2. Illustrate rational numbers on the number line;
3. Arrange rational numbers on the number line.

NOTE TO THE TEACHER:

Ask students to recall the relationship of the set of rational numbers to the set of integers and the set of non-integers (Lesson 4). This lesson gives students a challenge in their numerical estimation skills. How accurately can they locate rational numbers between two integers, perhaps, or between any two numbers?

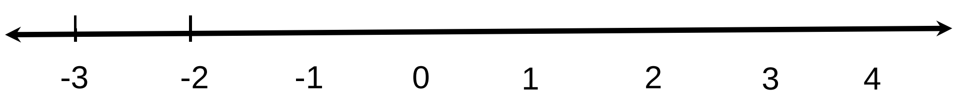
Lesson Proper

I. Activity

Determine whether the following numbers are rational numbers or not.

$- 2, \pi, \frac{1}{11}, \sqrt[3]{4}, \sqrt{16}, -1.89,$

Now, try to locate them on the real number line below by plotting:



NOTE TO THE TEACHER:

Give as many rational numbers as class time can allow. Give them in different forms: integers, fractions, mixed numbers, decimals, repeating decimals, etc.

II. Questions to Ponder

Consider the following examples and answer the questions that follow:

a. 7 ÷ 2 = 3 ½ ,
b. (-25) ÷ 4 = -6 ¼
c. (-6) ÷ (-12) = ½

1. Are quotients integers? **Not all the time. Consider $\frac{10}{7}$.**
2. What kind of numbers are they? **Quotients are rational numbers.**
3. Can you represent them on a number line? **Yes. Rational numbers are real numbers and therefore, they are found in the real number line.**

Recall what rational numbers are...

3 ½, -6 ¼, ½, are rational numbers. The word *rational* is derived from the word "ratio" which means quotient. Rational numbers are numbers which can be written as a quotient of two integers, $\frac{a}{b}$ where b ≠ 0.

The following are more examples of rational numbers:

$5 = \frac{5}{1}$ $\qquad$ $0.06 = \frac{6}{100}$ $\qquad$ $1.3 = \frac{13}{10}$

From the example, we can see that an integer is also a rational number and therefore, integers are a subset of rational numbers. Why is that?

Let's check on your work earlier. Among the numbers given, - 2, π, $\frac{1}{11}$, $\sqrt[3]{4}$, $\sqrt{16}$, -1.89, the numbers π and $\sqrt[3]{4}$ are the only ones that are not rational numbers. Neither can be expressed as a quotient of two integers. However, we can express the remaining ones as a quotient of two intergers:

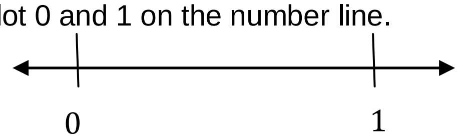
$$-2 = \tfrac{-2}{1},\ \sqrt{16} = 4 = \tfrac{4}{1},\ -1.89 = \frac{-189}{100}$$

Of course, $\frac{1}{11}$ is already a quotient by itself.

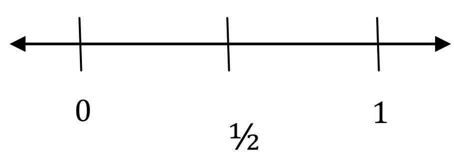
We can locate rational numbers on the real number line.

<u>Example 1</u>. Locate ½ on the number line.

a. Since 0 < ½ < 1, plot 0 and 1 on the number line.

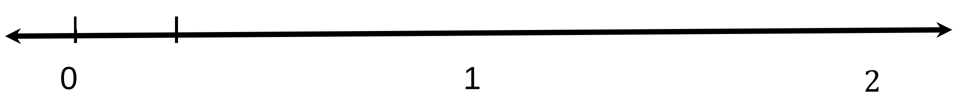


b. Get the midpoint of the segment from 0 to 1. The midpoint now corresponds to ½

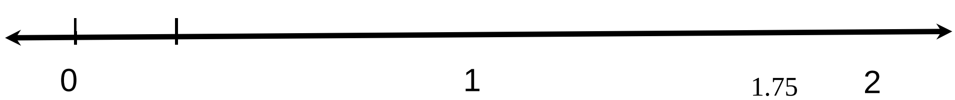


<u>Example 2</u>. Locate 1.75 on the number line.

a. The number 1.75 can be written as $\frac{7}{4}$ and, $1 < \frac{7}{4} < 2$. Divide the segment from 0 to 2 into 8 equal parts.

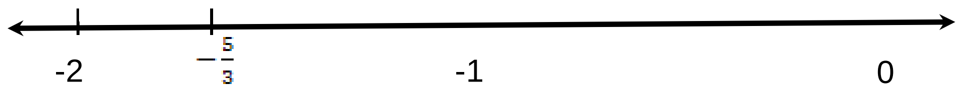


b. The 7th mark from 0 is the point 1.75.



Example 3. Locate the point $-\frac{5}{3}$ on the number line.

Note that -2 < $-\frac{5}{3}$ < -1. Dividing the segment from -2 to 0 into 6 equal parts, it is easy to plot $-\frac{5}{3}$. The number $-\frac{5}{3}$ is the 5th mark from 0 to the left.



Go back to the opening activity. You were asked to locate the rational numbers and plot them on the real number line. Before doing that, it is useful to arrange them in order from least to greatest. To do this, express all numbers in the same form –- either as similar fractions or as decimals. Because integers are easy to locate, they need not take any other form. It is easy to see that

$$-2 < -1.89 < \frac{1}{11} < \sqrt{16}$$

Can you explain why?

Therefore, plotting them by approximating their location gives

III. Exercises

1. Locate and plot the following on a number line (use only one number line).

a. $\frac{-10}{3}$

b. 2.07

c. $\frac{2}{5}$

d. 12

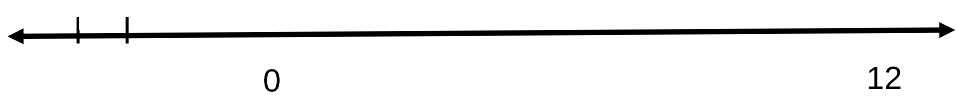
e. -0.01

f. $7\frac{1}{9}$

g. 0

h. $-\frac{1}{6}$

NOTE TO THE TEACHER:
You are given a number line to work on. Plot the numbers on this number line to serve as your answer key.



2. Name 10 rational numbers that are greater than -1 but less than 1 and arrange them from least to greatest on the real number line?

Examples are: $\frac{1}{10}, -\frac{3}{10}, -\frac{1}{2}, -\frac{1}{5}, -\frac{1}{100}, 0, \frac{1}{8}, \frac{2}{11}, \frac{8}{37}, \frac{9}{10}$

3. Name one rational number *x* that satisfies the descriptions below:

a. $-10 \le x < -9$,

Possible answers:

$$x = -\frac{46}{5},\ -\frac{48}{5},\ -9.75,\ -9\frac{8}{9}, -9.99$$

b. $\frac{1}{10} < x < \frac{1}{2}$

Possible answers:

$$x = -\frac{46}{5},\ -\frac{48}{5},\ -9.75,\ -9\frac{8}{9}, -9.99$$

c. $3 < x < \pi$

Possible answers:

$$x = 3.1,\ 3.01,\ 3.001,\ 3.12$$

d. $\frac{1}{4} < x < \frac{1}{3}$

Possible answers:

$$x = \frac{13}{50},\ 0.27,\ 0.28,\ \frac{299}{1000},\ \frac{3}{10}$$

e. $-\frac{1}{8} < x < -\frac{1}{9}$

Possible answers:

$$x = -\frac{3}{25},\ -0.124,\ -\frac{17}{144},\ -0.112$$

NOTE TO THE TEACHER:

In this exercise, you may allow students to use the calculator to check that their choice of x is within the range given. You may, as always also encourage them to use mental computation strategies if calculators are not readily available. The important thing is that students have a way of checking their answers and will not only rely on you to give the correct answers.

NOTE TO THE TEACHER:

End this lesson with a summary as well as a preview to what students will be expecting to learn about rational numbers, their properties, operations and uses.

Summary

In this lesson, you learned more about what rational numbers are and where they can be found in the real number line. By changing all rational numbers to equivalent forms, it is easy to arrange them in order, from least to greatest or vice versa.

Lesson 7: Forms of Rational Numbers and Addition and Subtraction of Rational Numbers
Time: 2 hours

Prerequisite Concepts: definition of rational numbers, subsets of real numbers, fractions, decimals

Objectives:
In this lesson, you are expected to:
1. Express rational numbers from fraction form to decimal form (terminating and repeating and non-terminating) and vice versa;
2. Add and subtract rational numbers;
3. Solve problems involving addition and subtraction of rational numbers.

> **NOTE TO THE TEACHER:**
> **The first part of this module is a lesson on changing rational numbers from one form to another, paying particular attention to changing rational numbers in non-terminating and repeating decimal form to fraction form. It is assumed that students know decimal fractions and how to operate on fractions and decimals.**

Lesson Proper:
A. Forms of Rational Numbers
I. Activity
1. Change the following rational numbers in fraction form or mixed number form to decimal form:

a. $-\frac{1}{4}$ = **-0.25**
b. $\frac{3}{10}$ = **0.3**
c. $3\frac{5}{100}$ = **3.05**
d. $\frac{5}{2}$ = **2.5**
e. $-\frac{17}{10}$ = **-1.7**
f. $-2\frac{1}{5}$ = **-2.2**

> **NOTE TO THE TEACHER:**
> **These should be treated as review exercises. There is no need to spend too much time on reviewing the concepts and algorithms involved here.**

2. Change the following rational numbers in decimal form to fraction form.

a. 1.8 = $\frac{9}{5}$
b. - 3.5 = $-\frac{7}{2}$
c. -2.2 = $-\frac{11}{5}$
d. -0.001 = $-\frac{1}{1000}$
e. 10.999 = $\frac{10999}{1000}$
f. $0.\overline{11}$ = $\frac{1}{9}$

> **NOTE TO THE TEACHER:**
> **The discussion that follows assumes that students remember why certain fractions are easily converted to decimals. It is not so easy to**

change fractions to decimals if they are not decimal fractions. Be aware of the fact that this is the time when the concept of a fraction becomes very different. The fraction that students remember as indicating a part of a whole or of a set is now a number (rational) whose parts (numerator and denominator) can be treated separately and can even be divided! This is a major shift in concept and students have to be prepared to understand how these concepts are consistent with what they know from elementary level mathematics.

II. Discussion

Non-decimal Fractions

There is no doubt that most of the above exercises were easy for you. This is because all except item 2f are what we call decimal fractions. These numbers are all parts of powers of 10. For example, $-\frac{1}{4} = \frac{25}{100}$ which is easily convertible to a decimal form, 0.25. Likewise, the number -3.5 = $-3\frac{5}{10} = -\frac{35}{10}$.

What do you do when the rational number is not a decimal fraction? How do you convert from one form to the other?

Remember that a rational number is a quotient of 2 integers. To change a rational number in fraction form, you need only to divide the numerator by the denominator.

Consider the number $\frac{1}{8}$. The smallest power of 10 that is divisible by 8 is 1000. But, $\frac{1}{8}$ means you are dividing 1 whole unit into 8 equal parts. Therefore, divide 1 whole unit first into 1000 equal parts and then take $\frac{1}{8}$ of the thousandths part. That is equal to $\frac{125}{1000}$ or 0.125.

Example: Change $\frac{1}{16}$, $\frac{9}{11}$ and $-\frac{1}{3}$ to their decimal forms.

The smallest power of 10 that is divisible by 16 is 10,000. Divide 1 whole unit into 10,000 equal parts and take $\frac{1}{16}$ of the ten thousandths part. That is equal to $\frac{625}{10000}$ or 0.625. You can obtain the same value if you perform the long division $1 \div 16$.

Do the same for $\frac{9}{11}$. Perform the long division $9 \div 11$ and you should obtain $0.\overline{81}$. Therefore, $\frac{9}{11}$ = $0.\overline{81}$. Also, $-\frac{1}{3} = -0.\overline{3}$. Note that both $\frac{9}{11}$ and $-\frac{1}{3}$ are non-terminating but repeating decimals.

To change rational numbers in decimal forms, express the decimal part of the numbers as a fractional part of a power of 10. For example, -2.713 can be changed initially to $-2\frac{713}{1000}$ and then changed to $-\frac{2173}{1000}$.

What about non-terminating but repeating decimal forms? How can they be changed to fraction form? Study the following examples:

Example 1: Change $0.\overline{2}$ to its fraction form.
Solution: Let

$$r = 0.222...$$
$$10r = 2.222...$$

Since there is only 1 repeated digit, multiply the first equation by 10.

Then subtract the first equation from the second equation and obtain

$$9r = 2.0$$
$$r = \frac{2}{9}$$

Therefore, $0.\overline{2}$ = $\frac{2}{9}$.

Example 2. Change $-1.\overline{35}$ to its fraction form.
Solution: Let

$$r = -1.353535...$$
$$100r = -135.353535...$$

Since there are 2 repeated digits, multiply the first equation by 100. In general, if there are n repeated digits, multiply the first equation by 10^n.

Then subtract the first equation from the second equation and obtain

$$99r = -134$$
$$r = -\frac{134}{99} = -1\frac{35}{99}$$

Therefore, $-1.\overline{35}$ = $-\frac{135}{99}$.

NOTE TO THE TEACHER:
Now that students are clear about how to change rational numbers from one form to another, they can proceed to learning how to add and subtract them. Students will realize soon that these skills are the same skills they learned back in elementary mathematics.

B. Addition and Subtraction of Rational Numbers in Fraction Form

I. Activity

Recall that we added and subtracted whole numbers by using the number line or by using objects in a set.

Using linear or area models, find the sum or difference.

a. $\frac{3}{5} + \frac{1}{5} =$ ______ c. $\frac{10}{11} - \frac{3}{11} =$ ______

b. $\frac{1}{8} + \frac{5}{8} =$ ______ d. $3\frac{6}{7} - 1\frac{2}{7} =$ ______

Without using models, how would you get the sum or difference?

Consider the following examples:

1. $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6}\ 0r\ \frac{2}{3}$
2. $\frac{6}{7} + \left(-\frac{2}{3}\right) = \frac{18}{21} + \left(-\frac{14}{21}\right) = \frac{4}{21}$
3. $-\frac{4}{3} + \left(-\frac{1}{5}\right) = -\frac{20}{15} + \left(-\frac{3}{15}\right) = -\frac{23}{15}\ or -1\frac{8}{15}$
4. $\frac{14}{5} - \frac{4}{7} = \frac{98}{35} - \frac{20}{35} = \frac{78}{35}\ or\ 2\frac{8}{35}$
5. $-\frac{7}{12} - \left(-\frac{2}{3}\right) = -\frac{7}{12} - \left(-\frac{8}{12}\right) = \frac{-7+8}{12} = \frac{1}{12}$
6. $-\frac{1}{6} - \left(-\frac{11}{20}\right) = -\frac{10}{60} - \left(-\frac{33}{60}\right) = \frac{-10+33}{60} = \frac{23}{60}$

Answer the following questions:
1. Is the common denominator always the same as one of the denominators of the given fractions?
2. Is the common denominator always the greater of the two denominators?
3. What is the least common denominator of the fractions in each example?
4. Is the resulting sum or difference the same when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Problem: Copy and complete the fraction magic square. The sum in each row, column, and diagonal must be 2.

a $^1/_2$		b
$^7/_5$	$^1/_3$	c
d	e	$^2/_5$

» What are the values of a, b, c, d and e? **a =** $\frac{1}{6}$, **b =** $\frac{4}{3}$, **c =** $\frac{4}{15}$, **d =** $\frac{13}{30}$, **e =** $\frac{7}{6}$

NOTE TO THE TEACHER:

The following pointers are not new to students at this level. However, if they had not mastered how to add and subtract fractions and decimals well, this is the time for them to do so.

Important things to remember

To Add or Subtract Fraction

- With the same denominator,

If a, b and c denote integers, and $b \neq 0$, then

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

- With different denominators, $\frac{a}{b}$ *and* $\frac{c}{d}$, where $b \neq 0$ and $d \neq 0$

If the fractions to be added or subtracted are dissimilar

» Rename the fractions to make them similar whose denominator is the least common multiple of *b* and *d*.

» Add or subtract the numerators of the resulting fractions.

» Write the result as a fraction whose numerator is the sum or difference of the numerators and whose denominator is the least common multiple of *b* and *d*.

Examples:

To Add:

a. $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{2}{7}$

b. $\frac{2}{5} + \frac{1}{4}$

LCM/LCD of 5 and 4 is 20

$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{8+5}{20} = \frac{13}{20}$

To Subtract:

a. $\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$

b. $\frac{4}{5} - \frac{1}{4}$

$\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{16-5}{20} = \frac{11}{20}$

NOTE TO THE TEACHER:

Below are the answers to the activity. Make sure that students clearly understand the answers to all the questions and the concepts behind each question.

II. Questions to Ponder (Post –Activity Discussion)

Let us answer the questions posed in activity.

You were asked to find the sum or difference of the given fractions.

a. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

b. $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$ *or* $\frac{3}{4}$

c. $\frac{10}{11} - \frac{3}{11} = \frac{7}{11}$

d. $3\frac{6}{7} - 1\frac{2}{7} = 2\frac{4}{7}$

Without using the models, how would you get the sum or difference?

You would have to apply the rule for adding or subtracting similar fractions.

1. Is the common denominator always the same as one of the denominators of the given fractions?

Not always. Consider $\frac{2}{5}+\frac{3}{4}$. *Their least common denominator is 20 not 5 or 4.*

2. Is the common denominator always the greater of the two denominators?

Not always. The least common denominator is always greater than or equal to one of the two denominators and it may not be the greater of the two denominators.

3. What is the least common denominator of the fractions in each example?

(1) *6* (2) *21* (3) *15* (4) *35* (5) *12* (6) *60*

4. Is the resulting sum or difference the same as when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Yes, for as long as the replacement fractions are equivalent to the original fractions.

NOTE TO THE TEACHER:

Answers in simplest form or lowest terms could mean both mixed numbers with the fractional part in simplest form or an improper fraction whose numerator and denominator have no common factor except 1. Both are acceptable as simplest forms.

III. Exercises

Do the following exercises.

a. Perform the indicated operations and express your answer in simplest form.

1. $\frac{2}{9}+\frac{3}{9}+\frac{1}{9}=\frac{2}{3}$
2. $\frac{6}{5}+\frac{3}{5}+\frac{4}{5}=\frac{13}{5}$
3. $\frac{2}{5}+\frac{7}{10}=\frac{11}{10}=1\frac{1}{10}$
4. $\frac{16}{24}-\frac{6}{12}=\frac{1}{6}$
5. $2\frac{5}{12}-\frac{2}{3}=\frac{7}{4}$
6. $8\frac{1}{4}+\frac{2}{7}=\frac{239}{28}=8\frac{15}{28}$
7. $3\frac{1}{4}+6\frac{2}{3}=9\frac{11}{12}$
8. $9\frac{5}{7}-3\frac{2}{7}=6\frac{3}{7}$
9. $\frac{7}{9}-\frac{1}{12}=\frac{25}{36}$
10. $11\frac{5}{9}-7\frac{5}{6}=\frac{67}{18}=3\frac{13}{18}$
11. $\frac{1}{4}+\frac{2}{3}-\frac{1}{2}=\frac{5}{12}$
12. $10-3\frac{5}{11}=\frac{72}{11}=6\frac{6}{11}$
13. $\frac{7}{20}+\frac{3}{8}+\frac{2}{5}=\frac{9}{8}$
14. $\frac{5}{12}+\frac{4}{9}-\frac{1}{4}=\frac{11}{18}$
15. $2\frac{5}{8}+\frac{1}{2}+7\frac{3}{4}=\frac{87}{8}=10\frac{7}{8}$

NOTE TO THE TEACHER:
You should give more exercises if needed. You, the teacher should probably use the calculator to avoid computing mistakes.

b. Give the number asked for.

1. What is three more than three and one-fourth? $6\frac{1}{4}$

2. Subtract from $15\frac{1}{2}$ the sum of $2\frac{1}{3}$ *and* $4\frac{2}{5}$. What is the result? $\frac{263}{30} = 8\frac{23}{30}$

3. Increase the sum of $6\frac{3}{14}$ *and* $2\frac{2}{7}$ *by* $3\frac{1}{2}$. What is the result? 12

4. Decrease $21\frac{3}{8}$ *by* $5\frac{1}{5}$. What is the result? $\frac{647}{40} = 16\frac{7}{40}$

5. What is $-8\frac{4}{5}$ *minus* $3\frac{2}{7}$? $-\frac{423}{35} = -12\frac{3}{35}$

NOTE TO THE TEACHER:
Note that the language here is crucial. Students need to translate the English phrases to the correct mathematical phrase or equation.

c. Solve each problem.

1. Michelle and Corazon are comparing their heights. If Michelle's height is $120\frac{3}{4}$ cm. and Corazon's height is $96\frac{1}{3}$ cm. What is the difference in their heights?

Answer: $24\frac{5}{12}$ **cm**

2. Angel bought $6\frac{3}{4}$ meters of silk, $3\frac{1}{2}$ meters of satin and $8\frac{2}{5}$ meters of velvet. How many meters of cloth did she buy? **Answer:** $18\frac{13}{20}$ **m**

3. Arah needs $10\frac{1}{4}$ kg. of meat to serve 55 guests, If she has $3\frac{1}{2}$ kg of chicken, a $2\frac{3}{4}$ kg of pork, and $4\frac{1}{4}$ kg of beef, is there enough meat for 55 guests? **Answer: Yes, she has enough. She has a total of** $10\frac{1}{2}$ **kilos.**

4. Mr. Tan has $13\frac{2}{5}$ liters of gasoline in his car. He wants to travel far so he added $16\frac{1}{2}$ liters more. How many liters of gasoline is in the tank? **Answer:** $29\frac{9}{10}$ **liters**

5. After boiling, the $17\frac{3}{4}$ liters of water was reduced to $9\frac{2}{3}$ liters. How much water has evaporated? **Answer:** $8\frac{1}{12}$ **liters**

NOTE TO THE TEACHER:
The last portion of this module is on the addition and subtraction of rational numbers in decimal form. This is mainly a review but emphasize that they are not just working on decimal numbers but with rational numbers. Emphasize that these decimal numbers are a result of the numerator being divided by the denominator of a quotient of two integers.

C. Addition and Subtraction of Rational Numbers in Decimal Form

There are 2 ways of adding or subtracting decimals.

1. Express the decimal numbers in fractions then add or subtract as described earlier.

Example:

Add: 2.3 + 7.21

$2\frac{3}{10} + 7\frac{21}{100}$

$2\frac{30}{100} + 7\frac{21}{100}$

$(2 + 7) + \left(\frac{30+21}{100}\right)$

$9 + \frac{51}{100} = 9\frac{51}{100}$ or 9.51

Subtract:: 9.6 – 3.25

$9\frac{6}{10} - 3\frac{25}{100}$

$9\frac{60}{100} - 3\frac{25}{100}$

$(9 - 3) + \frac{60-25}{100}$

$6 + \frac{35}{100} = 6\frac{35}{100}$ or 6.35

2. Arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract as with whole numbers.

Example:

Add: 2.3 + 7.21

$$\begin{array}{r} 2.3 \\ +\ \underline{7.21} \\ 9.51 \end{array}$$

Subtract: 9.6- 3.25

$$\begin{array}{r} 9.6 \\ \underline{-\ 3.25} \\ 6.35 \end{array}$$

Exercises:

1. Perform the indicated operation.
   1) 1,902 + 21.36 + 8.7 = **1,932.06**
   2) 45.08 + 9.2 + 30.545 = **84.825**
   6) 700 – 678.891 = **21.109**
   7) 7.3 – 5.182 = **2.118**

3) 900 + 676.34 + 78.003 = **1,654.343**

8) 51.005 – 21.4591 = **29.5459**

4) 0.77 + 0.9768 + 0.05301 = **1.79981**

9) (2.45 + 7.89) **–** 4.56 = **5.78**

5) 5.44 – 4.97 = **0.47**

10) (10 – 5.891) + 7.99 = **12.099**

2. Solve the following problems:
   a. Helen had P7500 for shopping money. When she got home, she had P132.75 in her pocket. How much did she spend for shopping? **P7367.25**
   b. Ken contributed P69.25, while John and Hanna gave P56.25 each for their gift to Teacher Daisy. How much were they able to gather altogether? **P181.75**
   c. Ryan said, "I'm thinking of a number N. If I subtract 10.34 from N, the difference is 1.34." What was Ryan's number? **11.68**
   d. Agnes said, "I'm thinking of a number N. If I increase my number by 56.2, the sum is 14.62." What was Agnes number? **– 41.58**
   e. Kim ran the 100-meter race in 135.46 seconds. Tyron ran faster by 15.7 seconds. What was Tyron's time for the 100-meter dash? **119.76**

NOTE TO THE TEACHER:
The summary is important especially because this is a long module. This lesson provided students with plenty of exercises to help them master addition and subtraction of rational numbers.

SUMMARY

This lesson began with some activities and instruction on how to change rational numbers from one form to another and proceeded to discuss addition and subtraction of rational numbers. The exercises given were not purely computational. There were thought questions and problem solving activities that helped in deepening one's understanding of rational numbers.

Lesson 8: Multiplication and Division of Rational Numbers
Time: 2 hours

Prerequisite Concepts: addition and subtraction of rational numbers, expressing rational numbers in different forms

Objectives:

In this lesson, you are expected to:

1. Multiply rational numbers;
2. Divide rational numbers;
3. Solve problems involving multiplication and division of rational numbers.

NOTE TO THE TEACHER:
This lesson reinforces what they learned in elementary mathematics. It starts with the visualization of the multiplication and division of rational numbers using the area model. Use different, yet appropriate shapes when illustrating using the area model. The opening activity encourages the students to use a model or drawing to help them solve the problem. Although, some students will insist they know the answer, it is a whole different skill to teach them to visualize using the area model.

Lesson Proper

A. Models for the Multiplication and Division

I. Activity:

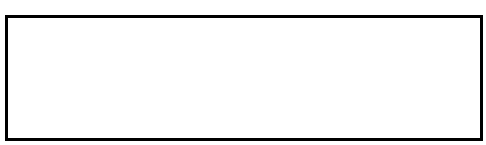
Make a model or a drawing to show the following:

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?
2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

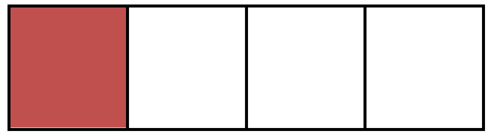
Can you make a model or a drawing to help you solve these problems?

A model that we can use to illustrate multiplication and division of rational numbers is the <u>area model</u>.

What is $\frac{1}{4} \times \frac{1}{3}$? Suppose we have one bar of chocolate represent 1 unit.

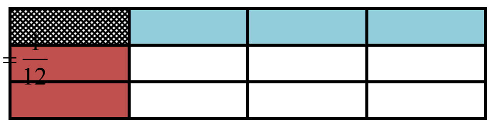


Divide the bar first into 4 equal parts vertically. One part of it is $\frac{1}{4}$



Then, divide each fourth into 3 equal parts, this time horizontally to make the divisions easy to see. One part of the horizontal division is $\frac{1}{3}$.

$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



There will be 12 equal-sized pieces and one piece is $\frac{1}{12}$. But, that one piece is $\frac{1}{3}$ of $\frac{1}{4}$, which we know from elementary mathematics to mean $\frac{1}{3} \times \frac{1}{4}$.

NOTE TO THE TEACHER

The area model is also used in visualizing division of rational numbers in fraction form. This can be helpful for some students. For others, the model may not be easily understandable. But, do not give up. It is a matter of getting used to. In fact, this is a good way to help them use a non-algorithmic approach to dividing rational numbers in fraction form: by using the idea that division is the reverse of multiplication.

What about a model for division of rational numbers?

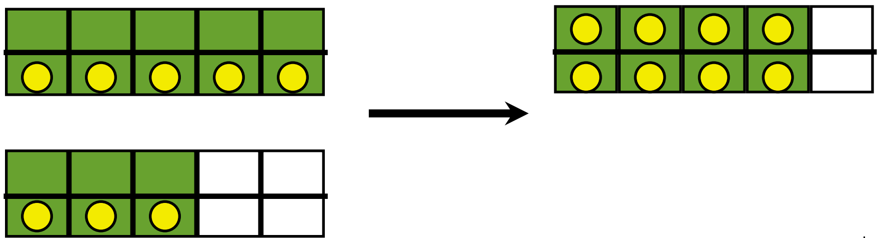
Take the division problem: $\frac{4}{5} \div \frac{1}{2}$. One unit is divided into 5 equal parts and 4 of them are shaded.

Each of the 4 parts now will be cut up in halves

Since there are 2 divisions per part (i.e. $\frac{1}{5}$) and there are 4 of them (i.e. $\frac{4}{5}$), then there will be 8 pieces out of 5 original pieces or $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$.

NOTE TO THE TEACHER

The solution to the problem $\frac{4}{5} \div \frac{1}{2}$ can be easily checked using the area model as well. Ask the students, what is $\frac{1}{2} \times \frac{8}{5}$. The answer can be obtained using the area model



$$\frac{1}{2} \times \frac{8}{5} \qquad = \qquad \frac{4}{5}$$

NOTE TO THE TEACHER:

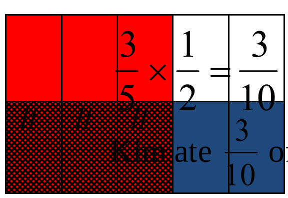
It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?

½



$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

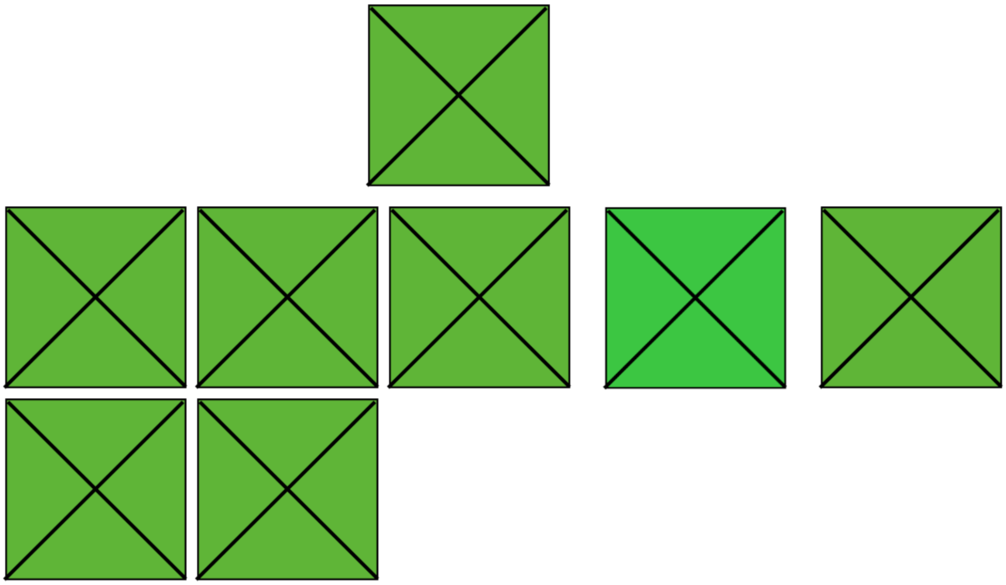
Kim ate $\frac{3}{10}$ of the whole pizza.

3/5

NOTE TO THE TEACHER

The area model works for multiplication of rational numbers because the operation is binary, meaning it is an operation done on two elements. The area model allows for at most "shading" or "slicing" in two directions.

2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?



The equation is $8 \div \frac{1}{4} = 32$. Since there are 4 fourths in one sandwich, there will be 4 x 8 = 32 triangular pieces and hence, 32 children will be fed.

How then can you multiply or divide rational numbers without using models or drawings?

NOTE TO THE TEACHER: **Below are important rules or procedures that the students must remember. From here on, be consistent in your rules so that your students will not be confused. Give plenty of examples.**

Important Rules to Remember

The following are rules that you must remember. From here on, the symbols to be used for multiplication are any of the following: ●, x, $\times$, or *x.*

1. To multiply rational numbers in fraction form simply multiply the numerators and multiply the denominators.

   In symbol, $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ where: b and d are NOT equal to zero, (b ≠ 0; d ≠ 0)

2. To divide rational numbers in fraction form, you take the reciprocal of the second fraction (called the divisor) and multiply it by the first fraction.

In symbol, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$ where: b, c, and d are NOT equal to zero.

<u>Example:</u>
Multiply the following and write your answer in simplest form

a. $\frac{3}{7} \bullet \frac{2}{5}$

$$\frac{3}{7} \bullet \frac{2}{5} = \frac{3 \, x \, 2}{7 \, x \, 5} = \frac{6}{35}$$

b. $4\frac{1}{3} \bullet 2\frac{1}{4}$

$$\frac{13}{3} \bullet \frac{9}{4} = \frac{13 \bullet 3 \bullet 3}{3 \bullet 4} = \frac{13 \bullet 3}{4}$$

$$= \frac{39}{4} \; or \; 9\frac{3}{4}$$

The easiest way to solve for this number is to change mixed numbers to an improper fraction and then multiply it. Or use prime factors or the greatest common factor, as part of the multiplication process.

Divide: $\frac{8}{11} \div \frac{2}{3}$

$$\frac{8}{11} \div \frac{2}{3} = \frac{8}{11} \bullet \frac{3}{2}$$

$$= \frac{2 \bullet 4}{11} \bullet \frac{3}{2}$$

$$\frac{4 \bullet 3}{11} = \frac{12}{11} \; or \; 1\frac{1}{11}$$

Take the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$ then multiply it with the first fraction. Using prime factors, it is easy to see that 2 can be factored out of the numerator then cancelled out with the denominator, leaving 4 and 3 as the remaining factors in the numerator and 11 as the remaining factors in the denominator.

III. Exercises.

Do the following exercises. Write your answer on the spaces provided:

1. Find the products. Express in lowest terms (i.e. the numerator and denominators do not have a common factor except 1). Mixed numbers are acceptable as well:

a. $\frac{5}{6} \bullet \frac{2}{3} = \frac{5}{9}$

b. $7 \bullet \frac{2}{3} = \frac{14}{3} = 4\frac{2}{3}$

c. $\frac{4}{20} \bullet \frac{2}{5} = \frac{2}{25}$

d. $10\frac{5}{6} \bullet 3\frac{1}{3} = \frac{325}{9} = 36\frac{1}{9}$

e. $-\frac{9}{20} \bullet \frac{25}{27} = -\frac{5}{12}$

f. $4\frac{1}{2} \bullet 5\frac{2}{3} = \frac{51}{2} = 25\frac{1}{2}$

g. $\frac{2}{15} \bullet \frac{3}{4} = \frac{1}{10}$

h. $\frac{1}{6} \bullet \frac{2}{3} \bullet \frac{1}{4} = \frac{1}{36}$

i. $-\frac{5}{6} \bullet \frac{2}{3} \bullet \left(-\frac{12}{15}\right) = \frac{4}{9}$

j. $\frac{9}{16} \bullet \frac{4}{15} \bullet (-2) = -\frac{3}{10}$

B. Divide:

1. $20 \div \frac{2}{3} = 30$
2. $\frac{5}{12} \div \left(-\frac{3}{4}\right) = -\frac{5}{9}$
3. $\frac{5}{50} \div \frac{20}{35} = \frac{7}{40}$
4. $5\frac{3}{4} \div 6\frac{2}{3} = \frac{69}{80}$
5. $\frac{9}{16} \div \frac{3}{4} \div \frac{1}{6} = \frac{9}{2} = 4\frac{1}{2}$
6. $\frac{8}{15} \div \frac{12}{25} = \frac{10}{9} = 1\frac{1}{9}$
7. $13\frac{1}{6} \div (-2) = -\frac{79}{12} = -6\frac{7}{12}$
8. $-\frac{5}{6} \div \left(-\frac{10}{14}\right) = \frac{7}{6} = 1\frac{1}{6}$
9. $-\frac{2}{9} \div \frac{11}{15} = -\frac{10}{33}$
10. $\frac{15}{6} \div \frac{2}{3} \div \frac{5}{8} = 6$

C. Solve the following:

1. Julie spent $3\frac{1}{2}$ hours doing her assignment. Ken did his assignment for $1\frac{2}{3}$ times as many hours as Julie did. How many hours did Ken spend doing his assignment? $\frac{35}{6} = 5\frac{5}{6}$ hours
2. How many thirds are there in six-fifths? $\frac{18}{5} = 3\frac{3}{5}$
3. Hanna donated $\frac{2}{5}$ of her monthly allowance to the Iligan survivors. If her monthly allowance is P3500, how much did she donate? P1,400.00
4. The enrolment for this school year is 2340. If $\frac{1}{6}$ are sophomores and $\frac{1}{4}$ are seniors, how many are freshmen or juniors? 1,365 students are freshmen or juniors
5. At the end of the day, a store had 2/5 of a cake leftover. The four employees each took home the same amount of leftover cake. How much of the cake did each employee take home? $\frac{1}{10}$ of the cake.

B. Multiplication and Division of Rational Numbers in Decimal Form

NOTE TO THE TEACHER
The emphasis here is on what to do with the decimal point when multiplying or dividing rational numbers in decimal form. Do not get stuck on the rules. Give a deeper explanation. Consider:

$$6.1 \times 0.08 = 6\frac{1}{10} \times \frac{8}{100} = \frac{488}{1000} = 0.488$$

The decimal places indicate the powers of 10 used in the denominators hence, the rule for determining where to place the decimal point in the product.

This unit will draw upon your previous knowledge of multiplication and division of whole numbers. Recall the strategies that you learned and developed when working with whole numbers.

Activity:

1. Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and explain where it should go and why.
   Example:

   215.2 x 3.2 = 68.864

2. Five students ordered buko pie and the total cost was P135.75. How much did each student have to pay if they shared the cost equally?

Questions and Points to Ponder:

1. In multiplying rational numbers in decimal form, note the importance of knowing where to place the decimal point in a product of two decimal numbers. Do you notice a pattern? *Take the sum of the decimal places in each of the multiplicand and the multiplier and that is the number of places in the product.*
2. In dividing rational numbers in decimal form, how do you determine where to place the decimal point in the quotient? *The number of decimal places in the quotient depends on the number of decimal places in the divisor and the dividend.*

NOTE TO THE TEACHER
Answer to the Questions and Points to Ponder is to be elaborated when you discuss the rules below.

Rules in Multiplying Rational Numbers in Decimal Form

1. Arrange the numbers in a vertical column.
2. Multiply the numbers, as if you are multiplying whole numbers.
3. Starting from the rightmost end of the product, move the decimal point to the left the same number of places as the sum of the decimal places in the multiplicand and the multiplier.

Rules in Dividing Rational Numbers in Decimal Form

1. If the divisor is a whole number, divide the dividend by the divisor applying the rules of a whole number. The position of the decimal point is the same as that in the dividend.
2. If the divisor is not a whole number, make the divisor a whole number by moving the decimal point in the divisor to the rightmost end, making the number seem like a whole number.

3. Move the decimal point in the dividend to the right the same number of places as the decimal point was moved to make the divisor a whole number.
4. Lastly divide the new dividend by the new divisor.

Exercises:

A. Perform the indicated operation

1. 3.5 ÷ 2 = 1.75
2. 78 x 0.4 = 31.2
3. 9.6 x 13 = 124.8
4. 3.24 ÷ 0.5 = 6.48
5. 1.248 ÷ 0.024 = 52
6. 27.3 x 2.5 = 68.25
7. 9.7 x 4.1 = 39.77
8. 3.415 ÷ 2.5 = 1.366
9. 53.61 x 1.02 = 54.6822
10. 1948.324 ÷ 5.96 = 326.9

B. Finds the numbers that when multiplied give the products shown.

1. $\square.\square \times \square = 10.6$

2. $\square.\square \times \square = 16.8$

3. $\square.\square \times \square = 21.6$

4. $\square.\square \times \square = 9.5$

5. $.\square\ \square\ \square \times \square = 21.98$

Answers: (1) 5.3 x 2 ; (2) 8.4 x 2 or 5.6 x 3; (3) 5.4 x 4; (4) 3.5 x 3; (5) 3.14 x 7

NOTE TO THE TEACHER: These are only some of the possible pairs. Be open to other pairs of numbers.

NOTE TO THE TEACHER

Give a good summary to this lesson emphasizing how this lesson was meant to deepen their understanding of rational numbers and develop better skills in multiplying and dividing rational numbers.

Summary

In this lesson, you learned to use the area model to illustrate multiplication and division of rational numbers. You also learned the rules for multiplying and dividing rational numbers in both the fraction and decimal forms. You solved problems involving multiplication and division of rational numbers.

Lesson 9: Properties of the Operations on Rational Numbers
Time: 1 hour

Pre-requisite Concepts: Operations on rational numbers

Objectives:

In this lesson, you are expected to

1. Describe and illustrate the different properties of the operations on rational numbers.
2. Apply the properties in performing operations on rational numbers.

NOTE TO THE TEACHER: **Generally, rational numbers appear difficult among students. The following activity should be fun and could help your students realize the importance of the properties of operations on rational numbers.**

Lesson Proper:

I. Activity

Pick a Pair

$\frac{2}{14}$	$\frac{3}{5}$	0	1	$\frac{13}{40}$
$\frac{13}{12}$	$\frac{1}{3}$	$\frac{3}{20}$		

From the box above, pick the correct rational number to be placed in the spaces provided to make the equation true.

1. $\frac{3}{14} + [\frac{2}{14}] = \frac{5}{14}$
2. $[\frac{2}{14}] + \frac{3}{14} = \frac{5}{14}$
3. $\frac{1}{3}\ x$ ___ $= 0$ [0]
4. $1 \times [\frac{3}{5}] = \frac{3}{5}$
5. $\frac{2}{3} + [0] = \frac{2}{3}$
6. $\left(\frac{1}{2} + \frac{1}{4}\right) + \frac{1}{3} =$ ___ $[\frac{13}{12}]$
7. $\frac{1}{2} + \left(\frac{1}{4} + \_\_\_\right) = \frac{13}{12}$ $[\frac{1}{3}]$
8. $\frac{2}{5} \times \left(\_\_\_ \times \frac{3}{4}\right) = \frac{3}{20}$ $[\frac{1}{2}]$
9. $\left(\frac{2}{5}\ x\ \frac{1}{2}\right) x\ \frac{3}{4} =$ ______ $[\frac{3}{20}]$
10. $\frac{1}{2}\ x\ \left(\frac{2}{5} + \frac{1}{4}\right) = \left(\frac{1}{2}\ x\ \frac{2}{5}\right) + \left(\frac{1}{2}\ x\ \frac{1}{4}\right) = [\frac{13}{40}]$

Answer the following questions:

1. What is the missing number in item 1?
2. How do you compare the answers in items 1 and 2
3. What about item 3? What is the missing number?
4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$?

5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number?
6. What is the missing number in items 6 and 7?
7. What can you say about the grouping in items 6 and 7?
8. What do you think are the answers in items 8 and 9?
9. What operation did you apply in item 10?

NOTE TO THE TEACHER
The follow-up problem below could make the points raised in the previous activity clearer.

Problem:
Consider the given expressions:

a. $\frac{1}{4}+\frac{1}{8}+\frac{1}{2}+\frac{2}{3} = \frac{1}{4}+\frac{1}{2}+\frac{2}{3}+\frac{1}{8}$

b. $\frac{2}{15} \bullet \frac{5}{6} = \frac{5}{6} \bullet \frac{2}{15}$

* Are the two expressions equal? If yes, state the property illustrated. *Yes, the expressions in item (a) are equal and so are the expressions in item (b). This is due to the Commutative Property of Addition and of Multiplication. The Commutative Property allows you to change the order of the addends or factors and the resulting sum or product, respectively, will not change.*

NOTE TO THE TEACHER
Discuss among your students the following properties. These properties make adding and multiplying of rational numbers easier to do.

PROPERTIES OF RATIONAL NUMBERS (ADDITION & MULTIPLICATION)

1. CLOSURE PROPERTY: For any two defined rational numbers. $\frac{a}{b}$ *and* $\frac{c}{d}$, their sum $\frac{a}{b}+\frac{c}{d}$ and product $\frac{a}{b}\ x\ \frac{c}{d}$ is also rational.

   For example:
   a. $\frac{3}{4}+\frac{2}{4} = \left(\frac{3+2}{4}\right) = \frac{5}{4}$

   b. $\frac{3}{4} \bullet \frac{2}{4} = \frac{6}{16}\ or\ \frac{3}{8}$

2. COMMUTATIVE PROPERTY: For any two defined rational numbers $\frac{a}{b}$ *and* $\frac{c}{d}$,

   i. $\frac{a}{b}+\frac{c}{d} = \frac{b}{d}+\frac{a}{b}$

   ii. $\frac{a}{b} \bullet \frac{c}{d} = \frac{c}{d}\ and\ \frac{a}{b}$

For example:

a. $\frac{2}{7} + \frac{1}{3} = \frac{1}{3} + \frac{1}{7}$

b. $\frac{6}{7} \bullet \frac{2}{3} = \frac{3}{3} \bullet \frac{6}{7}$

3. ASSOCIATIVE PROPERTY: For any three defined rational numbers $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$

   i. $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$

   ii. $\frac{a}{b} \bullet \left(\frac{c}{d} \bullet \frac{e}{f}\right) = \left(\frac{a}{b} \bullet \frac{c}{d}\right) \bullet \frac{e}{f}$

   For example:

   a. $\frac{3}{5} + \left(\frac{2}{3} + \frac{1}{4}\right) = \left(\frac{3}{5} + \frac{2}{3}\right) + \frac{1}{4}$

   b. $\frac{1}{4} \bullet \left(\frac{3}{4} \bullet \frac{2}{3}\right) = \left(\frac{1}{4} \bullet \frac{3}{4}\right) \bullet \frac{2}{3}$

4. DISTRIBUTIVE PROPERTY of multiplication over addition for rational numbers.

   If $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$ are any defined rational numbers, then $\frac{a}{b} \bullet \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \bullet \frac{c}{d}\right) + \left(\frac{a}{b} \bullet \frac{e}{f}\right)$

   For example: $\frac{3}{7} \bullet \left(\frac{2}{3} + \frac{7}{8}\right) = \left(\frac{3}{7} \bullet \frac{2}{3}\right) + \left(\frac{3}{7} \bullet \frac{7}{8}\right)$

5. DISTRIBUTIVE PROPERTY of multiplication over subtraction for rational numbers.
   If $\frac{a}{b}, \frac{c}{d}, and\ \frac{e}{f}$ are any defined rational numbers, then $\frac{a}{b} \bullet \left(\frac{c}{d} - \frac{e}{f}\right) = \left(\frac{a}{b} \bullet \frac{c}{d}\right) - \left(\frac{a}{b} \bullet \frac{e}{f}\right)$

   For example: $\frac{3}{10} \bullet \left(\frac{2}{3} - \frac{2}{8}\right) = \left(\frac{3}{10} \bullet \frac{2}{3}\right) - \left(\frac{3}{10} \bullet \frac{2}{8}\right)$

6. IDENTITY PROPERTY
   Addition: Adding 0 to a number will not change the identity or value of that number.
   $\frac{a}{b} + 0 = \frac{a}{b}$

   For example: $\frac{1}{2} + 0 = \frac{1}{2}$

Multiplication: Multiplying a number by 1 will not change the identity or value of that number.

$\frac{a}{b} \bullet 1 = \frac{a}{b}$

For example: $\frac{3}{5} \bullet 1 = \frac{3}{5}$

7. ZERO PROPERTY OF MULTIPLICATION: Any number multiplied by zero equals 0, i. e. $\frac{a}{b} \bullet 0 = 0$

   For example: $\frac{2}{7} \bullet 0 = 0$

II. Question to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER
Answer each question in the opening Activity thoroughly and discussed the concepts clearly. Allow students to express their ideas, their doubts and their questions. At this stage, they should really be able to verbalize what they understand or do not understand so that you the teacher may properly address any misconceptions they have. Give plenty of additional examples, if necessary.

Let us answer the questions posed in the opening activity.

1. What is the missing number in item1? » $\frac{2}{14}$
2. How do you compare the answers in items 1 and 2? » *The answer is the same, the order of the numbers is not important.*
3. What about item 3? What is the missing number? *» The missing number is 0. When you multiply a number with zero the product is zero.*
4. In item 4, what number did you multiply with 1 to get $\frac{3}{5}$? » $\frac{3}{5}$, *When you multiply a number by one the answer is the same.*
5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number? » 0, *When you add zero to any number, the value of the number does not change.*
6. What do you think is the missing number in items 6 and 7?» $\frac{13}{12}$
7. What can you say about the grouping in items 6 and 7? » *The groupings are different but they do not affect the sum.*
8. What do you think are the answers in items 8 and 9? » *The answer is the same in both items,* $\frac{3}{20}$.
9. What operation did you apply in item 10? » *The Distributive Property of Multiplication over Addition*

III. Exercises:

Do the following exercises. Write your answer in the spaces provided.
A. State the property that justifies each of the following statements.

1. $\frac{2}{3} + \frac{5}{8} = \frac{5}{8} + \frac{2}{3}$
Commutative Property

2. $1 \times \frac{9}{35} = \frac{9}{35}$
Identity Property for Multiplication

3. $\frac{4}{5} \bullet \left(\frac{3}{4} + \frac{2}{3}\right) = \left(\frac{4}{5} \bullet \frac{3}{4}\right) + \left(\frac{4}{5} \bullet \frac{2}{3}\right)$
Distributive Property of Multiplication over Addition

4. $\frac{3}{5} + \left(\frac{1}{2} + \frac{1}{4}\right) = \left(\frac{3}{5} + \frac{1}{2}\right) + \frac{1}{4}$
Associative Property

5. $\left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right) \bullet 1 = \left(\frac{2}{7} + \frac{1}{5} + \frac{2}{3}\right)$
Identity Property for Multiplication

6. $\left(\frac{3}{4} + 0\right) = \frac{3}{4}$
Identity Property for Addition

7. $\frac{1}{2} + \frac{5}{6} = \frac{4}{3}$
Closure Property

8. $\frac{3}{8} \bullet \frac{1}{4} \bullet \frac{2}{3} \bullet \frac{1}{2} = \frac{3}{8} \bullet \frac{2}{3} \bullet \frac{1}{4} \bullet \frac{1}{2}$
Commutative Property

9. $\frac{1}{4} \bullet \left(\frac{5}{7} - \frac{2}{3}\right) = \left(\frac{1}{4} \bullet \frac{5}{7}\right) - \left(\frac{1}{4} \bullet \frac{2}{3}\right)$
Distributive Property of Multiplication over Subtraction

10. $\left(\frac{2}{15} \bullet \frac{5}{7}\right) \bullet 0 = 0$
Zero Property for Multiplication

B. Find the value of N in each expression

1. $N + \frac{1}{45} = \frac{1}{45}$ **N = 0**

2. $\left(\frac{1}{4} \bullet N\right) \bullet \frac{2}{3} = \frac{1}{4} \bullet \left(\frac{6}{7} \bullet \frac{2}{3}\right)$ **N =** $\frac{6}{7}$

3. $\left(\frac{2}{15} + \frac{12}{30}\right) + \frac{1}{5} = \frac{2}{15} + \left(N + \frac{1}{5}\right)$ **N =** $\frac{12}{30}$

4. $0 + N = \frac{5}{18}$ **N =** $\frac{5}{18}$

6. $N \bullet \left(\frac{6}{14} + \frac{2}{7}\right) = \left(\frac{1}{6} \bullet \frac{6}{14}\right) + \left(\frac{1}{6} \bullet \frac{2}{7}\right)$ **N =** $\frac{1}{6}$

7. $\frac{8}{23} \bullet 1 = N$ **N =** $\frac{8}{23}$

8. $\frac{2}{9} + \frac{2}{3} = N$ **N =** $\frac{8}{9}$

NOTE TO THE TEACHER

You might want to add more exercises. When you are sure that your students have mastered the properties, do not forget to end your lesson with a summary.

Summary

This lesson is about the properties of operations on rational numbers. The properties are useful because they simplify computations on rational numbers. These properties are true under the operations addition and multiplication. Note that for the Distributive Property of Multiplication over Subtraction, subtraction is considered part of addition. Think of subtraction as the addition of a negative rational number.

Lesson 10: Principal Roots and Irrational Numbers
Time: 2 hours

Prerequisite Concepts: Set of rational numbers

Objectives:
In this lesson, you are expected to:
1. describe and define irrational numbers;
2. describe principal roots and tell whether they are rational or irrational;
3. determine between what two integers the square root of a number is;
4. estimate the square root of a number to the nearest tenth;
5. illustrate and graph irrational numbers (square roots) on a number line with and without appropriate technology.

> **NOTE TO THE TEACHER**
> **This is the first time that students will learn about irrational numbers. Irrational numbers are simply numbers that are not rational. However, they are not easy to determine, hence we limit our discussions to principal *nth* roots, particularly square roots. A lesson on irrational numbers is important because these numbers are often encountered. While the activities are meant to introduce these numbers in a non-threatening way, try not to deviate from the formal discussion on principal *nth* roots. The definitions are precise so be careful not to overextend or over generalize.**

Lesson Proper:
I. Activities
A. Take a look at the unusual wristwatch and answer the questions below.

1. Can you tell the time?

2. What time is shown in the wristwatch?

3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$?

4. How will you describe the result?

5. Can you take the exact value of $\sqrt{130}$?

6. What value could you get?

> **NOTE TO THE TEACHER**
> **In this part of the lesson, the square root of a number is used to introduce a new set of numbers called the irrational numbers. Take not of the two ways by which irrational numbers are described and defined.**

Taking the square root of a number is like doing the reverse operation of squaring a number. For example, both 7 and -7 are square roots of 49 since $7^2 = 49$ and $(-7)^2 = 49$. Integers such as 1, 4, 9, 16, 25 and 36 are called perfect squares. Rational numbers such as 0.16, $\frac{4}{100}$ and 4.84 are also, perfect squares. Perfect squares are numbers that have rational numbers as square roots. The square roots of perfect squares are rational numbers while the square roots of numbers that are not perfect squares are *irrational numbers*.

Any number that cannot be expressed as a quotient of two integers is an irrational number. The numbers $\sqrt{2}$, π, and the special number *e* are all irrational numbers. Decimal numbers that are non-repeating and non-terminating are irrational numbers.

> **NOTE TO THE TEACHER**
> **It does not hurt for students at this level to use a scientific calculator in obtaining principal roots of numbers. With the calculator, it becomes easier to identify as well irrational numbers.**

B. Activity

Use the $\sqrt[x]{\ }$ button of a scientific calculator to find the following values:

1. $\sqrt[6]{64}$ 2. $\sqrt[4]{-16}$ 3. $\sqrt[3]{90}$ 4. $\sqrt[5]{-3125}$ 5. $\sqrt{24}$

II. Questions to Ponder (Post-Activity Discussions)

Let us answer the questions in the opening activity.

1. Can you tell the time? *Yes*
2. What time is it in the wristwatch? *10:07*
3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$? *1, 2, 3, 4*
4. How will you describe the result? *They are all positive integers*.
5. Can you take the exact value of $\sqrt{130}$? *No*.
6. What value could you get? *Since the number is not a perfect square you could estimate the value to be between* $\sqrt{121}$ *and* $\sqrt{144}$*, which is about 11.4.*

Let us give the values asked for in Activity B. Using a scientific calculator, you probably obtained the following:

1. $\sqrt[6]{64}$ *= 2*
2. $\sqrt[4]{-16}$ *Math Error, which means not defined*
3. $\sqrt[3]{90}$ *= 4.481404747, which could mean non-terminating and non-repeating since the calculator screen has a limited size*
4. $\sqrt[5]{-3125}$ *= -5*
5. $\sqrt{24}$ *= 4.898979486, which could mean non-terminating and non-repeating since the calculator screen has a limited size*

> **NOTE TO THE TEACHER**
> **The transition from the concept of two square roots of a positive number to that of the principal *nth* root has always been a difficult one for students. The important and precisely stated concepts are in bold so that students pay attention to them. Solved problems that are meant to illustrate certain procedures and techniques in determining whether a principal root is rational or irrational, finding two consecutive integers between which the**

irrational number is found, estimating the value of irrational square roots to the nearest tenth, and plotting an irrational square root on a number line.

On Principal n^th Roots

Any number, say *a*, whose n^{th} power (n, a positive integer), is *b* is called the n^{th} root of b. Consider the following: $(-7)^2 = 49$, $2^4 = 16$ and $(-10)^3 = -1000$. This means that -7 is a 2nd or square root of 49, 2 is a 4th root of 16 and -10 is a 3rd or cube root of -1000.

However, we are not simply interested in any n^{th} root of a number; we are more concerned about the principal n^{th} root of a number. **The principal *n*^th root of a positive number is the positive *n*^th root. The principal *n*^th root of a negative number is the negative *n*^th root if *n* is odd. If *n* is even and the number is negative, the principal *n*^th root is not defined. The notation for the principal *n*^th root of a number *b* is $\sqrt[n]{b}$. In this expression, *n* is the index and *b* is the radicand. The *n*^th roots are also called radicals.**

Classifying Principal n^th Roots as Rational or Irrational Numbers

To determine whether a principal root is a rational or irrational number, determine if the radicand is a perfect n^{th} power of a number. If it is, then the root is rational. Otherwise, it is irrational.

Problem 1. Tell whether the principal root of each number is rational or irrational.

(a) $\sqrt[3]{225}$ (b) $\sqrt{0.04}$ (c) $\sqrt[5]{-111}$ (d) $\sqrt{10000}$ (e) $\sqrt[4]{625}$

Answers:

a) $\sqrt[3]{225}$ *is irrational*

(b) $\sqrt{0.04}$ *= 0.2 is rational*

(c) $\sqrt[5]{-111}$ *is irrational*

(d) $\sqrt{10000}$ *= 100 is rational*

(e) $\sqrt[4]{625}$ *= 5 is rational*

If a principal root is irrational, the best you can do for now is to give an estimate of its value. Estimating is very important for all principal roots that are not roots of perfect n^{th} powers.

Problem 2. The principal roots below are between two integers. Find the two closest such integers.

(a) $\sqrt{19}$ (b) $\sqrt[3]{101}$ (c) $\sqrt{300}$

Solution:

(a) $\sqrt{19}$

16 is a perfect integer square and 4 is its principal square root. 25 is the next perfect integer square and 5 is its principal square root. Therefore, $\sqrt{19}$ *is between 4 and 5.*

(b) $\sqrt[3]{101}$

64 is a perfect integer cube and 4 is its principal cube root. 125 is the next perfect integer cube and 5 is its principal cube root. Therefore, $\sqrt[3]{101}$ *is between 4 and 5.*

(c) $\sqrt{300}$

289 is a perfect integer square and 17 is its principal square root. 324 is the next perfect integer square and 18 is its principal square root. Therefore, $\sqrt{300}$ *is between 17 and 18.*

Problem 3. Estimate each square root to the nearest *tenth*.

(a) $\sqrt{40}$ (b) $\sqrt{12}$ (c) $\sqrt{175}$

Solution:

(a) $\sqrt{40}$

The principal root $\sqrt{40}$ *is between 6 and 7, principal roots of the two perfect squares 36 and 49, respectively. Now, take the square of 6.5, midway between 6 and 7. Computing,* $(6.5)^2 = 42.25$. *Since 42.25 > 40 then* $\sqrt{40}$ *is closer to 6 than to 7. Now, compute for the squares of numbers between 6 and 6.5:* $(6.1)^2 = 37.21$, $(6.2)^2 = 38.44$, $(6.3)^2 = 39.69$, *and* $(6.4)^2 = 40.96$. *Since 40 is close to 39.69 than to 40.96,* $\sqrt{40}$ *is approximately 6.3.*

(b) $\sqrt{12}$

The principal root $\sqrt{12}$ *is between 3 and 4, principal roots of the two perfect squares 9 and 16, respectively. Now take the square of 3.5, midway between 3 and 4. Computing* $(3.5)^2 = 12.25$. *Since 12.25 > 12 then* $\sqrt{12}$ *is closer to 3 than to 4. Compute for the squares of numbers between 3 and 3.5:* $(3.1)^2 = 9.61$, $(3.2)^2 = 10.24$, $(3.3)^2 = 10.89$, *and* $(3.4)^2 = 11.56$. *Since 12 is closer to 12.25 than to 11.56,* $\sqrt{12}$ *is approximately 3.5.*

(c) $\sqrt{175}$

The principal root $\sqrt{175}$ *is between 13 and 14, principal roots of the two perfect squares 169 and 196. The square of 13.5 is 182.25, which is greater than 175. Therefore,* $\sqrt{175}$ *is closer to 13 than to 14. Now:* $(13.1)^2 = 171.61$, $(13.2)^2 = 174.24$, $(13.3)^2 = 176.89$. *Since 175 is closer to 174.24 than to 176.89 then,* $\sqrt{175}$ *is approximately 13.2.*

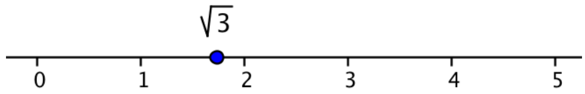
Problem 4. Locate and plot each square root on a number line.

(a) $\sqrt{3}$ (b) $\sqrt{21}$ (c) $\sqrt{87}$

Solution: You may use a program like Geogebra to plot the square roots on a number line.

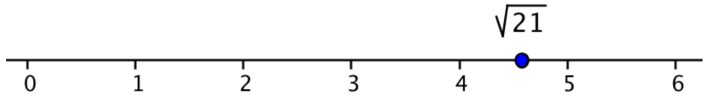
(a) $\sqrt{3}$

This number is between 1 and 2, principal roots of 1 and 4. Since 3 is closer to 4 than to 1, $\sqrt{3}$ is closer to 2. Plot $\sqrt{3}$ closer to 2.



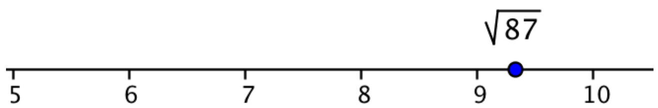
(b) $\sqrt{21}$

This number is between 4 and 5, principal roots of 16 and 25. Since 21 is closer to 25 than to 16, $\sqrt{21}$ is closer to 5 than to 4. Plot $\sqrt{21}$ closer to 5.



(c) $\sqrt{87}$

This number is between 9 and 10, principal roots of 81 and 100. Since 87 is closer to 81, then $\sqrt{87}$ is closer to 9 than to 10. Plot $\sqrt{87}$ closer to 9.



III. Exercises

A. Tell whether the principal roots of each number is rational or irrational.

1. $\sqrt{400}$
2. $\sqrt{64}$
3. $\sqrt{0.01}$
4. $\sqrt{26}$
5. $\sqrt{\frac{1}{49}}$
6. $\sqrt{13{,}689}$
7. $\sqrt{1000}$
8. $\sqrt{2.25}$
9. $\sqrt{39}$
10. $\sqrt{12.1}$

Answers:

1. rational
2. rational
3. rational
4. irrational
5. rational
6. rational
7. irrational
8. rational
9. irrational
10. irrational

B. Between which two consecutive integers does the square root lie?

1. $\sqrt{77}$
2. $\sqrt{700}$
3. $\sqrt{243}$
4. $\sqrt{444}$
5. $\sqrt{48}$
6. $\sqrt{90}$
7. $\sqrt{2045}$
8. $\sqrt{903}$
9. $\sqrt{1899}$
10. $\sqrt{100000}$

Answers:

1. 8 and 9	**6. 9 and 10**
2. 26 and 27	**7. 45 and 46**
3. 15 and 16	**8. 30 and 31**
4. 21 and 22	**9. 43 and 44**
5. 6 and 7	**10. 316 and 317**

C. Estimate each square root to the nearest *tenth* and plot on a number line.

1. $\sqrt{50}$	6. $\sqrt{250}$
2. $\sqrt{72}$	7. $\sqrt{5}$
3. $\sqrt{15}$	8. $\sqrt{85}$
4. $\sqrt{54}$	9. $\sqrt{38}$
5. $\sqrt{136}$	10. $\sqrt{101}$

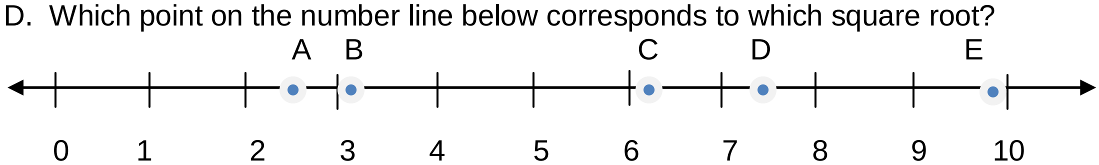
Answers:

1. 7.1	**5. 11.7**	**9. 6.2**
2. 8.5	**6. 15.8**	**10. 10.0**
3. 3.9	**7. 2.2**	
4. 7.3	**8. 9.2**	

NOTE TO THE TEACHER

You might think that plotting the irrational square roots on a number line is easy. Do not assume that all students understand what to do. Give them additional exercises for practice. Exercise D can be varied to include 2 or 3 irrational numbers plotted and then asking students to identify the correct graph for the 2 or 3 numbers.

D. Which point on the number line below corresponds to which square root?



1. $\sqrt{57}$ **D**
2. $\sqrt{6}$ **A**
3. $\sqrt{99}$ **E**
4. $\sqrt{38}$ **C**
5. $\sqrt{11}$ **B**

Summary

In this lesson, you learned about irrational numbers and principal n^{th} roots, particularly square roots of numbers. You learned to find two consecutive integers between which an irrational square root lies. You also learned how to estimate the square roots of numbers to the nearest tenth and how to plot the estimated square roots on a number line.

Lesson 11: The Absolute Value of a Number **Time: 1.5 hours**

Prerequisite Concepts: Set of real numbers

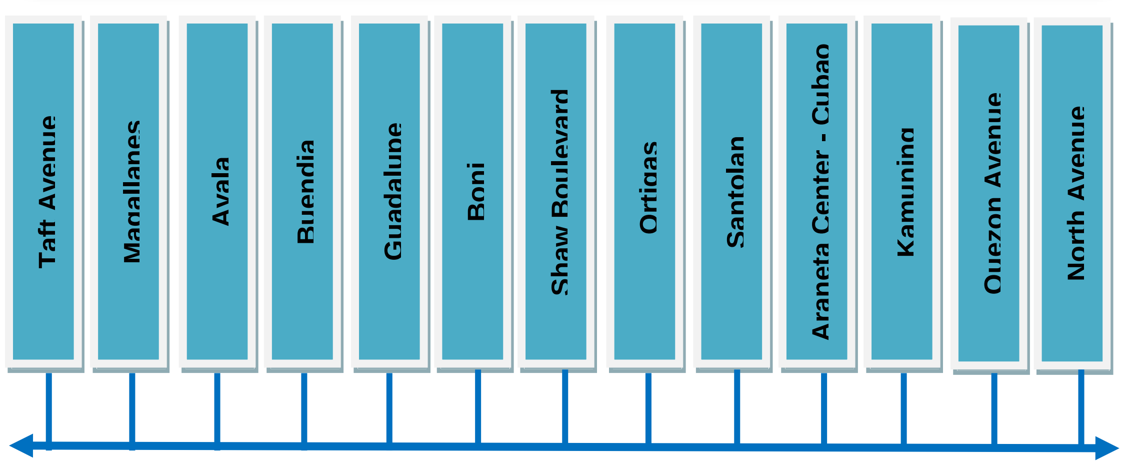
Objectives:
In this lesson, you are expected to describe and illustrate

a. the absolute value of a number on a number line.
b. the distance of the number from 0.

Lesson Proper:

I. Activity 1: THE METRO MANILA RAIL TRANSIT (MRT) TOUR

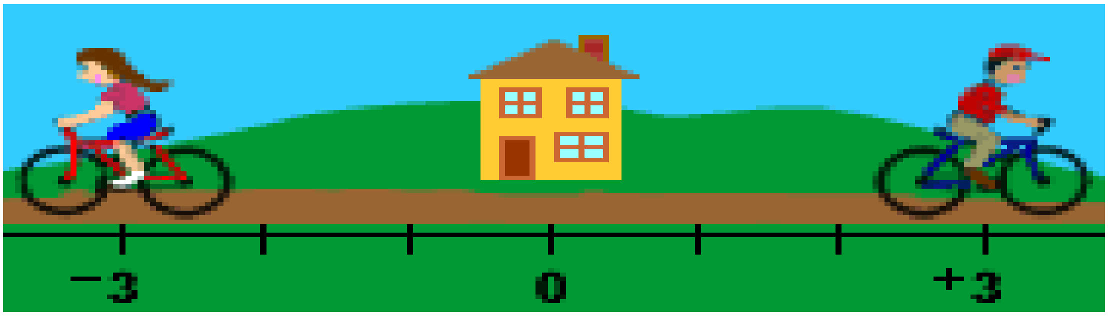
Suppose the MRT stations from Pasay City to Quezon City were on a straight line and were 500 meters apart from each other.

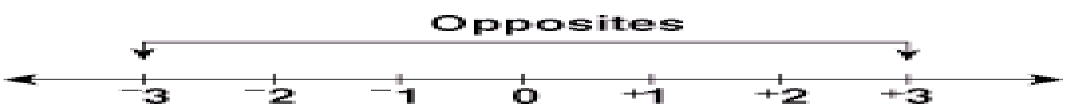


1. How far would the North Avenue station be from Taft Avenue? **6000 meters or 6 kilometers**
2. What if Elaine took the MRT from North Avenue and got off at the last station? How far would she have travelled? **6000 meters or 6 kilometers**
3. Suppose both Archie and Angelica rode the MRT at Shaw Boulevard and the former got off in Ayala while the latter in Kamuning. How far would each have travelled from the starting point to their destinations? **Archie travelled 2000 meters from Shaw Boulevard to Ayala. Angelica travelled 2000 meters from Shaw Boulevard to Kamuning.**
4. What can you say about the directions and the distances travelled by Archie and Angelica? **They went in opposite direction from the same starting point travelled the same distance.**

NOTE TO THE TEACHER:
This lesson focuses on the relationship between absolute value and distance. Point out to students that the absolute value of a number as a measure of distance will always be positive or zero since it is simply a magnitude, a measure. Students should realize the importance of the absolute value of a number in contexts such as transportation, weather, statistics and others.

Activity 2: THE BICYCLE JOY RIDE OF ARCHIEL AND ANGELICA





Problem: Archie and Angelica were at Aloys' house. Angelica rode her bicycle 3 miles west of Aloys' house, and Archie rode his bicycle 3 miles east of Aloys' house. Who travelled a greater distance from Aloys' house – Archie or Angelica?

Questions To Ponder:
1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

2. What are opposite numbers on the number line? Give examples and show on the number line.
3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?
4. What can you say about the absolute value of opposite numbers say -5 and +5?
5. How can we represent the absolute value of a number? What notation can we use?

NOTE TO THE TEACHER:
Below are important terminologies, notations and symbols that your students must learn and remember. From here on, be consistent in using these notations so as not to create confusion on the part of the students. Take note of the subtle difference in using the the absolute value bars from the parentheses.

Important Terms to Remember

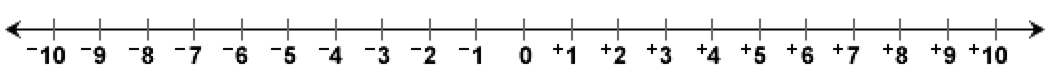
The following are terms that you must remember from this point on.

1. **Absolute Value –** of a number is the distance between that number and zero on the number line.
2. **Number Line –**is best described as a straight line which is extended in both directions as illustrated by arrowheads. A number line consists of three elements:
   a. set of positive numbers, and is located to the right of zero.
   b. set of negative numbers, and is located to the left of zero; and
   c. Zero.

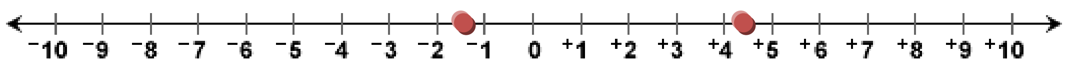
Notations and Symbols

The absolute value of a number is denoted by two bars $|\ |$.

Let's look at the number line:



The absolute value of a number, denoted "| |" is the distance of the number from zero. This is why the absolute value of a number is never negative. In thinking about the absolute value of a number, one only asks "how far?" not "in which direction?" Therefore, the absolute value of 3 and of -3 is the same, which is 3 because both numbers have the same distance from zero.



Warning: The absolute-value notation is *bars*, not parentheses or brackets. Use the proper notation; the other notations do *not* mean the same thing.

It is important to note that the absolute value bars do NOT work in the same way as do parentheses. Whereas – (–3) = +3, this is NOT how it works for absolute value:

Problem: Simplify – | –3 |.

Solution: Given – | – 3 |, first find the absolute value of – 3.

– | –3 | = – (3)

Now take the negative of 3. Thus, :

– | –3 | = – (3) = **–3**

This illustrates that if you take the negative of the absolute value of a number, you will get a negative number for your answer.

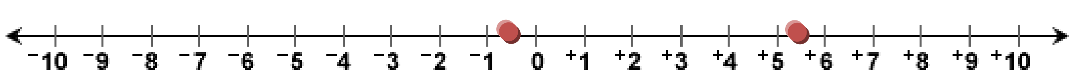
II. Questons to Ponder(Post-Activity Discussion)

NOTE TO THE TEACHER

It is important for you to examine and discuss the responses by your students to the questions posed in Activity 2. Pay particular attention to how to what they say and write. Always refer to practical examples so they can understand more. Encourage brainstorming, dialogues and arguments in the class. After the exchanges, see to it that all questions are answered and resolved.

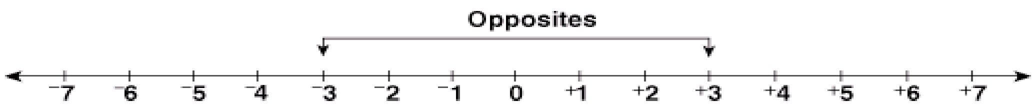
Let us answer the questions posed in Activity 2.

1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.
   The problem uses <u>integers</u>. Travelling 3 miles west can be represented by -3 (pronounced negative 3). Travelling 3 miles east can be represented by +3 (pronounced positive 3). Aloys' house can be represented by the integer 0.



2. What are opposite numbers on the number line? Give examples and show on the number line.

*Two integers that are the same distance from zero in opposite directions are called **opposites**. The integers $^{+}3$ and $^{-}3$ are opposites since they are each 3 units from zero.*



3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?
   The absolute value of a number is its distance from zero on the number line. The absolute value of +3 is 3, and the absolute value of -3 is 3.

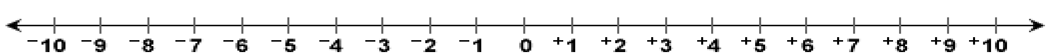
4. What can you say about the absolute value of opposite numbers say -5 and +5?
   Opposite numbers have the same absolute values.

5. How can we represent the absolute value of a number? What notation can we use?
   The symbol | | is used for the absolute value of a number.

III. Exercises

Carry out the following tasks. Write your answers on the spaces provided for each number.

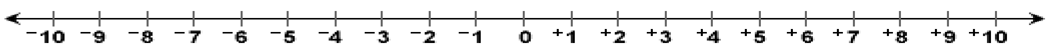
1. Find the absolute value of $^{+}3$, $^{-}3$, $^{+}7$, $^{-}5$, $^{+}9$, $^{-}8$, $^{+}4$, $^{-}4$. You may refer to the number line below. What should you remember when we talk about the absolute value of a number?



Solution: $\lvert {}^{+}3 \rvert = 3$	$\lvert {}^{+}9 \rvert = 9$
$\lvert {}^{-}3 \rvert = 3$	$\lvert {}^{-}8 \rvert = 8$
$\lvert {}^{+}7 \rvert = 7$	$\lvert {}^{+}4 \rvert = 4$
$\lvert {}^{-}5 \rvert = 5$	$\lvert {}^{-}4 \rvert = 4$

Remember that when we find the absolute value of a number, we are finding its distance from 0 on the number line. Opposite numbers have the same absolute value since they both have the same distance from 0. Also, you will notice that taking the absolute value of a number automatically means taking the positive value of that number.

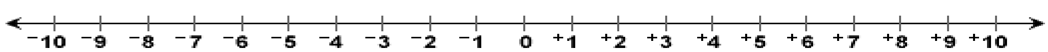
2. Find the absolute value of: $^{+}11$, $^{-}9$, $^{+}14$, $^{-}10$, $^{+}17$, $^{-}19$, $^{+}20$, $^{-}20$. You may extend the number line below to help you solve this problem.



Solution:

$\lvert {}^{+}11 \rvert = 11$	$\lvert {}^{+}17 \rvert = 17$
$\lvert {}^{-}9 \rvert = 9$	$\lvert {}^{-}19 \rvert = 19$
$\lvert {}^{+}14 \rvert = 14$	$\lvert {}^{+}20 \rvert = 20$
$\lvert {}^{-}10 \rvert = 10$	$\lvert {}^{-}20 \rvert = 20$

3. Use the number line below to find the value of N: $|N| = 5.1$



Solution: This problem asks us to find all numbers that are a distance of 5.1 units from zero on the number line. We let N represent all integers that satisfy this condition.

The number $^{+}5.1$ is 5.1 units from zero on the number line, and the number $^{-}5.1$ is also 5.1 units from zero on the number line. Thus both $^{+}5.1$ and $^{-}5.1$ satisfy the given condition.

4. When is the absolute value of a number equal to itself?

Solution: When the value of the number is positive or zero.

5. Explain why the absolute value of a number is never negative. Give an example that will support your answer.

Solution: *Let | N | = -4. Think of a number that when you get the absolute value will give you a negative answer. There will be no solution since the distance of any number from 0 cannot be a negative quantity.*

Enrichment Exercises:

A. Simplify the following.

1. $|7.04| =$ **7.04**
2. $|0| =$ **0**
3. $|-\frac{2}{9}| = \frac{2}{9}$
4. $-|15 + 6| =$ **-21**
5. $|-2\sqrt{2}| - |-3\sqrt{2}| = \mathbf{-}\sqrt{2}$

B. List at least two integers that can replace N such that.

1. $|N| = 4$ **{-4, 4}**
2. $|N| < 3$ **{-2, -1, 0, 1, 2}**
3. $|N| > 5$ **{…, -10, - 9, -8, -7, -6, 6, 7, …}**
4. $|N| \leq 9$ **{-9, -8, -7, …, 0, 1, …. 9}**
5. $0 < |N| < 3$ **{1, 2}**

NOTE TO THE TEACHER

There are several possible answers in each item. Be alert to the different answers of your students.

C. Answer the following.

1. Insert the correct relation symbol(>, =, <): $|-7|$ **>** $|-4|$.
2. If $|x - 7| = 5$, what are the possible values of x? **{2, 12}**
3. If $|x| = \frac{2}{7}$, what are the possible values of x? **{$-\frac{2}{7}$, $\frac{2}{7}$ }**
4. Evaluate the expression, $|x + y| - |y - x|$, if x = 4 and y = 7. **{8}**
5. A submarine navigates at a depth of 50 meters below sea level while exactly above it; an aircraft flies at an altitude of 185 meters. What is the distance between the two carriers? **235 meters**

Summary:

In this lesson you learned about the absolute value of a number, that it is a distance from zero on the number line denoted by the notation |N|. This notation is used for the absolute value of an unknown number that satisfies a given condition. You also learned that a distance can never be a negative quantity and absolute value pertains to the magnitude rather than the direction of a number.

GRADE 7 MATH TEACHING GUIDE

Lesson 12: SUBSETS OF REAL NUMBERS
Prerequisite Concepts: whole numbers and operations, set of integers, rational numbers, irrational numbers, sets and set operations, Venn diagrams
Objectives
In this lesson, you are expected to:

2. Describe and illustrate the real number system.
3. Apply various procedures and manipulations on the different subsets of the set of real numbers.
    a. Describe, represent and compare the different subsets of real number.
    b. Find the union, intersection and complement of the set of real numbers and its subsets

> **NOTE TO THE TEACHER:**
> **Many teachers claim that this lesson is quite simple because we use various kinds of numbers every day. Even the famous theorist of the Pythagorean Theorem, Pythagoras once said that, "All things are number." Truly, numbers are everywhere! But do we really know our numbers? Sometimes a person exists in our midst but we do not even bother to ask the name or identity of that person. It is the same with numbers. Yes, we are surrounded by these boundless figures but do we bother to know what they really are?**
> **In Activity 1, try to stimulate the students' interest in the lesson by drawing out their thoughts. The objective of Activities 2 and 3, is for you to ascertain your students' understanding of the different names of sets of numbers.**

Lesson Proper:
A.
I. Activity 1: Try to reflect on these . . .

It is difficult for us to realize that once upon a time there were no symbols or names for numbers. In the early days, primitive man showed how many animals he owned by placing an equal number of stones in a pile, or sticks in a row. Truly our number system underwent the process of development for hundreds of centuries.
Sharing Ideas! What do you think?

1. In what ways do you think did primitive man need to use numbers?
2. Why do you think he needed names or words to tell "how many"?
3. Was man forced to invent symbols to represent his number ideas?
4. Is necessity the root cause that led man to invent numbers, words and symbols?

> **NOTE TO THE TEACHER:**
> **You need to facilitate the sharing of ideas leading to the discussion of possible answers to the questions. Encourage students to converse, to contribute and to argue if necessary for better interactions.**

Activity 2: LOOK AROUND!

Fifteen different words/partitions of numbers are hidden in this puzzle. How many can you find? Look up, down, across, backward, and diagonally. Figures are scattered around that will serve as clues to help you locate the mystery words.

π, e, $\sqrt{3}$

0, 1, 2, 3, ...

$\frac{2}{3}, \frac{7}{5}$

-1, 0, 1, -$\frac{1}{2}$, $\frac{4}{3}$,
0.25, 0.1313...

1, 2, 3, ...

-4, -5, -6, ...

0

0.25,
0.33...

...,-3, -2, -1,
0, 1, 2, 3, ...

100%,
15%, 25%

N	A	F	R	A	C	T	I	O	N	S	I
S	P	B	A	C	C	D	Z	W	N	E	L
T	E	O	F	T	O	G	E	H	E	R	A
O	R	H	S	I	U	J	R	O	G	A	M
I	C	R	K	I	N	R	O	L	A	T	I
L	E	E	L	M	T	N	A	E	T	I	C
A	N	A	O	P	I	I	Q	L	I	O	E
R	T	L	R	S	N	T	V	U	V	N	D
U	I	N	T	E	G	E	R	E	E	A	A
T	I	R	R	A	T	I	O	N	A	L	I
A	N	O	N	I	N	T	E	G	E	R	S
N	N	U	M	N	U	M	B	E	R	S	S

Answer the following questions:

1. How many words in the puzzle were familiar to you? **<u>Expected Answers: Numbers, Fractions...</u>**
2. What word/s have you encountered in your early years? **<u>Expected Answer: Numbers...</u>**
   Define and give examples. **<u>Expected Answer: They are used to count things.</u>**
3. What word/s is/are still strange to you? **<u>Expected Answer: Irrational, ...</u>**

Activity 3: Determine what numbers/set of numbers will represent the following situations:

1. Finding out how many cows there are in a barn **<u>Counting Numbers</u>**
2. Corresponds to no more apples inside the basket **<u>Zero</u>**
3. Describing the temperature in the North Pole **<u>Negative Number</u>**
4. Representing the amount of money each member gets when "P200 prize is divided among 3 members **<u>Fraction, Decimal</u>**
5. Finding the ratio of the circumference to the diameter of a circle, denoted π (read "pi) **<u>Irrational Number</u>**

NOTE TO THE TEACHER: **You need to follow up on the preliminary activity. Students will definitely give varied answers. Be prepared and keep an open mind. Consequently, the next activity below is essential. In this phase, the students will be encouraged to use their knowledge of the real number system.**

The set of numbers is called **<u>the real number system</u>** that consists of different partitions/ **<u>subsets</u>** that can be represented graphically on a **<u>number line</u>**.

II. Questions to Ponder

Consider the activities done earlier and recall the different terms you encountered including the set of real numbers and together let us determine the various subsets. Let us go back to the first time we encountered the numbers...

Let's talk about the various subsets of real numbers.

Early Years...

1. What subset of real numbers do children learn at an early stage when they were just starting to talk? Give examples.
   Expected Answer: Counting Numbers or Natural Numbers

One subset is the **<u>counting (or natural) numbers</u>**. This subset includes all the numbers we use to count starting with "1" and so on. The subset would look like this: {1, 2, 3, 4, 5...}

In School at an Early Phase...

2. What do you call the subset of real numbers that includes zero (the number that represents nothing) and is combined with the subset of real numbers learned in the early years? Give examples.
   Expected Answer: Whole Numbers

Another subset is the **whole numbers**. This subset is exactly like the subset of counting numbers, with the addition of one extra number. This extra number is "0". The subset would look like this:{0, 1, 2, 3, 4...}

In School at Middle Phase...

3. What do you call the subset of real numbers that includes negative numbers (that came from the concept of "opposites" and specifically used in describing debt or below zero temperature) and is united with the whole numbers? Give examples.
   Expected Answer: Integers

A third subset is the **integers**. This subset includes all the whole numbers and their "opposites". The subset would look like this: {... -4, -3, -2, -1, 0, 1, 2, 3, 4...}

Still in School at Middle Period...

4. What do you call the subset of real numbers that includes integers and non-integers and are useful in representing concepts like ***"half a gallon of milk"***? Give examples.
   Expected Answer: Rational Numbers

The next subset is the **rational numbers**. This subset includes all numbers that "come to an end" or numbers that repeat and have a pattern. Examples of rational numbers are: 5.34, 0.131313..., $\frac{6}{7}$, $\frac{2}{3}$, 9

5. What do you call the subset of real numbers that is not a rational number but are physically represented like ***"the diagonal of a square"***?
   Expected Answer: Irrational Numbers

Lastly we have the set of **irrational numbers**. This subset includes numbers that cannot be exactly written as a decimal or fraction. Irrational numbers cannot be expressed as a ratio of two integers. Examples of irrational numbers are:

$\sqrt{2}$, $\sqrt[3]{101}$, and π

NOTE TO THE TEACHER:
Below are vital terms that must be remembered by students from here on. You, the other hand, must be consistent in the use of these terminologies so as not to puzzle or confuse your students. Give adequate examples and non-examples to further support the learning process of the students. As you discuss these terms, use terms related to sets, such as the union and intersection of sets.

Important Terms to Remember

The following are terms that you must remember from this point on.

1. **Natural/Counting Numbers** – are the numbers we use in counting things, that is {1, 2, 3, 4, . . . }. The three dots, called ellipses, indicate that the pattern continues indefinitely.

2. **Whole Numbers** – are numbers consisting of the set of natural or counting numbers and zero.

3. **Integers** – are the result of the union of the set of whole numbers and the negative of counting numbers.

4. **Rational Numbers** – are numbers that can be expressed as a quotient $\frac{a}{b}$ of two integers. The integer ***a*** is the numerator while the integer ***b***, which cannot be 0 is the denominator. This set includes fractions and some decimal numbers.

5. **Irrational Numbers** – are numbers that cannot be expressed as a quotient $\frac{a}{b}$ of two integers. Every irrational number may be represented by a decimal that neither repeats nor terminates.

6. **Real Numbers** – are any of the numbers from the preceding subsets. They can be found on the real number line. The union of rational numbers and irrational numbers is the set of real numbers.

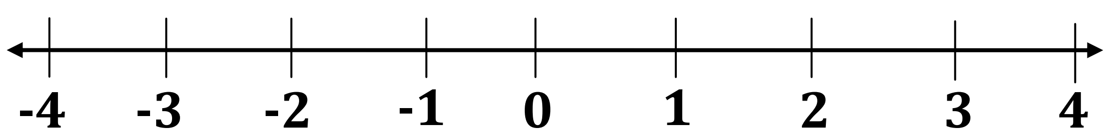
7. **Number Line** – a straight line extended on both directions as illustrated by arrowheads and is used to represent the set of real numbers. On the real number line, there is a point for every real number and there is a real number for every point.

III. Exercises

1. Locate the following numbers on the number line by naming the correct point.

$$\boxed{\mathbf{-2.66...}\ ,\ -1\tfrac{1}{2}\ ,\ \mathbf{-0.25}\ ,\ \tfrac{3}{4}\ ,\ \sqrt{2}\ ,\ \sqrt[3]{11}}$$

Answer:



2. Determine the subset of real numbers to which each number belongs. Use a tick mark (√) to answer.

Answer:

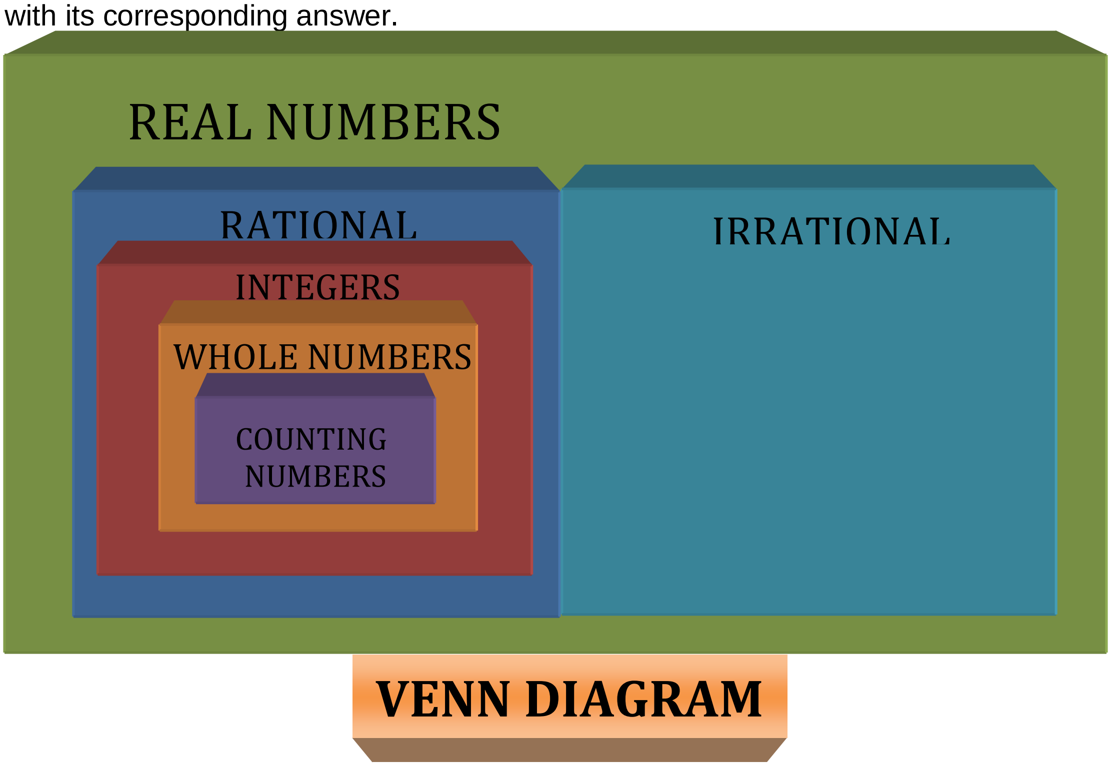
Number	Whole Number	Integer	Rational	Irrational
1. -86		√	√	
2. 34.74			√	
3. $\frac{4}{7}$			√	
4. $\sqrt{64}$	√	√	√	
5. $\sqrt{11}$				√
6. -0.125			√	
7. $-\sqrt{81}$		√	√	
8. e				√
9. -45.37			√	
10. -1.252525...			√	

B. Points to Contemplate

It is interesting to note that the set of rational numbers and the set of irrational numbers are disjoint sets; that is, their intersection is empty. The union of the set of rational numbers and the set of irrational numbers yields a set of numbers that is called the set of real numbers.

Exercise:

a. Based on the stated information, show the relationships among ***natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers*** and ***real numbers*** using the Venn diagram below. Fill each broken line with its corresponding answer.



b. Carry out the task being asked by writing your response on the space provided for each number.

1. Are all real numbers rational numbers? Prove your answer.

Expected Answer: *No, because the set of real numbers is composed of two subsets namely, rational numbers and irrational numbers. Therefore, it is impossible that all real numbers are rational numbers alone.*

2. Are all rational numbers whole numbers? Prove your answer.

Expected Answer: *No, because rational numbers is composed of two subsets namely, Integers where whole numbers are included and non-integers. Therefore, it is impossible that all rational numbers are whole numbers alone.*

3. Are $-\frac{1}{4}$ and $-\frac{2}{5}$ negative integers? Prove your answer.

Expected Answer: *They are negative numbers but not integers. An integer is composed of positive and negative whole numbers and not a signed fraction.*

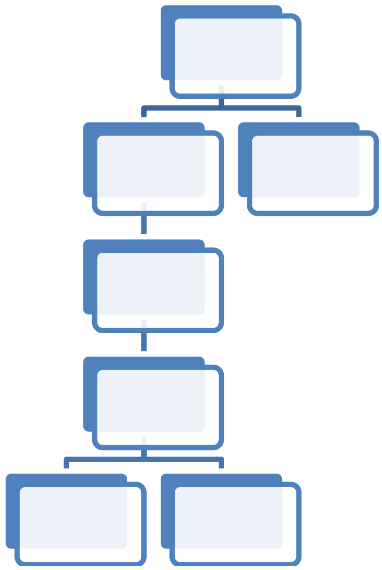
4. How is a rational number different from an irrational number?

Expected Answer: *Rational Numbers can be expressed as a quotient of two integers with a nonzero denominator while Irrational numbers cannot be written in this form.*

5. How do natural numbers differ from whole numbers?

Expected Answer: *Natural numbers are also known as counting numbers that will always start with 1. Once you include 0 to the set of natural numbers that becomes the set of whole numbers.*

c. Complete the details in the Hierarchy Chart of the Set of Real Numbers.



THE REAL NUMBER SYSTEM

NOTE TO THE TEACHER: **Make sure you summarize this lesson because there are many terms and concepts to remember.**

Summary

In this lesson, you learned different subsets of real numbers that enable you to name numbers in different ways. You also learned to determine the hierarchy and relationship of one subset to another that leads to the composition of the real number system using the Venn diagram and hierarchy chart. You also learned that it was because of necessity that led man to invent number, words and symbols.

Lesson 13: Significant Digits and the Scientific Notation OPTIONAL

Prerequisite Concepts: Rational numbers and powers of 10

Objectives:

In this lesson, you are expected to:

1. determine the significant digits in a given situation.
2. write very large and very small numbers in scientific notation

> **NOTE TO THE TEACHER**
> **This lesson may not be familiar to your students. The primary motivation for including this lesson is that they need these skills in their science course/s. You the teacher should make sure that you are clear about the many rules they need to learn.**

Lesson Proper:

I. A. Activity

The following is a list of numbers. The number of significant digits in each number is written in the parenthesis after the number.

234 (3)	0.0122 (3)
745.1 (4)	0.00430 (3)
6007 (4)	0.0003668 (4)
1.3×10^{2} (2)	10000 (1)
7.50×10^{-7} (3)	1000. (4)
0.012300 (5)	2.222×10^{-3} (4)
100.0 (4)	8.004×10^{5} (4)
100 (1)	6120. (4)
7890 (3)	120.0 (4)
4970.00 (6)	530 (2)

Describe what digits are not significant. ________________________________

> **NOTE TO THE TEACHER**
> **If this is the first time that your students will encounter this lesson then you have to be patient in explaining and drilling them on the rules. Give plenty of examples and exercises.**

Important Terms to Remember

Significant digits are the digits in a number that express the precision of a measurement rather than its magnitude. The number of significant digits in a given measurement depends on the number of significant digits in the given data. In calculations involving multiplication, division, trigonometric functions, for example, the number of significant digits in the final answer is equal to the least number of significant digits in any of the factors or data involved.

Rules for Determining Significant Digits

A. All digits that are not zeros are significant.

For example: 2781 has 4 significant digits
82.973 has 5 significant digits

B. Zeros may or may not be significant. Furthermore,

1. Zeros appearing between nonzero digits are significant.
   For example: 20.1 has 3 significant digits
   79002 has 5 significant digits
2. Zeros appearing in front of nonzero digits are not significant.
   For example: 0.012 has 2 significant digits
   0.0000009 has 1 significant digit
3. Zeros at the end of a number and to the right of a decimal are significant digits. Zeros between nonzero digits and significant zeros are also significant.
   For example: 15.0 has 3 significant digits
   25000.00 has 7 significant digits
4. Zeros at the end of a number but to the left of a decimal may or may not be significant. If such a zero has been measured or is the first estimated digit, it is significant. On the other hand, if the zero has not been measured or estimated but is just a place holder it is not significant. A decimal placed after the zeros indicates that they are significant
   For example: 560000 has 2 significant digits
   560000. has 6 significant digits

Significant Figures in Calculations

1. When multiplying or dividing measured quantities, round the answer to as many significant figures in the answer as there are in the measurement with the least number of significant figures.

2. When adding or subtracting measured quantities, round the answer to the same number of decimal places as there are in the measurement with the least number of decimal places.

   For example:

   a. 3.0 x 20.536 = 61.608

   Answer: 61 since the least number of significant digits is 2, coming from 3.0

   b. 3.0 + 20.536 = 23.536

   Answer: 23.5 since the addend with the least number of decimal places is 3.0

II. Questions to Ponder (Post-Activity Discussion)

> **NOTE TO THE TEACHER**
> **The difficult part is to arrive at a concise description of non-significant digits. Do not give up on this task. Students should be able to describe and define significant digits as well as non-significant digits.**

Describe what digits are not significant. *The digits that are not significant are the zeros before a non-zero digit and zeros at the end of numbers without the decimal point.*

Problem 1. Four students weigh an item using different scales. These are the values they report:

a. 30.04 g
b. 30.0 g
c. 0.3004 kg
d. 30 g

How many significant digits are in each measurement?

Answer: *30.04 has 4 significant; 30.0 has 3 significant digits; 0.3004 has 4 significant digits; 30 has 1 significant digit*

Problem 2. Three students measure volumes of water with three different devices. They report the following results:

Device	Volume
Large graduated cylinder	175 mL
Small graduated cylinder	39.7 mL
Calibrated buret	18.16 mL

If the students pour all of the water into a single container, what is the total volume of water in the container? How many digits should you keep in this answer?

Answer: The total volume is 232.86 mL. Based on the measures, the final answer should be 232.9 mL.

On the Scientific Notation

The speed of light is 300 000 000 m/sec, quite a large number. It is cumbersome to write this number in full. Another way to write it is 3.0×10^{8}. How about a very small number like 0.000 000 089? Like with a very large number, a very small number may be written more efficiently. 0.000 000 089 may be written as 8.9×10^{-8}.

Writing a Number in Scientific Notation

1. Move the decimal point to the right or left until after the first significant digit and copy the significant digits to the right of the first digit. If the number is a whole number and has no decimal point, place a decimal point after the first significant digit and copy the significant digits to its right.

For example, 300 000 000 has 1 significant digit, which is 3. Place a decimal point after 3.0

The first significant digit in 0.000 000 089 is 8 and so place a decimal point after 8, (8.9).

2. Multiply the adjusted number in step 1 by a power of 10, the exponent of which is the number of digits that the decimal point moved, positive if moved to the left and negative if moved to the right.

For example, 300 000 000 is written as 3.0×10^8 because the decimal point was moved past 8 places.

0.0 000 089 is written as 8.9×10^{-8} because the decimal point was moved 8 places to the right past the first significant digit 8.

III. Exercises

A. Determine the number of significant digits in the following measurements. Rewrite the numbers with at least 5 digits in scientific notation.

1. 0.0000056 L
2. 4.003 kg
3. 350 m
4. 4113.000 cm
5. 700.0 mL
6. 8207 mm
7. 0.83500 kg
8. 50.800 km
9. 0.0010003 m^3
10. 8 000 L

Answers: 1) 2; 2) 4; 3) 2; 4) 7; 5) 4; 6) 4; 7) 5; 8) 5; 9) 5; 10) 1

B. a. Round off the following quantities to the specified number of significant figures.

1. 5 487 129 m to three significant figures
2. 0.013 479 265 mL to six significant figures
3. 31 947.972 cm^2 to four significant figures
4. 192.6739 m^2 to five significant figures
5. 786.9164 cm to two significant figures

Answers: 1) 5 490 000 m; 2) 0.0134793 mL; 3) 31 950 cm^2; 4) 192.67 m2; 5) 790 cm

b. Rewrite the answers in (a) using the scientific notation

Answers: 1) 5.49×10^6; 2) 1.34793×10^{-2}; 3) 3.1950×10^4; 4) 1.9267×10^2; 5) 7.9×10^2

C. Write the answers to the correct number of significant figures

1. 4.5 X 6.3 ÷ 7.22 **= 3.9**
2. 5.567 X 3.0001 ÷ 3.45 **= 4.84**
3. (37 X 43) ÷ (4.2 X 6.0) **= 63**
4. (112 X 20) ÷ (30 X 63) **= 1**
5. 47.0 ÷ 2.2 **= 21**

D. Write the answers in the correct number of significant figures

1. 5.6713 + 0.31 + 8.123 **= 14.10**
2. 3.111 + 3.11 + 3.1 **= 9.3**
3. 1237.6 + 23 + 0.12 **= 1261**
4. 43.65 – 23.7 **= 20.0**
5. 0.009 – 0.005 + 0.013 **= 0.017**

E. Answer the following.

1. A runner runs the last 45m of a race in 6s. How many significant figures will the runner's speed have? **Answer: 2**

2. A year is 356.25 days, and a decade has exactly 10 years in it. How many significant figures should you use to express the number of days in two decades? **Answer: 1**

3. Which of the following measurements was recorded to 3 significant digits : 50 mL , 56 mL , 56.0 mL or 56.00 mL? **Answer: 56.0 mL**

4. A rectangle measures 87.59 cm by 35.1 mm. Express its area with the proper number of significant figures in the specified unit: **a.** in cm^2 **b.** in mm^2 **Answer: a. 307 cm^2 b. 30 700 mm^2**

5. A 125 mL sample of liquid has a mass of 0.16 kg. What is the density of the liquid in g/mL? **Answer: 1.3 g/mL**

Summary

In this lesson, you learned about significant digits and the scientific notation. You learned the rules in determining the number of significant digits. You also learned how to write very large and very small numbers using the scientific notation.

Lesson 14: More Problems Involving Real Numbers **Time: 1.5 hours**

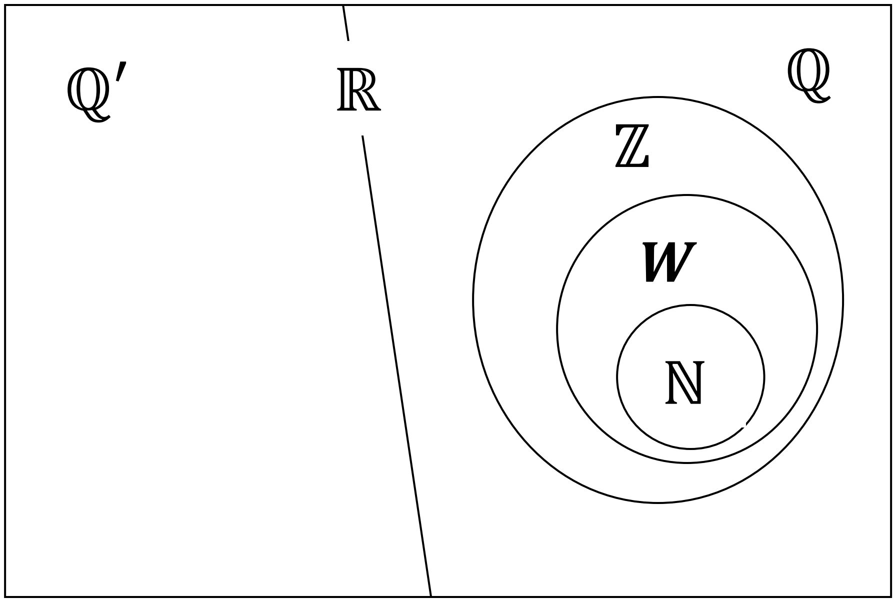
Prerequisite Concepts: Whole numbers, Integers, Rational Numbers, Real Numbers, Sets

Objectives:
In this lesson, you are expected to:
1. Apply the set operations and relations to sets of real numbers
2. Describe and represent real-life situations which involve integers, rational numbers, square roots of rational numbers, and irrational numbers
3. Apply ordering and operations of real numbers in modeling and solving real-life problems

NOTE TO THE TEACHER:
This module provides additional problems involving the set of real numbers. There will be no new concepts introduced, merely reinforcement of previously learned properties of sets and real numbers.

Lesson Proper:
Recall how the set of real numbers was formed and how the operations are performed. Numbers came about because people needed and learned to count. The set of counting numbers was formed. To make the task of counting easier, addition came about. Repeated addition then got simplified to multiplication. The set $\mathbb{N}$ of counting numbers is closed under both the operations of addition and multiplication. When the need to represent zero arose, the set ***W*** of whole numbers was formed. When the operation of subtraction began to be performed, the ***W*** was extended to the set $\mathbb{Z}$ or integers. $\mathbb{Z}$ is closed under the operations of addition, multiplication and subtraction. The introduction of division needed the expansion of $\mathbb{Z}$ to the set $\mathbb{Q}$ of rational numbers. $\mathbb{Q}$ is closed under all the four arithmetic operations of addition, multiplication, subtraction and division. When numbers are used to represent measures of length, the set $\mathbb{Q}$ or rational numbers no longer sufficed. Hence, the set $\mathbb{R}$ of real numbers came to be the field where properties work.

The above is a short description of the way the set of real numbers was built up to accommodate applications to counting and measurement and performance of the four arithmetic operations. We can also explore the set of real numbers by dissection – beginning from the big set, going into smaller subsets. We can say that $\mathbb{R}$ is the set of **all** decimals (positive, negative and zero). The set $\mathbb{Q}$ includes all the decimals which are repeating (we can think of terminating decimals as decimals in which all the digits after a finite number of them are zero). The set $\mathbb{Z}$ comprises all the decimals in which the digits to the right of the decimal point are all zero. This view gives us a clearer picture of the relationship among the different subsets of $\mathbb{R}$ in terms of inclusion.



We know that the *n*th root of any number which is not the *n*th power of a rational number is irrational. For instance, $\sqrt{2}$, $\sqrt{5}$, and $\sqrt[3]{9}$ are irrational.

Example 1. Explain why $3\sqrt{2}$ is irrational.

We use an argument called an indirect proof. This means that we will show why $3\sqrt{2}$ becoming rational will lead to an absurd conclusion. What happens if $3\sqrt{2}$ is rational? Because $\mathbb{Q}$ is closed under multiplication and $\frac{1}{3}$ is rational, then $3\sqrt{2} \times \frac{1}{3}$ is rational. However, $3\sqrt{2} \times \frac{1}{3} = \sqrt{2}$, which we know to be irrational. This is an absurdity. Hence we have to conclude that $3\sqrt{2}$ must be irrational.

Example 2. A deep-freeze compartment is maintained at a temperature of 12°C below zero. If the room temperature is 31°C, how much warmer is the room temperature than the temperature in the deep-freeze compartment.

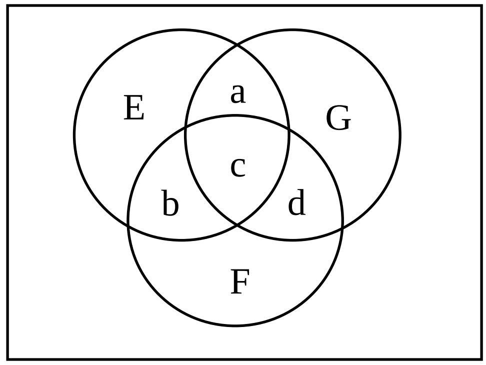
Get the difference between room temperature and the temperature inside the deep-freeze compartment

$31 - (-12) = 43$. *Hence, room temperature is 43°C warmer than the compartment.*

Example 3. Hamming Code

A mathematician, Richard Hamming developed an error detection code to determine if the information sent electronically is transmitted correctly. Computers store information using bits (binary digits, that is a 0 or a 1). For example, 1011 is a four-bit code. Hamming uses a Venn diagram with three "sets" as follows:

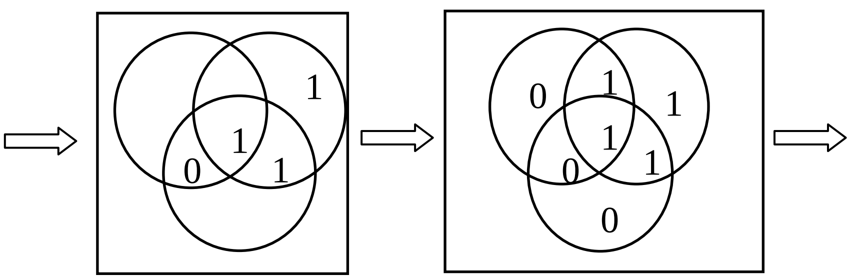
1. The digits of the four-bit code are placed in regions a, b, c, and d, in this order.



2. Three additional digits of 0's and 1's are put in the regions E, F, and G so that each "set" has an even number of 1's.
3. The code is then extended to a 7-bit code using (in order) the digits in the regions a, b, c, d, E, F, G.

For example, the code 1011 is encoded as follows:

1011



1011001

Example 4. Two students are vying to represent their school in the regional chess competition. Felix won 12 of the 17 games he played this year, while Rommel won 11 of the 14 games he played this year. If you were the principal of the school, which student would you choose? Explain.

The Principal will likely use fractions to get the winning ratio or percentage of each player. Felix has a $\frac{12}{17}$ winning ratio, while Rommel has a $\frac{11}{14}$ winning ratio. Since $\frac{11}{14} > \frac{12}{17}$, Rommel will be a logical choice.

Example 5. A class is having an election to decide whether they will go on a fieldtrip. They will have a fieldtrip if more than 50% of the class will vote Yes. Assume that every member of the class will vote. If 34% of the girls and 28% of the boys will vote Yes, will the class go on a fieldtrip? Explain.

Note to the Teacher

This is an illustration of when percentages cannot be added. Although $38 + 28 = 64 > 50$, less than half of the girls and less than half the boys voted Yes. This means that less than half of the students voted Yes. Explain that the percentages given are taken from two different bases (the set of girls and the set of boys in the class), and therefore cannot be added.

Example 6. A sale item was marked down by the same percentage for three years in a row. After two years the item was 51% off the original price. By how much was the price off the original price in the first year?

Since the price after 2 years is 51% off the original price, this means that the price is then 49% of the original. Since the percentage ratio must be multiplied to the original price twice (one per year), and $0.7 \times 0.7 = 0.49$, then the price per year is 70% of the price in the preceding year. Hence the discount is 30% off the original.

Note to the Teacher

This is again a good illustration of the non-additive property of percent. Some students will think that since the discount after 2 years is 51%, the discount per year is 25.5%. Explain the changing base on which the percentage is taken.

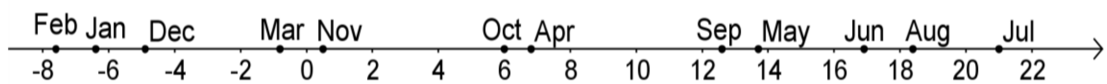
Exercises:

1. The following table shows the mean temperature in Moscow by month from 2001 to 2011

January	-6.4°C	May	13.7°C	September	12.6°C
February	-7.6°C	June	16.9°C	October	6.0°C
March	-0.8°C	July	21.0°C	November	0.5°C
April	6.8°C	August	18.4°C	December	-4.9°C

Plot each temperature point on the number line and list from lowest to highest.

Answer: List can be generated from the plot.



2. Below are the ingredients for chocolate oatmeal raisin cookies. The recipe yields 32 cookies. Make a list of ingredients for a batch of 2 dozen cookies.

1 ½ cups all-purpose flour
1 tsp baking soda
1 tsp salt
1 cup unsalted butter
¾ cup light-brown sugar
¾ cup granulated sugar
2 large eggs
1 tsp vanilla extract
2 ½ cups rolled oats
1 ½ cups raisins
12 ounces semi-sweet chocolate chips

Answer: Since 24/32 = ¾, we get ¾ of each item in the ingredients

1 1/8 cups all-purpose flour ¾ tsp baking soda ¾ tsp salt ¾ cup unsalted butter 9/16 cup light-brown sugar 9/16 cup granulated sugar 1 ½ large eggs ¾ tsp vanilla extract 1 7/8 cups rolled oats 1 1/8 cups raisins 9 ounces semi-sweet chocolate chips

3. In high-rise buildings, floors are numbered in increasing sequence from the ground-level floor to second, third, etc, going up. The basement immediately below the ground floor is usually labeled B1, the floor below it is B2, and so on. How many floors does an elevator travel from the 39th floor of a hotel to the basement parking at level B6?

Answer: We need to find the solution to 39 – *N* = –5. Hence $\boldsymbol{N = 39 - (-5) = 44}$.
Note that Level B6 is –5, not –6. This is because B1 is 0.

4. A piece of ribbon 25 m long is cut into pieces of equal length. Is it possible to get a piece with irrational length? Explain.

Answer: It is not possible to get an irrational length because the length is $\boldsymbol{\frac{25}{N}}$, where *N* is the number of pieces. This is clearly rational as it is the quotient of two integers.

5. Explain why $5 + \sqrt{3}$is irrational. (See Example 1.)

Solution:

What will happen if $\boldsymbol{5 + \sqrt{3}}$ is rational. Then since 5 is rational and the set of rationals is closed under subtraction, $\boldsymbol{5 + \sqrt{3} - 5 = \sqrt{3}}$ will become rational. This is clearly not true. Therefore, $\boldsymbol{5 + \sqrt{3}}$ cannot be rational.

Lesson 15: Measurement and Measuring Length **Time: 2.5 hours**

Prerequisite Concepts: Real Numbers and Operations

Objective
At the end of the lesson, you should be able to:
1. Describe what it means to measure;
2. Describe the development of measurement from the primitive to the present international system of unit;
3. Estimate or approximate length;
4. Use appropriate instruments to measure length;
5. Convert length measurement from one unit to another, including the English system;
6. Solve problems involving length, perimeter and area.

NOTE TO THE TEACHER:
This is a lesson on the English and Metric System of Measurement and using these systems to measure length. Since these systems are widely used in our community, a good grasp of this concept will help your students be more accurate in dealing with concepts involving length such as distance, perimeter and area. This lesson on measurement tackles concepts which your students have most probably encountered and will continue to deal with in their daily lives. Moreover, concepts and skills related to measurement are prerequisites to topics in Geometry as well as Algebra.

Lesson Proper
A.
I. Activity:
Instructions: Determine the dimension of the following using only parts of your arms. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER'S TABLE		CLASSROOM	
	Length	Width	Length	Width	Length	Width
Arm part used*						
Measurement						
Comparison to: (classmate's name)						

* For the arm part, please use any of the following only: the palm, the handspan and the forearm length

Important Terms to Remember:
>palm – the width of one's hand excluding the thumb
> handspan – the distance from the tip of the thumb to the tip of the little finger of one's hand with fingers spread apart.
> forearm length – the length of one's forearm: the distance from the elbow to the tip of the middle finger.

NOTE TO THE TEACHER:
The activities in this module involve measurement of actual objects and lengths found inside the classroom but you may modify the activity and include objects and distances outside the classroom. Letting the students use non-standard units of measurement first will give them the opportunity to appreciate our present measuring tools by emphasizing on the discrepancy of their results vis-a-vis their partner's results.

Answer the following questions:
1. What was your reason for choosing which arm part to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)
Let us answer the questions in the opening activity:
1. What is the appropriate arm part to use in measuring the length and width of the sheet of paper? of the teacher's table? Of the classroom? What was your reason for choosing which arm part to use? Why?
   - While all of the units may be used, there are appropriate units of measurement to be used depending on the length you are trying to measure.
   - For the sheet of paper, the palm is the appropriate unit to use since the handspan and the forearm length is too long.
   - For the teacher's table, either the palm or the handspan will do but the forearm length might be too long to get an accurate measurement.
   - For the classroom, the palm and handspan may be used but you may end up with a lot of repetitions. The best unit to use would be the forearm length.
2. Did you experience any difficulty when you were doing the actual measuring?
   The difficulties you may have experienced might include having to use too many repetitions.
3. Were there differences in your data and your classmate's data? Were the differences significant? What do you think caused those differences?
   If you and your partner vary a lot in height, then chances are your forearm length, handspan and palm may also vary, leading to different measurements of the same thing.

NOTE TO THE TEACHER:
This is a short introduction to the History of Measurement. Further research would be needed to widen to coverage of the concept. The questions that follow will help in enriching the discussion on this particular topic.

History of Measurement

One of the earliest tools that human beings invented was the unit of measurement. In olden times, people needed measurement to determine how long or wide things are; things they needed to build their houses or make their clothes. Later, units of measurement were used in trade and commerce. In the 3rd century BC Egypt, people used their body parts to determine measurements of things; the same body parts that you used to measure the assigned things to you.

The forearm length, as described in the table below, was called a cubit. The handspan was considered a half cubit while the palm was considered 1/6 of a cubit. Go ahead, check out how many handspans your forearm length is. The Egyptians came up with these units to be more accurate in measuring different lengths.

However, using these units of measurement had a disadvantage. Not everyone had the same forearm length. Discrepancies arose when the people started comparing their measurements to one another because measurements of the same thing differed, depending on who was measuring it. Because of this, these units of measurement are called non-standard units of measurement which later on evolved into what is now the inch, foot and yard, basic units of length in the English system of measurement.

III. Exercise:

1. Can you name other body measurements which could have been used as a non-standard unit of measurement? Do some research on other non-standard units of measurement used by people other than the Egyptians.
2. Can you relate an experience in your community where a non-standard unit of measurement was used?

B.

I. Activity

> **NOTE TO THE TEACHER:**
> **In this activity, comparisons of their results will underscore the advantages of using standard units of measurement as compared to using non-standard units of measurement. However, this activity may also provide a venue to discuss the limitations of actual measurements. Emphasize on the differences of their results, however small they may be.**

Instructions: Determine the dimension of the following using the specified English units only. Record your results in the table below. Choose a classmate and compare your results.

	SHEET OF INTERMEDIATE PAPER		TEACHER'S TABLE		CLASSROOM	
	Length	Width	Length	Width	Length	Width
Unit used*						
Measurement						
Comparison to: (classmate's name)						

For the unit used, choose which of the following SHOULD be used: inch or foot.

Answer the following questions:
1. What was your reason for choosing which unit to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:
1. What was your reason for choosing which unit to use? Why?
   - For the sheet of paper, the appropriate unit to use is inches since its length and width might be shorter than a foot.

   - For the table and the classroom, a combination of both inches and feet may be used for accuracy and convenience of not having to deal with a large number.

2. What difficulty, if any, did you experience when you were doing the actual measuring?
3. Were there differences in your data and your classmate's data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?
   - If you and your partner used the steel tape correctly, both your data should have little or no difference at all. The difference should not be as big or as significant as the difference when non-standard units of measurement were used. The slight difference might be caused by how accurately you tried to measure each dimension or by how you read the ticks on the steel tape. In doing actual measurement, a margin of error should be considered.

NOTE TO THE TEACHER:
The narrative that follows provides continuity to the development of the English system of measurement. The conversion factors stated herein only involve common units of length. Further research may include other English units of length.

History of Measurement (Continued)

As mentioned in the first activity, the inch, foot and yard are said to be based on the cubit. They are the basic units of length of the English System of Measurement, which also includes units for mass, volume, time, temperature and angle. Since the inch and foot are both units of length, each can be converted into the other. Here are the conversion factors, as you may recall from previous lessons:

1 foot = 12 inches
1 yard = 3 feet
For long distances, the mile is used:
1 mile = 1,760 yards = 5,280 feet

Converting from one unit to another might be tricky at first, so an organized way of doing it would be a good starting point. As the identity property of multiplication states, the product of any value and 1 is the value itself. Consequently, dividing a value by the same value would be equal to one. Thus, dividing a unit by its equivalent in another unit is equal to 1. For example:

1 foot / 12 inches = 1

3 feet / 1 yard = 1

These conversion factors may be used to convert from one unit to another. Just remember that you're converting from one unit to another so cancelling same units would guide you in how to use your conversion factors. For example:

1. Convert 36 inches into feet:

$$36\ \cancel{inches}\ x\ \frac{1\ foot}{12\ \cancel{inches}} = \mathbf{3\mathit{feet}}$$

2. Convert 2 miles into inches:

$$2\ \cancel{miles}\ x\ \frac{5280\ \cancel{feet}}{1\ \cancel{mile}}\ x\ \frac{12\ inches}{1\ \cancel{foot}} = \frac{2x5280x12}{1x1}\ inches = \mathbf{126,720\mathit{inches}}$$

Again, since the given measurement was multiplied by conversion factors which are equal to 1, only the unit was converted but the given length was not changed.
Try it yourself.

III. Exercise:

Convert the following lengths into the desired unit:

1. Convert 30 inches to feet **Solution:** $\mathbf{30\ \mathit{inches}\ \mathit{x}\frac{1\ \mathit{foot}}{12\ \mathit{inches}} = 2.5\ \mathit{feet}}$

2. Convert 130 yards to inches **Solution:** $\mathbf{130\ \mathit{yards}\ \mathit{x}\frac{3\ \mathit{feet}}{1\ \mathit{yard}}\mathit{x}\frac{12\ \mathit{inches}}{1\ \mathit{foot}} = 4,680\ \mathit{inches}}$

3. Sarah is running in a 42-mile marathon. How many more feet does Sarah need to run if she has already covered 64,240 yards?

Solution:

Step 1: $\mathbf{42\ \mathit{miles}\ \mathit{x}\frac{5,280\ \mathit{feet}}{1\ \mathit{mile}} = 221,760\ \mathit{feet}}$

Step 2: $\mathbf{64,240\ \mathit{yards}\ \mathit{x}\frac{3\ \mathit{feet}}{1\ \mathit{yard}} = 192,720\ \mathit{feet}}$

Step 3: 221,760 feet – 192,720 feet = 29,040 feet

Answer: Sarah needs to run 29,040 feet to finish the marathon

NOTE TO TEACHER:

In item 3, disregarding the units and not converting the different units of measurement into the same units of measurement is a common error.

C.

I. Activity:

NOTE TO THE TEACHER:

This activity introduces the metric system of measurement and its importance. This also highlights how events in Philippine and world

history determined the systems of measurement currently used in the Philippines.

Answer the following questions:
1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?
2. Which particular unit of height were you more familiar with? Why?

II. Questions to Ponder (Post-Activity Discussion)
Let us answer the questions in the activity above:
1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?
   - Chances are, you would find it difficult to answer the first question. As for the second question, a Filipina girl with a height of 5 feet, 7 inches would be considered tall by Filipino standards.

2. Which particular unit of height were you more familiar with? Why?
   - Again, chances are you would be more familiar with feet and inches since feet and inches are still being widely used in measuring and describing height here in the Philippines.

NOTE TO THE TEACHER:
The reading below discusses the development of the Metric system of measurement and the prefixes which the students may use of may encounter later on. Further research may include prefixes which are not commonly used as well as continuing efforts in further standardization of the different units.

History of Measurement (Continued)

The English System of Measurement was widely used until the 1800s and the 1900s when the Metric System of Measurement started to gain ground and became the most used system of measurement worldwide. First described by Belgian Mathematician Simon Stevin in his booklet, De Thiende (The Art of Tenths) and proposed by English philosopher, John Wilkins, the Metric System of Measurement was first adopted by France in 1799. In 1875, the General Conference on Weights and Measures (*Conférence générale des poids et mesures or CGPM)* was tasked to define the different measurements. By 1960, CGPM released the International System of Units (SI) which is now being used by majority of the countries with the biggest exception being the United States of America. Since our country used to be a colony of the United States, the Filipino people were schooled in the use of the English instead of the Metric System of Measurement. Thus, the older generation of Filipinos is more comfortable with English System rather than the Metric System although the Philippines have already adopted the Metric System as its official system of measurement.

The Metric System of Measurement is easier to use than the English System of Measurement since its conversion factors would consistently be in the decimal system, unlike the English System of Measurement where units of lengths have different conversion factors. Check out the units used in your steep tape measure, most likely they are inches and centimeters. The base unit for length is the meter and units longer or shorter than the meter would be achieved by adding prefixes to the base unit. These prefixes may also be used for the base units for mass, volume, time and other measurements. Here are the common prefixes used in the Metric System:

PREFIX	SYMBOL	FACTOR
tera	T	x 1,000,000,000,000
giga	G	x 1,000,000,000
mega	M	x 1,000,000
kilo	k	x 1,000
hecto	h	x 100
deka	da	x 10
deci	d	x 1/10
centi	c	x 1/100
milli	m	x 1/1,000
micro	µ	x 1/1,000,000
nano	n	x 1/1,000,000,000

For example:

1 kilometer = 1,000 meters
1 millimeter = 1/1,000 meter or 1,000 millimeters = 1 meter

These conversion factors may be used to convert from big to small units or vice versa. For example:

1. Convert 3 km to m:

$$3\ \cancel{km}\ x \frac{1{,}000\ m}{1\ \cancel{km}} = \mathbf{3{,}000m}$$

2. Convert 10 mm to m:

$$10\ \cancel{mm}\ x \frac{1\ m}{1000\ \cancel{mm}} = \frac{1}{100}\ \boldsymbol{or}\ \mathbf{0.01m}$$

As you can see in the examples above, any length or distance may be measured using the appropriate English or Metric units. In the question about the Filipina girl whose height was expressed in meters, her height can be converted to the more familiar feet and inches. So, in the Philippines where the official system of measurements is the Metric System yet the English System continues to be used, or as long as we have relatives and friends residing in the United States, knowing how to convert from the English System to the Metric System (or vice versa) would be useful. The following are common conversion factors for length:

1 inch = 2.54 cm
3.3 feet ≈ 1 meter

For example:

Convert 20 inches to cm:

$$20\ \cancel{in}\ x \frac{2.54\ cm}{1\ \cancel{in}} = \mathbf{50.8cm}$$

III. Exercise:

> **NOTE TO THE TEACHER:**
> **Knowing the lengths of selected body parts will help students in estimating lengths and distances by using these body parts and their measurements to estimate certain lengths and distances. Items 5 & 6 might require a review in determining the perimeter and area of common geometric figures.**

1. Using the tape measure, determine the length of each of the following in cm. Convert these lengths to meters.

	PALM	HANDSPAN	FOREARM LENGTH
Centimeters			
Meters			

2. Using the data in the table above, estimate the lengths of the following without using the steel tape measure or ruler:

	BALLPEN	LENGTH OF WINDOW PANE	LENGTH OF YOUR FOOT FROM THE TIP OF YOUR HEEL TO THE TIP OF YOUR TOES	HEIGHT OF THE CHALK BOARD	LENGTH OF THE CHALK BOARD
NON-STANDARD UNIT					
METRIC UNIT					

3. Using the data from table 1, convert the dimensions of the sheet of paper, teacher's table and the classroom into
Metric units. Recall past lessons on perimeter and area and fill in the appropriate columns:

	SHEET OF INTERMEDIATE PAPER				TEACHER'S TABLE				CLASSROOM			
	Length	Width	Peri-meter	Area	Length	Width	Peri-meter	Area	Length	Width	Peri-meter	Area
English units												
Metric Units												

4. Two friends, Zale and Enzo, run in marathons. Zale finished a 21-km marathon in Cebu while Enzo finished a 15-mile marathon in Los Angeles. Who between the two ran a longer distance? By how many meters?

Step 1: $\boldsymbol{21\ km\ x\ \frac{1{,}000\ m}{1\ km} = 21{,}000\ m}$

Step 2: $\boldsymbol{15\ mi\ x\ \frac{1.6\ km}{1\ mi}\ x\ \frac{1{,}000\ m}{1\ km} = 24{,}000\ m}$

Step 3: 24,000 m – 21,000 m = 3,000 m

Answer: Enzo ran a distance of 3,000 meters more.

5. Georgia wants to fence her square garden, which has a side of 20 feet, with two rows of barb wire. The store sold barb wire by the meter at P12/meter. How much money will Georgia need to buy the barb wire she needs?

Step 1: $\boldsymbol{20\ ft\ x\ 4\ sides\ x\ 2\ rows\ = 160\ ft}$

Step 2: $\boldsymbol{160\ ft\ x\ \frac{1\ m}{3.3\ ft} = 48.48m}$ ***rounded up to 49 m since the store sells barb wire by the m***

Step 3: 49 m x P12/meter = P 588

Answer: Georgia will need P 588 to buy 49 meters of barb wire

5. A rectangular room has a floor area of 32 square meters. How many tiles, each measuring 50 cm x 50 cm, are needed to cover the entire floor?

Step 1: $\boldsymbol{50\ cm\ x\ \frac{1\ m}{100\ cm} = 0.5\ m}$

Step 2: Area of 1 tile: 0.5 m x 0.5 m = 0.25 $\boldsymbol{m^2}$

Step 3: 32 $\boldsymbol{m^2}$ ***/ 0.25*** $\boldsymbol{m^2}$ ***= 128 tiles***

Answer: 128 tiles are needed to cover the entire floor

Summary

In this lesson, you learned: 1) that ancient Egyptians used units of measurement based on body parts such as the cubit and the half cubit. The cubit is the length of the forearm from the elbow to the tip of the middle finger; 2) that the inch and foot, the units for length of the English System of Measurement, are believed to be based on the cubit; 3) that the Metric System of Measurement became the dominant system in the 1900s and is now used by most of the countries with a few exceptions, the biggest exception being the United States of America; 4) that it is appropriate to use short base units of length for measuring short lengths and long units of lengths to measure long lengths or distances; 5) how to convert common English units of length into other English units of length using conversion factors; 6) that the Metric System of Measurement is based on the decimal system and is therefore easier to use; 7) that the Metric System of Measurement has a base unit for length (meter) and prefixes to signify long or short lengths or distances; 8) how to estimate lengths and distances using your arm parts and their equivalent Metric lengths; 9) how to convert common Metric units of length into other Metric units of length using the conversion factors based on prefixes; 10) how to convert common English units of length into Metric units of length (and vice versa) using conversion factors; 11) how to solve length, perimeter and area problems using English and Metric units.

Lesson 16: Measuring Weight/Mass and Volume **Time: 2.5 hours**

Prerequisite Concepts: Basic concepts of measurement, measurement of length

About the Lesson:

This is a lesson on measuring volume & mass/weight and converting its units from one to another. A good grasp of this concept is essential since volume & weight are commonplace and have practical applications.

Objectives:

At the end of the lesson, you should be able to:

1. estimate or approximate measures of weight/mass and volume;
2. use appropriate instruments to measure weight/mass and volume;
3. convert weight/mass and volume measurements from one unit to another, including the English system;
4. Solve problems involving weight/mass and volume/capacity.

Lesson Proper

A.

I. Activity:

Read the following narrative to help you review the concept of volume.

Volume

Volume is the amount of space an object contains or occupies. The volume of a container is considered to be the capacity of the container. This is measured by the number of cubic units or the amount of fluid it can contain and not the amount of space the container occupies. The base SI unit for volume is the cubic meter (m^3). Aside from cubic meter, another commonly used metric unit for volume of solids is the cubic centimeter (cm^3 or cc) while the commonly used metric units for volume of fluids are the liter (L) and the milliliter (mL).

Hereunder are the volume formulae of some regularly-shaped objects:

Cube: Volume = edge x edge x edge ($V = e^3$)

Rectangular prism: Volume = length x width x height (V = lwh)

Triangular prism: Volume = ½ x base of the triangular base x height of the triangular base x Height of the prism

$$(V = (\frac{1}{2}bh)H)$$

Cylinder: Volume = π x $(\text{radius})^2$ x height of the cylinder ($V = \pi r^2 h$)

Other common regularly-shaped objects are the different pyramids, the cone and the sphere. The volumes of different pyramids depend on the shape of its base. Here are their formulae:

Square-based pyramids: Volume = 1/3 x $(\text{side of base})^2$ x height of pyramid($V = 1/3\ s^2h$)

Rectangle-based pyramid: Volume=1/3 x length of the base x width of the base x height of pyramid(V=1/3 lwh)

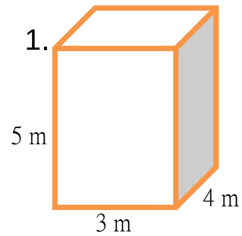
Triangle-based pyramid: Volume = 1/3 x ½ x base of the triangle x height of the triangle x Height of the pyramid

$$(V = \frac{1}{3}(\frac{1}{2}bh)H)$$

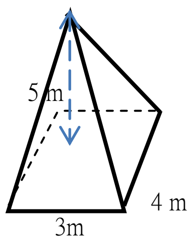
Cone: Volume = 1/3 x π x $(\text{radius})^2$ x height

Sphere: Volume = 4/3 x πx (radius)^3 (V = 4/3 πr^3)

Here are some examples:

1. 

V = lwh = 3 m x 4 m x 5 m
= (3 x 4 x 5) x (m x m x m) = 60 m^3

2. 

V = 1/3 lwh = 1/3 x 3 m x 4 m x 5 m
= (1/3 x 3 x 4 x 5) x (m x m x m) = 20 m^3

Answer the following questions:

1. Cite a practical application of volume.

2. What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.

3. What do you notice about the parts of the formulas that have been shaded? Come up with a general formula for the volume of all the given pyramids and for the cone.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the opening activity:

1. Cite a practical application of volume.

   Volume is widely used from baking to construction. Baking requires a degree of precision in the measurement of the ingredients to be used thus measuring spoons and cups are used. In construction, volume is used to measure the size of a room, the amount of concrete needed to create a specific column or beam or the amount of water a water tank could hold.

2. What do you notice about the parts of the formulas that have been underlined? Come up with a general formula for the volume of all the given prisms and for the cylinder.

   The formulas that have been underlined are formulas for area. The general formula for the volume of the given prisms and cylinder is just the area of the base of the prisms or cylinder times the height of the prism or cylinder ($V = A_{base}h$).

3. What do you notice about the parts of the formulas that have been shaded? Come up with a general formula for the volume of all the given pyramids and for the cone.

   The formulas that have been shaded are formulas for the volume of prisms or cylinders. The volume of the given pyramids is just 1/3 of the volume of a prism whose base and height are equal to that of the pyramid while the formula for the cone is just 1/3 of the volume of a cylinder with the same base and height as the cone ($V = 1/3\ V_{\text{prism or cylinder}}$).

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. How big is a Toblerone box (triangular prism) if its triangular side has a base of 3 cm and a height of 4.5 cm and the box's height is 25 cm?

Volume triangular prism : V=bh/2 H

= [(3 cm)(4.5 cm)/2][25 cm]

= 168.75 cm3

2. How much water is in a cylindrical tin can with a radius of 7 cm and a height of 20 cm if it is only a quarter full?

Step 1: Volumecylinder: V = πr2h

Step 2: ¼ V = ¼ (3080 cm3)

= (22/7)(7 cm)(7 cm)(20 cm)

= 770 cm3

= 3080 cm3

NOTE TO THE TEACHER **A common error in this type of problem is not noticing that the problem asks for the volume of water in the tank when it's only a quarter full.**

3. Which of the following occupies more space, a ball with a radius of 4 cm or a cube with an edge of 60 mm?

Step 1: Vsphere = 4/3 πr3 **Step 2: Vcube = e3**

Step 3: Since Vball > Vcube, then

= 4/3 (22/7)((4 cm)3) **= (6 cm)3** **the ball occupies more space**

= 268.19 cm3 **= 216 cm3** **than the cube.**

NOTE TO THE TEACHER **One of the most common mistakes involving this kind of problem is the disregard of the units used. In order to accurately compare two values, they must be expressed in the same units.**

B.

I. Activity

Materials Needed:

Ruler / Steel tape measure

Different regularly-shaped objects (brick, cylindrical drinking glass, balikbayan box)

Instructions: Determine the dimension of the following using the specified metric units only. Record your results in the table below and compute for each object's volume using the unit used to measure the object's dimensions. Complete the table by expressing/converting the volume using the specified units.

		BRICK			DRINKING GLASS		BALIKBAYAN BOX			CLASSROOM		
		Length	Width	Height	Radius	Height	Length	Width	Height	Length	Width	Height
Unit used*												
Measurement												
Volume	cm^3											
	m^3											
	in^3											
	ft^3											

For the unit used, choose ONLY one: centimeter or meter.

Answer the following questions:

1. What was your reason for choosing which unit to use? Why?
2. How did you convert the volume from cc to m^3 or vice versa?
3. How did you convert the volume from cc to the English units for volume?

Volume (continued)

The English System of Measurement also has its own units for measuring volume or capacity. The commonly used English units for volume are cubic feet (ft^3) or cubic inches (in^3) while the commonly used English units for fluid volume are the pint, quart or gallon. Recall from the lesson on length and area that while the Philippine government has mandated the use of the Metric system, English units are still very much in use in our society so it is an advantage if we know how to convert from the English to the Metric system and vice versa. Recall as well from the previous lesson on measuring length that a unit can be converted into another unit using conversion factors. Hereunder are some of the conversion factors which would help you convert given volume units into the desired volume units:

1 m^3 = 1 million cm^3
1 ft^3 = 1,728 in^3
1 in^3 = 16.4 cm^3
1 m^3 = 35.3 ft^3

1 gal = 3.79 L
1 gal = 4 quarts
1 quart = 2 pints
1 pint = 2 cups
1 cup = 16 tablespoons
1 tablespoon = 3 teaspoons

Since the formula for volume only requires length measurements, another alternative to converting volume from one unit to another is to convert the object's dimensions into the desired unit before solving for the volume.

For example:

1. How much water, in cubic centimeters, can a cubical water tank hold if it has an edge of 3 meters?

Solution 1 (using a conversion factor):

i. Volume = $e^3 = (3\ m)^3 = 27\ m^3$

ii. $27\ m^3$ x $^{1\ million\ cm3}/_{1\ m3}$ = **27 million cm^3**

Solution 2 (converting dimensions first):

i. 3 m x $^{100\ cm}/_{1\ m}$ = 300 cm

ii. Volume = $e^3 = (300\ cm)^3$ = **27 million cm^3**

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What was your reason for choosing which unit to use?

   Any unit on the measuring instrument may be used but the decision on what unit to use would depend on how big the object is. In measuring the brick, the glass and the balikbayan box, the appropriate unit to use would be centimeter. In measuring the dimensions of the classroom, the appropriate unit to use would be meter.

2. How did you convert the volume from cc to m^3 or vice versa?

   Possible answer would be converting the dimensions to the desired units first before solving for the volume.

3. How did you convert the volume from cc or m^3 to the English units for volume?

   Possible answer would be by converting the dimensions into English units first before solving for the volume.

III. Exercises:

Answer the following items. Show your solutions.

1. Convert 10 m^3 to ft^3

   10 m3 x 35.94 ft^3/1 m^3 = 359.4 ft^3

NOTE TO THE TEACHER

A common error in this type of problem is the use of the conversion factor for meter to feet instead of the conversion factor from m^3 to ft^3. This conversion factor may be arrived at by computing for the number of cubic feet in 1 cubic meter.

2. Convert 12 cups to mL

$$\mathbf{12\ cups\ x\ \frac{1\ pint}{2\ cups}\ x\ \frac{1\ quart}{2\ pints}\ x\ \frac{1\ gal}{4\ quarts}\ x\ \frac{3.79\ L}{1\ gal}\ x\ \frac{1000\ mL}{1\ L} = 2,842.5\ mL}$$

3. A cylindrical water tank has a diameter of 4 feet and a height of 7 feet while a water tank shaped like a rectangular prism has a length of 1 m, a width of 2 meters and a height of 2 meters. Which of the two tanks can hold more water? By how many cubic meters?

   Step 1: Vcylinder = πr2h

   Step 2: Vrectangular prism = lwh

   = (22/7)(0.61 m)2(2.135 m)

   = (1 m)(2 m)(2 m)

   = 2.5 m3

   = 4 m3

The rectangular water tank can hold 1.5 m3 more water than the cylindrical water tank.

> **NOTE TO THE TEACHER**
> **One of the most common mistakes involving this kind of problem is the disregard of the units used. In order to accurately compare two values, they must be expressed in the same units.**

C.

I. Activity:

Problem: The rectangular water tank of a fire truckmeasures 3 m by 4 m by 5 m. How many liters of water can the fire truck hold?

Volume (Continued)

While capacities of containers are obtained by measuring its dimensions, fluid volume may also be expressed using Metric or English units for fluid volume such as liters or gallons. It is then essential to know how to convert commonly used units for volume into commonly used units for measuring fluid volume.

While the cubic meter is the SI unit for volume, the liter is also widely accepted as a SI-derived unit for capacity. In 1964, after several revisions of its definition, the General Conference on Weights and Measures (CGPM) finally defined a liter as equal to one cubic decimeter. Later, the letter L was also accepted as the symbol for liter.

This conversion factor may also be interpreted in other ways. Check out the conversion factors below:

$1 \text{ L} = 1 \text{ dm}^3$
1 mL = 1 cc
$1{,}000 \text{ L} = 1 \text{ m}^3$

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

Step 1: V = lwh
= 3m x 4m x 5m
= 60 m^3

Step 2: $60 \text{ m}^3 \times {}^{1,000 \text{ L}} / {}_{1 \text{ m3}}$ = ***60,000 L***

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. A spherical fish bowl has a radius of 21 cm. How many mL of water is needed to fill half the bowl?

Vsphere = 4/3 πr3
= (4/3)(22/7)(21 cm)3
= 38,808 cm3 or cc
Since 1 cc = 1 mL, then 38,808 mL of water is needed to fill the tank

2. A rectangular container van needs to be filled with identical cubical balikbayan boxes. If the container van's length, width and height are 16 ft, 4 ft and 6ft, respectively, while each balikbayan box has an edge of 2 ft, what is the maximum number of balikbayan boxes that can be placed inside the van?

Step 1: Vvan = lwh

= (16 ft)(4 ft)(6 ft)
= 384 ft3 / 8 ft3
Step 2: Vbox = e3
= (2 ft)3
= 8 ft3
Step 3: Number of boxes = Vvan / Vbox
= 384 ft3
= 48 boxes

3. A drinking glass has a height of 4 in, a length of 2 in and a width of 2 in while a baking pan has a width of 4 in, a length of 8 in and a depth of 2 in. If the baking pan is to be filled with water up to half its depth using the drinking glass, how many glasses full of water would be needed?

Step 1: Vdrinking glass = lwh
= (4 in)(2 in)(2 in)
= 16 in^3
Step 2: Vbaking pan = lwh
= (4 in)(8 in)(2 in)
= 64 in^3 when full -> 32 in^3 when half full
Step 3: No. of glasses = (1/2)Vpan/Vglass
= 32 in^3/16 in^3 -> 2 glasses of water are needed to fill half the pan

D.
Activity:
Instructions: Fill the table below according to the column headings. Choose which of the available instruments is the most appropriate in measuring the given object's weight. For the weight, choose only one of the given units.

	INSTRUMENT*	WEIGHT		
		Gram	Kilogram	Pound
¢25-coin				
₱5-coin				
Small toy marble				
Piece of brick				
Yourself				

*Available instruments: triple-beam balance, nutrition (kitchen) scale, bathroom scale
Answer the following questions:
1. What was your reason for choosing which instrument to use?
2. What was your reason for choosing which unit to use?
3. What other kinds of instruments for measuring weight do you know?
4. What other units of weight do you know?

Mass/ Weight

In common language, mass and weight are used interchangeably although weight is the more popular term. Oftentimes in daily life, it is the mass of the given object which is called its weight. However, in the scientific community, mass and weight are two different measurements. Mass refers to the amount of matter an object has while weight is the gravitational force acting on an object.

Weight is often used in daily life, from commerce to food production. The base SI unit for weight is the kilogram (kg) which is almost exactly equal to the mass of one liter of water. For the English System of Measurement, the base unit for weight is the pound (lb). Since both these units are used in Philippine society, knowing how to convert from pound to kilogram or vice versa is important. Some of the more common Metric units are the gram (g) and the milligram (mg) while another commonly used English unit for weight is ounces (oz). Here are some of the conversion factors for these units:

1 kg = 2.2 lb	1 g = 1000 mg	1 metric ton = 1000 kg
1 kg = 1000 g	1 lb = 16 oz	

Use these conversion factors to convert common weight units to the desired unit. For example:

Convert 190 lb to kg: $19\cancel{0}\ lb\ x\ \frac{1\ kg}{2.\cancel{2}\ lb} = 86.18\ kg$

II. Questions to Ponder (Post-Activity Discussion)

1. What was your reason for choosing which instrument to use?
   Possible reasons would include how heavy the object to be weighed to the capacity of the weighing instrument.
2. What was your reason for choosing which unit to use?
   The decision on which unit to use would depend on the unit used by the weighing instrument. This decision will also be influenced by how heavy the object is.
3. What other kinds of instruments for measuring weight do you know?
   Other weighing instruments include the two-pan balance, the spring scale, the digital scales.
4. What other common units of weight do you know?
   Possible answers include ounce, carat and ton.

III. Exercise:

Answer the following items. Show your solution.

1. Complete the table above by converting the measured weight into the specified units.

2. When Sebastian weighed his balikbayan box, its weight was 34 kg. When he got to the airport, he found out that the airline charged $5 for each lb in excess of the free baggage allowance of 50 lb. How much will Sebastian pay for the excess weight?

   Step 1: 34 kg -> lb
   34 kg x 2.2 lb / 1 kg = 74.8 lb
   Step 2: 74.8 lb – 50 = 24.8 lb in excess
   Step 3: Payment = (excess lb)($5)
   = (24.8 lb)($5)
   = $124.00

3. A forwarding company charges P1,100 for the first 20 kg and P60 for each succeeding 2 kg for freight sent to Europe. How much do you need to pay for a box weighing 88 lb?

Step 1: 88 lb -> kg
88 lb x 1 kg / 2.2 lb = 40 kg
Step 2: (40 – 20)/2 = 10
Step 3: freight charge = ₱1,100 + (10)(₱60)
= ₱1,700.00

Summary

In this lesson, you learned: 1) how to determine the volume of selected regularly-shaped solids; 2) that the base SI unit for volume is the cubic meter; 3) how to convert Metric and English units of volume from one to another; 4) how to solve problems involving volume or capacity; 5) that mass and weight are two different measurements and that what is commonly referred to as weight in daily life is actually the mass; 6) how to use weighing intruments to measure the mass/weight of objects and people; 7) how to convert common Metric and English units of weight from one to another; 8) how to solve problems involving mass / weight.

Lesson 17: Measuring Angles, Time and Temperature **Time: 2.5 hours**

Prerequisite Concepts: Basic concepts of measurement, ratios

About the Lesson:

This lesson should reinforce your prior knowledge and skills on measuring angle, time and temperature as well as meter reading. A good understanding of this concept would not only be useful in your daily lives but would also help you in geometry and physical sciences.

Objectives:

At the end of the lesson, you should be able to:

1. estimate or approximate measures of angle, time and temperature;
2. use appropriate instruments to measure angles, time and temperature;
3. solve problems involving time, speed, temperature and utilities usage (meter reading).

Lesson Proper

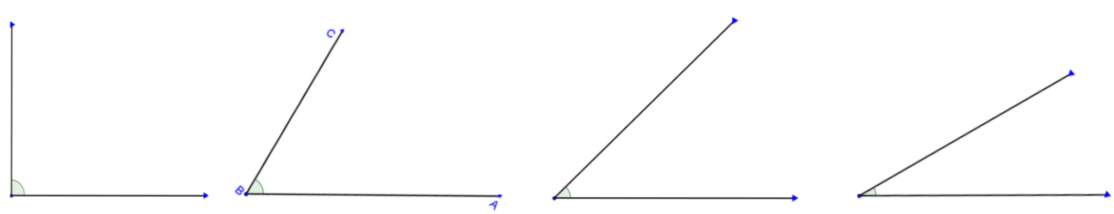
A.

I. Activity:

Material needed:

Protractor

Instruction: Use your protractor to measure the angles given below. Write your answer on the line provided.



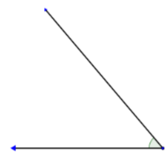
1.____________ 2.______________ 3._____________ 4.____________

Angles

Derived from the Latin word *angulus*, which means corner, an angle is defined as a figure formed when two rays share a common endpoint called the vertex. Angles are measured either in degree or radian measures. A protractor is used to determine the measure of an angle in degrees. In using the protractor, make sure that the cross bar in the middle of the protractor is aligned with the vertex and one of the legs of the angle is aligned with one side of the line passing through the cross bar. The measurement of the angle is determined by its other leg.

Answer the following items:

1. Estimate the measurement of the angle below. Use your protractor to check your estimate.



Estimate_______________
Measurement using the protractor_______

2. What difficulties did you meet in using your protractor to measure the angles?
3. What can be done to improve your skill in estimating angle measurements?

II. Questions to Ponder (Post-activity discussion):

1. Estimate the measurement of the angles below. Use your protractor to check your estimates.

   Measurement = 50°

2. What difficulties did you meet in using your protractor to measure the angles?

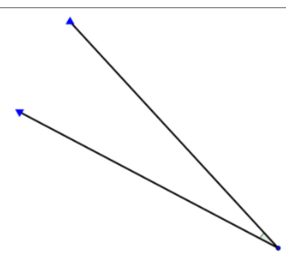
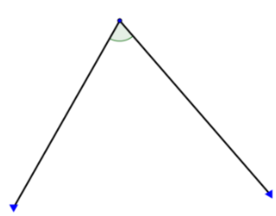
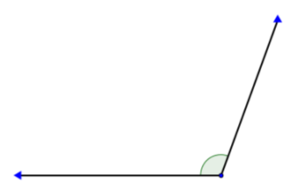
   One of the difficulties you may encounter would be on the use of the protractor and the angle orientation. Aligning the cross bar and base line of the protractor with the vertex and an angle leg, respectively, might prove to be confusing at first, especially if the angle opens in the clockwise orientation. Another difficulty arises if the length of the leg is too short such that it won't reach the tick marks on the protractor. This can be remedied by extending the leg.

3. What can be done to improve your skill in estimating angle measurements?

   You may familiarize yourself with the measurements of the common angles like the angles in the first activity and use these angles in estimating the measurement of other angles.

III. Exercise:

Instructions: Estimate the measurement of the given angles, then check your estimates by measuring the same angles using your protractor.

ANGLE	 A	 B	 C
ESTIMATE			
MEASUREMENT	**20°**	**70°**	**110°**

B.

I. Activity

Problem: An airplane bound for Beijing took off from the Ninoy Aquino International Airport at 11:15 a.m. Its estimated time of arrival in Beijing is at1550 hrs. The distance from Manila to Beijing is 2839 km.

1. What time (in standard time) is the plane supposed to arrive in Beijing?
2. How long is the flight?
3. What is the plane's average speed?

Time and Speed

The concept of time is very basic and is integral in the discussion of other concepts such as speed. Currently, there are two types of notation in stating time, the 12-hr notation (standard time) or the 24-hr notation (military or astronomical time). Standard time makes use of a.m. and p.m. to distinguish between the time from 12midnight to 12 noon (a.m. or *ante meridiem*) and from 12 noon to 12 midnight (p.m. or *post meridiem*). This sometimes leads to ambiguity when the suffix of a.m. and p.m. are left out. Military time prevents this ambiguity by using the 24-hour notation where the counting of the time continues all the way to 24. In this notation, 1:00 p.m. is expressed as 1300 hours or 5:30 p.m. is expressed as 1730 hours.

Speed is the rate of an object's change in position along a line. Average speed is determined by dividing the distance travelled by the time spent to cover the distance (Speed = $^{distance}/_{time}$ or S = $^{d}/_{t}$, read as "distance per time"). The base SI unit for speed is meters per second ($^{m}/_{s}$). The commonly used unit for speed is $^{Kilometers}/_{hour}$ (kph or $^{km}/_{h}$) for the Metric system and miles/hour (mph or $^{mi}/_{hr}$) for the English system.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. What time (in standard time) is the plane supposed to arrive in Beijing?
   3:50 p.m.
2. How long is the flight?
   1555 hrs – 1115 hrs = 4 hrs, 40 minutes or 4.67 hours
3. What is the plane's average speed?
   S = d/t
   = 2839 km / 4.67 hrs
   = 607.92 kph

III. Exercise:

Answer the following items. Show your solutions.

1. A car left the house and travelled at an average speed of 60 kph. How many minutes will it take for the car to reach the school which is 8 km away from the house?

t = d / S
= 8 km / 60 kph
= 2/15 hours = 8 minutes

NOTE TO THE TEACHER

One of the most common mistakes of the students is disregarding the units of the given data as well as the unit of the answer. In this particular case, the unit of time used in the problem is hours while the desired unit for the answer is in minutes.

2. Sebastian stood at the edge of the cliff and shouted facing down. He heard the echo of his voice 4 seconds after he shouted. Given that the speed of sound in air is 340 m / s, how deep is the cliff?

Let d be the total distance travelled by Sebastian's voice.

d = St
= (340 m/s)(4 sec)

= 1,360 m

Since Sebastian's voice has travelled from the cliff top to its bottom and back, the cliff depth is therefore half of d. Thus, the depth of the cliff is

d / 2 = 680 m

> **NOTE TO THE TEACHER**
>
> **One of the common mistakes students made in this particular problem is not realizing that 4 seconds is the time it took for Zale's voice to travel from the top of the cliff and back to Zale. Since it took 4 seconds for Sebastian's voice to bounce back to him, 1,360 m is twice the depth of the cliff.**

3. Maria ran in a 42-km marathon. She covered the first half of the marathon from 0600 hrs to 0715 hours and stopped to rest. She resumed running and was able to cover the remaining distance from 0720 hrs to 0935 hrs. What was Maria's average speed for the entire marathon?

Since the total distance travelled is 42 km and the total time used is 3:35 or 3 7/12 hrs. If S is the average speed of Maria, then

S = 42 km / (3 7/12 hours)

= 11.72 kph

> **NOTE TO THE TEACHER**
>
> **A common error made in problems such as this is the exclusion of the time Maria used to rest from the total time it took her to finish the marathon.**

C.

I. Activity:

Problem: Zale, a Cebu resident, was packing his suitcase for his trip to New York City the next day for a 2-week vacation. He googled New York weather and found out the average temperature there is 59°F. Should he bring a sweater? What data should Zale consider before making a decision?

Temperature

Temperature is the measurement of the degree of hotness or coldness of an object or substance. While the commonly used units are Celsius (°C) for the Metric system and Fahrenheit (°F) for the English system, the base SI unit for temperature is the Kelvin (K). Unlike the Celsius and Fahrenheit which are considered degrees, the Kelvin is considered as an absolute unit of measure and therefore can be worked on algebraically.

Hereunder are some conversion factors:

$°C = (^5/_9)(°F - 32)$

$°F = (^9/_5)(°C) + 32$

$K = °C + 273.15$

For example:

Convert 100°C to °F: $°F = (^9/_5)(100\ °C) + 32$

$= 180 + 32$

$= 212\ °F$

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the problem above:

1. What data should Zale consider before making a decision?

   In order to determine whether he should bring a sweater or not, Zale needs to compare the average temperature in NYC to the temperature he is used to which is the average temperature in Cebu. He should also express both the average temperature in NYC and in Cebu in the same units for comparison.

2. Should Zale bring a sweater?

   The average temperature in Cebu is between 24 – 32 °C. Since the average temperature in NYC is 59°F which is equivalent to 15°C, Zale should probably bring a sweater since the NYC temperature is way below the temperature he is used to. Better yet, he should bring a jacket just to be safe.

III. Exercise:

Instructions: Answer the following items. Show your solution.

1. Convert 14°F to K.

Step 1: $^{o}C = (5/9)(14^{o}F - 32)$
$= -10^{o}$

Step 2: $^{o}K = {}^{o}C + 273.15$
$= -10^{o} + 273.15$
$^{o}K = 263.15^{o}$

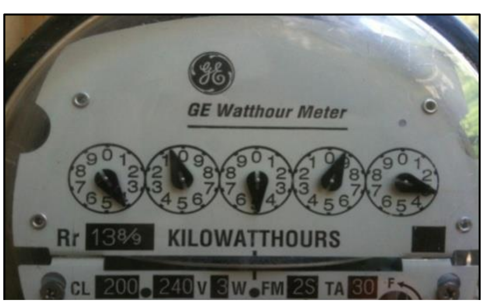
2. Maria was preparing the oven to bake brownies. The recipe's direction was to pre-heat the oven to 350°F but her oven thermometer was in °C. What should be the thermometer reading before Maria puts the baking pan full of the brownie mix in the oven?

$^{o}C = (5/9)(^{o}F – 32)$
$= (5/9)(350^{o}F – 32)$
$= (5/9)(318)$
$= 176.67^{o}$

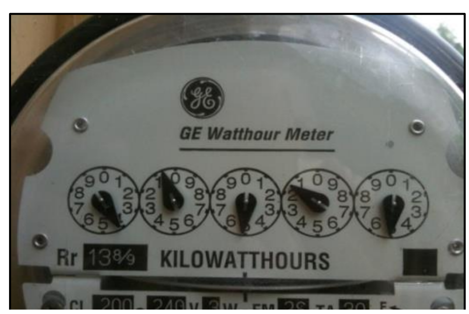
D.

Activity:

Instructions: Use the pictures below to answer the questions that follow.



Initial electric meter reading at 0812 hrs on 14 Feb 2012



Final electric meter reading at 0812 hrs on 15 Feb 2012

1. What was the initial meter reading? Final meter reading?
2. How much electricity was consumed during the given period?
3. How much will the electric bill be for the given time period if the electricity charge is P9.50 / kiloWatthour?

Reading Your Electric Meter

Nowadays, reading the electric meter would be easier considering that the newly-installed meters are digital but most of the installed meters are still dial-based. Here are the steps in reading the electric meter:

a. To read your dial-based electric meter, read the dials from left to right.

b. If the dial hand is between numbers,the smaller of the two numbers should be used. If the dial hand is on the number, check out the dial to the right. If the dial hand has passed zero, use the number at which the dial hand is pointing. If the dial hand has not passed zero, use the smaller number than the number at which the dial hand is pointing.

c. To determine the electric consumption for a given period, subtract the initial reading from the final reading.

NOTE TO THE TEACHER

The examples given here are simplified for discussion purposes. The computation reflected in the monthly electric bill is much more complicated than the examples given here. It is advisable to ask students to bring a copy of the electric bill of their own homes for a more thorough discussion of the topic.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions above:

1. What was the initial meter reading? final meter reading?
   The initial reading is 40493 kWh. For the first dial from the left, the dial hand is on the number 4 so you look at the dial immediately to the right which is the second dial. Since the dial hand of the second dial is past zero already, then the reading for the first dial is 4. For the second dial, since the dial hand is between 0 and 1 then the reading for the second dial is 0. For the third dial from the left, the dial hand is on the number 5 so you look at the dial immediately to the right which is the fourth dial. Since the dial hand of the fourth dial has not yet passed zero, then the reading for the third dial is 4. The final reading is 40515 kWh.
2. How much electricity was consumed during the given period?
   Final reading – initial reading = 40515 kWh – 40493 kWh = 22 kWh
3. How much will the electric bill be for the given time period if the electricity charge is ₱9.50 / kiloWatthour?
   Electric bill = total consumption x electricity charge
   = 22 kWh x P9.50 / kWh
   = P209

III. Exercise:

Answer the following items. Show your solution.

1. The pictures below show the water meter reading of Sebastian's house.

Initial meter reading at 0726 hrs
on 20 February 2012

Final meter reading at 0725 hrs
on 21 February 2012

If the water company charges P14 / cubic meter of water used, how much must Sebastian pay the water company for the given period?

Step 1: Water consumption = final meter reading – initial meter reading

Step 2:Payment = number of cubic meters of water consumed x rate

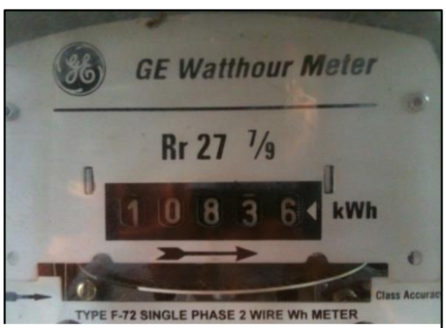
= 2393.5 m3 – 2392.7 m3

= 0.8 m3 x ₱14/m3

= 0.8 m3

= ₱11.20

2. The pictures below show the electric meter reading of Maria's canteen.



Initial meter reading at 1600 hrs on 20
Feb 2012

Final meter reading @ 1100 hrs on 22
Feb 2012

If the electric charge is P9.50 / kWh, how much will Maria pay the electric company for the given period?

Step 1: consumption = final meter reading – initial meter reading

Step 2: Payment = number of kWh consumed x rate

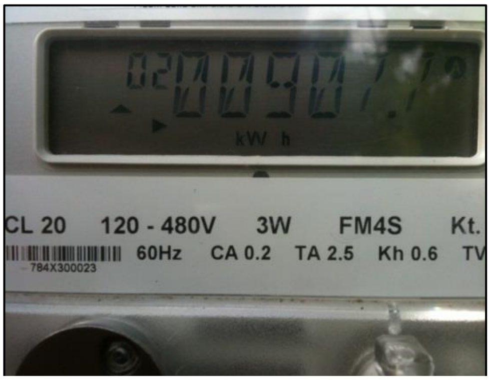
= 10860 kWh – 10836 kWh

= 24 kWh x ₱9.50/kWh

= 24 kWh

= ₱228.00

3. The pictures below show the electric meter reading of a school.



Initial meter reading @1700 hrs on 15 July 2012

Final meter reading @ 1200 hrs on 16 July 2012

Assuming that the school's average consumption remains the same until 1700 hrs of 15 August 2012 and the electricity charge is P9.50 / kWh, how much will the school be paying the electric company?

Average hourly electric consumption = (final meter reading – initial meter reading) / time
= (911.5 kWh – 907.7 kWh) / 19 hrs
= 0.2 kW

Electric consumption from 15 July 2012 to 15 August 2012 = average hourly consumption x number of hours
= 0.2 kW x 744 hrs
= 148.8 kWh

Payment = number of kWh consumed x rate
= 148.8 kWh x ₱9.50/kWh
= ₱1,413.60

Summary

In this lesson, you learned:

1. how to measure angles using a protractor;
2. how to estimate angle measurement;
3. express time in 12-hr or 24-hr notation;
4. how to measure the average speed as the quotient of distance over time;
5. convert units of temperature from one to the other;
6. solve problems involving time, speed and temperature;
7. read utilities usage.

Lesson 18: Constants, Variables and Algebraic Expressions

Prerequisite Concepts: Real Number Properties and Operations

Objectives:
At the end of the lesson, you should be able to:
1. Differentiate between constants and variables in a given algebraic expression
2. Evaluate algebraic expressions for given values of the variables

> **NOTE TO THE TEACHER**
> **This lesson is an introduction to the concept of constants, unknowns and variables and algebraic expressions. Familiarity with this concept is necessary in laying a good foundation for Algebra and in understanding and translating mathematical phrases and sentences, solving equations and algebraic word problems as well as in grasping the concept of functions. In this lesson, it is important that you do not assume too much. Many misconceptions have arisen from a hurried up discussion of these basic concepts. Take care in introducing the concept of a letter and its different uses in algebra and the concept of a term in an algebraic expression.**

Lesson Proper
I. Activity

A. Instructions: Complete the table below according to the pattern you see.

TABLE A		
ROW	1^{ST} TERM	2^{ND} TERM
a.	1	5
b.	2	6
c.	3	7
d.	4	
e.	5	
f.	6	
g.	59	
h.	Any number *n*	

B. Using Table A as your basis, answer the following questions:
1. What did you do to determine the 2^{nd} term for rows d to f?
2. What did you do to determine the 2^{nd} term for row g?
3. How did you come up with your answer in row h?
4. What is the relation between the 1^{st} and 2^{nd} terms?
5. Express the relation of the 1^{st} and 2^{nd} terms in a mathematical sentence.

> **NOTE TO THE TEACHER**
> **Encourage your students to talk about the task and to verbalize whatever pattern they see. Again, do not hurry them up. Many students do not get the same insight as the fast ones.**

II. Questions to Ponder (Post-Activity Discussion)

A. The 2^{nd} terms for rows d to f are 8, 9 and 10, respectively. The 2^{nd} term in row g is 63. The 2^{nd} term in row h is the sum of a given number *n* and 4.

B.

1. One way of determining the 2^{nd} terms for rows d to f is to add 1 to the 2^{nd} term of the preceding row (e.g 7 + 1 = 8). Another way to determine the 2^{nd} term would be to add 4 to its corresponding 1^{st} term (e.g. 4 + 4 = 8).

> **NOTE TO TEACHER:**
> **Most students would see the relation between terms in the same column rather than see the relation between the 1st and 2nd terms. Students who use the relation within columns would have a hard time determining the 2nd terms for rows g & h.**

2. Since from row f, the first term is 6, and from 6 you add 53 to get 59, to get the 2^{nd} term of row g, 10 + 53 = 63. Of course, you could have simply added 4 to 59.
3. The answer in row h is determined by adding 4 to *n*, which represents any number.
4. The 2^{nd} term is the sum of the 1^{st} term and 4.
5. To answer this item better, we need to be introduced to Algebra first.

Algebra

We need to learn a new language to answer item 5. The name of this language is Algebra. You must have heard about it. However, Algebra is not entirely a new language to you. In fact, you have been using its applications and some of the terms used for a long time already. You just need to see it from a different perspective.

Algebra comes from the Arabic word, *al-jabr (*which means restoration), which in turn was part of the title of a mathematical book written around the 820 AD by Arab mathematician, *Muhammad ibn Musa al-Khwarizmi*. While this book is widely considered to have laid the foundation of modern Algebra, history shows that ancient Babylonian, Greek, Chinese and Indian mathematicians were discussing and using algebra a long time before this book was published.

Once you've learned this new language, you'll begin to appreciate how powerful it is and how its applications have drastically improved our way of life.

III. Activity

> **NOTE TO THE TEACHER**
> **It is crucial that students begin to think algebraically rather than arithmetically. Thus, emphasis is placed on how one reads algebraic expressions. This activity is designed to allow students to realize the two meanings of some signs and symbols used in both Arithmetic and Algebra, such as the equal sign and the operators +, -, and now *x*, which has become a variable and not a multiplication symbol. Tackling these double meanings will help your students transition comfortably from Arithmetic to Algebra.**

Instructions: How do you understand the following symbols and expressions?

SYMBOLS / EXPRESSIONS	MEANING
1. *x*	
2. 2 + 3	
3. =	

IV. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the previous activity:

1. You might have thought of *x* as the multiplication sign. From here on, *x* will be considered a symbol that stands for any value or number.
2. You probably thought of 2 + 3 as equal to 5 and must have written the number 5. Another way to think of 2 + 3 is to read it as the sum of 2 and 3.
3. You must have thought, “Alright, what am I supposed to compute?” The sign “=” may be called the equal sign by most people but may be interpreted as a command to carry out an operation or operations. However, the equal sign is also a symbol for the relation between the expressions on its left and right sides, much like the less than “<” and greater than “>” signs.

The Language Of Algebra

The following are important terms to remember.

a. **constant** – a constant is a number on its own. For example, 1 or 127;

b. **variable** – a variable is a symbol, usually letters, which represent a value or a number. For example, *a* or *x*. In truth, you have been dealing with variables since pre-school in the form of squares (□), blank lines (___) or other symbols used to represent the unknowns in some mathematical sentences or phrases;

c. **term** – a term is a constant or a variable or constants and variables multiplied together. For example, 4, *xy* or 8*yz*. The term’s number part is called the **numerical coefficient** while the variable or variables is/are called the **literal coefficient/s**. For the term 8*yz*, the **numerical coefficient** is 8 and the **literal coefficients** are *yz*;

d. **expression** – an Algebraic expression is a group of terms separated by the plus or minus sign. For example, *x* – 2 or 4*x* + ½*y* – 45

Problem: Which of the following is/are equal to 5?

a. 2 + 3 b. 6 – 1 c. $^{10}/_{2}$ d. 1+4 e. all of these

Discussion: The answer is e since 2 + 3, 6 – 1, $^{10}/_{2}$ and 1 + 4 are all equal to 5.

NOTE TO TEACHER:

One of the most difficult obstacles is the transition from seeing say, an expression such as 2 + 3, as a sum rather than an operation to be carried out. A student of arithmetic would feel the urge to answer 5 instead

of seeing 2 + 3 as an expression which is another way of writing the number 5. Since the ability to see expressions as both a process and a product is essential in grasping Algebraic concepts, more exercises should be given to students to make them comfortable in dealing with expressions as products as well as processes.

Notation

Since the letter x is now used as a variable in Algebra, it would not only be funny but confusing as well to still use x as a multiplication symbol. Imagine writing the product of 4 and a value x as $4xx$! Thus, Algebra simplifies multiplication of constants and variables by just writing them down beside each other or by separating them using only parentheses or the symbol " $\bullet$" . For example, the product of 4 and the value x (often read as four x) may be expressed as $4x$, $4(x)$ or $4\bullet x$. Furthermore, division is more often expressed in fraction form. The division sign ÷ is now seldom used.

NOTE TO TEACHER:

A common misconception is viewing the equal sign as a command to execute an operational sign rather than regard it as a sign of equality. This may have been brought about by the treatment of the equal sign in arithmetic (5 + 2 = 7, 3 – 2 = 1, etc.). This misconception has to be corrected before proceeding to discussions on the properties of equality and solving equations since this will pose as an obstacle in understanding these concepts.

Problem: Which of the following equations is true?

a. $12 + 5 = 17$
b. $8 + 9 = 12 + 5$
c. $6 + 11 = 3(4 + 1) + 2$

Discussion: All of the equations are true. In each of the equations, both sides of the equal sign give the same number though expressed in different forms. In a) 17 is the same as the sum of 12 and 5. In b) the sum of 8 and 9 is 17 thus it is equal to the sum of 12 and 5. In c) the sum of 6 and 11 is equal to the sum of 2 and the product of 3 and the sum of 4 and 1.

NOTE TO THE TEACHER

The next difficulty is what to do with letters when values are assigned to them or when no value is assigned to them. Help students understand that letters or variables do not always have to have a value assigned to them but that they should know what to do when letters are assigned numerical values.

On Letters and Variables

Problem: Let x be any real number. Find the value of the expression $3x$ (the product of 3 and x, remember?) if

a) $x = 5$ b) $x = {}^{1}/_{2}$ c) $x = -0.25$

Discussion: The expression 3*x* means multiply 3 by any real number *x*. Therefore,
a) If $x = 5$, then $3x = 3(5) = 15$.
b) If $x = {}^{1}/_{2}$, then $3x = 3({}^{1}/_{2}) = {}^{3}/_{2}$
c) If $x = -0.25$, then $3x = 3(-0.25) = -0.75$

The letters such as *x*, *y*, *n*, etc. do not always have specific values assigned to them. When that is the case, simply think of each of them as any number. Thus, they can be added ($x + y$), subtracted ($x – y$), multiplied (xy), and divided (${}^{x}/_{y}$) like any real number.

Problem: Recall the formula for finding the perimeter of a rectangle, P = 2L + 2W. This means you take the sum of twice the length and twice the width of the rectangle to get the perimeter. Suppose the length of a rectangle is 6.2 cm and the width is ${}^{1}/_{8}$ cm. What is the perimeter?

Discussion: Let L = 6.2 cm and W = ${}^{1}/_{8}$ cm. Then,
$$P = 2(6.2) + 2({}^{1}/_{8}) = 12.4 + ¼ = 12.65 \text{ cm}$$

V. Exercises:
Note to teacher: Answers are in bold characters.

1. Which of the following is considered a constant?
a. *f* b. □ **c. 500** d. 42*x*
2. Which of the following is a term?
a. 23*m* + 5 **b. (2)(6*x*)** c. *x* – *y* + 2 d. ½ *x* – *y*
3. Which of the following is equal to the product of 27 and 2?
a. 29 b. 49 + 6 **c. 60 – 6** d. 11(5)
4. Which of the following makes the sentence 69 – 3 = ___ + 2 true?
a. 33 b. **64** c. 66 d. 68
5. Let *y* = 2*x* + 9. What is *y* when *x* = 5?
a. 118 b. 34 c. 28 **d. 19**

Let us now answer item B.5. of the initial problem using Algebra:

1. The relation of the 1st and 2nd terms of Table A is “the 2nd term is the sum of the 1st term and 4”. To express this using an algebraic expression, we use the letters *n* and *y* as the variables to represent the 1st and 2nd terms, respectively. Thus, if *n* represents the 1st term and *y* represents the 2nd term, then
$$y = n + 4.$$

FINAL PROBLEM:
A. Fill the table below:

TABLE B		
ROW	1ST TERM	2ND TERM
a.	10	23
b.	11	25
c.	12	27
d.	13	**29**
e.	15	**33**

f.	18	**39**
g.	37	**77**
h.	*n*	**2(*n*) + 3**

B. Using Table B as your basis, answer the following questions:
1. What did you do to determine the 2nd term for rows d to f?
2. What did you do to determine the 2nd term for row g?
3. How did you come up with your answer in row h?
4. What is the relation between the 1st and 2nd terms? **The 2nd term is the sum of twice the 1st term and 3**.
5. Express the relation of the 1st and 2nd terms using an algebraic expression. **Let y be the 2nd term and x be the 1st term, then y = 2x + 3.**

Summary

In this lesson, you learned about constants, letters and variables, and algebraic expressions. You learned that the equal sign means more than getting an answer to an operation; it just means that expressions on either side have equal values. You also learned how to evaluate algebraic expressions when values are assigned to letters.

Lesson 19: Verbal Phrases and Mathematical Phrases
Time: 2 hours

Prerequisite Concepts: Real Numbers and Operations on Real Numbers

Objectives
In this lesson, you will be able to translate verbal phrases to mathematical phrases and vice versa.

> **NOTE TO THE TEACHER**
> **Algebra is a language that has its own "letter", symbols, operators and rules of "grammar". In this lesson, care must be taken when translating because you still want to maintain the correct grammar in the English phrase without sacrificing the correctness of the equivalent mathematical expression.**

Lesson Proper
I. Activity 1

Directions: Match each verbal phrase under Column A to its mathematical phrase under Column B. Each number corresponds to a letter which will reveal a quotation if answered correctly. A letter may be used more than once.

	Column A		**Column B**
______ 1.	The sum of a number and three	A.	x + 3
______ 2.	Four times a certain number decreased by one	B.	3 + 4x
______ 3.	One subtracted from four times a number	E.	4 + x
______ 4.	A certain number decreased by two	I.	x + 4
______ 5.	Four increased by a certain number	L.	4x – 1
______ 6.	A certain number decreased by three	M.	x – 2
______ 7.	Three more than a number	N.	x – 3
______ 8.	Twice a number decreased by three	P.	3 – x
______ 9.	A number added to four	Q.	2 – x
______ 10.	The sum of four and a number	R.	2x – 3
______ 11.	The difference of two and a number	U.	4x + 3
______ 12.	The sum of four times a number and three		
______ 13.	A number increased by three		
______ 14.	The difference of four times a number and one		

> **NOTE TO THE TEACHER**
> **Make sure that all phrases in both columns are clear to the students.**

II. Question to Ponder (Post-Activity Discussion)

Which phrase was easy to translate? ____
Translate the mathematical expression *2(x-3)* in at least two ways.

____
____
____

Did you get the quote, "ALL MEN ARE EQUAL"? If not, what was your mistake?
____

III. Activity 2
Directions: Choose the words or expressions inside the boxes and write it under its respective symbol.

plus	more than	times	divided by	is less than
increased by	subtracted from	multiplied by	ratio of	is greater than or equal to
is greater than	the quotient of	of	is at most	is less than or equal to
the sum of	the difference of	diminished by	less than	added to
is at least	the product of	decreased by	is not equal to	minus

+	–	x	÷	<	>	≤	≥	≠
increased by	**decreased by**	**multiplied by**	**ratio of**	**is less than**	**is greater than**	**is less than or equal to**	**is greater than or equal to**	**is not equal to**
added to	**subtracted from**	**of**	**the quotient of**			**is at most**	**is at least**	
the sum of	**the difference of**	**the product of**						
more than	**less than**							
	diminished by							

IV. Question to Ponder (Post-Activity Discussion)

1. Addition would indicate an increase, a putting together, or combining. Thus, phrases like *increased by* and *added to* are addition phrases.
2. Subtraction would indicate a lessening, diminishing action. Thus, phrases like *decreased by*, *less*, *diminished by* are subtraction phrases.
3. Multiplication would indicate a multiplying action. Phrases like *multiplied by* or *n times* are multiplication phrases.
4. Division would indicate partitioning, a quotient, and a ratio. Phrases such as *divided by*, *ratio of*, and *quotient of* are common for division.
5. The inequalities are indicated by phrases such as *less than*, *greater than*, *at least*, and *at most*.
6. Equalities are indicated by phrases like *the same as* and *equal to*.

NOTE TO THE TEACHER

Emphasize to students that these are just some common phrases. They should not rely too much on the specific phrase but rely instead on the meaning of the phrases.

V. THE TRANSLATION OF THE "=" SIGN

Directions: The table below shows two columns, A and B. Column A contains mathematical sentences while Column B contains their verbal translations. Observe the items under each column and compare. Answer the proceeding questions.

Column A Mathematical Sentence	Column B Verbal Sentence
x + 5 = 4	The sum of a number and 5 is 4.
2x – 1 = 1	Twice a number decreased by 1 is equal to 1.
7 + x = 2x + 3	Seven added by a number x is equal to twice the same number increased by 3.
3x = 15	Thrice a number x yields 15.
x – 2 = 3	Two less than a number x results to 3.

VI. Question to Ponder (Post-Activity Discussion)

1) Based on the table, what do you observe are the common verbal translations of the "=" sign? **"is", "is equal to"**

2) Can you think of other verbal translations for the "=" sign? **"results in", "becomes"**

3) Use the phrase "is equal to" on your own sentence.

4) Write your own pair of mathematical sentence and its verbal translation on the last row of the table.
4 - x < 5: Four decreased by a certain number is less than 5.

VII. Exercises:

A. Directions: Write your responses on the space provided.

1. Write the verbal translation of the formula for converting temperature from Celsius (C) to Fahrenheit (F) which is $F = \frac{9}{5}C + 32$.

   The temperature in Fahrenheit (F) is nine-fifths of the temperature in Celsius (C) increased by (plus) 32.
   The temperature in Fahrenheit (F) is 32 more than nine-fifths of the temperature in Celsius (C).

2. Write the verbal translation of the formula for converting temperature from Fahrenheit (F) to Celsius (C) which is $C = \frac{5}{9}(F - 32)$.

The temperature in Celsius (C) is five-ninths of the difference of the temperature in Fahrenheit (F) and32.

3. Write the verbal translation of the formula for simple interest: *I* = *PRT*, where *I* is simple interest, *P* is Principal Amount, *R* is Rate and *T* is time in years.

The simple interest (I) is the product of the Principal Amount (P), Rate (R) and time (T) in years.

4. The perimeter (P) of a rectangle is twice the sum of the length (L) and width (W). Express the formula of the perimeter of a rectangle in algebraic expressions using the indicated variables.
   Answer: P = 2 (L + W)
5. The area (A) of a rectangle is the product of length (L) and width (W).
   Answer: A = LW
6. The perimeter (P) of a square is four times its side (S).
   Answer: P = 4S
7. Write the verbal translation of the formula for Area of a Square (A): $A = s^2$, where *s* is the length of a side of a square.
   The Area of a Square (A) is the square of side (s).
8. The circumference (C) of a circle is twice the product of π and radius (r).
   Answer: C = 2πr
9. Write the verbal translation of the formula for Area of a Circle (A): $A = \pi r^2$, where *r* is the radius.
   The Area of a Circle (A) is the product π and the square of radius (r).
10. The midline (k) of a trapezoid is half the sum of the bases (a and b) or the sum of the bases (a and b) divided by 2.
    Answer: k = $\frac{1}{2}(a+b)$
11. The area (A) of a trapezoid is half the product of the sum of the bases (a and b) and height (h).
    A=$\frac{1}{2}(a+b)h$
12. The area (A) of a triangle is half the product of the base (b) and height (h).
    A = $\frac{1}{2}bh$
13. The sum of the angles of a triangle (A, B and C) is 180^0.
    A + B + C = 180^0
14. Write the verbal translation of the formula for Area of a Rhombus (A): $A = \frac{1}{2}d_1d_2$, where d_1 *and* d_2 are the lengths of diagonals.
    The Area of a Rhombus (A) is half the product of the diagonals, d_1 and d_2.
15. Write the verbal translation of the formula for the Volume of a rectangular parallelepiped (V): $A = lwh$, where *l* is the length, *w* is the width and *h* is the height.
    The Volume of a regular parallelepiped (V) is the product of the length (l), width (w) and height (h).
16. Write the verbal translation of the formula for the Volume of a sphere (V): $V = \frac{4}{3}\pi r^3$, where *r* is the radius.

The Volume of a sphere (V) is four-thirds of the product of π and the square of radius (r).

17. Write the verbal translation of the formula for the Volume of a cylinder (V): $V = \pi r^2 h$, where r is the radius and h is the height.

    The Volume of a cylinder (V) is the product of π, the square of radius (r) and height (h).

18. The volume of the cube (V) is the cube of the length of its edge (a). Or the volume of the cube (V) is the length of its edge (a) raised to 3. Write its formula.

    $\boldsymbol{V = a^3}$

NOTE TO THE TEACHER **Allow students to argue and discuss, especially since not all are well versed in the English language.**

B. Directions: Write as many verbal translations as you can for this mathematical sentence.

$$\mathbf{3x - 2 = -4}$$

Possible answers are

1. Three times (Thrice) a number x decreased by (diminished by) two is (is equal to/ results to/ yields to) – 4.
2. 2 less than three times (Thrice) a number x is (is equal to/ results to/ yields to) – 4.
3. 2 subtracted from three times (Thrice) a number x is (is equal to/ results to/ yields to) – 4.
4. The difference of Three times (Thrice) a number x and two is (is equal to/ results to/ yields to) – 4.

C. REBUS PUZZLE
Try to answer this puzzle!
What number must replace the letter x?

x + ("  " – "b") = "  " – "kit"

Answer: ***x* + 1 = 10 → x = 9**

SUMMARY

In this lesson, you learned that verbal phrases can be written in both words and in mathematical expressions. You learned common phrases associated with addition, subtraction, multiplication, division, the inequalities and the equality. With this lesson, you must realize by now that mathematical expressions are also meaningful.

Lesson 20: Polynomials **Time: 1.5 hours**

Pre-requisite Concepts: Constants, Variables, Algebraic expressions

Objectives:

In this lesson, the students must be able to:

1) Give examples of polynomials, monomials, binomials, and trinomials;
2) Identify the base, coefficient, terms and exponent sin a given polynomial.

Lesson Proper:

I. A. Activity 1: Word Hunt

Find the following words inside the box.

BASE	CUBIC
COEFFICIENT	LINEAR
DEGREE	QUADRATIC
EXPONENT	QUINTIC
TERM	QUARTIC
CONSTANT	
BINOMIAL	
MONOMIAL	
POLYNOMIAL	
TRINOMIAL	

P	C	I	T	N	I	U	Q	Y	N	E	T
P	M	E	X	P	O	N	E	N	T	S	C
C	O	E	F	F	I	C	I	E	N	T	O
Q	N	L	I	N	E	A	R	B	D	R	N
U	O	C	Y	A	P	M	R	A	E	I	S
A	M	R	I	N	L	M	T	S	G	N	T
D	I	U	N	B	O	Q	U	N	R	O	A
R	A	E	Q	P	U	M	V	T	E	M	N
A	L	S	O	B	D	C	I	R	E	I	T
T	A	A	C	U	B	I	N	A	S	A	A
I	U	B	I	N	O	M	I	A	L	L	C
C	I	T	R	A	U	Q	R	T	I	C	B

NOTE TO THE TEACHER:

These words may be given as assignment before the teaching day so that the students can participate actively during the activity. The easier way of defining the terms is by giving an example.

Definition of Terms

In the algebraic expression $3x^2 - x + 5$, $3x^2$, $-x$ and 5 are called the terms.

Term is a constant, a variable or a product of constant and variable.

In the term $3x^2$, 3 is called the numerical coefficient and x^2 is called the **literal coefficient**.

In the term $-x$ has a **numerical coefficient** which is -1 and a literal coefficient which is x.

The term 5 is called the ***constant,*** which is usually referred to as the term without a variable.

Numerical coefficient is the constant/number.

Literal coefficient is the variable including its exponent.

The word ***Coefficient*** alone is referred to as the numerical coefficient.

In the literal coefficient x^2, x is called the ***base*** and 2 is called the ***exponent***.

Degree is the highest exponent or the highest sum of exponents of the variables in a term.

In $3x^2 - x + 5$, the degree is 2.

In $3x^2y^3 - x^4y^3$ the degree is 7.

Similar Terms are terms having the same literal coefficients.

$3x^2$ and $-5x^2$ are similar because their literal coefficients are the same.

$5x$ and $5x^2$ are NOT similar because their literal coefficients are NOT the same.

$2x^3y^2$ and $-4x^2y^3$ are NOT similar because their literal coefficients are NOT the same.

NOTE TO THE TEACHER:

Explain to the students that a constant term has no variable, hence the term <u>constant</u>. Its value does not change.

A ***polynomial*** is a kind of algebraic expression where each term is a constant, a variable or a product of a constant and variable in which the variable has a whole number (non-negative number) exponent. A polynomial can be a monomial, binomial, trinomial or a multinomial.

An algebraic expression is **NOT** a polynomial if

1) the exponent of the variable is NOT a whole number {0, 1, 2, 3..}.

2) the variable is inside the radical sign.

3) the variable is in the denominator.

NOTE TO THE TEACHER:

Explain to the students the difference between multinomial and polynomial. Give emphasis on the use of the prefixes mono, bi, tri and multi or poly.

Kinds of Polynomial according to the number of terms

1) Monomial – is a polynomial with only one term
2) Binomial – is polynomial with two terms
3) Trinomial – is a polynomial with three terms
4) Polynomial – is a polynomial with four or more terms

B. Activity 2

Tell whether the given expression is a polynomial or not. If it is a polynomial, determine its degree and tell its kind according to the number of terms. If it is NOT, explain why.

1)	$3x^2$	6)	$x^{\frac{1}{2}} - 3x + 4$
2)	$x^2 - 5xy$	7)	$\sqrt{2}\,x^4 - x^7 + 3$
3)	10	8)	$3x^2\sqrt{2x} - 1$
4)	$3x^2 - 5xy + x^3 + 5$	9)	$\frac{1}{3}x - \frac{3x^3}{4} + 6$
5)	$x^3 - 5x^{-2} + 3$	10)	$\frac{3}{x^2} - x^2 - 1$

NOTE TO THE TEACHER:

We just have to familiarize the students with these terms so that they can easily understand the different polynomials. This is also important in solving polynomial equations because different polynomial equations have different solutions.

Kinds of Polynomial according to its degree

1) Constant – a polynomial of degree zero
2) Linear – a polynomial of degree one
3) Quadratic – a polynomial of degree two
4) Cubic – a polynomial of degree three
5) Quartic – a polynomial of degree four
6) Quintic – a polynomial of degree five

* The next degrees have no universal name yet so they are just called “polynomial of degree ____.”

A polynomial is in ***Standard Form*** if its terms are arranged from the term with the highest degree, up to the term with the lowest degree.

If the polynomial is in standard form the first term is called the ***Leading Term***, the numerical coefficient of the leading term is called the ***Leading Coefficient*** and the exponent or the sum of the exponents of the variable in the leading term the ***Degree*** of the polynomial.

The standard form of $2x^2 - 5x^5 - 2x^3 + 3x - 10$ is $\boldsymbol{-5x^5 - 2x^3 + 2x^2 + 3x - 10}$.
The terms $-5x^5$ is the leading term, -5 is its leading coefficient and 5 is its degree.
It is a quintic polynomial because its degree is 5.

C. Activity 3
Complete the table.

Given	Leading Term	Leading Coefficient	Degree	Kind of Polynomial according to the no. of terms	Kind of Polynomial According to the degree	Standard Form
1) $2x + 7$	**$2x$**	**2**	**1**	**monomial**	**linear**	**$2x + 7$**
2) $3 - 4x + 7x^2$	**$7x^2$**	**7**	**2**	**trinomial**	**quadratic**	**$7x^2 - 4x + 3$**
3) 10	**10**	**10**	**0**	**monomial**	**constant**	**10**
4) $x^4 - 5x^3 + 2x - x^2 - 1$	**x^4**	**1**	**4**	**multinomial**	**quartic**	**$x^4 - 5x^3 - x^2 + 2x - 1$**
5) $5x^5 + 3x^3 - x$	**$5x^5$**	**5**	**5**	**Trinomial**	**Quintic**	**$5x^5 + 3x^3 - x$**
6) $3 - 8x$	**$-8x$**	**-8**	**1**	**Binomial**	**Linear**	**$-8x + 3$**
7) $x^2 - 9$	**x^2**	**1**	**2**	**Binomial**	**Quadratic**	**$x^2 - 9$**
8) $13 - 2x + x^5$	**x^5**	**1**	**5**	**Trinomial**	**Quintic**	**$x^5 - 2x + 13$**
9) $100x^3$	**$100x^3$**	**100**	**3**	**Monomial**	**Cubic**	**$100x^3$**
10) $2x^3 - 4x^2 + 3x^8 - 6$	**$3x^8$**	**3**	**8**	**Multinomial**	**Polynomial of degree 8**	**$3x^8$+2x3 $- 4x^2 - 6$**

Summary

In this lesson, you learned about the terminologies in polynomials: term, coefficient, degree, similar terms, polynomial, standard form, leading term, leading coefficient.

Lesson 21: Laws of Exponents **Time: 1.5 hours**

Pre-requisite Concepts:

The students have mastered the multiplication.

Objectives:

In this lesson, the students must be able to:

1) define and interpret the meaning of a^n where n is a positive integer;
2) derive inductively the Laws of Exponents (restricted to positive integers)
3) illustrate the Laws of Exponents.

Lesson Proper

I. Activity 1

Give the product of each of the following as fast as you can.

1) 3 x 3 = ________ **Ans. *9***
2) 4 x 4 x 4 = ________ **Ans. *64***
3) 5 x 5 x 5 = ________ **Ans. *125***
4) 2 x 2 x 2 = ________ **Ans. *8***
5) 2 x 2 x 2 x 2 = ________ **Ans. *16***
6) 2 x 2 x 2 x 2 x 2 = ________ **Ans. *32***

II. Development of the Lesson

Discovering the Laws of Exponent

NOTE TO THE TEACHER:
You can follow up this activity by telling the students that 3 x 3 x 3 = 3^3, 4 x 4 x 4 = 4^3 and so on. From here, you can now explain the very first and basic law of exponent. The elementary teachers have discussed this already.

A) a^n = a x a x a x a ….. (n times) In ***a^n*, *a*** is called the base and ***n*** is called the exponent

NOTE TO THE TEACHER:
We have to emphasize that violation of a law means a wrongdoing. So tell them that there is no such thing as multiplying the base and the exponent as stated in the very first law.

Exercises

1) Which of the following is/are correct?

a) 4^2 = 4 x 4 = 16 b) 2^4 = 2 x 2 x 2 x 2 = 8
c) 2^5 = 2 x 5 = 10 d) 3^3 = 3 x 3 x 3 = 27

Sample Ans. ***CORRECT*** ***INCORRECT***
INCORRECT ***CORRECT***

2) Give the value of each of the following as fast as you can.

	a) 2^3	b) 2^5	c) 3^4	d) 10^6
Sample Ans.	***8***	***32***	***81***	***1,000,000***

NOTE TO THE TEACHER:
It is important to tell the students to use "dot" or "parenthesis" as a symbol for multiplication because at this stage, we are already using x as a variable.

Let the students explore on the next activities. If they can't figure out what you want them to see, guide them. Throw more questions. If it won't work, do the lecture. The "What about these" are follow-up questions. The students should be the one to answer it.

Activity 2

Evaluate the following by applying the law that we have discussed. Investigate the result. Make a simple conjecture on it. The first two are done for you.

1) $(2^3)^2 = 2^3 \bullet 2^3 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 64$
2) $(x^4)^3 = x^4 \bullet x^4 \bullet x^4 = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x = x^{12}$
3) $(3^2)^2 =$ **Ans. *81***
4) $(2^2)^3 =$ **Ans. *64***
5) $(a^2)^5=$ **Ans.** $\boldsymbol{a^{10}}$

Did you notice something?
What can you conclude about $(\boldsymbol{a^n})^{\boldsymbol{m}}$? What will you do with ***a***, ***n*** and ***m***?
B) $\boldsymbol{(a^n)^m = a^{nm}}$

What about these?
1) $(x^{100})^3=$ **Ans.** $\boldsymbol{x^{300}}$
2) $(y^{12})^5=$ **Ans.** $\boldsymbol{y^{60}}$

Activity 3

Evaluate the following. Notice that the bases are the same. The first example is done for you.

1) $(2^3)(2^2) = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 2^5 = 32$
2) $(x^5)(x^4) =$ **Ans.** $\boldsymbol{x^9}$
3) $(3^2)(3^4) =$ **Ans. *729***
4) $(2^4)(2^5) =$ **Ans. *512***
5) $(x^3)(x^4) =$ **Ans.** $\boldsymbol{x^7}$

Did you notice something?

What can you conclude about $\boldsymbol{a^n \bullet a^m}$? What will you do with ***a***, ***n*** and ***m***?

C) $\boldsymbol{a^n \bullet a^m = a^{n+m}}$

What about these?

1) $(x^{32})(x^{25})$ **Ans.** $\boldsymbol{x^{57}}$
2) $(y^{59})(y^{51})$ **Ans.** $\boldsymbol{y^{110}}$

Activity 4

Evaluate each of the following. Notice that the bases are the same. The first example is done for you.

1) $\frac{2^7}{2^3} = \frac{128}{8} = 16$ → remember that 16 is the same as 2^4
2) $\frac{3^5}{3^3} =$ **Ans.** ***9***
3) $\frac{4^3}{4^2} =$ **Ans.** ***4***
4) $\frac{2^8}{2^6} =$ **Ans.** ***4***

Did you notice something?

What can you conclude about $\frac{a^n}{a^m}$? What will you do with ***a, n*** and ***m***?

D) $\frac{a^n}{a^m} = \boldsymbol{a^{n-m}}$

What about these?

1) $\frac{x^{20}}{x^{13}}$ **Ans.** $\boldsymbol{x^7}$
2) $\frac{y^{105}}{y^{87}}$ **Ans.** $\boldsymbol{y^{18}}$

NOTE TO THE TEACHER:

After they finished the discovery of the laws of exponent, it is very important that we summarize those laws. Don't forget to tell them that there are still other laws of exponent, which they will learn on the next stage (second year).

Laws of exponents

1) $\boldsymbol{a^n = a \bullet a \bullet a \bullet a \bullet a}$….. (n times)
2) $\boldsymbol{(a^n)^m = a^{nm}}$ ***power of powers***
3) $\boldsymbol{a^n \bullet a^m = a^{m+n}}$ ***product of a power***
4) $\frac{a^n}{a^m} = \boldsymbol{a^{n-m}}$ ***quotient of a power***

NOTE TO THE TEACHER:

The next two laws of exponent are for you to discuss with your students.

5) $a^0 = 1$ where $a \neq 0$ ***law for zero exponent***

Ask the students. "If you divide number by itself, what is the answer?"

Follow it up with these: (Do these one by one)

No.	Result	Applying a law of Exponent	***GIVEN (Start here)***	ANSWER	REASON
1)	5^0 ←	5^{1-1}	$\frac{5}{5}$ ←	1 →	Any number divided by itself is equal to 1.
2)	100^0 ←	100^{1-1}	$\frac{100}{100}$ ←	1 →	
3)	x^0 ←	x^{1-1}	$\frac{x}{x}$ ←	1 →	
4)	a^0 ←	a^{5-5}	$\frac{a^5}{a^5}$ ←	1 →	

You can draw the conclusion from the students. As they will see, all numbers that are raised to zero is equal to 1. But take note, the *base* should not be equal to zero because division by zero is not allowed.

What about these?

a) $(7,654,321)^0$ **Ans.** ***1***

b) $3^0 + x^0 + (3y)^0$ **Ans.** ***3***

6) $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ ***law for negative exponent***

You can start the discussion by showing this to the students.

a) $\frac{2}{4} = \frac{1}{2}$ then show that $\frac{2}{4} = \frac{2^1}{2^2} = 2^{1-2}$

which means $2^{1-2} = 2^{-1} = \frac{1}{2}$

b) $\frac{4}{32} = \frac{1}{8}$ then show that $\frac{4}{32} = \frac{2^2}{2^5} = 2^{2-5}$

which means $2^{2-5} = 2^{-3} = \frac{1}{8}$

c) $\frac{27}{81} = \frac{1}{3}$ then show that $\frac{27}{81} = \frac{3^3}{3^4} = 3^{3-4}$

which means $3^{3-4} = 3^{-1} = \frac{1}{3}$

Now ask them.

What did you notice?

What about these?

d) x^{-2} **Ans.** $\frac{1}{x^2}$

e) 3^{-3} **Ans.** $\frac{1}{27}$

f) $(5-3)^{-2}$ **Ans.** $\frac{1}{4}$

Now, explain them the rule. If you can draw it from them, better.

III. Exercises

A. Evaluate each of the following.

1) 2^8	**Ans.** ***256***	6) $(2^3)^3$	**Ans.** ***512***
2) 8^2	**Ans.** ***64***	7) $(2^4)(2^3)$	**Ans.** ***128***
3) 5^{-1}	**Ans.** ***1/5***	8) $(3^2)(2^3)$	**Ans.** ***72***
4) 3^{-2}	**Ans.** ***1/9***	9) $x^0 + 3^{-1} – 2^2$	**Ans.** ***-8/3***
5) 18^0	**Ans.** 1	10) $[2^2 – 3^3 + 4^4]^0$	**Ans.** ***1***

B. Simplify each of the following.

1) $(x^{10})(x^{12})$ **Ans.** x^{22}

2) $(y^{-3})(y^8)$ **Ans.** y^5

3) $(m^{15})^3$ **Ans.** m^{45}

4) $(d^{-3})^2$ **Ans.** $1/d^6$

5) $(a^{-4})^{-4}$ **Ans.** a^{16}

6) $\frac{z^{23}}{z^{15}}$ **Ans.** z^8

7) $\frac{b^8}{b^{12}}$ **Ans.** $1/b^4$

8) $\frac{c^3}{c^{-2}}$ **Ans.** c^5

9) $\frac{x^7 y^{10}}{x^3 y^5}$ **Ans.** x^4y^5

10) $\frac{a^8 b^2 c^0}{a^5 b^5}$ **Ans.** a^3/b^3

11) $\frac{a^8 a^3 b^{-2}}{a^{-1} b^{-5}}$ **Ans.** $a^{12}b^3$

Summary:

In these lessons, you have learned some laws of exponent.

Lesson 22: Addition and Subtraction of Polynomials **Time: 2 hours**

Pre-requisite Concepts: Similar Terms, Addition and Subtraction of Integers

About the Lesson: This lesson will teach students how to add and subtract polynomials using tiles at first and then by paper and pencil after.

Objectives:

In this lesson, the students are expected to:

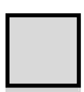
1) add and subtract polynomials;
2) solve problems involving polynomials.

> **NOTE TO THE TEACHER**
> **It is possible that at this point, some of your students still cannot relate to x's and y's. If that is so, then they will have difficulty moving on with the next lessons. The use of Tiles in this lesson is a welcome respite for students who are struggling with variables, letters, and expressions. Take advantage and use these tiles to the full. You may make your own tiles.**

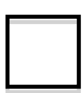
Lesson Proper:

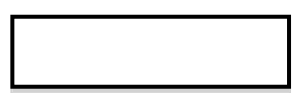
I. Activity 1

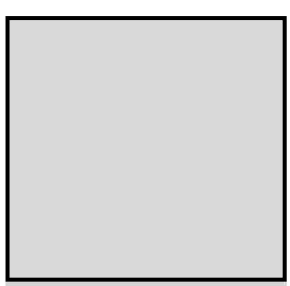
Familiarize yourself with the tiles below:

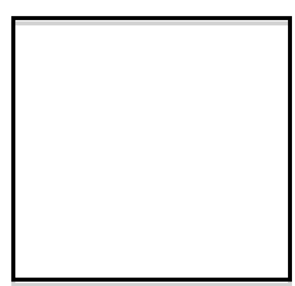

Stands for (+1)


Stands for ($+x$)


Stands for (-1)


Stands for ($-x$)


Stands for ($+x^2$)


Stands for ($-x^2$)

Can you represent the following quantities using the above tiles?

1. $x - 2$
2. $4x + 1$

Activity 2.

Use the tiles to find the sum of the following polynomials;

1. $5x + 3x$
2. $(3x - 4) - 6x$
3. $(2x^2 - 5x + 2) + (3x^2 + 2x)$

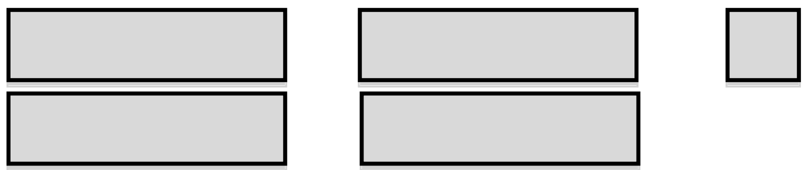
Can you come up with the rules for adding polynomials?

II. Questions/Points to Ponder (Post-Activity Discussion)
The tiles can make operations on polynomials easy to understand and do.

Let us discuss the first activity.

1. To represent $x – 2$, we get one ($+x$) tile and two (-1) tiles.

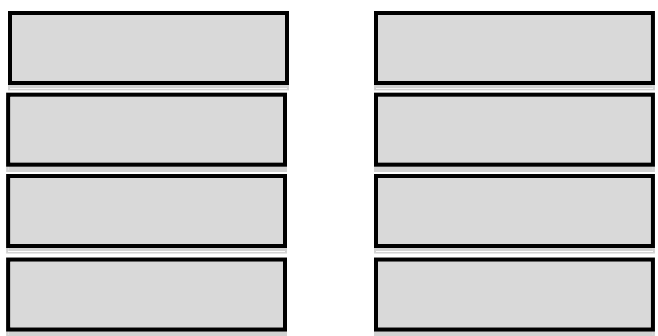
2. To represent $4x +1$, we get four ($+x$) tiles and one (+1) tile.



What about the second activity? Did you pick out the correct tiles?

1. $5x + 3x$

Get five (+x tiles) and three more (+x) tiles. How many do you have in all?

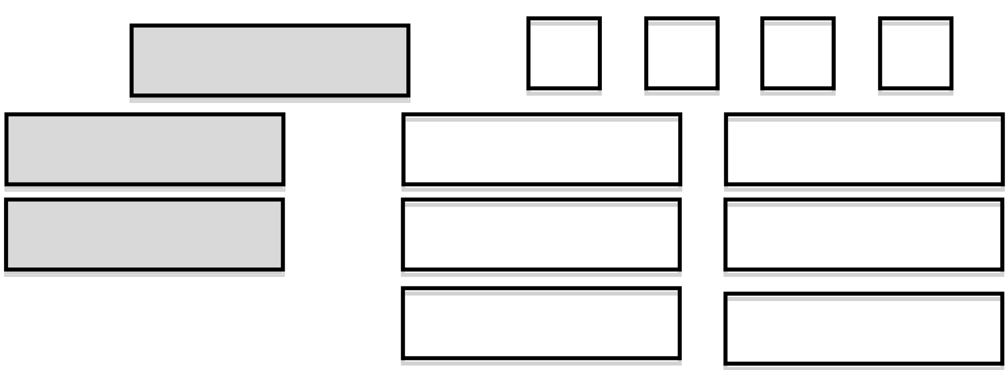


There are eight (+x) altogether. Therefore, $5x + 3x = 8x$.

2. $(3x - 4) - 6x$

Get three ($+x$) tiles and four (-1) tiles to represent ($3x$ - 4). Add six ($-x$) tiles.

[Recall that subtraction also means adding the negative of the quantity.]



Now, recall further that a pair of one (+x) and one (-x) is zero. What tiles do you have left?
That's right, if you have with you three (-x) and four (-1), then you are correct. That means the sum is ($-3x - 4$).

> **NOTE TO THE TEACHER**
> **At this point, encourage your students to work on the problems without using Tiles if they are ready. Otherwise, let them continue using the tiles.**

3. $(2x^2 - 5x + 2) + (3x^2 + 2x)$

What tiles would you put together? You should have two (+x^2), five (-x) and two (+1) tiles then add three (+x^2) and two (+x) tiles. Matching the pairs that make zero, you have in the end five (+x^2), three (-x), and two (+1) tiles. The sum is $5x^2 - 3x + 2$.

Or, using your pen and paper,

$$(2x^2 - 5x + 2) + (3x^2 + 2x) = (2x^2+3x^2) + (- 5x + 2x) + 2 = 5x^2 - 3x + 2$$

> **NOTE TO THE TEACHER**
> **Make sure your students can verbalize what they do to add polynomials so that it is easy for them to remember the rules.**

Rules for Adding Polynomials

To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column.

> **NOTE TO THE TEACHER:**
> **You may give as many examples as you want if you think that your students need it. Your number of examples may vary on the kind of students that you have. If you think that the students understand it after two examples, you may let them work on the next examples.**

Do you think you can add polynomials now without the tiles?

Perform the operation.

1) Add $4a - 3b + 2c$, $5a + 8b - 10c$ and $-12a + c$.

$$\begin{array}{r} 4a - 3b + 2c \\ 5a + 8b - 10c \\ \underline{+\ -12a \qquad\quad + c} \\ \mathbf{-3a + 5b - 7c} \end{array}$$

2) Add $13x^4 - 20x^3 + 5x - 10$ and $-10x^2 - 8x^4 - 15x + 10$.

$$\begin{array}{l} \quad 13x^4 - 20x^3 \qquad\qquad + 5x - 10 \\ \underline{+\ -8x^4 \qquad\qquad - 10x^2 - 15x + 10} \\ \\ \quad \mathbf{5x^4 - 20x^3 - 10x^2 - 10x} \end{array}$$

Rules for Subtracting Polynomials
To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule. Also, remember what subtraction means. It is adding the negative of the quantity.

Perform the operation.

1) $5x – 13x = 5x + (-5x) + (-8x) = -8x$
2) $$\begin{array}{r} 2x^2 – 15x + 25 \\ \underline{-\ \ 3x^2 + 12x – 18} \end{array} \qquad\qquad \begin{array}{r} 2x^2 – 15x + 25 \\ \underline{+ -3x^2 – 12x + 18} \end{array}$$
3) $(30x^3 – 50x^2 + 20x – 80) – (17x^3 + 26x + 19)$
$$\begin{array}{r} 30x^3 – 50x^2 + 20x – 80 \\ \underline{+ -17x^3 \qquad\quad - 26x – 19} \end{array}$$

III. Exercises

A. Perform the indicated operation, first using the tiles when applicable, then using paper and pen.

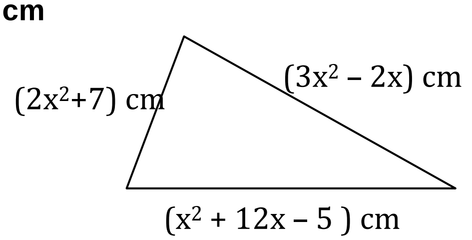
1) $3x + 10x$
2) $12y – 18y$
3) $14x^3 + (-16x^3)$
4) $-5x^3 -4x^3$
5) $2x – 3y$
6) $10xy – 8xy$
7) $20x^2y^2 + 30x^2y^2$
8) $-9x^2y + 9x^2y$
9) $10x^2y^3 – 10x^3y^2$
10) $5x – 3x – 8x + 6x$

Answers: 1) $13x$; 2) $-6y$; 3) $-2x^3$; 4) $-9x^3$; 5) $2x – 3y$; 6) $2xy$; 7) $50x^2y^2$; 8) 0; 9) $10x^2y^3 – 10x^3y^2$; 10) 0

NOTE TO THE TEACHER: You may do this in the form of a game.

B. Answer the following questions. Show your solution.

1) What is the sum of $3x^2 – 11x + 12$ and $18x^2 + 20x – 100$? **$21x^2 + 9x – 88$**
2) What is $12x^3 – 5x^2 + 3x + 4$ less than $15x^3 + 10x + 4x^2 – 10$? **$3x^3 + 9x^2 + 7x – 14$**
3) What is the perimeter of the triangle shown at the right? **$(6x^2 + 10x + 2)$ cm**



4) If you have $(100x^3 – 5x + 3)$ pesos in your wallet and you spent $(80x^3 – 2x^2 + 9)$ pesos in buying foods, how much money is left in your pocket? **$(20x^3 + 2x^2 – 5x – 6)$ pesos**
5) What must be added to $3x + 10$ to get a result of $5x – 3$? **$2x – 13$**

NOTE TO THE TEACHER:
The summary of the lesson should be drawn from the students (as much as possible). Let the students re-state the rules. This is a way of checking what they have learned and how they understand the lesson.

Summary

In this lesson, you learned about tiles and how to use them to represent algebraic expressions. You learned how to add and subtract terms and polynomials using these tiles. You were also able to come up with the rules in adding and subtracting polynomials. To add polynomials, simply combine similar terms. To combine similar terms, get the sum of the numerical coefficients and annex the same literal coefficients. If there is more than one term, for convenience, write similar terms in the same column. To subtract polynomials, change the sign of the subtrahend then proceed to the addition rule.

Lesson 23: Multiplying Polynomials **Time: 3 hours**

Pre-requisite Concepts: Laws of exponents, Adding and Subtracting Polynomials, Distributive Property of Real Numbers

Objectives:

In this lesson, you should be able to:

1) multiply polynomials such as;
   a) monomial by monomial,
   b) monomial by polynomial with more than one term,
   c) binomial by binomial,
   d) polynomial with more than one term to polynomial with three or more terms.
2) solve problems involving multiplying polynomials.

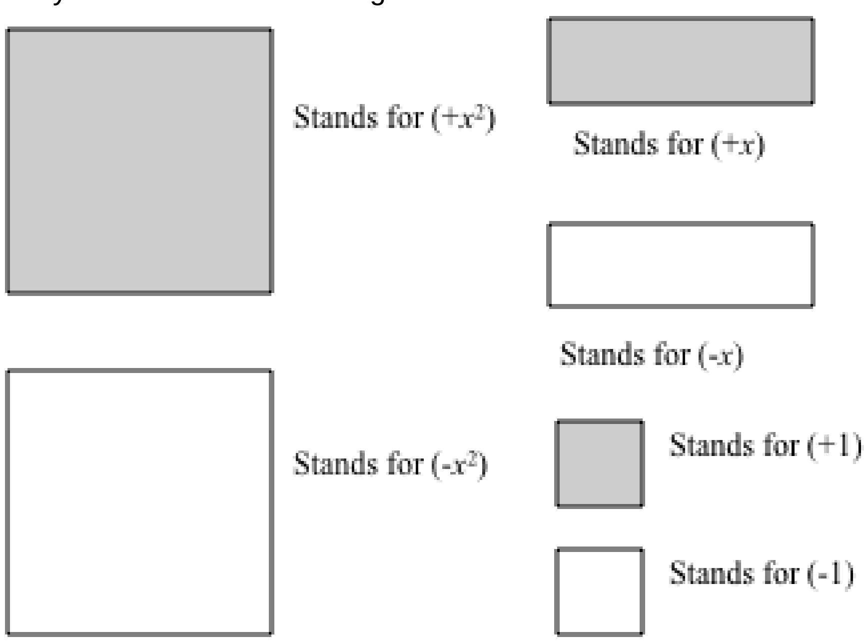
NOTE TO THE TEACHER

Give students the chance to work with the Tiles. These tiles not only help provide a context for multiplying polynomials, they also help students learn special products in the future. Give your students time to absorb and process the many steps and concepts involved in multiplying polynomials.

Lesson Proper

I. Activity

Familiarize yourself with the following tiles:

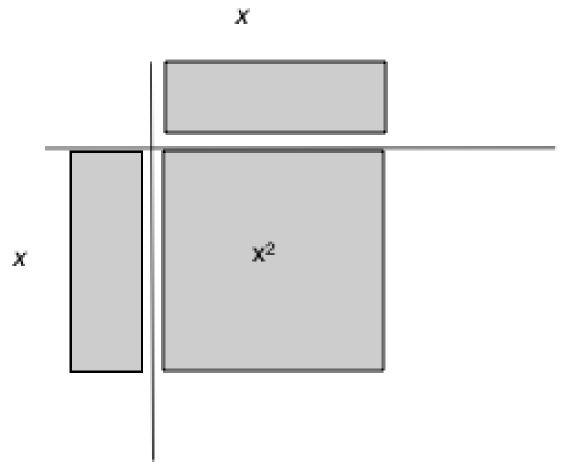


Now, find the following products and use the tiles whenever applicable:

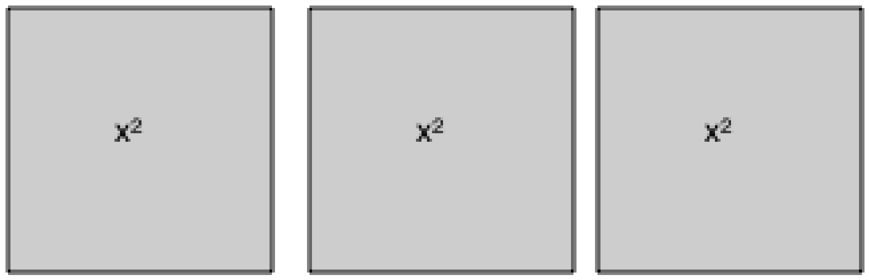
1) $(3x)(x)$ 2) $(-x)(1+x)$ 3) $(3-x)(x+2)$

Can you tell what the algorithms are in multiplying polynomials?

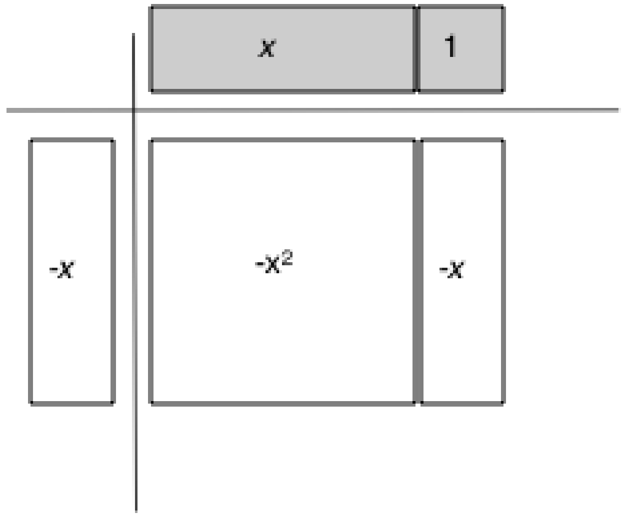
II. Questions/Points to Ponder (Post-Activity Discussion)
Recall the Laws of Exponents. The answer to item (1) should not be a surprise. By the Laws of Exponents, $(3x)(x) = 3x^2$. Can you use the tiles to show this product?



So, $3x^2$ is represented by three of the big shaded squares.

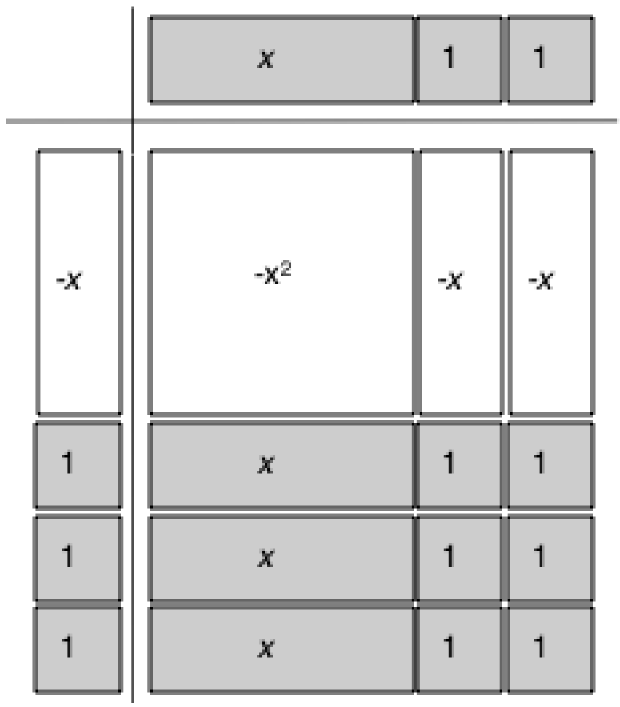


What about item (2)? The product $(-x)(1+x)$ can be represented by the following.



The picture shows that the product is $\left(-x^2\right)+\left(-x\right)$. Can you explain what happened? Recall the sign rules for multiplying.

The third item is (3 - *x*)(*x* + 2). How can you use the Tiles to show the product?



$(-x^2)+(-2x)+3x+6=(-x^2)+x+6$

Rules in Multiplying Polynomials

NOTE TO THE TEACHER:
Emphasize to the students that the most important thing that they have to remember in multiplying polynomials is the "*distributive property.*"

A. To multiply a monomial by another monomial, simply multiply the numerical coefficients then multiply the literal coefficients by applying the basic laws of exponent.

Examples:

1) $(x^3)(x^5) = x^8$
2) $(3x^2)(-5x^{10}) = -15x^{12}$
3) $(-8x^2y^3)(-9xy^8) = 72x^3y^{11}$

NOTE TO THE TEACHER:
You may give first the examples and let them think of the rule or do it the other way around. Also, if you think that they can easily understand

it, let them do the next few examples. Ask for volunteers. Give additional exercises for them to do on the board.

B. To multiply monomial by a polynomial, simply apply the distributive property and follow the rule in multiplying monomial by a monomial.

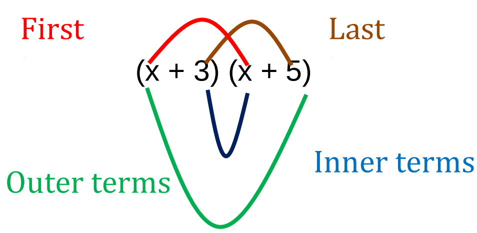
Examples:

1) $3x(x^2 – 5x + 7) = 3x^3 – 15x^2 + 21x$
2) $-5x^2y^3 (2x^2y – 3x + 4y^5) = -10x^4y^4 + 15x^3y^3 – 20x^2y^8$

C. To multiply binomial by another binomial, simply distribute the first term of the first binomial to each term of the other binomial then distribute the second term to each term of the other binomial and simplify the results by combining similar terms. This procedure is also known as the F-O-I-L method or Smile method. Another way is the vertical way of multiplying which is the conventional one.

Examples

1) $(x + 3)(x + 5) =$ $\mathbf{x^2 + 8x + 15}$



F –> $(x)(x) = x^2$

O –> $(x)(5) = 5x$

I –> $(3)(x) = 3x$

L –> $(3)(5)= 15$

Since 5x and 3x are similar terms we can combine them. $5x + 3x = 8x$. The final answer is

$x^2 + 8x + 15$

2) $(x - 5)(x + 5) = x^2 + 5x – 5x – 25 =$ ***$x^2 – 25$***
3) $(x + 6)^2 = (x + 6)(x + 6) = x^2 + 6x + 6x + 36 =$ ***$x^2 + 12x + 36$***
4) $(2x + 3y)(3x – 2y) = 6x^2 – 4xy + 9xy – 6y^2 =$ ***$6x^2 + 5xy – 6y^2$***
5) $(3a – 5b)(4a + 7) =$ ***$12a^2 + 21a – 20ab – 35b$***

There are no similar terms so it is already in simplest form.

Guide questions to check whether the students understand the process or not

If you multiply $(2x + 3)$ and $(x – 7)$ by F-O-I-L method,

a) the product of the first terms is $\mathbf{2x^2}$.
b) the product of the outer terms is **-14x**.
c) the product of the inner terms is **3x**.
d) the product of the last terms is **-21**.
e) Do you see any similar terms? What are they? **-14x and 3x**
f) What is the result when you combine those similar terms? **-11x**
g) The final answer is $\mathbf{2x^2 -11x -21}$

Another Way of Multiplying Polynomials

1) Consider this example.

$$\begin{array}{r} 78 \\ \times\ \underline{59} \\ 702 \\ \underline{390} \\ \boldsymbol{4602} \end{array}$$

This procedure also applies the distributive property.

2) Now, consider this.

$$\begin{array}{r} 2x + 3 \\ \underline{x - 7} \\ 14x + 21 \\ \underline{2x^2 + 3x} \\ \boldsymbol{2x^2 + 17x + 21} \end{array}$$

This one looks the same as the first one.

NOTE TO THE TEACHER:
Be very careful in explaining the second example because the aligned terms are not always similar.

Consider the example below.

$$\begin{array}{r} 3a - 5b \\ \underline{4a + 7} \\ 21a - 35b \\ \underline{12a^2 - 20ab} \\ \boldsymbol{12a^2 - 20ab + 21a - 35b} \end{array}$$

In this case, although 21a and -20ab are aligned, you cannot combine them because they are not similar.

D. To multiply a polynomial with more than one term by a polynomial with three or more terms, simply distribute the first term of the first polynomial to each term of the other polynomial. Repeat the procedure up to the last term and simplify the results by combining similar terms.

Examples:

1) $(x + 3)(x^2 - 2x + 3) = x(x^2 - 2x + 3) - 3(x^2 - 2x + 3)$
$= x^3 - 2x^2 + 3x - 3x^2 + 6x - 9$
$= \mathbf{x^3 - 5x^2 + 9x - 9}$

2) $(x^2 + 3x - 4)(4x^3 + 5x - 1) = x^2(4x^3 + 5x - 1) + 3x(4x^3 + 5x - 1) - 4(4x^3 + 5x - 1)$
$= 4x^5 + 5x^3 - x^2 + 12x^4 + 15x^2 - 3x - 16x^3 - 20x + 4$
$= \mathbf{4x^5 + 12x^4 - 11x^3 + 14x^2 - 23x + 4}$

3) $(2x - 3)(3x + 2)(x^2 - 2x - 1) = (6x^2 - 5x - 6)(x^2 - 2x - 1)$
$= 6x^4 - 17x^3 - 22x^2 + 17x + 6$

**Do the distribution one by one.*

NOTE TO THE TEACHER:
We cannot finish this lesson in one day. The first two (part A and B) can be done in one session. We can have one or two sessions (distributive

property and FOIL method) for part C because if the students can master it, they can easily follow part D. Moreover, this is very useful in factoring.

III. Exercises

A. Simplify each of the following by combining like terms.

1) $6x + 7x$ = $\mathbf{13x}$
2) $3x – 8x$ = $\mathbf{-5x}$
3) $3x – 4x – 6x + 2x$ = $\mathbf{-5x}$
4) $x^2 + 3x – 8x + 3x^2$ = $\mathbf{4x^2 – 5x}$
5) $x^2 – 5x + 3x – 15$ = $\mathbf{x^2 -2x – 15}$

B. Call a student or ask for volunteers to recite the basic laws of exponent but focus more on the "product of a power" or "multiplying with the same base". Give follow up exercises through flashcards.

1) $x^{12} \div x^5$ = $\mathbf{x^7}$
2) $a^{-10} \bullet a^{12}$ = $\mathbf{a^2}$
3) $x^2 \bullet x^3$ = $\mathbf{x^5}$
4) $2^2 \bullet 2^3$ = $\mathbf{2^5}$
5) $x^{100} \bullet x$ = $\mathbf{x^{101}}$

C. Answer the following.

1) Give the product of each of the following.
   a) $(12x^2y^3z)(-13ax^3z^4)$ = $\mathbf{-156ax^5y^3z^5}$
   b) $2x^2(3x^2 – 5x – 6)$ = $\mathbf{6x^4 - 10x^3 - 12x^2}$
   c) $(x – 2)(x^2 – x + 5)$ = $\mathbf{x^3 - 3x^2 + 7x – 10}$

2) What is the area of the square whose side measures (2x – 5) cm? (*Hint: Area of the square =* s^2) **($4x^2 – 20x + 25$) cm²**

3) Find the volume of the rectangular prism whose length, width and height are (x + 3) meter, (x – 3) meter and (2x + 5) meter. *(Hint: Volume of rectangular prism = l x w x h)* **($2x^3 + 5x^2 – 18x – 45$) cubic meters**

4) If I bought (3x + 5) pencils which cost (5x – 1) pesos each, how much will I pay for it? **($15x^2 + 22x – 5$) pesos**

Summary

In this lesson, you learned about multiplying polynomials using different approaches: using the Tiles, using the FOIL, and using the vertical way of multiplying numbers.

Lesson 24: Dividing Polynomials **Time: 3 hours**

Pre-requisite Concepts: Addition, Subtraction, and Multiplication of Polynomials

About the Lesson: In this lesson, students will continue to work with Tiles to help reinforce the association of terms of a polynomial with some concrete objects, hence helping them remember the rules for dividing polynomials.

Objectives:

In this lesson, the students must be able to:

1) divide polynomials such as:
   a) polynomial by a monomial and
   b) polynomial by a polynomial with more than one term.
2) solve problems involving division of polynomials.

Lesson Proper

I. Activity 1:

Decoding

" I am the father of Archimedes." Do you know my name? Find it out by decoding the hidden message below.

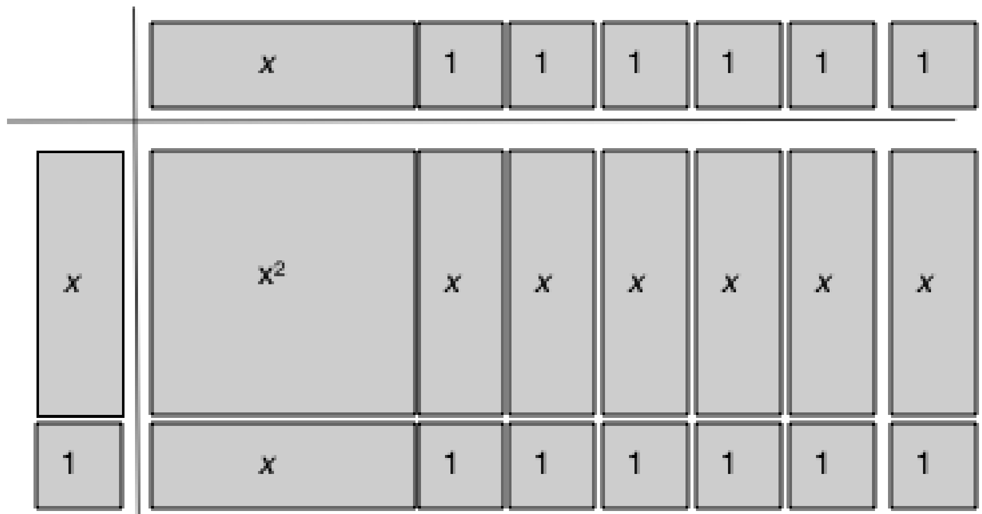
Match Column A with its answer in Column B to know the name of Archimedes' father. Put the letter of the correct answer in the space provided below.

Column A (Perform the indicated operation)			Column B
1)	$(3x^2 - 6x - 12) + (x^2 + x + 3)$	S	$4x^2 + 12x + 9$
2)	$(2x - 3)(2x + 3)$	H	$4x^2 - 9$
3)	$(3x^2 + 2x - 5) - (2x^2 - x + 5)$	I	$x^2 + 3x - 10$
4)	$(3x^2 + 4) + (2x - 9)$	P	$4x^2 - 5x - 9$
5)	$(x + 5)(x - 2)$	A	$2x^2 - 3x + 6$
6)	$3x^2 - 5x + 2x - x^2 + 6$	E	$4x^2 - 6x - 9$
7)	$(2x + 3)(2x + 3)$	D	$3x^2 + 2x - 5$
		V	$5x^3 - 5$

P	H	I	D	I	A	S
1	2	3	4	5	6	7

Activity 2.

Recall the Tiles. We can use these tiles to divide polynomials of a certain type. Recall also that division is the reverse operation of multiplication. Let's see if you can work out this problem using Tiles: $\left(x^2 + 7x + 6\right) \div \left(x + 1\right)$



The answer is $x + 6$.

II. Questions/Points to Ponder (Post-Activity Discussion)

The answer to Activity 1 is PHIDIAS. Di you get it? If not, what went wrong?

In Activity 2, note that the dividend is under the horizontal bar similar to the long division process on whole numbers.

Rules in Dividing Polynomials

To divide polynomial by a monomial, simply divide each term of the polynomial by the given divisor.

Examples:

1) Divide $12x^4 – 16x^3 + 8x^2$ by $4x^2$

a) $$\frac{12x^4 - 16x^3 + 8x^2}{4x^2}$$
$$= \frac{12x^4}{4x^2} - \frac{16x^3}{4x^2} + \frac{8x^2}{4x^2}$$
$$= 3x^2 - 4x + 2$$

b. $$\begin{array}{r l} & 3x^2 - 4x + 2 \\ 4x^2 \big) & \overline{12x^4 - 16x^3 + 8x^2} \\ & \underline{12x^4} \\ & \qquad -16x^3 \\ & \qquad \underline{-16x^3} \\ & \qquad\qquad\quad 8x^2 \\ & \qquad\qquad\quad \underline{8x^2} \\ & \qquad\qquad\quad\; 0 \end{array}$$

2) Divide $15x^4y^3 + 25x^3y^3 – 20x^2y^4$ by $-5x^2y^3$ $= \frac{15x^4y^3}{-5x^2y^3} + \frac{25x^3y^3}{-5x^2y^3} - \frac{20x^2y^4}{-5x^2y^3}$

$= -3x^2 – 5x + 4y$

To divide polynomial by a polynomial with more than one term (by long division), simply follow the procedure in dividing numbers by long division.

These are some suggested steps to follow:

1) Check the dividend and the divisor if it is in standard form.
2) Set-up the long division by writing the division symbol where the divisor is outside the division symbol and the dividend inside it.
3) You may now start the Division, Multiplication, Subtraction and Bring Down cycle.
4) You can stop the cycle when:
   a) the quotient (answer) has reached the constant term.
   b) the exponent of the divisor is greater than the exponent of the dividend

NOTE TO THE TEACHER:

Better start the examples with whole numbers but you have to be very cautious with the differences in procedure in bringing down number or terms. With the whole numbers, you can only bring down numbers one at a time. With the polynomials, you may or you may not bring down all terms altogether. It is also important that you familiarize the students with the divisor, dividend and quotient.

Examples:

1) Divide 2485 by 12.

$$\begin{array}{r} 207 \\ 12\overline{)2485} \\ \underline{24} \\ 8 \\ 0 \\ 85 \\ 84 \\ 1 \end{array}$$

r. 1 or $207\frac{1}{12}$

2) Divide $x^2 – 3x – 10$ by $x + 2$

$$\begin{array}{r} x-5 \\ x+2\overline{)x^2-3x-10} \\ \underline{x^2+2x} \\ -5x-10 \\ \underline{-5x-10} \\ 0 \end{array}$$

1) divide x^2 by x and put the result on top
2) multiply that result to x + 2
3) subtract the product to the dividend
4) bring down the remaining term/s
5) repeat the procedure from 1.

3) Divide $x^3 + 6x^2 + 11x + 6$ by $x - 3$

$$
\begin{array}{r}
x^2-3x+2 \\
x-3 \overline{\smash{\big)}\, x^3-6x^2+11x-6} \\
\underline{x^3-3x^2} \qquad\qquad \\
-3x+11x \qquad \\
\underline{-3x+9x} \qquad \\
2x-6 \\
\underline{2x-6} \\
0
\end{array}
$$

4) Divide $2x^3 - 3x^2 - 10x - 4$ by $2x - 1$

$$
\begin{array}{l}
\qquad\qquad x^2-2x-4-\dfrac{2}{2x+1} \\
2x+1 \overline{\smash{\big)}\, 2x^3-3x^2-10x-6} \\
\qquad\quad \underline{2x^3+x^2} \\
\qquad\qquad -4x^2-10x \\
\qquad\qquad \underline{-4x^2-2x} \\
\qquad\qquad\qquad -8x-6 \\
\qquad\qquad\qquad \underline{-8x-4} \\
\qquad\qquad\qquad\qquad -2
\end{array}
$$

5) Divide $x^4 - 3x^2 + 2$ by $x^2 - 2x + 3$

NOTE TO THE TEACHER:
In this example, it is important that we explain to the students the importance of inserting missing terms.

$$
\begin{array}{l}
\qquad\qquad\qquad x^2+2x-2+\dfrac{-10x+18}{x^2-2x+3} \\
x^2-2x+3 \overline{\smash{\big)}\, x^4+0x^3-3x^2+0x+12} \\
\qquad\qquad \underline{x^4-2x^3+3x^2} \\
\qquad\qquad\quad 2x^3-6x^2+0x \\
\qquad\qquad\quad \underline{2x^3-4x^2+6x} \\
\qquad\qquad\qquad -2x^2-6x+12 \\
\qquad\qquad\qquad -\underline{2x^2+4x-6} \\
\qquad\qquad\qquad\qquad -10x+18
\end{array}
$$

III. Exercises

Answer the following.

1) Give the quotient of each of the following.

a) $30x^3y^5$ divided by $-5x^2y^5$ **= -6x**

b) $\dfrac{13x^3 - 26x^5 - 39x^7}{13x^3}$ = **$1 - 2x^2 - 3x^4$**

c) Divide $7x + x^3 - 6$ by $x - 2$ = **$x^2 + 2x + 11$ r. 16**

2) If I spent $(x^3 + 5x^2 - 2x - 24)$ pesos for $(x^2 + x - 6)$ pencils, how much does each pencil cost? **(x + 4) pesos**

3) If 5 is the number needed to be multiplied by 9 to get 45, what polynomial is needed to be multiplied to $x + 3$ to get $2x^2 + 3x - 9$? **(2x – 3)**

4) The length of the rectangle is x cm and its area is $(x^3 - x)$ cm^2. What is the measure of its width? **$(x^2 - 1)$ cm**

NOTE TO THE TEACHER:
If you think that the problems are not suitable to your students, you may construct a simpler problem solving that they can solve.

Summary:

In this lesson, you have learned about dividing polynomials first using the Tiles then using the long way of dividing.

Lesson 25: Special Products **Time: 3.5 hours**

Pre-requisite Concepts: Addition and Multiplication of Polynomials

Objectives:

In this lesson, you are expected to:

find (a) inductively, using models and (b) algebraically the

1. product of two binomials
2. product of a sum and difference of two terms
3. square of a binomial
4. cube of a binomial
5. product of a binomial and a trinomial

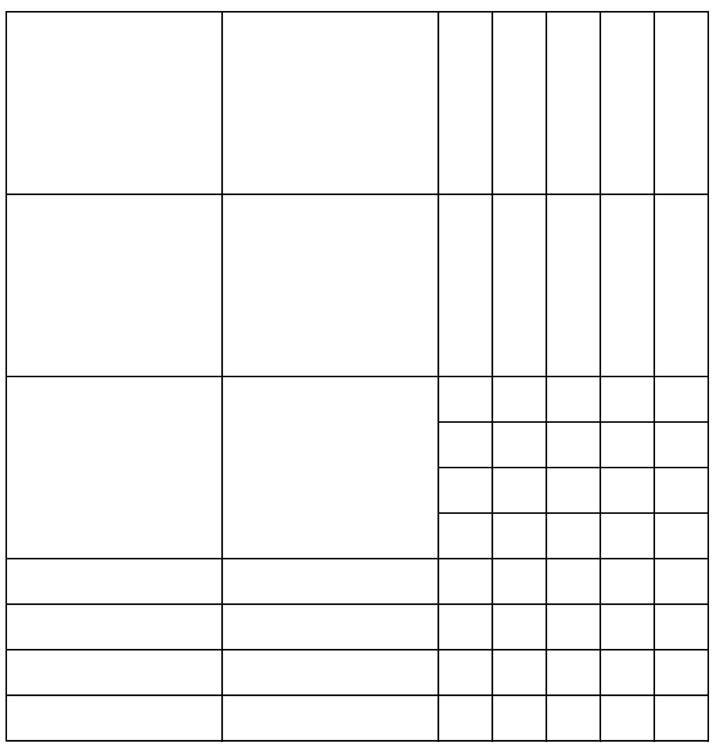
Lesson Proper:

A. *Product of two binomials*

I. Activity

Prepare three sets of algebra tiles by cutting them out from a page of newspaper or art paper. If you are using newspaper, color the tiles from the first set black, the second set red and the third set yellow.

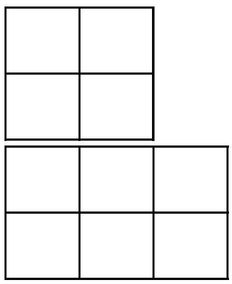
This activity uses algebra tiles to find a general formula for the product of two binomials. Have the students bring several pages of newspaper and a pair of scissors in class. Ask them to cut at least 3 sheets of paper in the following pattern. Have them color the pieces from one sheet black, the second red and the last one yellow.



Problem:

1. What is the area of a square whose sides are 2cm?
2. What is the area of a rectangle with a length of 3cm and a width of 2cm?
3. Demonstrate the area of the figures using algebra tiles.

Solution:

1. 2cm x 2cm = 4cm^2
2. 3cm x 2cm = 6cm^2
3. 

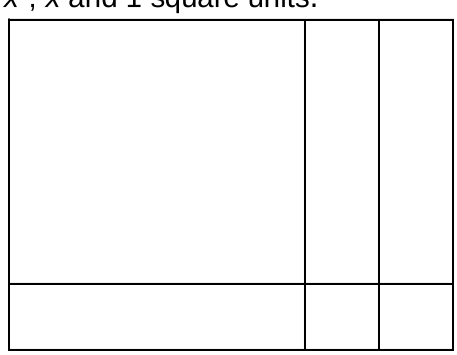
Tell the students that the large squares have dimensions of *x* units, the rectangles are *x* units by 1 unit and the small squares have a side length of 1 unit.

Review with the students the area of a square and a rectangle. Have them determine the area of the large square, the rectangle and the small square.

Problem:

1. What are the areas of the different kinds of algebra tiles?
2. Form a rectangle with a length of $x + 2$ and a width of $x + 1$ using the algebra tiles. What is the area of the rectangle?

Solution:

1. x^2, x and 1 square units.
2. 

   The area is the sum of all the areas of the algebra tiles.
   Area = $x^2 + x + x + x + 1 + 1 = x^2 + 3x + 2$

Ask the students what the product of $x + 1$ and $x + 2$ is. Once they answer $x^2 + 3x + 2$, ask them again, why is it the same as the area of the rectangle. Explain that the area of a rectangle is the product of its length and its width and if the dimensions are represented by binomials, then the area of the rectangle is equivalent to the product of the two binomials.

Problem:

1. Use algebra tiles to find the product of the following:
   a. $(x+2)(x+3)$
   b. $(2x+1)(x+4)$
   c. $(2x+1)(2x+3)$
2. How can you represent the difference $x - 1$ using algebra tiles?

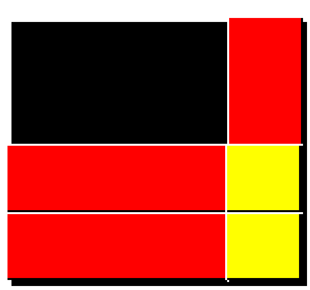
Solution:

1.
   a. x^2+5x+6
   b. $2x^2+9x+4$
   c. $4x^2+8x+3$
2. You should use black colored tiles to denote addition and red colored tiles to denote subtraction.

Problem:

1. Use algebra tiles to find the product of the following:
   a. $(x-1)(x-2)$
   b. $(2x-1)(x-1)$
   c. $(x-2)(x+3)$
   d. $(2x-1)(x+4)$

Solution:



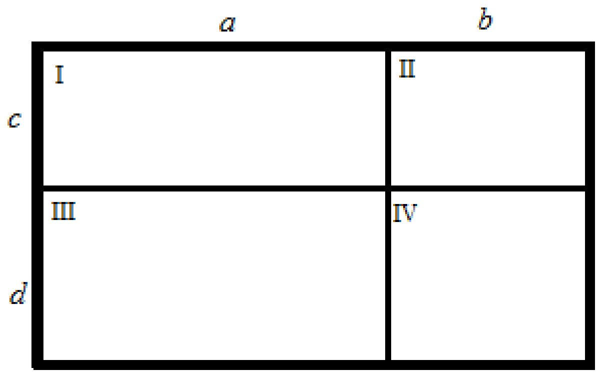
1. x^2-3x+2

   The students should realize that the yellow squares indicate that they have subtracted that area twice using the red figures and they should “add them” back again to get the product.

2. $2x^2 - 3x + 1$
3. $x^2 + x - 6$
4. $2x^2 + 7x - 4$

II. Questions to Ponder

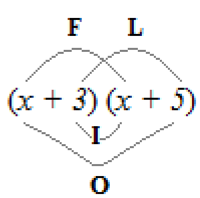
1. Using the concept learned in algebra tiles what is the area of the rectangle shown below?



2. Derive a general formula for the product of two binomials $(a+b)(c+d)$.

The area of the rectangle is equivalent to the product of $(a+b)(c+d)$ which is $ac+ad+bc+cd$. This is the general formula for the product of two binomials $(a+b)(c+d)$. This general form is sometimes called the **FOIL** method where the letters of **FOIL** stand for first, outside, inside, and last.

Example: Find the product of (x + 3) (x + 5)



First: $x \cdot x = x^2$
Outside: $x \cdot 5 = 5x$
Inside: $3 \cdot x = 3x$
Last: $3 \cdot 5 = 15$

$(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$

III. Exercises

Find the product using the **FOIL** method. Write your answers on the spaces provided:

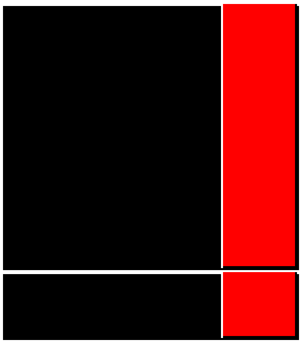
1. $(x + 2)(x + 7)$ $x^2 + 9x + 14$
2. $(x + 4)(x + 8)$ $x^2 + 12x + 32$
3. $(x - 2)(x - 4)$ $x^2 - 6x + 24$
4. $(x - 5)(x + 1)$ $x^2 - 4x - 5$
5. $(2x + 3)(x + 5)$ $2x^2 + 13x + 15$
6. $(3x - 2)(4x + 1)$ $12x^2 - 5x - 2$
7. $(x^2 + 4)(2x - 1)$ $2x^3 - x^2 + 8x - 4$
8. $(5x^3 + 2x)(x^2 - 5)$ $5x^5 - 23x^3 - 10x$
9. $(4x + 3y)(2x + y)$ $8x^2 + 10xy + 3y^2$
10. *10.* $(7x - 8y)(3x + 5y)$ $21x^2 + 11xy - 40y^2$

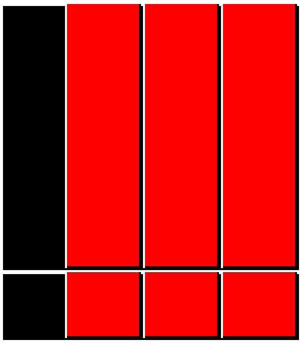
B. product of a sum and difference of two terms

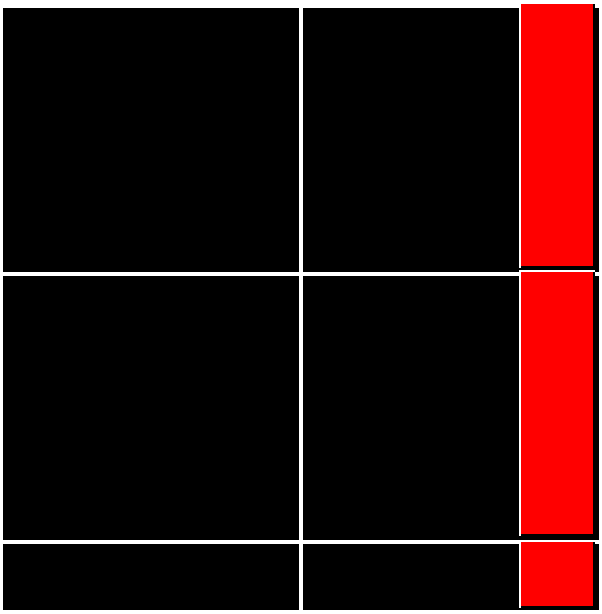
I. Activity

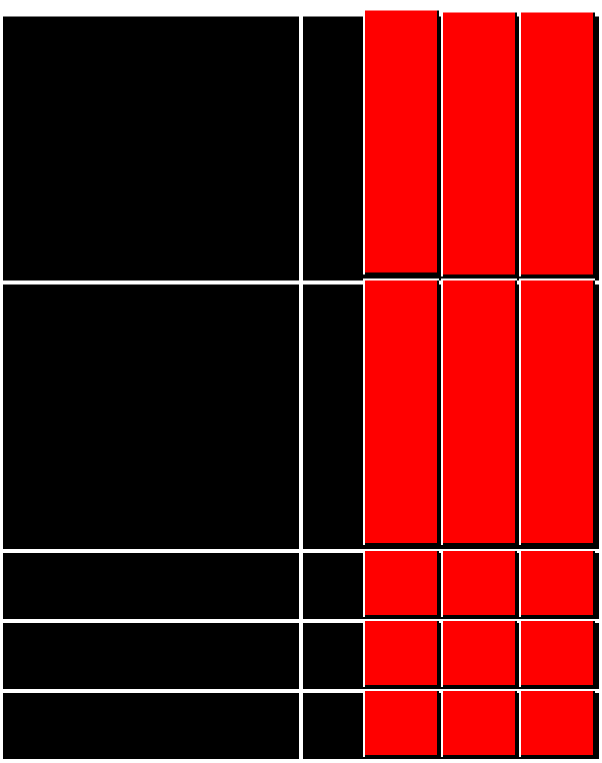
1. Use algebra tiles to find the product of the following:
   a. $(x + 1)(x - 1)$
   b. $(x + 3)(x - 3)$
   c. $(2x - 1)(2x + 1)$
   d. $(2x - 3)(2x + 3)$
2. Use the **FOIL** method to find the products of the above numbers.

The algebra tiles should be arranged in this form.









The students should notice that for each multiplication there are an equal number of black and red rectangles. This means that they “cancel” out each other. Also, the red small squares form a bigger square whose dimensions are equal to the last term in the factors.

Answers

1. $x^2 + x - x - 1 = x^2 - 1$
2. $x^2 + 3x - 3x - 9 = x^2 - 9$
3. $4x^2 + 2x - 2x - 1 = 4x^2 - 1$
4. $4x^2 + 6x - 6x - 9 = 4x^2 - 9$

II. Questions to Ponder

1. What are the products?
2. What is the common characteristic of the factors in the activity?
3. Is there a pattern for the products for these kinds of factors? Give the rule.

Concepts to Remember

The factors in the activity are called the sum and difference of two terms. Each binomial factor is made up of two terms. One factor is the **sum** of the terms and the other factor being their **difference**. The general form is (a + b) (a – b).

The product of the sum and difference of two terms is given by the general formula
$(a + b) (a - b) = a^2 - b^2$.

III. Exercises

Find the product of each of the following:

1. $(x - 5) (x + 5)$ $x^2 - 25$
2. $(x + 2) (x - 2)$ $x^2 - 4$
3. $(3x - 1) (3x + 1)$ $9x^2 - 1$
4. $(2x + 3) (2x - 3)$ $4x^2 - 9$
5. $(x + y^2) (x - y^2)$ $x^2 - y^4$
6. $(x^2 - 10)(x^2 + 10)$ $x^4 - 100$
7. $(4xy + 3z^3) (4xy - 3z^3)$ $16x^2y^2 - 9z^6$
8. $(3x^3 - 4)(3x^3 + 4)$ $9x^6 - 16$
9. $[(x + y) - 1] [(x + y) + 1]$ $(x + y)^2 - 1 = x^2 + 2xy + y^2 - 1$
10. $(2x + y - z) (2x + y + z)$ $(2x + y)^2 - z^2 = 4x^2 + 4xy + y^2 - z^2$

C. square of a binomial

I. Activity

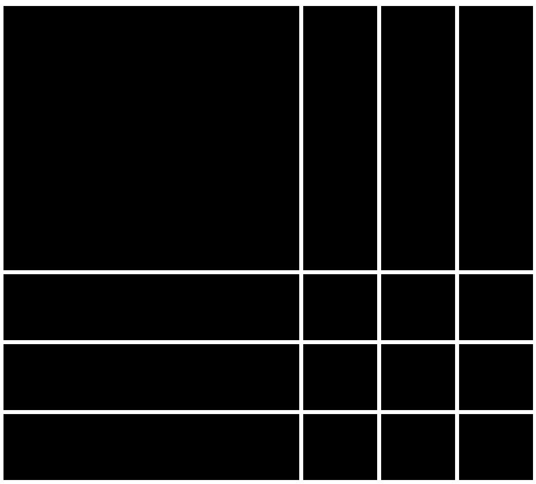
1. Using algebra tiles, find the product of the following:
   a. $(x + 3) (x + 3)$
   b. $(x - 2) (x - 2)$
   c. $(2x + 1) (2x + 1)$

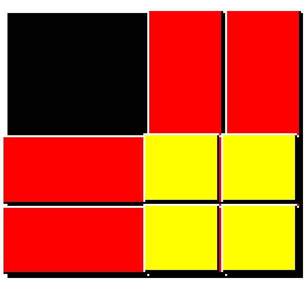
d. (2x – 1) (2x – 1)

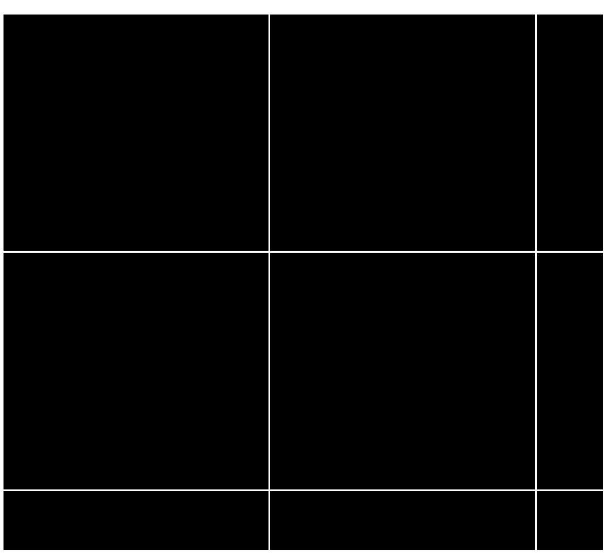
2. Use the **FOIL** method to find their products.

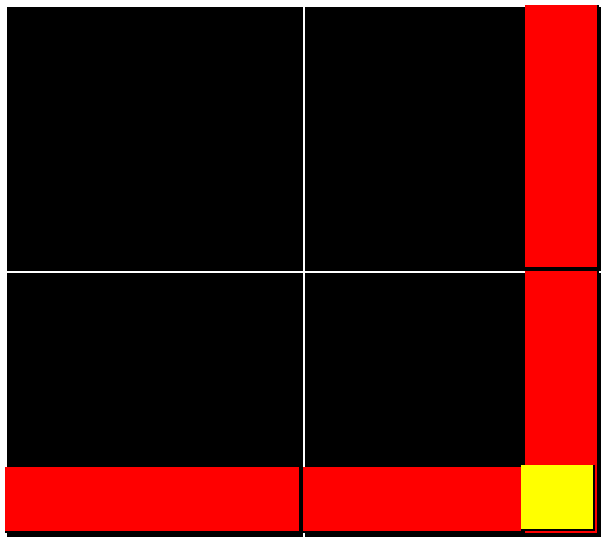
Answers:

1. $x^2 + 6x + 9$
2. $x^2 – 4x + 4$
3. $4x^2 + 4x + 1$
4. $4x^2 - 4x + 1$









II. Questions to Ponder

1. Find another method of expressing the product of the given binomials.
2. What is the general formula for the square of a binomial?
3. How many terms are there? Will this be the case for all squares of binomials? Why?
4. What is the difference between the square of the sum of two terms from the square of the difference of the same two terms?

Concepts to Remember

The square of a binomial $(a \pm b)^2$ is the product of a binomial when multiplied to itself. The square of a binomial has a general formula, $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

The students should know that the outer and inner terms using the FOIL method will be identical and can be combined to form one term. This means that the square of a binomial will always have three terms. Furthermore, they should realize that the term b^2 is always positive while the sign of the middle term 2ab depends on whether the binomials are sums or differences.

III. Exercises

Find the squares of the following binomials.

1. $(x + 5)^2$ $x^2 + 10x + 25$
2. $(x - 5)^2$ $x^2 – 10x + 25$
3. $(x + 4)^2$ $x^2 + 8x + 16$
4. $(x – 4)^2$ $x^2 – 8x + 16$
5. $(2x + 3)^2$ $4x^2 + 12x + 9$
6. $(3x - 2)^2$ $9x^2 – 12x + 4$

7. $(4 – 5x)^2$ $16 – 40x + 25x^2$
8. $(1 + 9x)^2$ $1 + 18x + 81x^2$
9. $(x^2 + 3y)^2$ $x^4 + 6x^2y + 9y^2$
10. $(3x^3 – 4y^2)^2$ $9x^6 – 24x^6y^4 + 16y^4$

D. Cube of a binomial

I. Activity

A. The cube of the binomial $(x + 1)$ can be expressed as $(x + 1)^3$. This is equivalent to
$(x + 1)(x + 1)(x + 1)$.

1. Show that $(x + 1)^2 = x^2 + 2x + 1$.
2. How are you going to use the above expression to find $(x + 1)^3$?
3. What is the expanded form of $(x + 1)^3$?

Answers:

1. By using special products for the square of a binomial, we can show that $(x + 1)^2 = x^2 + 2x + 1$.
2. $(x + 1)^3 = (x + 1)^2(x + 1) = (x^2 + 2x + 1)(x + 1)$
3. $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$

B. Use the techniques outlined above, to find the following:

1. $(x + 2)^2$
2. $(x – 1)^2$
3. $(x – 2)^2$

Answers:

1. $x^3 + 6x^2 + 12x + 8$
2. $x^3 – 3x^2 + 3x – 1$
3. $x^3 – 6x^2 + 12x – 8$

This activity is meant to present the students with several simple examples of finding the cube of a binomial. They should then analyze the answers to identify the pattern and the general rule in finding the cube of a binomial.

II. Questions to Ponder

1. How many terms are there in each of the cubes of binomials?
2. Compare your answers in numbers 1 and 2?
   a. What are similar with the first term? How are they different?
   b. What are similar with the second terms? How are they different?
   c. What are similar with the third terms? How are they different?
   d. What are similar with the fourth terms? How are they different?

3. Craft a rule for finding the cube of the binomial in the form $(x + a)^3$. Use this rule to find $(x + 3)^3$. Check by using the method outlined in the activity.
4. Compare numbers 1 and 3 and numbers 2 and 4.
    a. What are the similarities for each of these pairs?
    b. What are their differences?
5. Craft a rule for finding the cube of a binomial in the form $(x – a)^3$. Use this rule to find $(x – 4)^3$.
6. Use the method outlined in the activity to find $(2x + 5)^3$. Can you apply the rule you made in number 3 for getting the cube of this binomial? If not, modify your rule and use it to find
$(4x + 1)^3$.

Answers:

1. The cube of a binomial has four terms.
2. First, make sure that the students write the expanded form in standard form.
    a. The first terms are the same. They are both x^3.
    b. The second terms have the same degree, x^2. Their coefficients are different. (3 and 6).
    c. The third terms have the same degree, x. Their coefficients are 3 and 12.
    d. The fourth terms are both constants. The coefficients are 1 and 8.

Make sure that the students notice that the ratio of the coefficients of the terms are 1, 2, 4 and 8. These correspond to the powers of the second term 2^0, 2^1, 2^2, and 2^3.

3. $(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$. Thus,
$(x+3)^3 = x^3 + 3(3)x^2 + 3(3^2)x + 3^3 = x^3 + 9x^2 + 27x + 27$
4. The pairs have similar terms, except that the second and fourth terms of $(x-a)^3$ are negative while those of $(x+a)^3$ are positive.
5. $(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$. Thus,
$(x-4)^3 = x^3 - 3(4)x^2 + 3(4^2)x - 4^3 = x^3 - 12x^2 + 48x - 64$.
6. From numbers, 3 and 5, we can generalize the formula to
$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$. In $(2x + 5)^3$, a = 2x and b = 5. Thus,
$(2x+5)^3 = (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + 5^3 = 8x^3 + 60x^2 + 150x + 125$

Concepts to Remember

The cube of a binomial has the general form, $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$.

III. Exercises

Expand.

1. $(x+5)^3$
2. $(x-5)^3$

3. $(x+7)^3$
4. $(x-6)^3$
5. $(2x+1)^3$
6. $(3x-2)^3$
7. $(x^2-1)^3$
8. $(x+3y)^3$
9. $(4xy+3)^3$
10. $(2p-3q^2)^3$

Answers

1. $x^3+15x^2+75x+125$
2. $x^3-15x^2+75x-125$
3. $x^3+21x^2+147x+343$
4. $x^3-18x^2+108x-216$
5. $8x^3+12x^2+6x+1$
6. $27x^3-54x^2+36x-8$
7. $x^6-3x^4+3x^2-1$
8. $x^3+9x^2y+27xy^2+27y^3$
9. $64x^3y^3+144x^2y^2+108xy+27$
10. $8p^3-36p^2q^2+54pq^4-27q^6$

D. Product of a binomial and a trinomial

I. Activity

In the previous activity, we have tried multiplying a trinomial with a binomial. The resulting product then had four terms. But, the product of a trinomial and a binomial **does not always** give a product of four terms.

1. Find the product of x^2-x+1 and $x+1$.
2. How many terms are in the product?

Answers:

The product is x^3+1 and it has two terms. Tell the students that the product is a sum of two cubes and can be written as x^3+1^3.

3. What trinomial should be multiplied to $x-1$ to get x^3-1?

Answers

The other factor should be $x^2 + x + 1$. This question can be done step-by-step analytically. First, ask the students what the first term should be and why. They should realize that the first term can only be x^2, since multiplying it by x from $(x – 1)$ is the only way to get x^3. Then, ask them what the last term should be and why. The only possible answer is 1, since that is the only way to get -1 in $(x^3 – 1)$ by multiplying by -1 in $(x – 1)$. They should then be able to get that the middle term should be +x.

4. Is there a trinomial that can be multiplied to $x – 1$ to get $x^3 + 1$?

Answers

There is none. To get the sum of two cubes, one of the factors should be the sum of the terms. Similarly, explain that to get the difference of two cubes, one of the factors should be the difference of the terms.

5. Using the methods outlined in the previous problems, what should be multiplied to $x + 2$ to get
$x^3 + 8$? Multiplied to $x – 3$ to get $x^3 – 27$?

II. Questions to Ponder

Answers

$(x^2 – 2x + 4)(x + 2) = x^3 + 8$ and $(x^2 + 3x + 9)(x - 3) = x^3 – 27$

1. What factors should be multiplied to get the product $x^3 + a^3$? $x^3 – a^3$?

Answers

$$\left(x \mp ax + a^2\right)\left(x \pm a\right) = x^3 \pm a^3$$

2. What factors should be multiplied to get $27x^3 + 8$?

Answers

Make the students discover that the previous formula can be generalized to $\left(a^2 \mp ab + b^2\right)\left(a \pm b\right) = a^3 \pm b^3$. $27x^3 + 8 = (3x)^3 + 2^3$; a = 3x and b = 2. Thus,
$[(3x)^2 – (3x)(2) + 2^2](3x + 2) = (9x^2 – 6x + 4)(3x + 2) = 27x^3 + 8$

Concepts to Remember

The product of a trinomial and a binomial can be expressed as the sum or difference of two cubes if they are in the following form.

$$\left(a^2 - ab + b^2\right)\left(a + b\right) = a^3 + b^3$$

$$(a^2+ab+b^2)(a-b)=a^3-b^3$$

III. Exercises

A. Find the product.

1. $(x^2-3x+9)(x+3)$
2. $(x^2+4x+16)(x-4)$
3. $(x^2-6x+36)(x+6)$
4. $(x^2+10x+100)(x-10)$
5. $(4x^2+10x+25)(2x-5)$
6. $(9x^2+12x+16)(3x-4)$

B. What should be multiplied to the following to get a sum/difference of two cubes? Give the product.

1. $(x-7)$
2. $(x+8)$
3. $(4x+1)$
4. $(5x-3)$
5. (x^2+2x+4)
6. $(x^2-11x+121)$
7. $(100x^2+30x+9)$
8. $(9x^2-21x+49)$

Answers

A.

1. x^3+27
2. x^3-64
3. x^3+216
4. x^3-1000
5. $8x^3-125$
6. $27x^3-64$

B.

1. $x^2+7x+49$; x^3-343
2. $x^2-8x+64$; x^3+512
3. $16x^2-4x+1$; $64x^3+1$
4. $25x^2+15x+9$; $125x^3-27$
5. $x-2$; x^3-8
6. $x+11$; x^3+1331
7. $10x-3$; $1000x^3-27$
8. $3x+7$; $27x^3+343$

Summary: You learned plenty of special products and techniques in solving problems that require special products.

GRADE 7 MATH TEACHING GUIDE

Lesson 26: Solving Linear Equations and Inequalities in One Variable Using Guess and Check **Time: 1 hour**

Prerequisite Concepts: Evaluation of algebraic expressions given values of the variables

About the Lesson: This lesson will deal with finding the unknown value of a variable that will make an equation true (or false). You will try to prove if the value/s from a replacement set is/are solution/s to an equation or inequality. In addition, this lesson will help you think logically via guess and check even if rules for solving equations are not yet introduced.

Objective:

In this lesson, you are expected to:

1. Find the solution of an equation and inequality involving one variable from a given replacement set by guess and check.

Lesson Proper:

I. Activity

A mathematical expression may contain variables that can take on many values. However, when a variable is known to have a specific value, we can substitute this value in the expression. This process is called *evaluating a mathematical expression.*

Instructions: Evaluate each expression under Column A if x = 2. Match it to its value under Column B and write the corresponding letter on the space before each item. A passage will be revealed if answered correctly.

COLUMN A

_____ 1. $3 + x$
_____ 2. $3x - 2$
_____ 3. $x - 1$
_____ 4. $2x - 9$
_____ 5. $\frac{1}{2}x + 3$
_____ 6. $5x$
_____ 7. $x - 5$
_____ 8. $1 - x$
_____ 9. $-4 + x$
_____ 10. $3x$
_____ 11. $14 - 5x$
_____ 12. $-x + 1$
_____ 13. $1 - 3x$

COLUMN B

A. –3
C. –1
E. –5
F. 1
H. –2
I. 4
L. 5
O. 6
S. 10

Answer: "**<u>__LIFE IS A CHOICE_</u>**"

‘

II. Activity

Mental Arithmetic: How many can you do orally?

1)	$2(5) + 2$	6)	$5(4)$
2)	$3(2 – 5)$	7)	$2(5 + 1)$
3)	$6(4 + 1)$	8)	$– 9 + 1$
4)	$–(2 – 3)$	9)	$3 + (–1)$
5)	$3 + 2(1 + 1)$	10)	$2 – (–4)$

Answers:	(1) 12	(2) –9	(3) 30	(4) 1	(5) 7
	(6) 20	(7) 12	(8) –8	(9) 2	(10) 6

III. Activity

Directions: The table below shows two columns, A and B. Column A contains mathematical expressions while Column B contains mathematical equations. Observe the items under each column and compare. Answer the questions that follow.

Column A	**Column B**
Mathematical Expressions	Mathematical Equations
$x + 2$	$x + 2 = 5$
$2x – 5$	$4 = 2x – 5$
x	$x = 2$
7	$7 = 3 – x$
___________	___________
___________	___________

1) How are items in Column B different from Column A? **[Possible answers: One mathematical expression is given in Column A, while items in column B consist of two mathematical expressions that are connected with an equal sign; Column B contains an equal sign.]**

2) What symbol is common in all items of Column B? **[Answer: The equal sign “=”]**

3) Write your own examples (at least 2) on the blanks provided below each column. **[Answers: Column A: ensure that students give mathematical expressions (these should not contain any statement or equality or inequality (such as =, <, ≥, or ≠). Column**

B: students should give statements of equality so their examples should contain “=”)

Directions: In the table below, the first column contains a mathematical expression, and a corresponding mathematical equation is provided in the third column. Answer the questions that follow.

Mathematical Expression	**Verbal Translation**	**Mathematical Equation**	**Verbal Translation**
$2x$	five added to a number	$2x = x + 5$	Doubling a number gives the same value as adding five to the number.
$2x - 1$	twice a number decreased by 1	$1 = 2x - 1$	1 is obtained when twice a number is decreased by 1.
$7 + x$	seven increased by a number	$7 + x = 2x + 3$	Seven increased by a number is equal to twice the same number increased by 3.
$3x$	thrice a number	$3x = 15$	Thrice a number x gives 15.
$x - 2$	two less than a number	$x - 2 = 3$	Two less than a number x results to 3.

5) What is the difference between the verbal translation of a mathematical expression from that of a mathematical equation? **[The verbal translation of a mathematical expression is a phrase while the verbal translation of a mathematical equation is a sentence.]**

6) What verbal translations for the "=" sign do you see in the table? [**gives the same value as, is, is equal to, gives, results to]** What other words can you use? **[yields, is the same as]**

7) Can we evaluate the first mathematical expression ($x + 5$) in the table when $x = 3$? **[Yes.]** What happens if we substitute $x = 3$ in the corresponding mathematical equation ($x + 5 = 2x$)? **[The equation is not satisfied; it is false]**

8) Can a mathematical equation be true or false? **[Yes.]** What about a mathematical expression? **[No.]**

9) Write your own example of a mathematical expression and equation (with verbal translations) in the last row of the table. **[Answers may vary. Just ensure that students give a phrase for a mathematical expression and a sentence for a mathematical equation.]**

NOTE TO THE TEACHER

Emphasize the difference between a mathematical expression and a mathematical equation. A mathematical expression such as $3x - 1$ just represents a value—it cannot be true or false. However, a mathematical equation such as $3x - 1 = 11$ may be true or false, depending on the value of x.

IV. Activity

From the previous activities, we know that a mathematical equation with one variable is similar to a complete sentence. For example, the equation $x - 3 = 11$ can be expressed as, “Three less than a number is eleven.” This equation or statement may or may not be true, depending on the value of x. In our example, the statement $x - 3 = 11$ is true if $x = 14$, but not if $x = 7$. We call $x = 14$ a *solution* to the mathematical equation $x - 3 = 11$.

In this activity, we will work with mathematical inequalities which, like a mathematical equation, may either be true or false. For example, $x - 3 < 11$ is true when $x = 5$ or when $x = 0$ but not when $x = 20$ or when $x = 28$. We call all possible x values (such as 5 and 0) that make the inequality true *solutions* to the inequality.

Complete the following table by placing a check mark on the cells that correspond to x values that make the given equation or inequality true.

	$x = -4$	$x = -1$	$x = 1$	$x = 2$	$x = 3$	$x = 8$
$0 = x - 2$				√		
$3x + 1 < 0$	√	√				
$1 \geq 2 - x$			√	√	√	√
$\frac{1}{2}(x - 1) = -1$		√				

1) In the table, are there any examples of linear equations that have more than one solution? **[No.]**

2) Do you think that there can be more than one solution to a linear inequality in one variable? Why or why not? **[Yes. Some examples in the table show that a linear inequality may have more than one solution. There are several numbers that may be less than or greater than any given number.]**

NOTE TO THE TEACHER

Emphasize that an inequality may have more than one solution because there are infinitely many numbers that are greater than (or less than) a given number. This is not the same for equations. For example, for $x + 1 < 3$, all numbers less than 2 will satisfy the inequality. But for $x + 1 = 3$, only $x = 2$ will satisfy the equation.

V. Questions/Points to Ponder

In the previous activity, we saw that linear equations in one variable may have a unique solution, but linear inequalities in one variable may have many solutions. The following examples further illustrate this idea.

Example 1. Given, $x + 5 = 13$, prove that only one of the elements of the replacement set $\{-8, -3, 5, 8, 11\}$ satisfies the equation.

$x + 5 = 13$				
For $x = -8$: $-8 + 5 = -3$ $-3 \neq 13$ Therefore –8 is not a solution.	For $x = -3$: $-3 + 5 = 2$ $2 \neq 13$ Therefore –3 is not a solution.	For $x = 5$: $5 + 5 = 10$ $10 \neq 13$ Therefore 5 is not a solution.	For $x = 8$: $8 + 5 = 13$ $13 = 13$ Therefore 8 is a solution.	For $x = 11$: $11 + 5 = 16$ $16 \neq 13$ Therefore 11 is not a solution.

Based on the evaluation, only $x = 8$ satisfied the equation while the rest did not. Therefore, we proved that only one element in the replacement set satisfies the equation.

We can also use a similar procedure to find solutions to a mathematical inequality, as the following example shows.

Example 2. Given, $x - 3 \leq 5$, determine the element/s of the replacement set $\{-8, -3, 5, 8, 11\}$ that satisfy the inequality.

$x - 3 \leq 5$				
For $x = -8$: $-8 - 3 = -11$ $-11 \leq 5$ Therefore –8 is a solution.	For $x = -3$: $-3 - 3 = -6$ $-6 \leq 5$ Therefore –3 is a solution.	For $x = 5$: $5 - 3 = 2$ $2 \leq 5$ Therefore 5 is a solution.	For $x = 8$: $8 - 3 = 5$ $5 \leq 5$ Therefore 8 is a solution.	For $x = 11$: $11 - 3 = 8$ $8 \nleq 13$ Therefore 11 is not a solution.

Based on the evaluation, the inequality was satisfied if $x = -8, -3, 5$, or 8. The inequality was not satisfied when $x = 11$. Therefore, there are 4 elements in the replacement set that are solutions to the inequality.

IV. Exercises

Given the replacement set $\{-3, -2, -1, 0, 1, 2, 3\}$, determine the solution/s for the following equations and inequalities. Show your step-by-step computations to prove your conclusion.

1) $x + 8 < 10$
2) $2x + 4 = 3$
3) $x - 5 > -3$
4) $x > -4$ and $x \leq 2$
5) $x < 0$ and $x > 2.5$

Answers: (1) –3, –2, –1, 0, 1 (2) none (3) 3 (4) –3, –2, –1, 0, 1, 2 (5) none

Solve for the value of x to make the mathematical sentence true. You may try several values for x until you reach a correct solution.

1) $x + 6 = 10$
2) $x - 4 = 11$
3) $2x = 8$
4) $\frac{1}{5}x = 3$
5) $5 - x = 3$
6) $4 + x = 9$
7) $-4x = -16$
8) $-\frac{2}{3}x = 6$
9) $2x + 3 = 13$
10) $3x - 1 = 14$

Answers:	(1) 4	(2) 15	(3) 4	(4) 15
	(5) 2	(6) 5	(7) 4	(8) –9
	(9) 5	(10) 5		

V. Activity

Match the solutions under Column B to each equation or inequality in one variable under Column A. Remember that in inequalities there can be more than one solution.

	COLUMN A
_____ 1.	$3 + x = 4$
_____ 2.	$3x - 2 = 4$
_____ 3.	$x - 1 < 10$
_____ 4.	$2x - 9 \geq -7$
_____ 5.	$\frac{1}{2}x + 3 = -3$
_____ 6.	$2x > -10$
_____ 7.	$x - 5 = 13$
_____ 8.	$1 - x = 11$
_____ 9.	$-3 + x > 1$
_____ 10.	$-3x = 15$
_____ 11.	$14 - 5x \leq -1$
_____ 12.	$-x + 1 = 10$
_____ 13.	$1 - 3x = 13$

	COLUMN B
A.	-9
B.	-1
C.	-5
D.	1
E.	-2
F.	4
G.	-4
H.	6
I.	10
J.	2
K.	18
L.	11
M.	-10
N.	3
O.	-12

Answers: (1) D (2) J
(3) L (4) D
(5) O (6) C
(7) K (8) M
(9) F (10) C
(11) N (12) A
(13) G

VI. Activity

Scavenger Hunt. You will be given only 5-10 minutes to complete this activity. Go around the room and ask your classmates to solve one task. They should write the correct answer and place their signature in a box. Each of your classmates can sign in at most two boxes. You cannot sign on own paper. Also, when signing on your classmates' papers, you cannot always sign in the same box.

.

Find someone who

Can give the value of x so that $x + 3 = 5$ is a true equation.	**Can determine the smallest integer value for x that can hold $x > 1.5$ true.**	**Can solve by guess and check for the solution of $9x–1=8$.**	**Can give the value of $3x–1$ if $x = 3$.**
Can give the numerical value of $3(2^2 – 3^2)$.	**Knows which is greater between x^3 and 3^x when $x = 2$.**	**Can translate the phrase 'a number x increased by 3 is 2' to algebraic expression.**	**Can determine which of these {0,1, 2,…, 8, 9} is/are solution/s of $3x < 9$.**
Can write an inequality for which all positive numbers are NOT solutions.	**Knows the largest integer value of x that can satisfy the inequality $2x – 1 < 3$?**	**Knows what Arabic word is known to be the origin of the word Algebra.**	**Can write an equation that is true when $x = 4$.**
Can write a simple inequality that will is satisfied by the elements in the set {–1, 0, 1.1, $\sqrt{2}$, 3, 4, …}.	**Can name the set of numbers satisfying the inequality $x < 0$.**	**Can explain what an open sentence is.**	**Can give the positive integer values of x that can satisfy $x + 3 < 6$.**

Summary

In this lesson, you learned how to evaluate linear equations at a specific value of *x*. You also learned to determine whether particular values of *x* are solutions to linear equations and inequalities in one variable.

Lesson 27: Solving Linear Equations and Inequalities Algebraically **Time: 2 hours**

Prerequisite Concepts: Operations on polynomials, verifying a solution to an equation

About the Lesson: This lesson will introduce the properties of equality as a means for solving equations. Furthermore, simple word problems on numbers and age will be discussed as applications to solving equations in one variable.

Objectives:

In this lesson, you are expected to:

1. Identify and apply the properties of equality
2. Find the solution of an equation involving one variable by algebraic procedure using the properties of equality
3. Solve word problems involving equations in one variable

Lesson Proper:

I. Activity1

The following exercises serve as a review of translating between verbal and mathematical phrases, and evaluating expressions.

Instructions: Answer each part neatly and promptly.

A. Translate the following verbal sentences to mathematical equation.
   1. The difference between five and two is three.
      Answer: 5 – 2 = 3
   2. The product of twelve and a number *y* is equal to twenty-four.
      Answer: 12y = 24
   3. The quotient of *a* number *x* and twenty-five is one hundred.
      Answer: $\frac{x}{25} = 100$
   4. The sum of five and twice y is fifteen.
      Answer: 5 + 2y = 15
   5. Six more than a number *x* is 3.
      Answer: x+6 = 3

B. Translate the following equations to verbal sentences using the indicated expressions.
   1. *a* + 3 = 2, “the sum of” ***Answer: The sum of a number a and 3 is 2.***
   2. x – 5 = 2, “subtracted from” ***Answer: Five subtracted from a number x is 2.***

3. $\frac{2}{3}x = 5$, "of" **_Answer: Two-thirds of a number x is 5._**

4. $3x + 2 = 8$, "the sum of" **_Answer: The sum of thrice (or three times) a number x and 2 is 8._**

5. $6b = 36$, "the product of" **_Answer: The product of six times a number b is 36._**

C. Evaluate $2x + 5$ if:

1. $x = 5$ **_Answer: 2(5) + 5 = 10 + 5 = 15_**
2. $x = -4$ **_Answer: 2(–4) + 5 = –8 + 5 = –3_**
3. $x = -7$ **_Answer: 2(–7) + 5 = –14 + 5 = –9_**
4. $x = 0$ **_Answer: 2(0) + 5 = 0 + 5 = 5_**
5. $x = 13$ **_Answer: 2(13) + 5 = 26 + 5 = 31_**

II. Activity

The Properties of Equality. To solve equations algebraically, we need to use the various properties of equality. **Create your own examples for each property.**

A. Reflexive Property of Equality

For each real number a, $a = a$.

Examples: $3 = 3$ $-b = -b$ $x + 2 = x + 2$

B. Symmetric Property of Equality

For any real numbers a and b, if $a = b$ then $b = a$.

Examples: If $2 + 3 = 5$, then $5 = 2 + 3$.
If $x - 5 = 2$, then $2 = x - 5$.

C. Transitive Property of Equality

For any real numbers a, b, and c,
If $a = b$ and $b = c$, then $a = c$

Examples: If $2 + 3 = 5$ and $5 = 1 + 4$, then $2 + 3 = 1 + 4$.
If $x - 1 = y$ and $y = 3$, then $x - 1 = 3$.

D. Substitution Property of Equality

For any real numbers a and b: If $a = b$, then a may be replaced by b, or b may be replaced by a, in any mathematical sentence without changing its meaning.

Examples: If $x + y = 5$ and $x = 3$, then $3 + y = 5$.
If $6 - b = 2$ and $b = 4$, then $6 - 4 = 2$.

E. Addition Property of Equality (APE)

For all real numbers a, b, and c,
$a = b$ if and only if $a + c = b + c$.

If we add the same number to both sides of the equal sign, then the two sides remain equal.

Example: 10 + 3 = 13 is true if and only if 10 + 3 **+ 248** = 13**+ 248** is also true (because the same number, 248, was added to both sides of the equation).

F. Multiplication Property of Equality(MPE)

For all real numbers *a*, *b*, and *c*, where $c \neq 0$,

$a = b$ if and only if $ac = bc$.

If we multiply the same number to both sides of the equal sign, then the two sides remain equal.

Example: 3 · 5 = 15 is true if and only if (3 · 5) **· 2** = 15 **· 2** is also true (because the same number, 2, was multiplied to both sides of the equation).

NOTE TO THE TEACHER
Emphasize why there is no Subtraction or Division Property of Equality, as explained below.

Why is there no Subtraction or Division Property of Equality? Even though subtracting or dividing the same number from both sides of an equation preserves equality, these cases are already covered by APE and MPE. Subtracting the same number from both sides of an equality is the same as *adding* a negative number to both sides of an equation. Also, dividing the same number from both sides of an equality is the same as *multiplying* the reciprocal of the number to both sides of an equation.

III. Exercises

Directions: Answer the following exercises neatly and promptly.

A. Identify the property shown in each sentence.
   1. If 3 · 4 = 12 and 12 = 2 · 6. then 3 · 4 = 2 · 6 **Answer: Transitive Property**
   2. 12 = 12 **Answer: Reflexive Property**
   3. If *a* + 2 = 8, then a + 2 + (–2) = 8 + (–2). **Answer: Addition Property**
   4. If 1 + 5 = 6, then 6 = 1 + 5. **Answer: Symmetric Property**
   5. If 3*x* = 10, then $\frac{1}{3}(3x) = \frac{1}{3}(10)$ **Answer: Multiplication Property**

B. Fill-in the blanks with correct expressions indicated by the property to be used.
   1. If 2 + 5 = 7, then 7 = ____ (Symmetric Property)
      Answer: 5 + 2
   2. (80 + 4) · 2 = 84 · ____ (Multiplication Property) **Answer: 2**
   3. 11 + 8 = 19 and 19 = 10 + 9, then 11 + 8 = _____ (Transitive Property) **Answer: 19**
   4. (3 + 10) + (–9) = 13 + ____ (Addition Property)
      Answer: –9
   5. 3 = ____ (Reflexive Property)
      Answer: 3

IV. Questions/Points to Ponder

Finding solutions to equations in one variable using the properties of equality. Solving an equation means finding the values of the unknown (such as *x*) so that the equation becomes true. Although you may solve equations using Guess and Check, a more systematic way is to use the properties of equality as the following examples show.

Example 1.	Solve $x - 4 = 8$.	
Solution	$x - 4 = 8$	Given
	$x - 4 \mathbf{+ 4} = 8 \mathbf{+ 4}$	APE (Added 4 to both sides)
	$x = 12$	

Checking the solution is a good routine after solving equations. The Substitution Property of Equality can help. This is a good practice for you to check mentally.

$$x = 12 \qquad \begin{aligned} x - 4 &= 8 \\ 12 - 4 &= 8 \\ 8 &= 8 \end{aligned}$$

Since 8 = 8 is true, then the $x = 12$ is a correct solution to the equation.

Example 2.	Solve $x + 3 = 5$.	
Solution	$x + 3 = 5$	Given
	$x + 3 \mathbf{+ (-3)} = 5 \mathbf{+ (-3)}$	APE (Added –3 to both sides)
	$x = 2$	

Example 3.	Solve $3x = 75$.	
Solution	$3x = 75$	Given
	$3x \cdot \left(\frac{1}{3}\right) = 75 \cdot \left(\frac{1}{3}\right)$	MPE (Multiplied $\frac{1}{3}$ to both sides)
	$x = 25$	

Note also that multiplying $\frac{1}{3}$ to both sides of the equation is the same as dividing by 3, so the following solution may also be used:

$3x = 75$ Given

$\frac{3x}{3} = \frac{75}{3}$ MPE (Divided both sides of the equation by 3)

$x = 25$

In Examples 1-3, we saw how the properties of equality may be used to solve an equation and to check the answer. Specifically, the properties were used to "isolate" *x*, or make one side of the equation contain only *x*.

In the next examples, there is an *x* on both sides of the equation. To solve these types of equations, we will use the properties of equality so that all the *x*'s will be on one side of the equation only, while the constant terms are on the other side.

Example 4. Solve $4x + 7 = x - 8$.

Solution	$4x + 7 = x - 8$	Given
	$4x + 7 \mathbf{+ (-7)} = x - 8 \mathbf{+ (-7)}$	APE
	$4x = x - 15$	
	$4x \mathbf{+ (-x)} = x - 15 \mathbf{+ (-x)}$	APE
	$3x = -15$	
	$\frac{3x}{\mathbf{3}} = \frac{-15}{\mathbf{3}}$	MPE (Multiplied by $\frac{1}{3}$)
	$x = -5$	

Example 5. Solve $\frac{x}{3} + \frac{x-2}{6} = 4$.

Solution	$\frac{x}{3} + \frac{x-2}{6} = 4$	Given
	$\left(\frac{x}{3} + \frac{x-2}{6}\right) \cdot \mathbf{6} = 4 \cdot \mathbf{6}$	MPE (Multiplied by the LCD: 6)
	$2x + (x - 2) = 24$	
	$2x + x - 2 = 24$	
	$3x - 2 \mathbf{+ 2} = 24 \mathbf{+ 2}$	APE
	$3x = 26$	
	$\frac{3x}{\mathbf{3}} = \frac{26}{\mathbf{3}}$	MPE (Multiplied by $\frac{1}{3}$)
	$x = \frac{26}{3}$	

NOTE TO THE TEACHER:
Emphasize that when solving linear equations, it is usually helpful to use the properties of equality to combine all terms involving *x* on one side of the equation, and all constant terms on the other side.

V. Exercises:

Solve the following equations, and include all your solutions on your paper.

1. $-6y - 4 = 16$ **Answer: $x = -10/3$**
2. $3x + 4 = 5x - 2$ **Answer: $x = 3$**
3. $x - 4 - 4x = 6x + 9 - 8x$ **Answer: $x = -13$**
4. $5x - 4(x - 6) = -11$ **Answer: $x = -35$**
5. $4(2a + 2.5) - 3(4a - 1) = 5(4a - 7)$ **Answer: $a = 2$**

VI. Questions/Points to Ponder

To solve the equation $-14 = 3a - 2$, a student gave the solution below. Read the solution and answer the following questions.

$$\begin{aligned} -14 &= 3a - 2 \\ -14 + 2 &= 3a - 2 + 2 \\ -12 &= 3a \\ -12 + (-3a) &= 3a + (-3a) \\ -12 - 3a &= 0 \\ -12 - 3a + 12 &= 0 + 12 \\ \frac{-3a}{-3} &= \frac{12}{-3} \\ a &= -4 \end{aligned}$$

Is this a correct solution?

What suggestions would you have in to shorten the method used to solve the equation?

Answer: The student could have used MPE in Line 3 of the solution. He/she could have multiplied both sides of the equation with 1/3 to obtain $a = -4$

1) Is this a correct solution?

2) What suggestions would you have in terms of shortening the method used to solve the equation?

Do equations always have exactly one solution? Solve the following equations and answer the questions.

Let students answer the following questions. Discuss the responses with the whole class.

A) $3x + 5 = 3(x - 2)$

Guide Questions

1) Did you find the value of the unknown?
2) By guess and check, can you think of the solution? **[The equation actually has no solution. Do not be surprised if no student could produce a solution.]**
3) This is an equation that has no solution or a null set, can you explain why? **[If $-3x$ is added to both sides of the equation, we will obtain $5 = -6$. This equation is false. Regardless of what x is, we will still get a false statement, so there is no solution.]**
4) Give another equation that has no solution and prove it.

B) $2(x - 5) = 3(x + 2) - x - 16$

Guide Questions

1) Did you find the value of the unknown?
2) Think of 2 or more numbers to replace the variable x and evaluate, what do you notice? **[All real numbers will actually make the given equation true. Do not be surprised if students come up with several solutions.]**
3) This is an equation that has many or infinite solutions, can you explain why? **[By adding like terms on each side of the equation, we get $2x - 10 = 2x - 10$. This equation is true no matter what we substitute for x because both sides of the equation are exactly the same.]**
4) Give another equation that has many or infinite solution and prove it.

C) Are the equations $7 = 9x - 4$ and $9x - 4 = 7$ equivalent equations? Defend your answer. **[Yes, both have 11/9 as the unique solution.]**

VII. Questions/Points to Ponder

Solving word problems involving equations in one variable. The following is a list of suggestions when solving word problems.

1. Read the problem cautiously. Make sure that you understand the meanings of the words used. Be alert for any technical terms used in the statement of the problem.
2. Read the problem twice or thrice to get an overview of the situation being described.
3. Draw a figure, a diagram, a chart or a table that may help in analyzing the problem.
4. Select a meaningful variable to represent an unknown quantity in the problem (perhaps t, if time is an unknown quantity) and represent any other unknowns in terms of that variable (since the problems are represented by equations in one variable).
5. Look for a guideline that you can use to set up an equation. A guideline might be a formula, such as *distance equals rate times time*, or a statement of a relationship, such as "The sum of the two numbers is 28."
6. Form an equation that contains the variable and that translates the conditions of the guideline from verbal sentences to equations.
7. Solve the equation, and use the solution to determine other facts required to be solved.
8. Check answers to the **original statement of the problem and not on the equation formulated.**

Example 1. **NUMBER PROBLEM**

Find five consecutive odd integers whose sum is 55.

Solution Let

x = 1st odd integer
$x + 2$ = 2nd odd integer
$x + 4$ = 3rd odd integer
$x + 6$ = 4th odd integer
$x + 8$ = 5th odd integer

$$x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 55$$
$$5x + 20 = 55$$
$$5x + 20 + (-20) = 55 + (-20)$$
$$\frac{5x}{5} = \frac{35}{5}$$

$x = 7$ The 1st odd integer
$x + 2 = 7 + 2 = 9$ 2nd odd integer
$x + 4 = 7 + 4 = 11$ 3rd odd integer
$x + 6 = 7 + 6 = 13$ 4th odd integer
$x + 8 = 7 + 8 = 15$ 5th odd integer

The five consecutive odd integers are 7, 9, 11, 13, and 15. We can check that the answers are correct if we observe that the sum of these integers is 55, as required by the problem.

Example 2. **AGE PROBLEM**

Margie is 3 times older than Lilet. In 15 years, the sum of their ages is 38 years. Find their present ages.

Representation:

	Age now	**In 15 years**
Lilet	x	x + 15
Margie	3x	3x + 15

In 15 years, the sum of their ages is 38 years.

Equation: $(x + 15) + (3x + 15) = 38$

Solution:
$$4x + 30 = 38$$
$$4x = 38 - 30$$
$$4x = 8$$
$$x = 2$$

Answer: Lilet's age now is 2 while, Margie's age now is 3(2) or 6.

Checking: Margie is 6 which is 3 times older than Lilet who's only 2 years old. In 15 years, their ages will be 21 and 17. The sum of these ages is 21 + 17 = 38.

VIII. Exercises:

1. The sum of five consecutive integers is 0. Find the integers.
   Answers: –2, –1, 0, 1, and 2
2. The sum of four consecutive even integers is 2 more than five times the first integer. Find the smallest integer.
   Answers: 10
3. Find the largest of three consecutive even integers when six times the first integer is equal to five times the middle integer.
   Answers: 14
4. Find three consecutive even integers such that three times the first equals the sum of the other two.
   Answers: 6, 8, and 10
5. Five times an odd integer plus three times the next odd integer equals 62. Find the first odd integer.
   Answers: 7
6. Al's father is 45. He is 15 years older than twice Al's age. How old is Al?
   Answer: Al is 15 years old.
7. Karen is twice as old as Lori. Three years from now, the sum of their ages will be 42. How old is Karen?
   Answer: Karen is 24 years old.
8. John is 6 years older than his brother. He will be twice as old as his brother in 4 years. How old is John now?
   Answer: John is 8 years old.
9. Carol is five times as old as her brother. She will be three times as old as her brother in two years. How old is Carol now?
   Answer: Carol is 10 years old.
10. Jeff is 10 years old and his brother is 2 years old. In how many years will Jeff be twice as old as his brother?
    Answer: 6 years

IX. Activity
Solution Papers (Individual Transfer Activity)

Each student will be assigned to two word problems on number and age. They will prepare two solution papers for the problems following the format below.

Name:	Date Submitted:
Year and Section:	Score:
YOUR OWN TITLE FOR THE PROBLEM:	
Problem: ______________________________________ ____ ________________________	
Representation:	
Solution: Conclusion:	
Checking:	

A rubric will be used to judge each solution paper.

Solution Paper Rubric

	Title	Correctness/ Completeness	Neatness
3 (Exemplary)	The display contains a title that **clearly and specifically** tells what the data shows.	**All data** is **accurately** represented on the graph. All parts are complete.	The solution paper is very **neat and easy to read.**
2 (Proficient)	The display contains a title that **generally** tells what the data shows.	All parts are complete. Data representation contains **minor errors.**	The solution paper is **generally** neat and readable.
1 (Revision Needed)	The title **does not reflect** what the data shows.	All parts are complete. However, the data is not accurately represented, contains **major errors.** **Or** Some parts are missing and there are minor errors.	The solution paper is **sloppy and difficult** to read.
0 (No Credit)	The title is **missing**.	Some parts and data are **missing**.	The display is a **total mess**.

Summary

This lesson presented the procedure for solving linear equations in one variable by using the properties of equality. To solve linear equations, use the properties of equality to isolate the variable (or *x*) to one side of the equation.

In this lesson, you also learned to solve word problems involving linear equations in one variable. To solve word problems, define the variable as the unknown in the problem and translate the word problem to a mathematical equation. Solve the resulting equation.

Lesson 28: Solving First Degree Inequalities in One Variable Algebraically
Time: 2 hours

Pre-requisite Concepts: Definition of Inequalities, Operation on Integers, Order of Real Numbers

About the Lesson: This lesson discusses the properties of inequality and how these may be used to solve linear inequalities.

Objectives:
In this lesson, you are expected to:
1. State and apply the different properties of inequality;
2. Solve linear inequalities in one variable algebraically; and
3. Solve problems that use first-degree inequality in one variable.

NOTE TO TEACHER:
This lesson needs the students' understanding on the operation on integers, solving linear equation and the order of the real numbers. You may give drill exercises before giving exercises on solving linear inequalities.

Lesson Proper:
I. Activity
A. Classify each statement as true or false and explain your answer. (You may give examples to justify your answer.)
   1. Given any two real numbers *x* and *y*, exactly one of the following statements is true: $x > y$ or $x < y$.
   2. Given any three real numbers *a*, *b*, and *c*. If $a < b$ and $b < c$, then $a < c$.
   3. From the statement "10 > 3", if a positive number is added to both sides of the inequality, the resulting inequality is correct.
   4. From the statement "–12 < –2", if a negative number is added to both sides of the inequality, the resulting inequality is correct.

B. Answer the following questions. Think carefully and multiply several values before giving your answer.
   1. If both sides of the inequality 2 < 5 are multiplied by a non-zero number, will the resulting inequality be true or false?
   2. If both sides of the inequality –3 < 7 are multiplied by a non-zero number, will the resulting inequality be true or false?

II. Questions/Points to Ponder

Properties of Inequalities

The following are the properties of inequality. These will be helpful in finding the solution set of linear inequalities in one variable.

1. Trichotomy Property

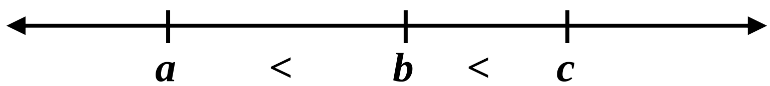
For any number a and b, one and only one of the following is true: $a < b$, $a = b$, or $a > b$.

This property may be obvious, but it draws our attention to this fact so that we can recall it easily next time.

2. Transitive Property of Inequality

For any numbers a, b and c, (a) if $a < b$ and $b < c$, then $a < c$, and
(b) if $a > b$ and $b > c$, then $a > c$.

The transitive property can be visualized using the number line:

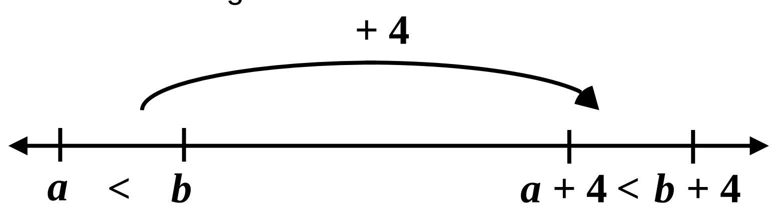


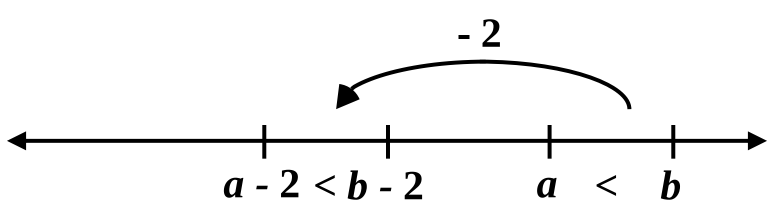
If a is to the left of b, and b is to the left of c, then a is to the left of c.

3. Addition Property of Inequality (API)

For all real numbers a, b and c: (a) if $a < b$, then $a + c < b + c$, and
(b) if $a > b$, then $a + c > b + c$.

Observe that adding the same number to both a and b **will not** change the inequality. Note that this is true whether we add a **positive** or **negative** number to both sides of the inequality. This property can also be visualized using the number line:

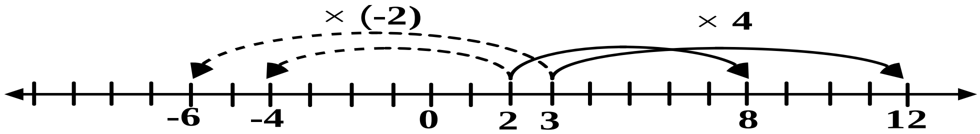




4. Multiplication Property of Inequality (MPI)

For all real numbers a, b and c, then all the following are true:
(a) if $c > 0$ and $a < b$, then $ac < bc$;
(b) if $c > 0$ and $a > b$, then $ac > bc$.
(c) if $c < 0$ and $a < b$, then $ac > bc$;
(d) if $c < 0$ and $a > b$, then $ac < bc$.

Observe that multiplying a **positive** number to both sides of an inequality **does not** change the inequality. However, multiplying a **negative** number to both sides of an inequality **reverses** the inequality. Some applications of this property can be visualized using a number line:



In the number line, it can be seen that if 2 < 3, then 2(4) < 3(4), but 2(–2) **>** 3(–2).

NOTE TO THE TEACHER:
Emphasize the points below, which relate to the reasons why there is no Subtraction and Division Property of Inequality, and why we cannot multiply (or divide) a variable to (from) both sides of an inequality.

- **Subtracting numbers.** The API also covers subtraction because subtracting a number is the same as adding its negative.
- **Dividing numbers.** The MPI also covers division because dividing by a number is the same as multiplying by its reciprocal.
- **Do not multiply (or divide) by a variable.** The MPI shows that the direction of the inequality depends on whether the number multiplied is positive or negative. However, a variable may take on positive or negative values. Thus, it would not be possible to determine whether the direction of the inequality will be retained not.

III. Exercises

A. Multiple-Choice. Choose the letter of the best answer.

[Answers are in bold font.]

1. What property of inequality is used in the statement If *m* > 7 and 7 > *n*, then *m* > *n*"?
   A. Trichotomy Property **C. Transitive Property of Inequality**
   B. Addition Property of Inequality D. Multiplication Property of Inequality
2. If *c* > *d* and *p* < 0, then *cp* ? *dp*.
   A. < B. > C. = D. Cannot be determined
3. If *r* and *t* are real numbers and *r* < *t*, which one of the following **must** be true?
   A. –*r* < –*t* **B. –*r* > –*t*** C. *r* < –*t* D. –*r* > *t*

If *w* < 0 and *a* + *w* > *c* + *w*, then what is the relationship between *a* and *c*?
A. *a* > *c* B. *a* = *c* C. *a* < *c* D. The relationship cannot be determined.

5. If *f* < 0 and *g* > 0, then which of the following is true?
A. *f* + *g* < 0 C. *f* + *g* > 0 B. *f* + *g* = 0 **D. The relationship between *a* and *b* cannot be determined**.

B. Fill in the blanks by identifying the property of inequality used in each of the following:

1. $x + 11 \geq 23$ Given

$x + 11 + (-11) \geq 23 + (-11)$ ____________

Answer : Addition Property of Inequality (API)

$x \geq 12$

2. $5x < -15$ Given

$(5x)\frac{1}{5} < (-15)\frac{1}{5}$ ____________

Answer : Multiplication Property of Inequality (MPI)

$x < -3$

3. $3x - 7 > 14$ Given

$3x - 7 + 7 > 14 + 7$ ____________

Answer : Addition Property of Inequality (API)

$(3x)\frac{1}{3} > (21)\frac{1}{3}$ ____________

Multiplication Property of Inequality (MPI)

$x > 7$

$x \geq 12$

NOTE TO THE TEACHER: To check if the students really understand the different properties of inequality, ask them to answer the following questions.

IV. Activity

Answer each exercise below. After completing all the exercises, compare your work with a partner and discuss.

From the given replacement set, find the solution set of the following inequalities.

1. $2x + 5 > 7$; $\{-6, -3, 4, 8, 10\}$

   Answer: {4, 8, 10}
2. $5x + 4 < -11$; $\{-7, -5, -2, 0\}$

   Answer: {–7, –5}
3. $3x - 7 \geq 2$; $\{-2, 0, 3, 6\}$

   Answer: {3, 6}
4. $2x \leq 3x - 1$; $\{-5, -3, -1, 1, 3\}$

   Answer: {1, 3}
5. $11x + 1 < 9x + 3$; $\{-7, -3, 0, 3, 5\}$

   Answer: { –7, –3, 0}

Answer the following exercises in groups of five.

What number/expression must be placed in the box to make each statement correct?

What number/expression must be placed in the box to make each statement correct?

1. $x - 20 < -12$ Given

$x - 20 + \square < -12 \square$ API

Answer : **20**

$x < 8$

2. $-7x \geq 49$ Given

$(-7x)\square \geq (49)\square$ MPI

Answer : $-\frac{1}{7}$

$x \leq -7$

3. $\frac{x}{4} - 8 > 3$ Given

$\frac{x}{4} - 8\square > 3\square$ API

Answer : 8

$\frac{x}{4} > 11$

$\square\left(\frac{x}{4}\right) > (11)\square$ MPI

Answer : 4

$x > 12$

4. $13x + 4 < -5 + 10x$ Given

$13x + 4 + \square < -5 + 10x\square$ API

Answer : –10*x*

$3x + 4 < -5$

$3x + 4 + \square < -5\ \square$ API

Answer : –4

$3x < -9$

$\square\ 3x < (-9)\square$ MPI

Answer : $\frac{1}{3}$

$x < -3$

NOTE TO THE TEACHER

- **The last statement in each item in the preceding set of exercises is the solution set of the given inequality. For example, in #4, the solution to 13*x*+ 4 < –5 + 10*x* consists of all numbers less than –3 (or *x* < –3). This solution represents all numbers that make the inequality true.**
- **The solution can be written using set notation as {*x* | *x* < –3}. This is read as the "set of all numbers *x* such that *x* is less than –3".**
- **Emphasize that when solving linear inequalities in one variable, isolate the variable that you are solving for on one side of the inequality by applying the properties of inequality.**

V. Questions/Points to Ponder

Observe how the properties of inequality may be used to find the solution set of linear inequalities:

1. $b + 14 \geq 17$

$$b + 14 - 14 \geq 17 - 14$$

$$b \geq 3$$

Solution Set: $\{b \mid b \geq 3\}$

2. $4t - 17 < 51$

$$4t - 17 + 17 < 51 + 17$$

$$4t < 68$$

$$\frac{4t}{4} < \frac{68}{4}$$

$$< 17$$

Solution Set: $\{t \mid t < 17\}$

3. $2r - 32 \geq 4r + 12$

$$2r - 32 - 4r \geq 4r + 12 - 4r$$

$$-2r - 32 \geq 12$$

$$-2r - 32 + 32 \geq 12 + 32$$

$$-2r \geq 44$$

$$\frac{-2r}{-2} \geq \frac{44}{-2}$$

$$r \leq -22$$

Solution Set: $\{r \mid r \leq -22\}$

NOTE TO TEACHER:

Emphasize to the students that they can also subtract a positive number instead of adding a negative number to both sides of the inequality. Likewise, they can also divide both sides of the inequality by an integer instead of multiplying them by a fraction. Thus, subtraction property and division property of inequality are already covered by the API and MPI.

VI. Exercises

Find the solution set of the following inequalities.

1. $b - 19 \leq 15$ **Answer: $\{b \mid b \leq 34\}$**
2. $9k \leq -27$ **Answer: $\{k \mid k \leq -3\}$**
3. $-2p > 32$ **Answer: $\{p \mid p < -16\}$**
4. $3r - 5 \geq 4$ **Answer: $\{r \mid r \geq 3\}$**
5. $2(1 + 5x) < 22$ **Answer: $\{x \mid x < 2\}$**
6. $3w + 10 > 5w + 24$ **Answer: $\{w \mid w < -7\}$**
7. $12x - 40 \geq 11x - 50$ **Answer: $\{x \mid x \geq -10\}$**
8. $7y + 8 < 17 + 4y$ **Answer: $\{y \mid y < 3\}$**
9. $h - 9 < 2(h - 5)$ **Answer: $\{h \mid h > 1\}$**
10. $10u + 3 - 5u > -18 - 2u$ **Answer: $\{u \mid u > -3\}$**

NOTICE TO TEACHER:

Provide drill exercises in translating verbal sentences into mathematical statements involving linear inequalities.

VII. Questions/Points to Ponder

Match the verbal sentences in column A with the corresponding mathematical statements in column B.

	COLUMN A		COLUMN B
d	1) x is less than or equal to 28.	a)	$2x < 28$
c	2) Two more than x is greater than 28.	b)	$x + 2 \geq 28$
b	3) The sum of a number x and 2 is at least 28.	c)	$x + 2 > 28$
a	4) Twice a number x is less than 28.	d)	$x \leq 28$
e	5) Two less than a number x is at most 28.	e)	$x - 2 \leq 28$

Being familiar with translating between mathematical and English phrases will help us to solve word problems, as the following discussion will show.

SOLVING PROBLEMS INVOLVING FIRST-DEGREE INEQUALITY

There are problems in real life that require several answers. Those problems use the concept of inequality. Here are some points to remember when solving word problems that use inequality.

POINTS TO REMEMBER:

- ✓ Read and understand the problem carefully.
- ✓ Represent the unknowns using variables.
- ✓ Formulate an inequality.
- ✓ Solve the inequality formulated.
- ✓ Check or justify your answer.

Example 1. Keith has P5,000.00 in a savings account at the beginning of the summer. He wants to have at least P2,000.00 in the account by the end of the summer. He withdraws P250.00 each week for food and transportation. How many weeks can Keith withdraw money from his account?

Solution:

Step 1: Let w be the number of weeks Keith can withdraw money.

Step 2: $\boxed{50000} \quad - \quad \boxed{250w} \quad \geq \quad 2000$

amount at the beginning of the summer — withdraw — 250 each week — at least — amount at the end of the summer

Step 3:
$$50000 - 250w \geq 2000$$
$$-250w \geq 2000 - 5000$$
$$-250\,w \geq -3000$$
$$w \leq 12$$

Therefore, Keith can withdraw money from his account not more than 12 weeks. We can check our answer as follows. If Keith withdraws P250 per month for 12 months, then the total money withdrawn is P3000. Since he started with P5000, then he will still have P2000 at the end of 12 months.

VIII. Exercises

Solve the following problems on linear inequalities.

1. Kevin wants to buy some pencils at a price of P4.50 each. He has no more than P55.00. What is the greatest number of pencils can Kevin buy? **Answer**:12 pencils
2. In a pair of consecutive even integers, five times the smaller is less than four times the greater. Find the largest pair of integers satisfying the given condition. **Answer:** 6 and 8

NOTICE TO THE TEACHER:
End the lesson with a good summary.

Summary

In this lesson, you learned about the different properties of linear inequality and the process of solving linear inequalities.

- Many simple inequalities can be solved by adding, subtracting, multiplying or dividing both sides until you are left with the variable on its own.
- The direction of the inequality can change when:
  - Multiplying or dividing both sides by a **negative** number
  - Swapping left and right hand sides
- Do not multiply or divide by a **variable** (unless you know it is always positive or always negative).
- While the procedure for solving linear inequalities is similar to that for solving linear equations, the solution to a linear inequality in one variable usually consists of a range of values rather than a single value.

Lesson 29: Solving Absolute Value Equations and Inequalities

Time: 2.5 hours

Pre-requisite Concepts: Properties of Equations and Inequalities, Solving Linear Equations, Solving Linear Inequalities

About the Lesson: This lesson discusses solutions to linear equations and inequalities that involve absolute value.

Objectives:

In this lesson, the students are expected to:

1. solve absolute value equations;
2. solve absolute value inequalities; and
3. solve problems involving absolute value.

> **NOTE TO THE TEACHER:**
> **This lesson is an integration of the students' skills on solving linear equation and inequality. To check students' prior knowledge on absolute value, you may give some drill exercises like evaluating absolute value expressions.**

Lesson Proper

I. Activity

Previously, we learned that the absolute value of a number *x* (denoted by $|x|$) is the distance of the number from zero on the number line. The absolute value of zero is zero. The absolute value of a positive number is itself. The absolute value of a negative number is its opposite or positive counterpart.

Examples are:

$|0| = 0$ $|4| = 4$ $|-12| = 12$ $|7 - 2| = 5$
$|2 - 7| = 5$

Is it true that the absolute value of any number can **never** be negative? Why or why not?

II. Questions/Points to Ponder

1) $|a| = 11$ **Answer**: **±11** 6) $|b| + 2 = 3$
Answer: ±1
2) $|m| = 28$ **Answer: ±28** 7) $|w - 10| = 1$
Answer: 11, 9
3) $|r| = \frac{12}{17}$ **Answer:** $\pm \frac{12}{17}$ 8) $\frac{|u|}{2} = 4$
Answer: ±8
4) $|y| + 1 = 3$ **Answer: ±2** 9) $2|x| = 22$
Answer: ±11
5) $|p| - 1 = 7$ **Answer: ± 8** 10) $3|c + 1| = 6$
Answer: 1, –3

Many absolute value equations are not easy to solve by the guess-and-check method. An easier way may be to use the following procedure.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.
Step 2: Is the number on the other side of the equation negative? If it is, then the equation has no solution. (Think, why?) If it is not, then proceed to step 3.
Step 3: If the absolute value of an expression is equal to a positive number, say a, then the expression inside the absolute value can either be a or $-a$ (Again, think, why?). Equate the expression inside the absolute value sign to a and to $-a$, and solve both equations.

Example 1: Solve $|3a - 4| - 9 = 15$.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	$|3a - 4| - 11 = 15$ $|3a - 4| = 26$	
Step 2: Is the number on the other side of the equation negative?	No, it's a positive number, 26, so proceed to step 3	
Step 3: To satisfy the equation, the expression inside the absolute value can either be +26 or –26. These correspond to two equations.	$3a - 4 = 26$	$3a - 4 = -26$
Step 4: Solve both equations.	$3a - 4 = 26$ $3a = 30$ $a = 10$	$3a - 4 = -26$ $3a = -22$ $a = -\frac{22}{3}$

We can check that these two solutions make the original equation true. If $a = 10$, then $|3a - 4| - 9 = |3(10) - 4| - 9 = 26 - 9 = 15$. Also, if $a = -22/3$, then $|3a - 4| - 9 = |3(-22/3) - 4| - 9 = |-26| - 9 = 15$.

Example 2: Solve $|5x + 4| + 12 = 4$.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	$|5x + 4| + 12 = 4$ $|5x + 4| = -8$
Step 2: Is the number on the other side of the equation negative?	Yes, it's a negative number, –8. There is no solution because $|5x + 4|$ can never be negative, no matter what we substitute for x.

Example 3: Solve $|c - 7| = |2c - 2|$.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	Done, because the expression on the left already consists only of a single absolute value expression.	
Step 2: Is the number on the other side of the equation negative?	No, because $\|2c - 2\|$ is surely **not** negative (the absolute value of a number can never be negative). Proceed to Step 3.	
Step 3: To satisfy the equation, the expression inside the first absolute value, $c - 7$, can either be $+(2c - 2)$ or $-(2c - 2)$. These correspond to two equations. [Notice the similarity to Step 3 of Example 1.]	$c - 7 = +(2c - 2)$	$c - 7 = -(2c - 2)$
Step 4: Solve both equations.	$c - 7 = +(2c - 2)$ $c - 7 = 2c - 2$ $-c - 7 = -2$ $-c = 5$ $c = -5$	$c - 7 = -(2c - 2)$ $c - 7 = -2c + 2$ $3c - 7 = 2$ $3c = 9$ $c = 3$

Again, we can check that these two values for c satisfy the original equation.

Example 4: Solve $|b + 2| = |b - 3|$

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	Done, because the expression on the left already consists only of a single absolute value expression.	
Step 2: Is the number on the other side of the equation negative?	No, because $\|b - 3\|$ is surely **not** negative (the absolute value of a number can never be negative). Proceed to Step 3.	
Step 3: To satisfy the equation, the expression inside the first absolute value, $b + 2$, can either be equal to $+(b - 3)$ or $-(b - 3)$. These correspond to two equations. [Notice the similarity to Step 3 of Example 1.]	$b + 2 = +(b - 3)$	$b + 2 = -(b - 3)$

Step 4: Solve both equations.	$b + 2 = +(b - 3)$ $b + 2 = b - 3$ $2 = -3$ This is false. There is no solution from this equation	$b + 2 = -(b - 3)$ $b + 2 = -b + 3$ $2b + 2 = 3$ $2b = 1$ $b = \frac{1}{2}$

Since the original equation is satisfied even if only of the two equations in Step 3 were satisfied, then this problem has a solution: $b = \frac{1}{2}$. This value of b will make the original equation true.

Example 5: Solve $|x - 4| = |4 - x|$.

Step 1: Let the expression on one side of the equation consist only of a single absolute value expression.	Done, because the expression on the left already consists only of a single absolute value expression.	
Step 2: Is the number on the other side of the equation negative?	No, because $|4 - x|$ is surely **not** negative (the absolute value of a number can never be negative). Proceed to Step 3.	
Step 3: To satisfy the equation, the expression inside the first absolute value, $x - 4$, can either be equal to $+(4 - x)$ or $-(4 - x)$. These correspond to two equations. [Notice the similarity to Step 3 of Example 1.]	$x - 4 = +(4 - x)$	$x - 4 = -(4 - x)$
Step 4: Solve both equations.	$x - 4 = +4 - x$ $2x - 4 = 4$ $2x = 8$ $x = 4$	$x - 4 = -(4 - x)$ $x - 4 = -4 + x$ $-3 = -3$ This is true no matter what value x is. All real numbers are solutions to this equation

Since the original equation is satisfied even if only of the two equations in Step 3 were satisfied, then the solution to the absolute value equation is the set of all real numbers.

III. Exercises

Solve the following absolute value equations.

1. $|m| - 3 = 37$ **Answer: $m = -40, 40$** 6.
   $|2n - 9| = |n + 6|$ **Answer: $n = 1, 15$**
2. $|2v| - 4 = 28$ **Answer: $v = -16, 16$** 7.
   $|5y + 1| = |3y - 7|$ **Answer: $y = -4, \frac{3}{4}$**
3. $|5z + 1| = 21$ **Answer**: $z = -\frac{22}{5}, 4$ 8. $|2t + 3|$
   $= |2t - 4|$ **Answer: $t = \frac{1}{4}$**
4. $|4x + 2| - 3 = -7$ **Answer: no solution** 9.
   $|6w - 2| = |6w + 18|$ **Answer: $w = -\frac{4}{3}$**
5. $|3a - 8| + 4 = 11$ **Answer: $a = \frac{1}{3}, 5$** 10. $|10 -$
   $u| = |u - 10|$ **Answer: $\{u | u \in \mathbb{R}\}$**

IV. Activity

Absolute Value Inequalities.

You may recall that when solving an absolute value equation, you came up with one, two or more solutions. You may also recall that when solving linear inequalities, it was possible to come up with an interval rather than a single value for the answer.

Now, when solving absolute value inequalities, you are going to combine techniques used for solving absolute value equations as well as first-degree inequalities.

Directions: From the given options, identify which is included in the solution set of the given absolute value inequality. You may have one or more answers in each item.

NOTE TO THE TEACHER:

In solving absolute value inequalities, you may present to the students the different forms of writing the solution set (i.e. set notation, interval notation).

Directions: From the given options, identify which is included in the solution set of the given absolute value inequality. You may have one or more answers in each item.

1. $|x - 2| < 3$ a) 5 b) –1 c) 4 d) 0 e) –2 **Answer: c and d**
2. $|x + 4| \geq 41$ a) –50 b) –20 c) 10 d) 40 e) 50 **Answer: a, d and e**
3. $\left|\frac{x}{2}\right| > 9$ a) –22 b) –34 c) 4 d) 18 e) 16 **Answer: a and b**
4. $|2a - 1| \leq 19$ a) 14 b) 10 c) –12 d) –11 e) –4 **Answer: b and e**
5. $2|u - 3| < 16$ a) –3 b) –13 c) 7 d) 10 e) 23 **Answer: a, c, and d**
6. $|m + 12| - 4 > 32$ a) –42 b) –22 c) –2 d) 32 e) 42 **Answer: d and e**
7. $|2z + 1| + 3 \leq 6$ a) –4 b) –1 c) 3 d) 0 e) 5 **Answer: b and d**
8. $|2r - 3| - 4 \geq 11$ a) –7 b) –11 c) 7 d) 11 e) 1 **Answer: a, b and d**
9. $|11 - x| - 2 > 4$ a) 15 b) 11 c) 2 d) 4 e) 8 **Answer: c and d**
10. $\left|\frac{x}{3} + 1\right| < 10$ a) –42 b) –36 c) –30 d) –9 e) 21 **Answer: c, d and e**

V. Questions/Points to Ponder

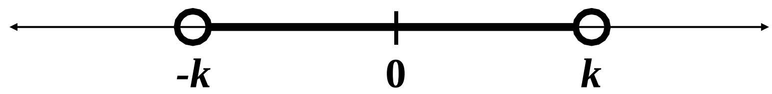
Think about the inequality $|x| < 7$. This means that the expression in the absolute value symbols needs to be less than 7, but it also has to be greater than –7. So answers like 6, 4, 0, –1, as well as many other possibilities will work. With $|x| < 7$, any real number between –7 and 7 will make the inequality true. The solution consists of all numbers satisfying the double inequality $-7 < x < 7$.

Suppose our inequality had been $|x| > 7$. In this case, we want the absolute value of x to be larger than 7, so obviously any number larger than 7 will work (8, 9, 10, etc.). But numbers such as –8, –9, –10 and so on will also work since the absolute value of all those numbers are positive and larger than 7. Thus, the solution or this problem is the set of all x such that $x < -7$ or $x > 7$.

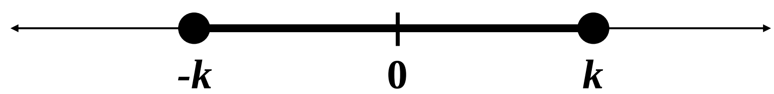
With so many possibilities, is there a systematic way of finding all solutions? The following discussion provides an outline of such a procedure.

In general, an absolute value inequality may be a “less than” or a “greater than” type of inequality (either **|*x*| < *k*** or **|*x*| > *k***). They result in two different solutions, as discussed below.

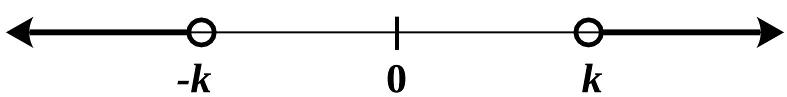
1. Let k be a positive number. Given $|x| < k$, then $-k < x < k$.
   The solution may be represented on the number line. Observe that the solution consists of all numbers whose distance from 0 is less than k.



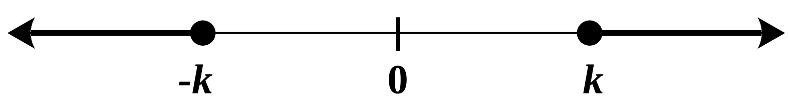
- If the inequality involves ≤ instead of <, then $\pm k$ will now be part of the solution, which gives $-k \le x \le k$. This solution is represented graphically below.



Let k be a positive number. Given $|x| > k$, then $x < -k$ **or** $x > k$.
The solution may be represented on a number line. Observe that the solution consists of all numbers whose distance from 0 is greater than k.



- If the inequality involves ≥ instead of >, then $\pm k$ will now be part of the solution, which gives $x \le -k$ **or** $x \ge k$. This solution represented graphically below.



Example 1: Solve $|x – 4| < 18$.

Step 1: This is a “less than” absolute value inequality. Set up a double inequality.	$-18 < x - 4 < 18$
Step 2: Solve the double inequality.	$-18 + 4 < x < 18 + 4$ $-14 < x < 22$

Therefore, the solution of the inequality is $\{x \mid -14 < x < 22\}$. We can check that choosing a number in this set will make the original inequality true. Also, numbers outside this set will not satisfy the original inequality.

Example 2: Solve $|2x + 3| > 13$.

Step 1: This is a "greater than" absolute value inequality. Set up two separate inequalities	$2x + 3 < -13$	$2x + 3 > 13$
Step 2: Solve the two inequalities.	$2x + 3 - 3 < -13 - 3$ $2x < -16$ $x < -8$	$2x + 3 - 3 > 13 - 3$ $2x > 10$ $x > 5$

Therefore, the solution of the inequality is $\{x \mid x < -8 \text{ or } x > 5\}$. This means that all x values less than –8 **or** greater than 5 will satisfy the inequality. By contrast, any number between –8 and 5 (including –8 and 5) will not satisfy the inequality. How do you think will the solution change if the original inequality was ≥ instead of >?

Example 3: Solve $|3x - 7| - 4 \geq 10$

Step 1: Isolate the absolute value expression on one side.	$|3x - 7| \geq 14$	
Step 2: This is a "greater than" absolute value inequality. Set up a two separate inequalities.	$3x - 7 \leq -14$	$3x - 7 \geq 14$
Step 3: Solve the two inequalities	$3x - 7 \leq -14$ $3x + 7 \leq -14 + 7$ $3x \leq -7$ $x \leq -\frac{7}{3}$	$3x - 7 \geq 14$ $3x - 7 + 7 \geq 14 + 7$ $3x \geq 21$ $x \geq 7$

Therefore, the solution of the inequality is $\{x \mid x \leq -\frac{7}{3} \text{ or } x \geq 7\}$.

VI. EXERCISES

Directions: Solve the following absolute value inequalities and choose the letter of the correct answer from the given choices.

1. What values of a satisfy the inequality $|4a + 1| > 5$?

A. $\{a \mid a < -\frac{3}{2} \text{ or } a > 1\}$ B. $\{a \mid a > -\frac{3}{2} \text{ or } a > 1\}$

Answer: A

C. $\{a \mid a > -\frac{3}{2} \text{ or } a < 1\}$ D. $\{a \mid a < -\frac{3}{2} \text{ or } a < 1\}$

2. Solve for the values of y in the inequality $|y – 20| \leq 4$.

A. $\{y \mid 16 \geq y \leq 24\}$ B. $\{y \mid 16 \geq y \geq 24\}$

Answer: C

C. $\{y \mid 16 \leq y \leq 24\}$ D. $\{y \mid 16 \leq y \geq 24\}$

3. Find the solution set of $|b – 7| < 6$.

A. $\{b \mid –13 < b < 13\}$ B. $\{b \mid 1 < b < 13\}$

Answer: B

C. $\{b \mid 1 > b > 13\}$ D. $\{b \mid –13 > b > 13\}$

4. Solve for c: $|c + 12| + 3 > 17$

A. $\{c \mid c > –2 \text{ or } c < 2\}$ B. $\{c \mid c > –26 \text{ or } c < 2\}$

Answer: D

C. $\{c \mid c < –2 \text{ or } c > 2\}$ D. $\{c \mid c < –26 \text{ or } c > 2\}$

5. Solve the absolute value inequality: $|1 – 2w| < 5$

A. $\{c \mid 3 < c < –2\}$ B. $\{c \mid –3 < c < 2\}$

Answer: C

C. $\{c \mid 3 > c > –2\}$ D. $\{c \mid –3 > c > 2\}$

VII. Questions/Points to Ponder

Solve the following problems involving absolute value.

1. You need to cut a board to a length of 13 inches. If you can tolerate no more than a 2% relative error, what would be the boundaries of acceptable lengths when you measure the cut board? *(Hint: Let x = actual length, and set up an inequality involving absolute value.)*
   Answer: 2% of 13 inches is 0.26 inches. Set up the inequality $|x – 13| = 0.26$ (or $|13 – x| \leq 0.26$). The solution to both these equations is $12.74 \leq x \leq 13.26$. Thus, the acceptable lengths are from 12.74 inches to 13.26 inches.
2. A manufacturer has a 0.6 oz tolerance for a bottle of salad dressing advertised as 16 oz. Write and solve an absolute value inequality that describes the acceptable volumes for “16 oz” bottles. *(Hint: Let x = actual amount in a bottle, and set up an inequality involving absolute value.)*
   Answer: $|x – 16| \leq 0.6$ (or $|16 – x| \leq 0.6$), both of which has the solution $15.4 < x < 16.6$. Thus, the bottle can range from 15.4 oz to 16.6 oz, inclusive.

NOTE TO THE TEACHER: **End the lesson with a good summary.**

Summary

In this lesson you learned how to solve absolute value equations and absolute value inequalities. If a is a positive number, then the solution to the absolute value equation $|x| = a$ is $x = a$ or $x = -a$.

There are two types of absolute value inequalities, each corresponding to a different procedure. If **$|x| < k$**, then **$-k < x < k$**. If **$|x| > k$**, then **$x < -k$ or $x > k$**. These principles work for any positive number k.

Lesson 30: Basic Concepts and Terms in Geometry

About the Lesson:

This lesson focuses on plane figures. Included in the discussion are the basic terms used in geometry such as points, lines and planes. The focus of this section is the different ways of describing and representing the basic objects used in the study of geometry.

Objectives:

In this lesson, the participants are expected to:

1. describe the undefined terms;
2. give examples of objects that maybe used to represent the undefined terms;
3. name the identified point(s), line(s) and plane(s) in a given figure;
4. formulate the definition of parallel lines, intersecting lines, concurrent lines, skew lines, segment, ray, and congruent segments;
5. perform the set operations on segments and rays.

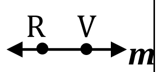
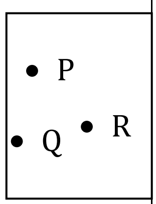
Lesson Proper

A. Introduction to the Undefined Terms:

In any mathematical system, definitions are important. Elements and objects must be defined precisely. However, there are some terms or objects that are the primitive building blocks of the system and hence cannot be defined independently of other objects. In geometry, these are **point, line**, **plane**, and **space**. There are also relationships like **between** that are not formally defined but are merely described or illustrated.

In Euclidean Geometry, the geometric terms point, line, and plane are all undefined terms and are purely mental concepts or ideas. However, we can use concrete objects around us to represent these ideas. Thus, these undefined terms can only be described.

Term	Figure	Description	Notation
Point	• A	A point suggests an exact location in space. It has no dimension. We use a capital letter to name a point.	point A

Line	 	A line is a set of points arranged in a row. It is extended endlessly in both directions. It is a one-dimensional figure. Two points determine a line. That is, two distinct points are contained by exactly one line. We use a lower case letter or any two points on the line to name the line.	line ***m*** or $\overleftrightarrow{RV}$
Plane	 	A plane is a set of points in an endless flat surface. The following determine a plane: (a) three non-collinear points; (b) two intersecting lines; (c) two parallel lines; or (d) a line and a point not on the line. We use a lower case letter or three points on the plane to name the plane.	plane PQR or □PQR

I. **Activity 1**

II.

Objects Representing the Undefined Terms

1. These are some of the objects around us that could represent a point or line or plane. Place each object in its corresponding column in the table below.

Blackboard	Corner of a table	intersection of a side wall and ceiling	tip of a needle
Laser	Electric wire		
		Intersection of the front wall, a side wall and ceiling	surface of a table
Tip of a ballpen	Wall	Edge of a table	Paper

Objects that could represent a point	Objects that could represent a line	Objects that could represent a plane

Answers:

The following can represent a point: corner of a table; tip of a needle; intersection of the front wall, side wall and ceiling; tip of a ballpen.
The following can represent a line: intersection of a side wall and ceiling, laser, electric wire edge of a table.
The following can represent a plane: blackboard, surface of a table, wall,

II. Questions to Ponder:

1. Consider the stars in the night sky. Do they represent points?
2. Consider the moon in its fullest form. Would you consider a full moon as a representation of a point?
3. A dot represents a point. How big area dot that represents a point and a dot that represents a circular region?
4. A point has no dimension. A line has a dimension. How come that a line composed of dimensionless points has a dimension?
5. A pencil is an object that represents a line. Does a pencil extend infinitely in both directions? Is a pencil a line?

Note to the Teacher:
The questions above are not meant to generate "correct" answers. They are used to emphasize that point, line and plane are abstract geometric concepts and are not to be found in material things around us. It is good to constantly remember that representations of geometric objects are imperfect and are to be differentiated from the actual objects they represent.

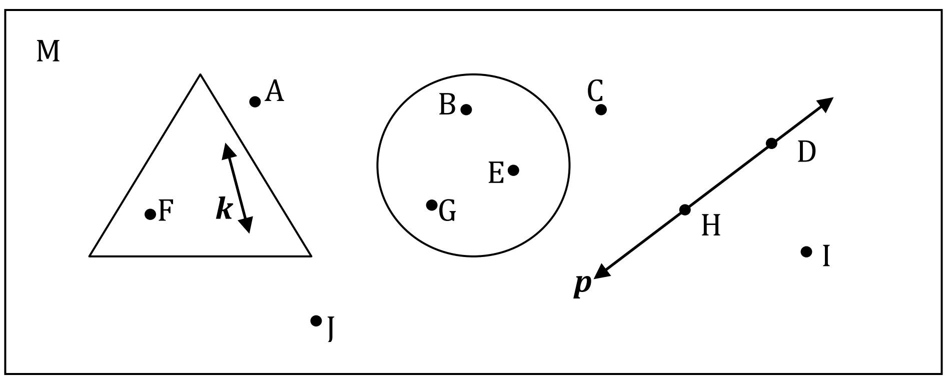
III. Exercises

1. List down 5 other objects that could represent
   a. a point.
   b. a line.
   c. a plane.

Sample Answers:

Point	Line	Plane
Dot on the letter I grain of sand corner of a sheet of paper point of a knife light made by laser pointed on the wall	Edge of a knife washing line edge of a sidewalk flagpole queue/line formed by pupils	Window pane floor surface of a tray stage platform cardboard

2. Use the figure below, identify what is being asked.



a) Name the point(s) in the interior region of the circle.

b) Name the point(s) in the interior region of the triangle.

c) Name the line(s) in the interior region of the triangle.

d) Give other name(s) for line ***p*.**

e) Name the plane that can be formed by the three points in the interior of the circle.

f) Name the plane formed by line ***p***and point I.

g) Name the points outside the circular region.

h) Name the points outside the region bounded by the triangle.

i) Name the points of plane M.

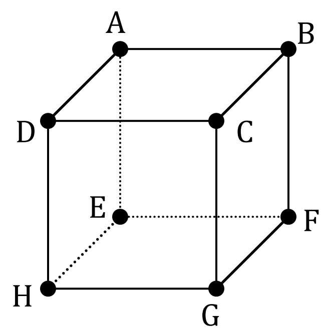
j) Give other names for plane M.

Answers:

(a) B, G, E; **(b)** F; **(c)** No line can be in the interior of a triangle because lines have infinite length; **(d)** $\overleftrightarrow{HD}$ or $\overleftrightarrow{DH}$; **(e)** Plane M; **(f)** Plane M; **(g)** A, C, D, F, H, I, J;
(h) A, B, C, D, E, G, H, I, J; **(i)** A, B, C, D, E, F, G, H, I, J; **(j)** Plane ABC, FGH (any three letters can determine the plane)

C. Recall:
   (a) Two points determine a line.
   (b) Three points not on the same line determine a plane.
   (c) Two intersecting lines determine a plane.
   (d) Two parallel lines determine a plane.
   (e) A line and a point not on the line determine a plane.

Given: The points A, B, C, D, E, F, G, H are corners of a box shown below:



Answer the following:

1. How many lines are possible which can be formed by these points? (Hint: There are more than 20.) Refer to statement (a) above. _________

2. What are the lines that contain the point A? (Hint: There are more than 3 lines.) __________________________

3. Identify the different planes which can be formed by these points. (Hint: There are more than six. Refer to statement (d) above. ______________

4. What are the planes that contain line DC? _________

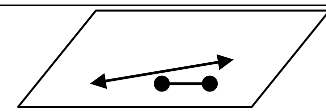
5. What are the planes that intersect at line BF? _______

Answers:
1.26 lines; **2.** AB, AC, AD, AE, AF, AG, AH; **3.** ABC, ADE, ABE, CDH, BCG, EFG, ABG, BCE, CDE, ADF, ACF, ACH, BDE, BDG, BEG, DEG, AFH, BFH, ACE, BDF;**4.** ABC, CDH, CDE; **5.** ABF, BCF, BDF

B. Other basic geometric terms on points and lines

The three undefined terms in Plane Geometry are ***point, line*** and ***plane***.

Relationships between the above objects are defined and described in the activities that follow.

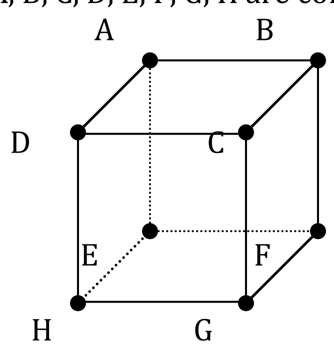
Geometric Terms	Illustration
Collinear points are points on the same line.	
Coplanar points/lines are points/lines on the same plane.	

The following activity sheet will help us develop the definitions of the other relationships.

I. Activity 2

Other Geometric Terms on Lines

Refer to the figure below:
Given: The points A, B, C, D, E, F, G, H are corners of a box as shown:



Intersecting Lines

Lines DH and DC intersect at point D. They are intersecting lines.
Lines CG and GF intersect at point G. They are also intersecting lines.

1. What other lines intersect with line DH? __________
2. What other lines intersect with line CG? __________
3. What lines intersect with EF? _______________

Possible Answers:
1. AD, BD, ED, DG, DF, EH, GH, FH, AH, BH; **2.** AC, BC, CD, CE, CF, CH, AG, BG, DG, EG, GH; **3.** AE, BE, CE, DE, EG, EH, AF, BF, CF, DF, FG, FH

Parallel Lines

Lines AB and DC are ***parallel***.

Lines DH and CG are ***parallel***.

4. What other lines are parallel to line AB? _________
5. What other lines are parallel to line CG? _________
6. What lines are parallel to line AD? ___________

How would you describe parallel lines?

> **Possible Answers:**
> **4.** EF, HG; **5.** AE, BF; **6.** BC, EH, FG; Parallel lines do not meet, the distance between them does not change, they go in the same direction

Concurrent Lines

Lines AD, AB, and AE are concurrent at point A.

Lines GH, GF, and GC are concurrent at point G.

7. Name if possible, other lines that are concurrent at point A. __________
8. Name if possible, other lines that are concurrent at point G. __________
9. What lines are concurrent at point F? _________

What do you think are concurrent lines? How would you distinguish concurrent lines from intersecting lines?

> **Possible Answers:**
> **7.** AC, AH, AF, AG are also concurrent at A.
> **8.** AG, BG, DG, EG are also concurrent at A.
> **9.** AF, BF, CF, DF, EF, FG, FH are concurrent at F.
> Concurrent lines are lines that intersect at a point. We usually use the term concurrent for three or more lines passing through a common point.

Skew Lines

Lines DH and EF are two lines which are neither intersecting nor parallel. These two lines do not lie on a plane and are called ***skew lines***. Lines AE and GF are also skew lines. The lines DH, CG, HE and GF are ***skew to*** AB.

10. What other lines are skew to DH? ___________
11. What other lines are skew to EF? ____________
12. What lines are skew to BF? ________________

> **Possible Answers:**
> **10.** AB, BC, EF, FG, AC, EG, AF, AG, BE, BG, CE, CF are skew to DH.
> **11.** AC, AD, AG, AH, BC, BD, BG, BH, CG, CH, DG are skew to EF.
> **12.** AC, AD, AG, AH, BC, BD, BG, BH, CG, CH, DG, DH are skew to EF.

Remember:

- Two lines are ***intersecting*** if they have a common point.
- Three or more lines are ***concurrent*** if they all intersect at only one point.
- ***Parallel lines*** are coplanar lines that do not meet.
- ***Skew lines*** are lines that do not lie on the same plane

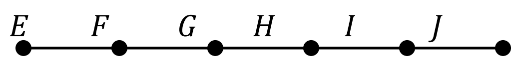
C. ***Subsets of Lines***

The ***line segment*** and the ***ray*** are some of the subsets of a line. A segment has two endpoints while a ray has only one endpoint and is extended endlessly in one direction. The worksheets below will help you formulate the definitions of segments and rays.

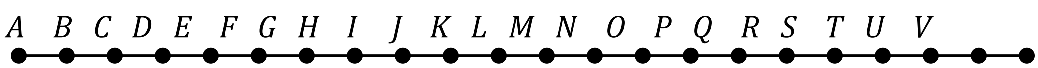
Activity 3
Definition of a Line Segment

ABCD

AD is a line segment. The points *A*, *B*, *C*, and *D*are on line segment *AD*. In notation, we write $\overline{AD}$ or simply *AD*. We can also name it as $\overline{DA}$ or *DA.*



FH is a segment. The points *F*, *G*, and *H* are on line segment *FH*. The points *E*, *I*, and *J* are not on line segment *FH*. In notation, we write $\overline{FH}$. We can also name it as $\overline{HF}$ or *HF*.



The points *E*, *F* ,*G*, and *J* are on line segment *EQ* or segment *QE*.

The points *C*, *D*, *T*, and *U* are not on line segment *EQ*.
Answer the following:

1. Name other points which are on line segment *EQ*. _______________

2. Name other points which are not on line segment *EQ*. _______________

Complete the following statements:

3. A line segment is part of a line that has _________.

4. Line segment *EQ* consists of the points ___________________.

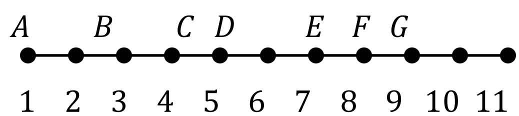
Possible Answers:

1. *H, I, K, L, M, N, O, P* are on *EQ*
2. *A, B, R, S, V* are not on *EQ*
3. Endpoints, finite length
4. *E*, *Q*, and all points between *E* and *Q*.

The line segment. A ***line segment*** is part of a line that has two endpoints. We define a line segment $\overline{AB}$ as a subset of line $\overleftrightarrow{AB}$ consisting of the points *A* and *B* and all the points between them. If the line to which a line segment belongs is given a scale so that it turns into the real line, then the length of the segment can be determined by getting the distance between its endpoints.

Activity 4
Congruent Segments

Given the points on the number line:



1. Determine the length of the following:

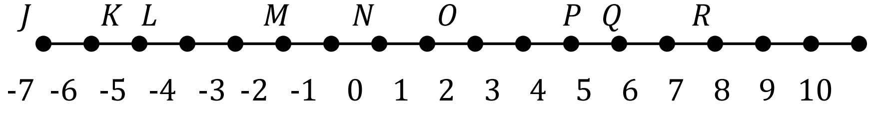
a) *AB* = ______
b) *DE* = ______
c) *BD*= ______
d) *DF* = ______

e) *AC*= ________
f) *DG* = ________
g) *BE* = ________
h) *CG* = ________

2. The following segments are congruent: *AB* and *DE*; *BD* and *DF*; *AC* and *DG*, *BE* and *CG*.

3. The following pairs of segments are not congruent: *AB* and *CF*; *BD* and *AE*; *AC* and *BF*; *BG* and *AD*.

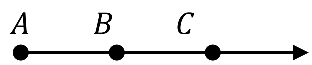
4. Using the figure below, which segments are congruent?



Define congruent segments: Congruent segments are segments _______________.

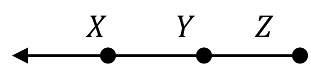
Remember:
Segments are **congruent** if they have the same length.

Activity 5
Definition of a Ray



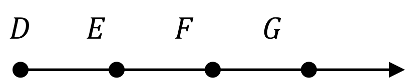
This is ray *AB*. We can also name it as ray *AC*.
In symbol, we write $\overrightarrow{AC}$.

The points *A*, *B*, *C* are on ray *AC*.



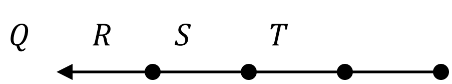
This is ray *ZY*. We can also name it as ray *ZX*.
In symbol, we write $\overrightarrow{ZX}$. We do NOT write it as $\overleftarrow{XZ}$.

The points *X*, *Y*, *Z* are on ray *ZY*.



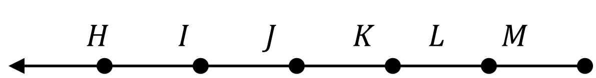
This is ray *DE*. We can also name it as ray *DF* or ray *DG*.

The points *D, E, F, G* are on ray *DE*.



This is ray *TS*. We can also name it as ray *TR* or ray *TQ*.

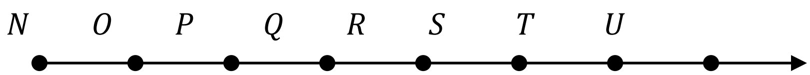
The points *Q, R, S, T* are on ray *TS*.



This is ray *ML*.

1. How else can you name this ray? ________

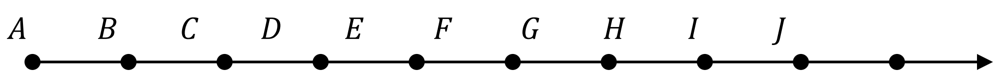
2. What are the points on ray *ML*? _______________



The points *Q, R, S, T, U* are on ray *QR*.

The points *N, O, P* are not on ray *QR*.

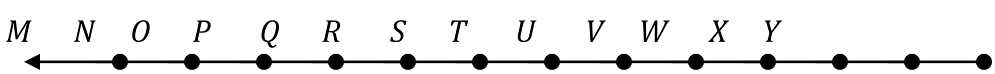
3. How else can you name ray *QR*? ______________.



4. What are the points on ray *DE*? ______________

5. What are the points not on ray *DE*? ___________

5. How else can you name ray *DE*? ________________



7. What are the points on ray *QT*?

8. What are the points on ray *PQ*?

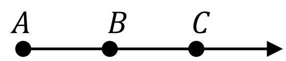
9. What are the points on ray *XU*?

10. What are the points on ray *SP*?

In general, how do you describe the points on any ray *AC*?

____

The ray. A ray is also a part of a line but has only one endpoint, and extends endlessly in one direction. We name a ray by its endpoint and one of its points. We always start on the endpoint. The figure is ray AB or we can also name it as ray AC. ***It is not correct to name it as ray BA or ray CA.*** In notation, we write $\overrightarrow{AB}$ or $\overrightarrow{AC}$.



The points A, B, C are on ray AC.
However, referring to another ray $\overrightarrow{BC}$, the point A is not on ray $\overrightarrow{BC}$.

Remember:
Ray $\overrightarrow{AB}$ is a subset of the line AB. The points of $\overrightarrow{AB}$ are the points on segment AB and all the points X such that B is between A and X.

We say:

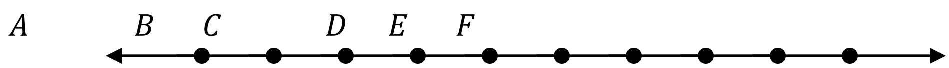
$$\left.\begin{array}{l} AB \text{ is parallel to } CD \\ \overrightarrow{AB} \text{ is parallel to } CD \\ \overrightarrow{AB} \text{ is parallel to } \overleftrightarrow{CD} \\ \overleftrightarrow{AB} \text{ is parallel to } CD \end{array}\right\} \text{ if the lines } \overleftrightarrow{AB} \text{ and } \overleftrightarrow{CD} \text{ are parallel.}$$

D. Set operations involving line and its subsets

Since the lines, segments and rays are all sets of points, we can perform set operations on these sets.

Activity 6
The Union/Intersection of Segments and Rays

Use the figure below to determine the part of the line being described by the union or intersection of two segments, rays or segment and ray:



Example: $\overrightarrow{DE} \cup \overrightarrow{CF}$is the set of all points on the ray DE and segment CF. Thus, all these points determine ray $\overrightarrow{CD}$.

$\overrightarrow{BC} \cap \overrightarrow{ED}$is the set of all points common to ray $\overrightarrow{BC}$ and ray $\overrightarrow{ED}$. The common points are the points on the segment BE.

Answer the following:

1) $\overline{AB} \cup \overline{BE} =$ __________
2) $\overrightarrow{DF} \cup \overline{BD} =$ __________
3) $\overrightarrow{CB} \cup \overline{CE} =$ __________
4) $\overrightarrow{DE} \cup \overline{BD} =$ __________
5) $\overrightarrow{CA} \cup \overrightarrow{CD} =$ __________
6) $\overline{BF} \cap \overline{AD} =$ __________
7) $\overrightarrow{FD} \cap \overline{AB} =$ __________
8) $\overrightarrow{FE} \cap \overrightarrow{CD} =$ __________
9) $\overrightarrow{CA} \cap \overrightarrow{CE} =$ __________
10) $\overline{BC} \cap \overrightarrow{CE} =$ __________

Possible Answers:
1. $\overline{AE}$; **2.** $\overrightarrow{BF}$; **3.** $\overrightarrow{EB}$; **4.** $\overrightarrow{BE}$; **5.** $\overleftrightarrow{CD}$; **6.** BD; **7.** $\overline{AB}$; **8.** $\overline{CF}$; **9.** C; **10.** C

Summary

In this lesson, you learned about the basic terms in geometry which are point, line, plane, segment, and ray. You also learned how to perform set operations on segments and rays.

Lesson 31: Angles

Prerequisite Concepts: Basic terms and set operation on rays

About the Lesson:

This lesson is about angles and angle pairs, and the angles formed when two lines are cut by a transversal.

Objectives:

In this lesson, you are expected to:

1. Define angle, angle pair, and the different types of angles
2. Classify angles according to their measures
3. Solve problems involving angles.

Lesson Proper

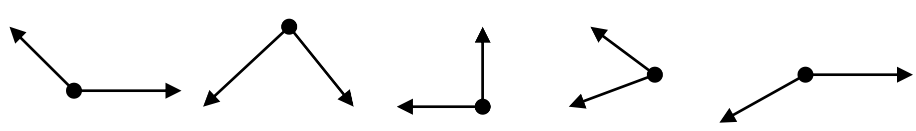
We focus the discussion on performing set operations on rays. The worksheet below will help us formulate a definition of an angle.

A. Definition of Angle

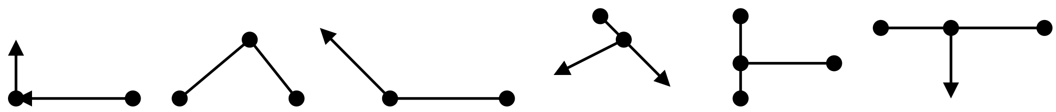
I. Activity

Activity 7
Definition of an Angle

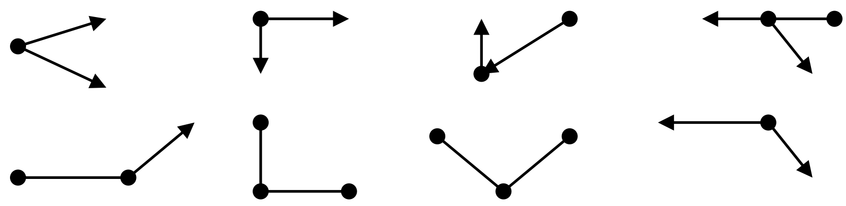
The following are angles:



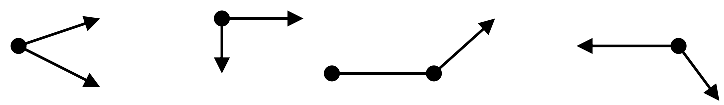
The following are not angles:



Which of these are angles?



The following are angles:



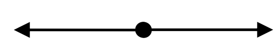
How would you define an angle?

An angle is ____.

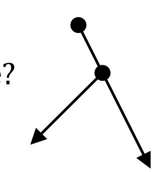
An **angle** is a union of two non-collinear rays with common endpoint. The two non-collinear rays are the ***sides*** of the angle while the common endpoint is the ***vertex***.

II. Questions to ponder:

1. Is this an angle?

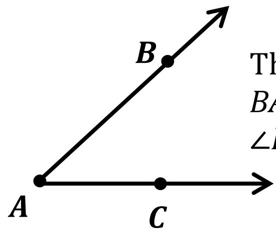


2. Why is this figure, taken as a whole, not an angle?



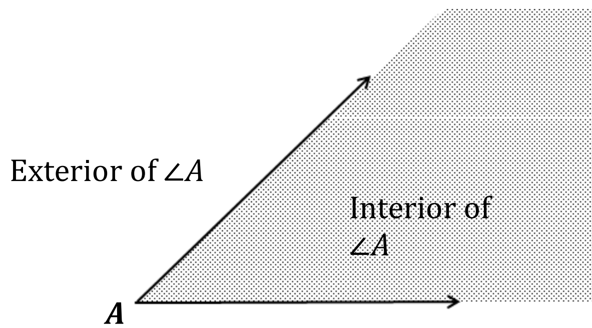
Possible Answers:
1. The rays that form an angle must be noncollinear.
2. The rays that form an angle must meet at their endpoints.

If no confusion will arise, an angle can be designated by its vertex. If more precision is required three letters are used to identify an angle. The middle letter is the vertex, while the other two letters are points one from each side (other than the vertex) of the angle. For example:



The angle on the left can be named angle A or angle BAC, or angle CAB. The mathematical notation is $\angle A$, or $\angle BAC$, or $\angle CAB$.

An angle divides the plane containing it into two regions: the interior and the exterior of the angle.



B. Measuring and constructing angles

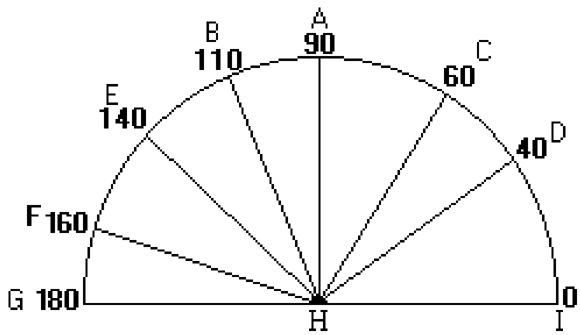
I. Activity

A protractor is an instrument used to measure angles. The unit of measure we use is the degree, denoted by °. Angle measures are between 0°and 180°. The measure of $\angle A$ is denoted by m$\angle A$, or simply $\angle A$.

Activity 8
Measuring an Angle

a) Construct angles with the following measures: 90^o, 60^o , 30^o , 120^o

b) From the figure, determine the measure of each angle.



1) $\angle EHC$ = ________ 6) $\angle CHB$ = ________ 11) $\angle BHE$ = ________

2) $\angle CHF$ = ________ 7) $\angle DHG$ = ________ 12) $\angle CHI$ = ________

3) $\angle IHA$ = ________ 8) $\angle FHI$ = ________ 13) $\angle BHG$ = ________

4) $\angle BHD$ = ________ 9) $\angle EHF$ = ________ 14) $\angle CHD$ = ________

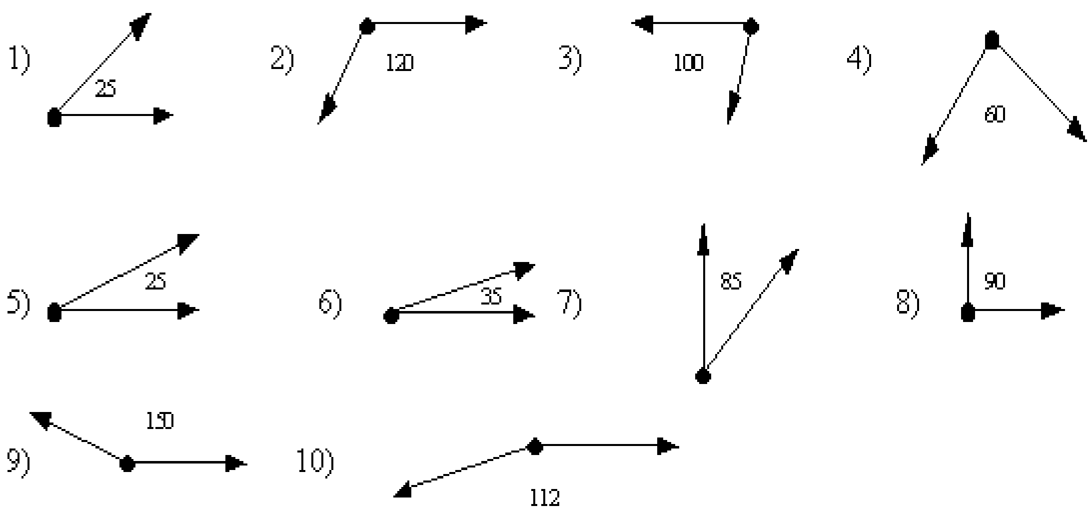
5) $\angle AHG$ = ________ 10) $\angle DHI$ = ________ 15) $\angle BHI$ = ________

Answers:

$\angle EHC = 80°$	$\angle CHB = 50°$	$\angle BHE = 30°$
$\angle CHF = 100°$	$\angle DHG = 140°$	$\angle CHI = 60°$
$\angle IHA = 90°$	$\angle FHI = 160°$	$\angle BHG = 70°$
$\angle BHD = 70°$	$\angle EHF = 20°$	$\angle CHD = 20°$
$\angle AHG = 90°$	$\angle DHI = 40°$	$\angle BHI = 110°$

Exercise 9.Estimating Angle Measures

A. In the drawings below, some of th
e indicated measures of angles are correct and some are obviously wrong. Using estimation, state which measures are correct and which are wrong. The measures are given in degrees. ***You are not expected to measure the angles.***



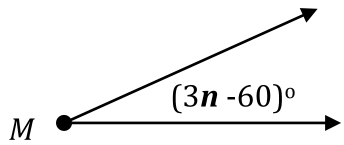
The following are good estimates: 2, 6, 8, 9,

Discussion:

The three different types of angles are acute, right and obtuse angles. An ***acute angle*** measures more than 0° but less than 90°; a ***right angle*** measures exactly 90° while an ***obtuse angle*** measures more than 90° but less than 180°. If two lines or segments intersect so that they form a right angle, then they are ***perpendicular***. In fact, two perpendicular lines meet to form four right angles.

Note that we define angle as a union of two non-collinear rays with a common endpoint. In trigonometry, an angle is sometimes defined as the rotation of a ray about its endpoint. Here, there is a distinction between the initial position of the ray and its terminal position. This leads to the designation of the initial side and the terminal side. The measure of an angle is the amount of rotation. If the direction of the rotation is considered, negative angles might arise. This also generates additional types of angles: the zero, straight, reflex and perigon angles. A zero angle measures exactly 0°; a straight angle measures exactly 180°; a reflex angle measures more than 180° but less than 360° and aperigon angle measures exactly 360°.

II. Question to ponder:



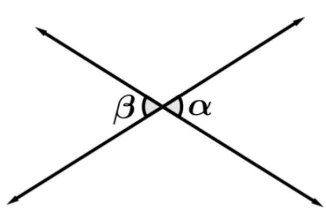
If $\angle M$ is an acute angle, what are the possible values of ***n***?

$0 < 3n - 60 < 90$, so $60 < 3n < 120$. This gives us $20 < n < 40$, or n is between 20 and 40.

A. On Angle Pairs:

I. Definitions

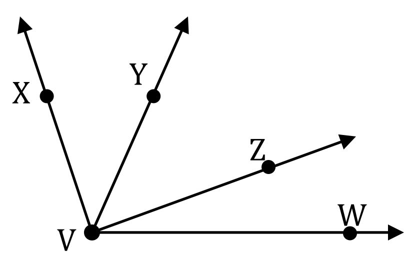
Two angles are ***adjacent*** if they are coplanar, have common vertex and a common side but have no common interior points.
Two angles are ***complementary*** if the sum of their measures is 90^0.
Two angles are ***supplementary*** if the sum of their measures is 180^0.
Two angles form a ***linear pair*** if they are both adjacent and supplementary.
Vertical angles are the opposite angles formed when two lines intersect. Vertical angles are congruent.



In the figure, $\angle\alpha$ and $\angle\beta$ are vertical angles.

II. Activity

Exercise 10: Parts of an Angle



Use the given figure to identify the following:

1) The sides of ∠YVW ___________
2) The sides of ∠XVY ___________
3) The angle(s) adjacent to ∠ZVW ___________
4) The angle(s) adjacent to ∠ XVZ ___________
5) The angle(s) adjacent to ∠YVZ ___________
6) The side common to ∠ XVY and ∠YVZ ___________
7) The side common to ∠XVZ and∠ZVW ___________
8) The side common to ∠XVZ and∠ZVY ___________
9) The side common to ∠XVY and ∠YVW ___________
10) The common vertex. ___________

Answers:
1. $\overrightarrow{VY}$ and $\overrightarrow{VW}$; **2.** $\overrightarrow{VX}$ and $\overrightarrow{VY}$; **3.** $\angle ZVY$ and $\angle ZVX$; **4.** $\angle ZVW$; **5.** $\angle YVX$ and $\angle ZVW$; **6.** $\overrightarrow{VY}$; **7.** $\overrightarrow{VZ}$; **8.** $\overrightarrow{VZ}$; **9.** $\overrightarrow{VY}$; **10.** V

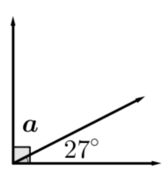
III. Question to Ponder:

Why are the angles ∠ XVZ and ∠YVZ not considered to be adjacent angles?

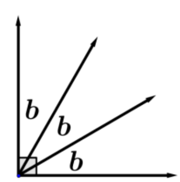
$\angle XVZ$ and $\angle YVZ$ are not adjacent because their interiors are not disjoint.

Exercise 11:

A. Determine the measures of the angles marked with letters. (Note: Figures are not drawn to scale.)



1.



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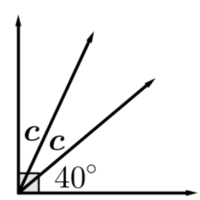

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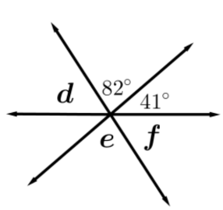

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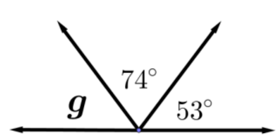

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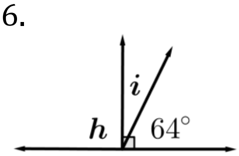


B. Determine whether the statement is true or false. If false, explain why.

7. 20^o, 30^o,40^o are complementary angles.

8. 100^o, 50^o, 30^o are supplementary angles.

Answers: **1.** $\angle a = 63°$; **2.**$\angle b = 30°$; **3.**$\angle c = 25°$; **4.**$\angle d = 57°$, $\angle e = 82°$; $\angle f = 57°$; **5.** $\angle g = 53°$; **6.**$\angle h = 90°$, $\angle i = 26°$; **7.** Not complementary; **8.** Not supplementary

Note that only pairs of angles are complementary or supplementary to each other. Hence, the angles measuring 20°, 30° and 40° are not complementary. Similarly, the angles measuring 100°, 50° and 30° are not supplementary.

B. Angles formed when two lines are cut by a transversal.

I. Discussion

Given the lines x and y in the figure below. The line z is a transversal of the two lines. A ***transversal*** is a line that intersects two or more lines. The following angles are formed when a transversal intersects the two lines:

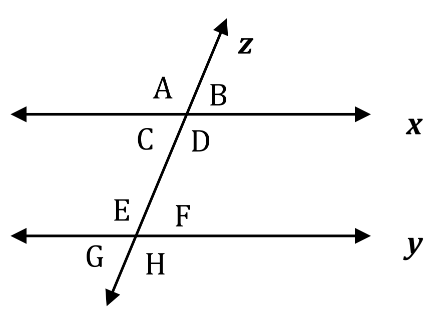
The ***interior angles*** are the four angles formed between the lines x and y. In the figure, these are $\angle C$, $\angle D$, $\angle E$, and $\angle F$.

The ***exterior angles*** are the four angles formed that lie outside the lines x and y. These are $\angle A$, $\angle B$, $\angle G$, and $\angle H$.

The ***alternate interior angles*** are two interior angles that lie on opposite sides of a transversal. The angle pairs $\angle C$and $\angle F$ are alternate interior angles. So are $\angle D$and $\angle E$.

The ***alternate exterior angles*** are two exterior angles that lie on opposite sides of the transversal. In the figure, $\angle A$ and $\angle H$ are alternate exterior angles, as well as $\angle B$and $\angle G$.

The ***corresponding angles*** are two angles, one interior and the other exterior, on the same side of the transversal. The pairs of corresponding angles are$\angle A$and $\angle E$, $\angle B$ and $\angle F$, $\angle C$ and $\angle G$, and $\angle D$ and $\angle H$.

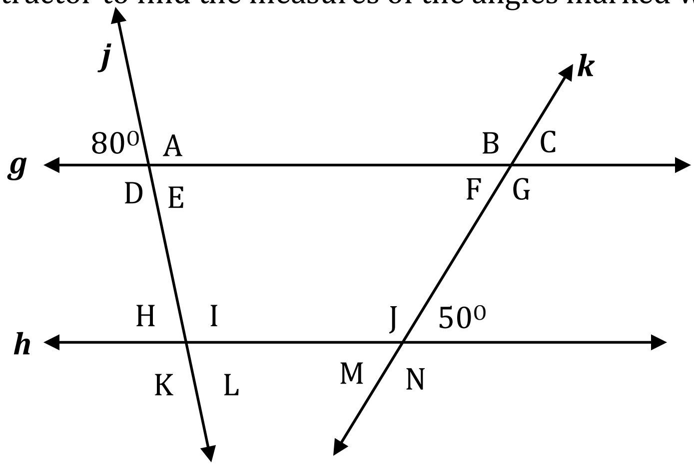


II. Activity12

Angles Formed when Two Parallel Lines are Cut by a Transversal

Draw parallel lines g and h. Draw a transversal ***j*** so that it forms an 80º angle line with ***g*** as shown. Also, draw a transversal ***k*** so that it forms a 50º angle with line ***h*** as shown.

Use your protractor to find the measures of the angles marked with letters.



$\angle A =$ ______ $\angle E =$ ______ $\angle B =$ ______ $\angle C =$ ______

$\angle D =$ ______ $\angle H =$ ______ $\angle G =$ ______ $\angle F =$ ______

$\angle I =$ ______ $\angle L =$ ______ $\angle J =$ ______ $\angle M =$ ______

$\angle K =$ ______ $\angle N =$ ______

Answers:

$\angle A = 120°$	$\angle E = 80°$	$\angle B = 130°$	$\angle C = 50°$
$\angle D = 120°$	$\angle H = 80°$	$\angle G = 130°$	$\angle F = 50°$
$\angle I = 120°$	$\angle L = 80°$	$\angle J = 130°$	$\angle M = 50°$
$\angle K = 120°$		$\angle N = 130°$	

Compare the measures of all the:

a) corresponding angles
b) alternate interior angles
c) alternate exterior angles.

What do you observe? ______________________

Complete the statements below:

When two parallel lines are cut by a transversal, then

a) The corresponding angles are ________________.
b) The alternate interior angles are ______________.
c) The alternate exterior angles are ____________.

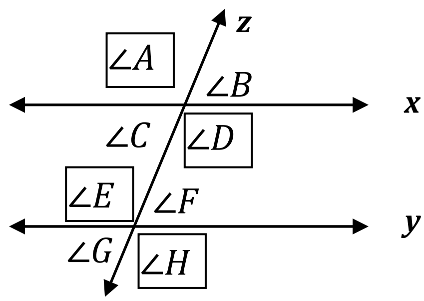
III. Questions to ponder:

Use the figure below to answer the following questions:

1. If lines ***x*** and ***y***are parallel and z is a transversal, what can you say about
   a) any pair of angles that are boxed?
   b) one boxed and one unboxed angle?

2. If $\angle B = (2m - 20)°$and $\angle C = (130 - m)°$, what is the value of ***m***?

$\boxed{m = 50}$

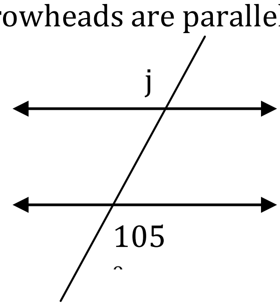


Remember:

When two parallel lines are cut by a transversal as shown, the boxed angles are congruent. Also, corresponding angles are congruent, alternate interior angles are congruent and alternate exterior angles are congruent. Moreover, linear pairs are supplementary, interior angles on the same side of the transversal are supplementary, and exterior angles on the same side of the transversal are supplementary.

Exercise 13. Determine the measures of the angles marked with letters. Lines with arrowheads are parallel. (Note: Figures are not drawn to scale.)

1.



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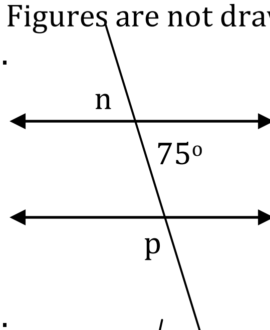

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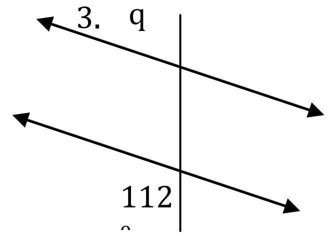

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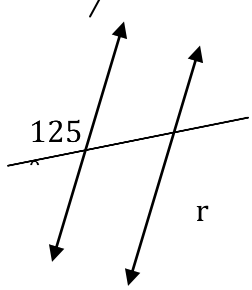

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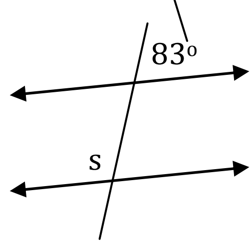

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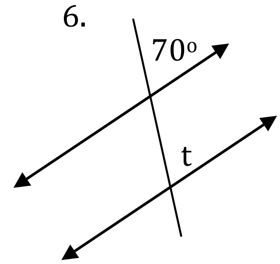

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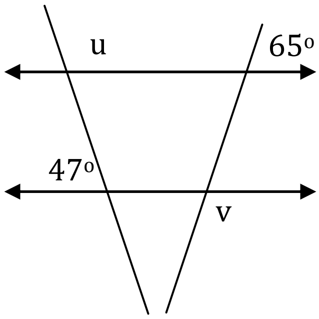

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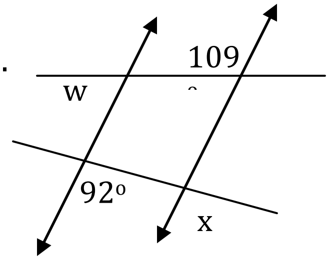

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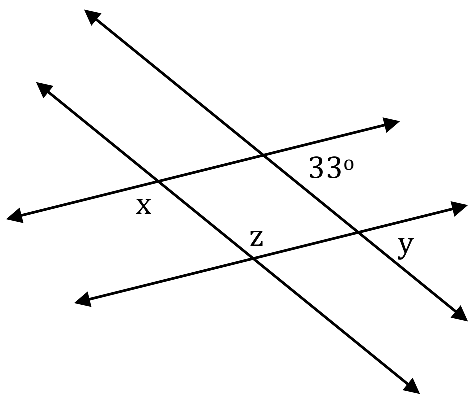

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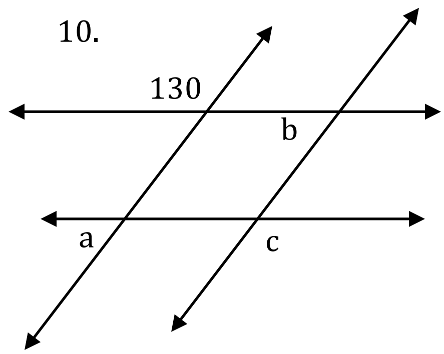


Answers:
1. $\angle j = 105°$; **2.**$\angle n = 75°, \angle p = 105°$; **3.**$\angle q = 68°$; **4.**$\angle r = 125°$; **5.**$\angle s = 97°$; **6.**$\angle t = 70°$; **7.**$\angle u = 133°, \angle v = 115°$; **8.**$\angle w = 71°, \angle x = 92°$; **9.**$\angle v = 33°, \angle x = 147°, \angle z = 147°$; **10.**$\angle a = 50°, \angle b = 50°, \angle c = 130°$

Summary:

In this lesson, you learned about angles, constructing angles with a given measure, measuring a given angle; types of angles and angle pairs.

Lesson 32: Basic Constructions
About the Lesson:

This lesson is about geometric constructions using only a compass and straightedge.

Objectives:

In this lesson, you are expected to:

1. Perform basic constructions in geometry involving segments, midpoints, angles and angle bisectors
2. Sketch an equilateral triangle accurately.

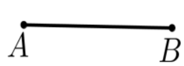
> **Note to the Teacher:** In this module, the students will learn how to do basic constructions in geometry using straight edge and compass. The focus is exposure to some of the terms (bisector, perpendicular) and familiarity with the instruments (compass and straight edge). The justification or formal proof for each construction will be kept for later year levels. This does not mean however that "why" questions must be avoided. It might be good to ask good students to formulate his or her own justification intuitively and informally.

Lesson Proper

Using only the compass and straightedge, we can perform the basic constructions in geometry. We use a straightedge to construct a line, ray, or segment when two points are given. The marks indicated in the ruler may not be used for measurement. We use a compass to construct an arc (part of a circle) or a circle, given a center point and a radius length.

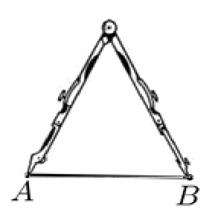
Construction 1. To construct a segment congruent to a given segment

Given: Line segment *AB*:

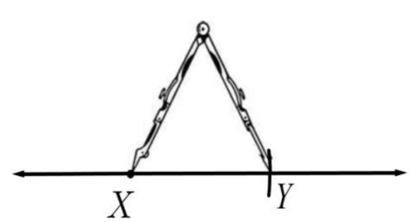


Construct: Line segment *XY* congruent to *AB*.

Use the straight edge to draw a line and indicate a point *X* on the line.



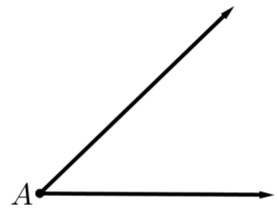
Fix compass opening to match the length of *AB.*



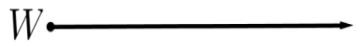
Mark on the line the point *Y* with distance *AB* from *X*.

Construction 2. To construct an angle congruent to a given angle.

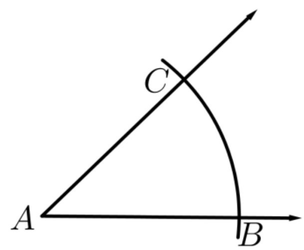
Given: $\angle A$



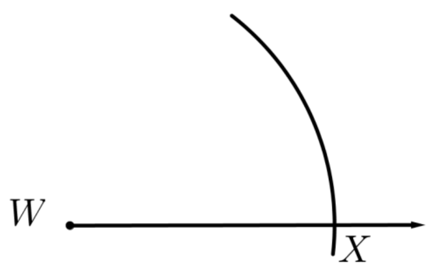
Construct: $\angle W$ congruent to $\angle A$.



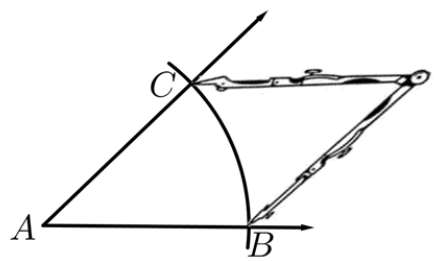
Draw a ray with endpoint W.



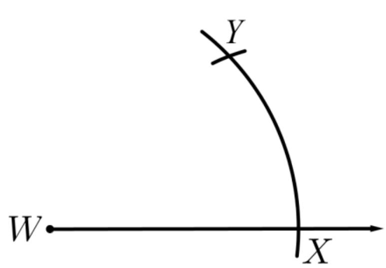
Draw a circular arc (part of a circle) with center at A and cutting the sides of $\angle A$ at points B and C, respectively.



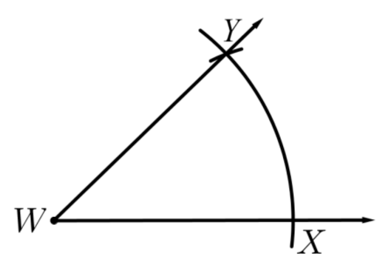
Draw a similar arc using center W and radius AB, intersecting the ray at X.



Set the compass opening to length BC.



Using X as center and BC as radius, draw an arc intersecting the first arc at point Y.



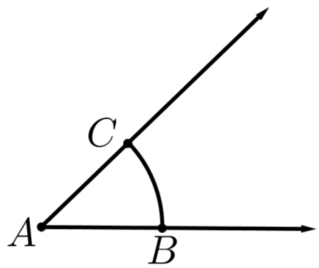
Draw ray $\overrightarrow{WY}$ to complete $\angle W$ congruent to $\angle A$.

Construction 3. To construct the bisector of a given angle.

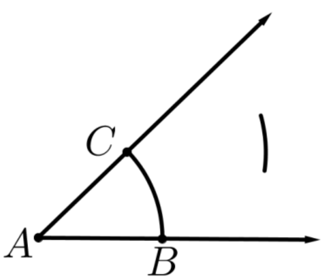
The ***bisector*** of an angle is the ray through the vertex and interior of the angle which divides the angle into two angles of equal measures.

Given: $\angle A$

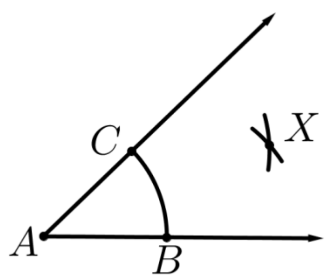
Locate points B and C one on each side of $\angle A$ so that $AB = AC$. This can be done by drawing an arc of a circle with center at A.



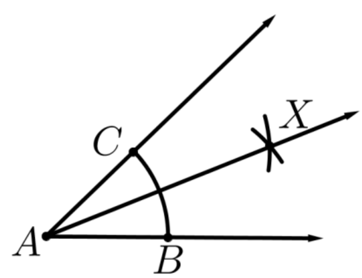
Construct: Ray $\overrightarrow{AX}$ such that X is in the interior of $\angle BAC$ and $\angle BAX = \angle XAC$



Using C as center and any radius r which is more than half of BC, draw an arc of a circle in the interior of $\angle A$.

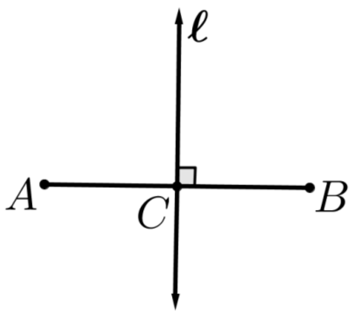


Then using B as center, construct an arc of the circle with the same radius r and intersecting the arc in the preceding step at point X.



Ray $\overrightarrow{AX}$ is the bisector of $\angle BAC$.

The ***midpoint*** of a line segment is the point on the line segment that divides it into two equal parts. This means that the midpoint of the segment AB is the point C on AB such that $AC = CB$. The ***perpendicular bisector*** of a line segment is the line perpendicular to the line segment at its midpoint.

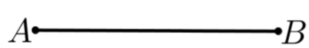


In the figure, C is the midpoint of AB. Thus,
$AC = CB$.
The line ℓ is the perpendicular bisector of AB.

You will learn and prove in your later geometry lessons that the perpendicular bisector of a segment is exactly the set of all points ***equidistant*** (with the same distance) from the two endpoints of the segment. This property is the principle behind the construction we are about to do.

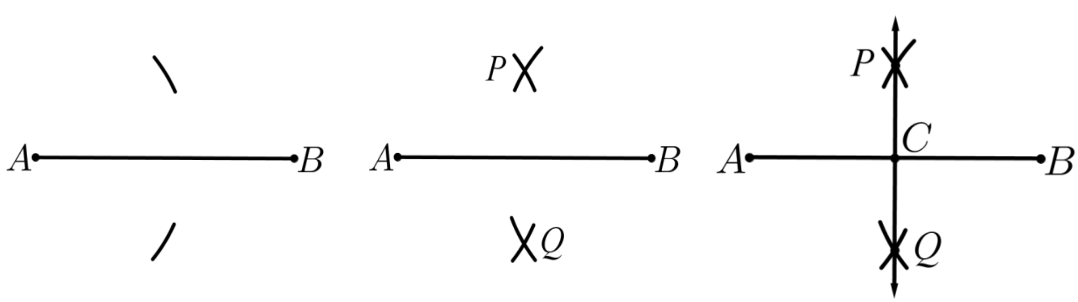
Construction 5. To constructs the midpoint and perpendicular bisector of a segment.

Given: Segment *AB*



Construct: The midpoint *C* of *AB*and the perpendicular bisector of *AB*.

As stated above, the idea in the construction of the perpendicular bisector is to locate two points which are equidistant from *A* and *B*. Since there is only one line passing through any two given points, the perpendicular bisector can be drawn from these two equidistant points.



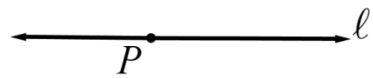
Using center$A$ and radius r which is more than half of AB, draw two arcs on both sides of AB.

Using center$B$ and radius r, draw arcs crossing the two previously drawn arcs at points P and Q.

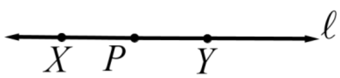
Line PQ is the perpendicular bisector of AB and the intersection of PQ with AB is the midpoint of AB.

Construction 6. To constructs the perpendicular to a given line through a given point on the line.

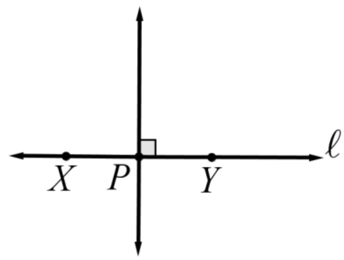
Given: Line ℓ and point P on ℓ



Construct: Line through P perpendicular to ℓ



Using center$P$ and any radius, locate two points, X and Y, on the circle which are on ℓ.



The perpendicular bisector of XY is the perpendicular to ℓ that passes through P.
Can you see why?

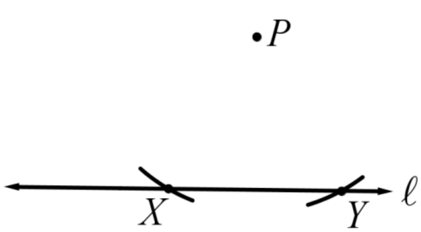
Answer to the question: Since PX and PY are equal,P is the midpoint of XY. Hence the perpendicular bisector of XY contains P and clearly is perpendicular to ℓ.

Construction 7. To construct the perpendicular to a given line through a given point not on the line

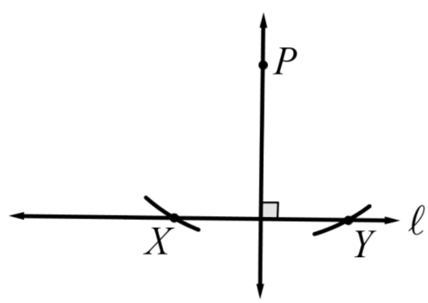
Given: Line ℓ and point P which is not on ℓ.

Construct: Line through P perpendicular to ℓ.

The technique used in Construction 6 will be utilized.



Using Pas center draw arcs of circle with big enough radius to cross the line ℓ. Mark on ℓ the two points (X and Y)crossed by the circle.

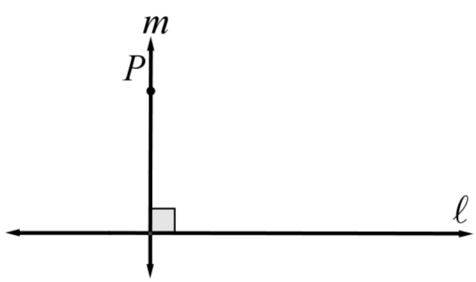


The perpendicular bisector of XY passes through P and is the line we want.

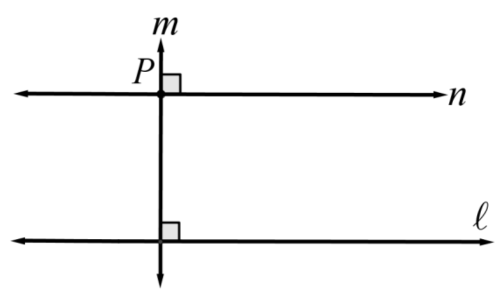
Construction 8. To construct a line parallel to a given line and though a point not on the given line

Given: Line ℓ and point P not on ℓ.

Construct: Line through P parallel to ℓ.



From P, draw the perpendicular$m$to ℓ.



Through P, draw the perpendicular to m (Construction 6).

Why is n parallel to ℓ?

Answer: Since two corresponding angles are equal (both right angles), the lines are parallel.

II. Exercises

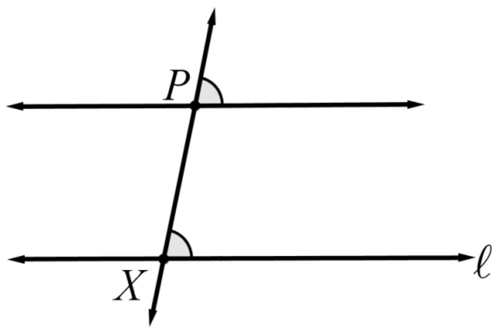
Draw ΔABC such that AB = 6 cm, BC = 8 cm and AC = 7 cm long. Use a ruler for this.
Do the following constructions using ΔABC.

1. Bisect the side BC.
2. Bisect the interior ∠B.
3. Construct the altitude from vertex C. (The perpendicular from $C$to $\overleftrightarrow{AB}$.)
4. Construct a line through B which is parallel to side AC.
5. Construct an equilateral triangle PQR so that PR and the altitude from vertex C have equal lengths.

6. Congruent angle construction can be used to do the parallel line construction (Construction 8) instead of perpendicular construction. How can this be done? What result are we applying in the parallel line construction?

Note to the Teacher:
Most of the constructions in the exercises are a repetition of the basic constructions in this lesson.

5. Construct segment PR congruent to the altitude from C. The third vertex, Q must be equidistant from P and R. Hence Q is any point on the perpendicular bisector of PR.
6. Draw any line through P intersecting the given line ℓ at point X. Construct at P an angle congruent to $\angle X$, with $\overrightarrow{PX}$ as one side. You are constructing congruent corresponding angles. This means that the other side of the angle at P is parallel to ℓ.



V. Summary

In this lesson, basic geometric constructions were discussed.

Lesson 33: Polygons
Prerequisite Concepts: Basic geometric terms
About the Lesson: This lesson is about polygons. Included in the discussion are its parts, classifications, and properties involving the sum of the measures of the interior and exterior angles of a given polygon.

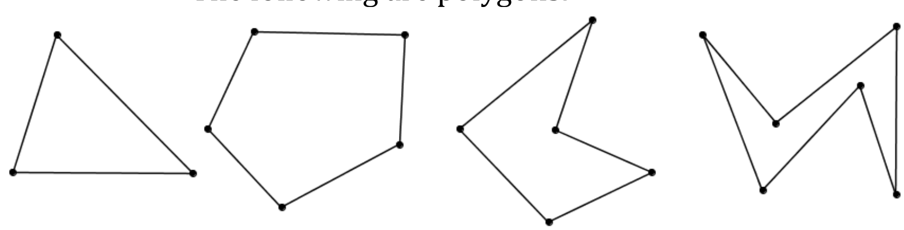
Objective:
In this lesson; you are expected to:
1. Define a polygon.
2. Illustrate the different parts of a polygon.
3. State the different classifications of a polygon.
4. Determine the sum of the measures of the interior and exterior angles of a convex polygon.

I. Lesson Proper

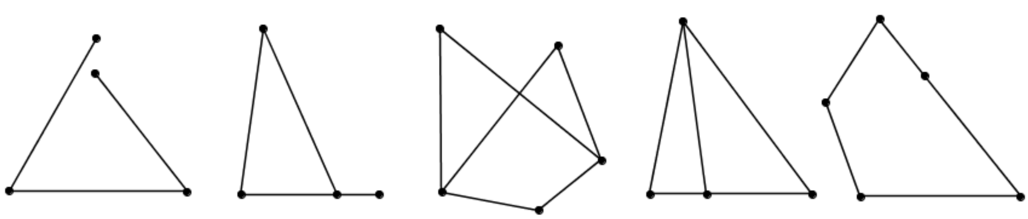
We first define the term polygon. The worksheet below will help us formulate a definition of a polygon.

Activity 15
Definition of a Polygon

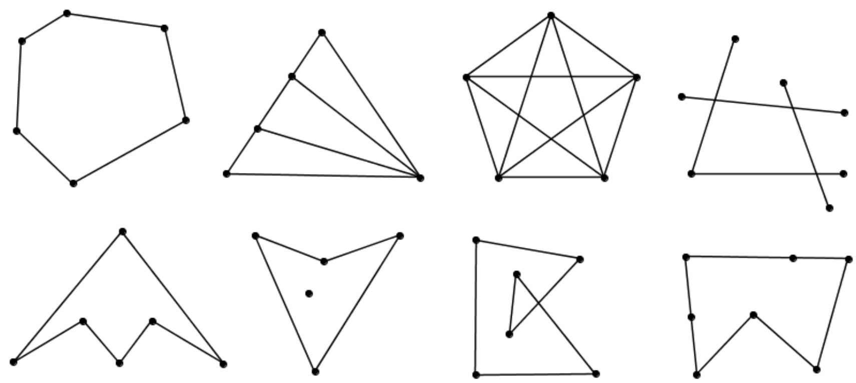
The following are polygons:



The following are not polygons:



Which of these are polygons?



What is then a polygon?

A. Definition, Parts and Classification of a Polygon

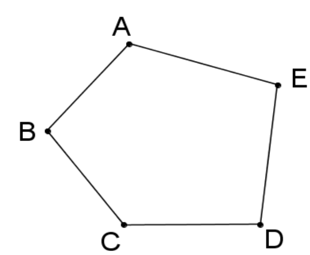
Use the internet to learn where the word "polygon" comes from.

The word "polygon" comes from the Greek words "poly**"**, which means "many," and "gon," which means "angles."

A **polygon** is a union of non-collinear segments, **the sides**, on a plane that meet at their endpoints, **the vertices**, so that each endpoint (vertex) is contained by exactly two segments (sides).

Go back to Activity 15 to verify the definition of a polygon.

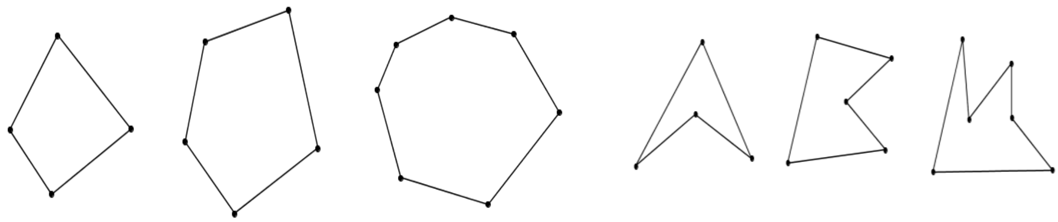
Polygons are named by writing their consecutive vertices in order, such as ABCDE or AEDCB or CDEAB or CBAED for the figure on the right.



A polygon separates a plane into three sets of points: the polygon itself, points in the interior (inside) of the polygon, and points in the exterior (outside) of the polygon.

Consider the following sets of polygons:

Set A **Set B**

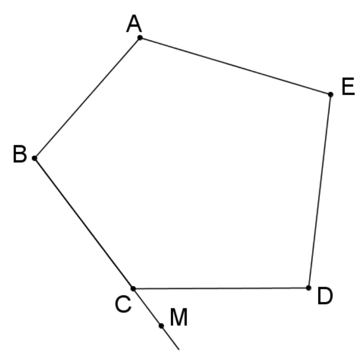


Can you state a difference between the polygons in Set A and in Set B?

Polygons in Set A are called convex, while the polygons in Set B are non-convex. A polygon is said to be **convex** if the lines containing the sides of the polygon do not cross the interior of the polygon.

There are two types of angles associated with a convex polygon: exterior angle and interior angle. **An exterior angle** of a convex polygon is an angle that is both supplement and adjacent to one of its interior angles.

In the convex polygon ABCDE, $\angle A$, $\angle B$, $\angle BCD$, $\angle D$, and $\angle E$ are the interior angles, while $\angle MCD$ is an exterior angle.

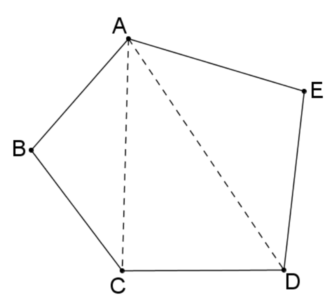


Consecutive vertices are vertices on the same side of the polygon. **Consecutive sides** are sides that have a common vertex. A **diagonal** is a segment joining non-consecutive vertices.

In the polygon ABCDE, some consecutive vertices are A and B, B and C.

Some consecutive sides are $\overline{AE}$ and $\overline{ED}$; $\overline{AB}$ and $\overline{BC}$

Some diagonals are $\overline{AC}$ and $\overline{AD}$.



The different types of polygons in terms of congruency of parts are equilateral, equiangular and regular. A polygon is **equilateral** if all its sides are equal; **equiangular** if all its angles are equal; and **regular** if it is both equilateral and equiangular.

Polygons are named according to the number of sides.

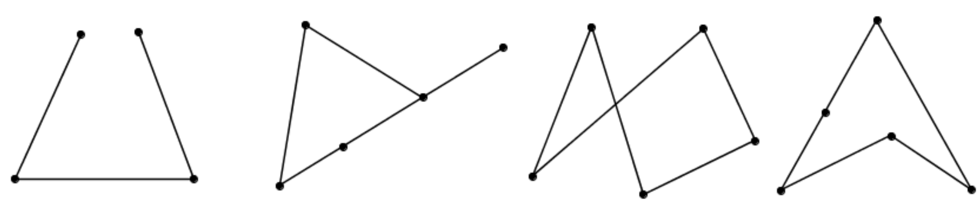
Name of Polygon	***Number of Sides***	***Name of Polygon***	***Number of sides***
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	Undecagon	11
Heptagon	7	Dodecagon	12

B. Questions to ponder:

1. Can two segments form a polygon? If yes, draw the figure. If no, explain why.

2. What is the minimum number of non-collinear segments needed to satisfy the definition of polygon above?

3. Why are the following figures not considered as polygons?



C. Properties of a Polygon

Activity 16
Number of Vertices and Interior Angles of a Polygon

Materials needed: match sticks, paste or glue, paper
Consider each piece of matchstick as the side of a polygon.

(Recall: A polygon is __________________________.)

Procedure:

1) Using three pieces of matchsticks form a polygon. Paste it on a piece of paper.

a) How many sides does it have? ________
b) How many vertices does it have? ______
c) How many interior angles does it have? ______

2) Using four pieces of match sticks form a polygon. Paste it on a piece of paper.

a) How many sides does it have? ________
b) How many vertices does it have? ______
c) How many interior angles does it have? ______

3) Using five pieces of matchsticks form a polygon. Paste it on a piece of paper.

a) How many sides does it have? ________
b) How many vertices does it have? ______
c) How many interior angles does it have? ______

4) Using six pieces of matchsticks form a polygon. Paste it on a piece of paper.

a) How many sides does it have? ________
b) How many vertices does it have? ______
c) How many interior angles does it have? ______

Were you able to observe a pattern? ___________

Complete the sentence below:

A polygon with n sides has __ number of vertices and _____ number of interior angles.

Activity 17
Types of Polygon

Recall:

A polygon is _______________________________________.
A polygon is equilateral is ____________________________.
A polygon is equiangular if ___________________________.
A polygon is regular if _______________________________.

1. Determine if a figure can be constructed using the given condition. If yes, sketch a figure. If no, explain why it cannot be constructed.

a) A triangle which is equilateral but not equiangular.
b) A triangle which is equiangular but not equilateral
c) A triangle which is regular
d) A quadrilateral which is equilateral but not equiangular.
e) A quadrilateral which is equiangular but not equilateral
f) A quadrilateral which is regular.

2. In general,

a) Do all equilateral polygons equiangular? If no, give a counterexample.
b) Do all equiangular polygons equilateral? If no, give a counterexample.
c) Do all regular polygons equilateral? If no, give a counterexample.
d) Do all regular polygons equiangular? If no, give a counterexample.
e) Do all equilateral triangles equiangular?
f) Do all equiangular triangles equilateral?

Activity 18
Sum of the Interior Angles of a Convex Polygon

Materials needed: pencil, paper, protractor

Procedures:

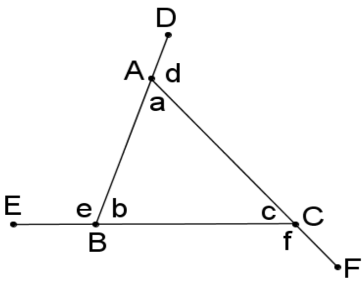
1) Draw a triangle. Using a protractor, determine the measure of its interior angles and determine the sum of the interior angles.
2) Draw a quadrilateral. Then fix a vertex and draw diagonals from this vertex. Then answer the following:
    a) How many diagonals are drawn from the fixed vertex?
    b) How many triangles are formed by this/these diagonal(s)?
    c) Without actually measuring, can you determine the sum of the interior angles of a quadrilateral?
3) Draw a pentagon. Then fix a vertex and draw diagonals from this vertex. Then answer the following:
    a) How many diagonals are drawn from the fixed vertex?
    b) How many triangles are formed by this/these diagonal(s)?
    c) Without actually measuring, can you determine the sum of the interior angles of a pentagon?
4) Continue this with a hexagon and heptagon.
5) Search for a pattern and complete the table below:

No. of sides	No. of diagonals from a fixed vertex	No. of triangles formed by the diagonals drawn from a fixed vertex	Sum of the interior angles
3			
4			
5			
6			
7			
8			
9			
10			
n			

6. Complete this: The sum of the interior angles of a polygon with n sides is ____.

Activity 19
The Sum of the Exterior Angles of Polygon

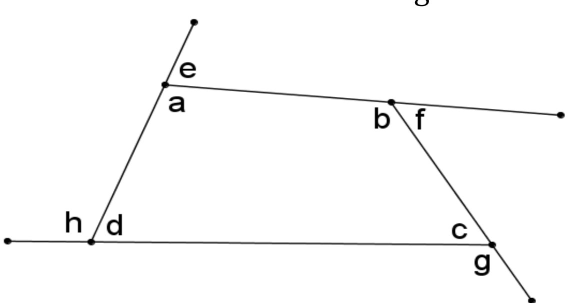
1. Given ΔABC with the exterior angle on each vertex as shown:



Let the interior angles at A, B, C measure a, b, c respectively while the exterior angles measure d, e, f.
Determine the following sum of angles:

a + d = ________
b + e = ________
c + f = ________
(a + d) + (b + e) + (c + f) = ________
(a + b+ c) + (d + e + f) = ________
a + b + c = ________
d + e + f = ________

2. Given the □ ABCD and the exterior angle at each vertex as shown:



Determine the following sum:

a + e = ________
b + f = ________
c + g = ________
d + h
(a + e) + (b + f) + (c + g) + (d + h) = ________
(a + b+ c + d) + (e + f + g + h) = ________
a + b + c + d = ________
e + f + g + h = ________

The sum of the exterior angles of a quadrilateral is _____________.

3. Do the same thing with convex pentagon, hexagon and heptagon. Then complete the following:

The sum of the exterior angles of a convex pentagon is __________.
The sum of the exterior angles of a convex hexagon is __________.
The sum of the exterior angles of a convex heptagon is __________.

4. What conclusion can you formulate about the sum of the exterior angles of a convex polygon?

I. Exercise 20

1. For each regular polygon, determine the measure of an exterior angle.
   a. quadrilateral b. hexagon c. nonagon

2. Determine the sum of the interior angles of the following convex polygons:
   a. pentagon b. heptagon c. octagon

3. Each exterior angle of a regular polygon measures 20^{o}. Determine the sum of its interior angles.

Summary: In this lesson, we learned about polygon, its parts and the different classifications of a polygon. We also performed some activities that helped us determine the sum of the interior and exterior angles of a convex polygon.

Answers

Questions to ponder:
1. No, because the segments are then collinear, which is not allowed in a polygon.
2. Three
3. The first figure is not closed; the second figure has collinear segments and not closed; the third figure has two intersecting segments; and the fourth figure has collinear segments.

Activity 17
1) a) No, because if two sides are equal in length, then the opposite angles are also equal in measure. b) No, equilateral triangles are equiangular. c) Yes d) Yes e) Yes f) Yes

2) a) No, a rhombus is a counterexample. b) No, a non-square rectangle is a counterexample. c) Yes d) Yes e) Yes f) Yes

Activity 18

Number of diagonals from a fixed vertex of an n-gon = $n - 3$

Number of triangles formed by the diagonals from a fixed vertex = 0 if n=3, and n-2 if n ≥ 4

Sum of interior angles = (n-2)x180°

Activity 19

1) a + d = b + e = c + f = 180°, a + b + c = 180°, d + e + f = 360°
2) a + e = b + f = c + g = d + h = 180°, a + b + c + d = 360°, e + f + g + h = 360°
3) 360° to each case
4) The sum of the measures of the exterior angles of a convex polygon is 360°.

Exercise 20

1) a) 90° b) 60° c) 40°
2) a) 540° b) 720° c) 1030°
3) 2880°

Lesson 34: Triangles
Prerequisite Concepts: Polygons
About the Lesson: This lesson is about triangles, its classifications and properties.

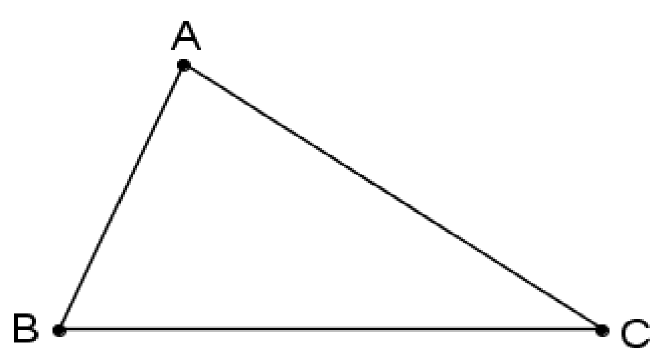
Objective:
In this lesson, you are expected to:

5. Define and illustrate the different terms associated with a triangle.
6. Classify triangles according to their angles and according to their sides. .
7. Derive relationships among sides and angles of a triangle.

II. Lesson Proper

A. Terms associated with a Triangle

Given ΔABC, its parts are the three vertices A, B, C; the three sides $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ and the three interior angles ∠A, ∠B and ∠C.



We discuss other terms associated with ΔABC.

Exterior angle – an angle that is adjacent and supplement to one of the interior angles of a triangle.

Remote interior angles of an exterior angle – Given an exterior angle of a triangle, the two remote interior angles of this exterior angle are the interior angles of the triangle that are not adjacent to the given exterior angle.

Angle bisector – This is a segment, a ray or a line that bisects an interior angle.

Altitude – This is a segment from a vertex that is perpendicular to the line containing the opposite side.

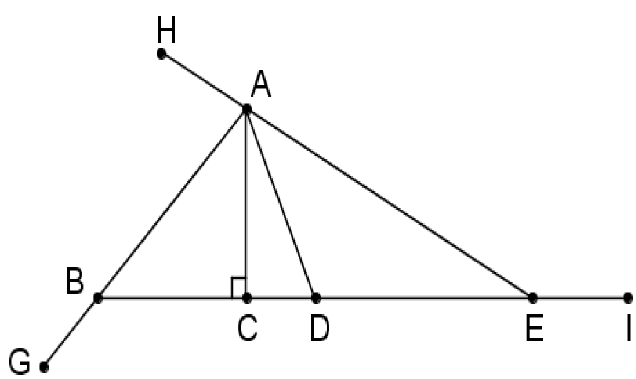
Median – This is a segment joining a vertex and the midpoint of the opposite side.

Perpendicular bisector of a side – Given a side of a triangle, a perpendicular bisector is a segment or a line that is perpendicular to the given side and passes through the midpoint of the given side.

Exercise 21
Parts of a Triangle

Given ΔABE with AC ⊥BE and BD = DE, identify the following parts of the triangle.

1) vertices ______________
2) sides ______________
3) interior angles ______________
4) exterior angles ______________
5) the remote interior angles of ∠ AEI ______________
6) the remote interior angles of ∠EBG ______________
7) altitude ______________
8) median ______________



B. The lengths of the sides of a triangle

Activity 22
Lengths of Sides of a Triangle

Materials Needed: coconut midribs or barbecue sticks, scissors, ruler

Procedure:

1. Cut pieces of midribs with the indicated measures. There are three pieces in each set.

2. With each set of midribs, try to form a triangle. Complete the table below:

Lengths of midribs (in cm)	Do they form a triangle or not?
3, 3, 7	
3, 3, 5	
4, 6, 10	
4, 6, 9	
5, 5, 10	
5, 5, 8	
6, 7, 11	
6, 7, 9	
4, 7, 12	
4, 7, 10	

3. For each set of lengths, add the two shortest lengths. Then compare the sum with the longest length.

What pattern did you observe? ____

C. Classification of Triangles

Triangles can be classified according to their interior angles or according to the number of congruent sides.

According to the interior angles:
Acute triangle is a triangle with three acute interior angles.
Right triangle is a triangle with one right angle.
Obtuse triangle is a triangle with one obtuse angle.

According to the number of congruent sides:
Scalene triangle is a triangle with no two sides congruent.
Isosceles triangle is a triangle with two congruent sides.
Equilateral triangle is a triangle with three congruent sides.

In an isosceles triangle, the angles opposite the congruent sides are also congruent. Meanwhile, in an equilateral triangle, all angles are congruent.

D. Some Properties of a Triangle

Activity 23
Pythagorean Triples

1. In a graphing paper, sketch the right triangles with the specified lengths (in cm) of legs. Then measure the hypotenuse. Let x and y be the legs and let z be the hypotenuse of the triangle.

2. Complete the first table.

Leg (x)	Leg (y)	Hypotenuse (z)		Leg (x)	Leg (y)	Hypotenuse (z)
3	4			10	24	
6	8			8	15	
9	12			20	21	
5	12			15	20	

3. Compute for x^2 , y^2 , and z^2 , and $x^2 + y^2$ and complete the second table.

x^2	y^2	z^2	$x^2 + y^2$		x^2	y^2	z^2	$x^2 + y^2$

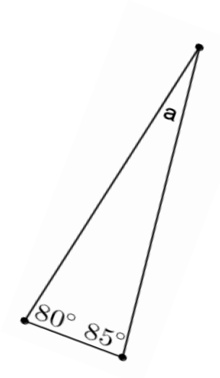
4. Compare the values of $x^2 + y^2$ with z^2. What did you observe?
   ____.
5. Formulate your conjecture about the lengths of the sides of a right triangle.
   ____

II. Exercise 24

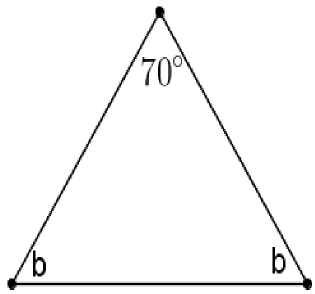
A. True or False
   1. A triangle can have exactly one acute angle.
   2. A triangle can have two right angles.
   3. A triangle can have two obtuse interior angles.
   4. A right triangle can be an isosceles triangle.
   5. An isosceles triangle can have an obtuse interior angle.
   6. An acute triangle can be an isosceles triangle.
   7. An obtuse triangle can be an scalene triangle.
   8. An acute triangle can be an scalene triangle.
   9. A right triangle can be an equilateral triangle.
   10. An obtuse triangle can be an isosceles triangle.

B. Determine the measure of the angles marked with letters. Lines with arrowheads are parallel.

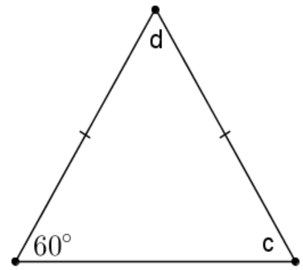
1)



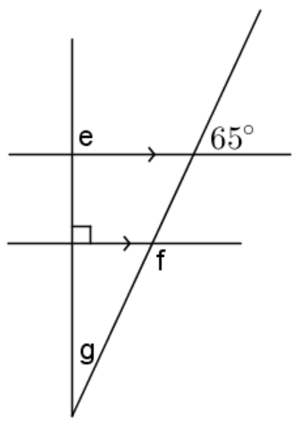
2)



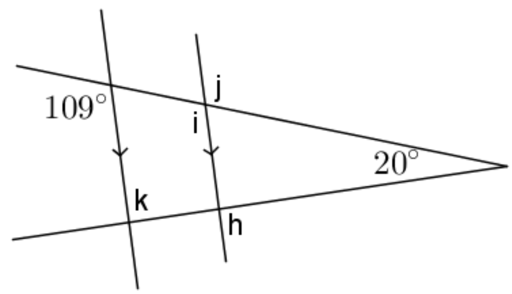
3)



4)



5)



C. Construct the following:

7. Construct a triangle whose sides are 5 cm, 8 cm, and 10 cm long.
8. Construct ΔPQR such that PQ = 5 cm, QR = 8 cm, and m∠Q = 60°.
9. Construct ΔWXY such that WX = 8 cm, m∠W = 15°, and m∠X = 60°.

D. Construct 4 different scalene triangles.

1. In the first triangle, construct all the perpendicular bisectors of the sides.
2. In the second triangle, construct all the angle bisectors.
3. In the third triangle, construct all the altitudes.
4. In the fourth triangle, construct a line passing through a vertex and parallel to the opposite side of the chosen vertex.

III. Question to ponder:

Try to construct a triangle whose sides are 4 cm, 6 cm and 11 cm. What did you observe? Could you explain why?

IV. Discuss the following properties of a triangle:

1. The perpendicular bisectors of the sides of a triangle are concurrent at a point. This point is called the **circumcenter** of the given triangle.
2. The medians of a triangle are concurrent at a point. This point is called the **centroid** of the given triangle.
3. The interior angle bisectors of a triangle are concurrent at a point. This point is called the **incenter** of the given triangle.
4. The altitudes of a triangle are concurrent at a point. This point is called the **orthocenter** of the given triangle.

V. Summary

In this lesson, we learned about triangles, its parts and its properties. The construction is used to illustrate some properties of a triangle involving the perpendicular bisectors of its sides, medians, bisectors of its interior angles and its altitudes.

Answers

Exercise 21

1) A, B, E 2) AB, BE, AE 3) $\angle ABE$, $\angle BAE$, $\angle AEB$ 4) $\angle BAH$, $\angle AEI$, $\angle EBG$ 5) $\angle BAE$, $\angle ABE$ 6) $\angle BAE$, $\angle AEB$
7) AC 8) AD

Activity 22

2. Yes, No, No, Yes, No, Yes, Yes, Yes, No, Yes

Exercise 24

A. 1) True 2) False 3) False 4) True 5) True 6) True 7) True 8) True 9) False
10) True

B. 1) $a = 15^o$ 2) $b = 55^o$ 3) $c = d = 60^o$ 4) $e = 90^o$, $f = 115^o$, $g = 25^o$ 5) $h = 91^o$, $i = j = 109^o$, $k = 89^o$

C. 1) Draw line segment AB with length 5 cm. With A as the center, draw a circle with radius 8 cm. With B as the center, draw another circle with radius 10 cm. Let C be one of the points of intersection of these circles. Then ABC is a desired triangle.

2) Draw the segment PQ. With Q as the center, draw a circle with radius 8 cm. Construct an equilateral triangle APQ. Extend QA to meet the circle just drawn at R. Then PQR is the desired triangle.

3) Construct an equilateral triangle ZWX. Then divide $\angle ZWX$ into four equal smaller angles (use bisection of an angle twice), and let Y be on side ZX such that $\angle YWX = 15^o$. Then WXY is the desired triangle.

Lesson 35: Quadrilaterals
Prerequisite Concepts: Polygons
About the Lesson: This lesson is about the quadrilateral, its classifications and properties.
Objective:
In this lesson, you are expected to:

8. Classify quadrilaterals
9. State the different properties of parallelogram.

I. Lesson Proper

A. Learning about quadrilaterals

A **quadrilateral** is a polygon with four sides.

1. Some special quadrilaterals:

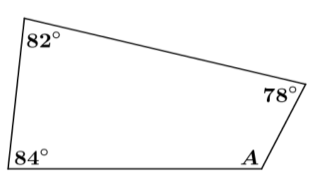
Trapezoid is a quadrilateral with exactly one pair of opposite sides parallel to each other. The parallel sides are called the **bases**, while the non-parallel sides are called the **legs.**

If the legs of a trapezoid are congruent (that is, equal in length), then the trapezoid is an **isosceles trapezoid.** Consequently, the base angles are congruent, and the remaining two angles are also congruent.

Parallelogram is a quadrilateral with two pairs of opposite sides parallel to each other.

Exercise 25. Angles in Quadrilateral

Find the angles marked with letters. (Note: Figures are not drawn to scales.)

1)


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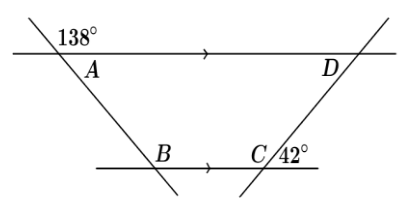

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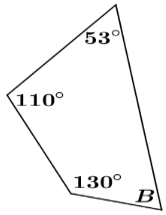

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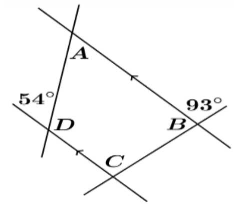

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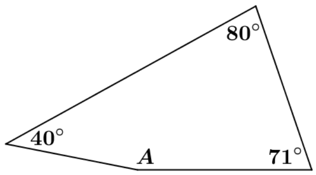

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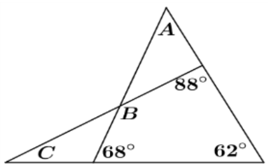

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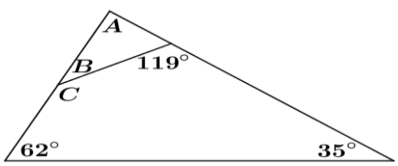

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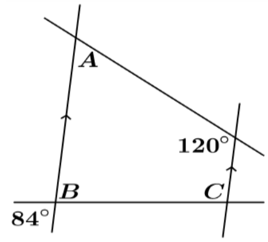

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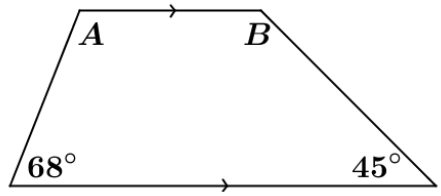

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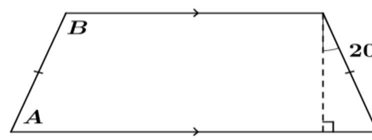


A. On Parallelograms

Activity 26
Vertices of a Parallelogram

Using a graphing paper, plot the three given points. Then find the three possible points for the fourth vertex so that the figure formed is a parallelogram. Sketch the figure.

Given vertices	Possible fourth vertex
A (2, 3), B (2, -3), C (4, 2)	
E (-8, 3), F(-2, 5), G(-4, 1)	
H(-3, 7), I(-6, 5), J(-1, 4)	
K(6, 3), L(7, 5), M(2, 6)	
N(6, -3), O(2, -4), P(5, -7)	

3)

B. On Properties of a Parallelogram

4)

Activity 27

Materials: Pair of scissors, ruler, cardboards or papers

Procedures:

A. Prepare five models of parallelograms. (Or use the attached sketch of parallelograms.)

Name the parallelogram a□ ABCD.

1. For the first parallelogram: cut the parallelogram into two so that you can compare ∠A and ∠C; ∠B and ∠D. What do you observe?

Opposite angles of a parallelogram are ______________________.

2. For the second parallelogram: cut the angles and arrange any two consecutive angles about a point. What do you observe about the sum of any two consecutive angles of a parallelogram?

Consecutive angles of a parallelogram are ___________________.

3. For the third parallelogram: cut the figure along the diagonal AC. Compare the two triangles formed. Can they be coincided with each other?

   For the fourth parallelogram: cut the figure along the diagonal BD. Compare the two triangles formed. Can they be coincided with each other?

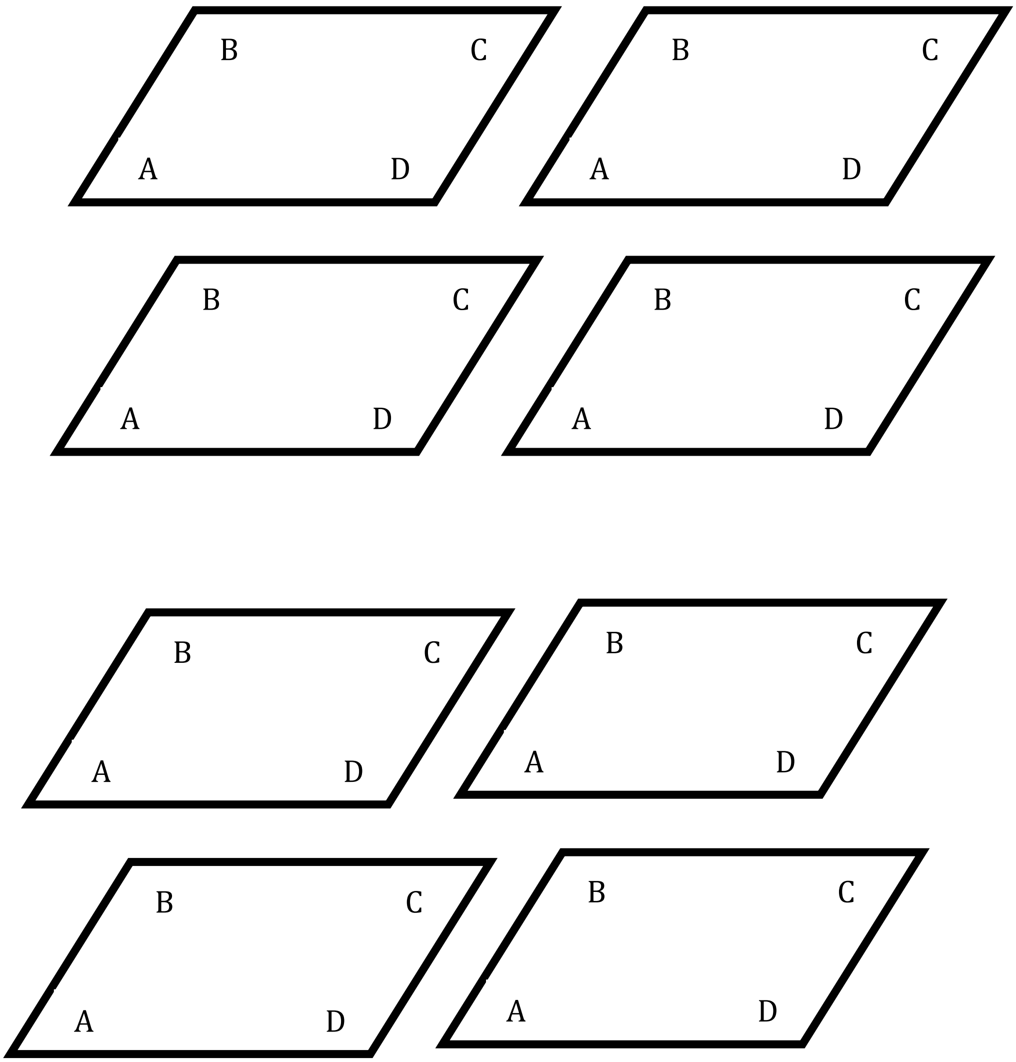
   In both parallelograms, what do you observe about the triangles formed by the diagonals?

Diagonals of a parallelogram divide the parallelogram into __________.

4. For the fifth parallelogram: cut the figure along the two diagonals. Then compare the partitioned diagonals. How did one diagonal divide the other diagonal?

Diagonals of a parallelogram ________________________.

Models for Activity 27



Activity 28
Special Properties of Rectangle, Square, Rhombus

Materials: two sets of models of rectangles, squares, rhombus. Name each as ABCD.
(see attached sheet for the models)

Procedure:

1.) Cut the rectangle along the diagonal AC. What type/s of triangle(s) is/are formed?
2.) Cut the rhombus along the diagonal AC. What type/s of triangle(s) is/are formed?
3.) Cut the square along the diagonal AC. What type/s of triangle(s) is/are formed?

In which parallelogram does the diagonal divide the parallelogram into two congruent right triangles? ___________________________

4.) In each figure, draw diagonals AC and BD and let the intersection be point O. In each figure, measure the lengths of the diagonals.

In which parallelogram are the diagonals congruent? ______________

5.) In each figure, draw diagonals AC and BD and let the intersection be point O. Then measure $\angle$AOD, $\angle$DOC, $\angle$COB, $\angle$BOA. What do you observe?

In which parallelogram are the diagonals perpendicular? ________________

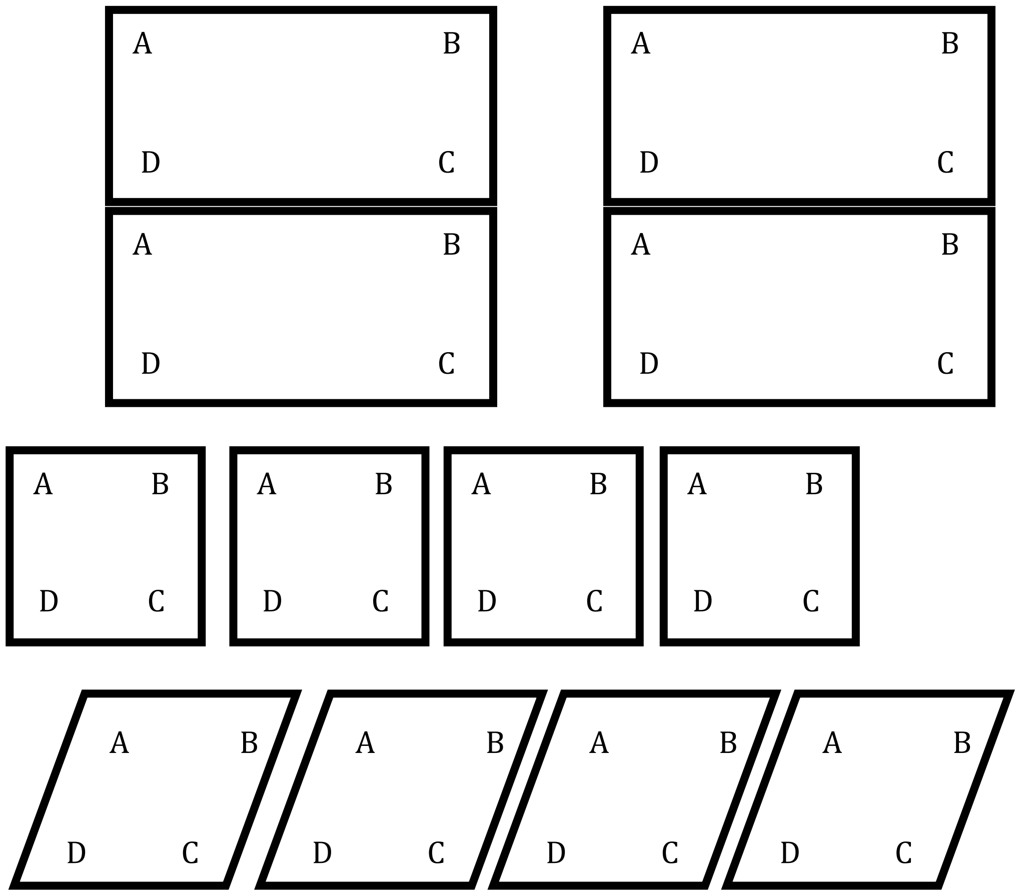
6.) From the results of # 4-5, complete the statements below:

Diagonals of a rhombus are ___________________.

Diagonals of a rectangle are ___________________.

Diagonals of a square are ______________________.

Models for Activity 28



Discussion:
Properties of a parallelogram:

Opposite angles of a parallelogram are congruent.
Consecutive angles of a parallelogram are supplementary.
Diagonals of a parallelogram divide the parallelogram into two congruent triangles.
Diagonals of a parallelogram bisect each other.

A diagonal of a rectangle divides the rectangle into two congruent right triangles.
A diagonal of a square divides the square into two congruent isosceles right triangles.
Diagonals of a rectangle are congruent.
Diagonals of a rhombus are perpendicular.
Diagonals of a square are both congruent and perpendicular.

Summary
In this lesson, we learned about quadrilaterals and the different types of quadrilaterals.. We also learned about parallelogram and its properties.
Answers:

Exercise 25

1. A = 116° 2. B = 67° 3. A = 169° 4. A = 50°, B = 142°, C = 30° 5. A = 83°, B = 36°, C = 144°
6. A = D = 42°, B = C = 138° 7. A = 54°, B = 87°, C = 93°, D = 126° 8. A = 60°, B = 84°, C = 96° 9. A = 112°, B = 135°
10. A = 70°, B = 110°

Activity 26

Given vertices	Possible fourth vertex
A (2, 3), B (2, -3), C (4, 2)	(0,-1), (4,-5), (4,7)
E (-8, 3), F(-2, 5), G(-4, 1)	(2,3), (-10,-1), (-6,7)
H(-3, 7), I(-6, 5), J(-1, 4)	(2,6), (-4,2), (-8,8)
K(6, 3), L(7, 5), M(2, 6)	(3,8), (11,2), (1,4)
N(6, -3), O(2, -4), P(5, -7)	(9,-6), (3,0), (1,-8)

For *Activity 28*, see **Discussion** after the activity.

Lesson 36: Circles

Prerequisite Concepts: Distance, angle measures

About the Lesson: Students have been introduced to circles as early as Grade 1, and they may easily recognize circles from a drawing, even without knowing how points on the circle are defined. This lesson extends students' visual understanding of circles by introducing them to its mathematical definition. Definitions of terms related to the circle also developed.

Objective:

In this lesson; you are expected to:

3. Define a circle and its parts.
4. Apply the definition to solve problems.

Lesson Proper:

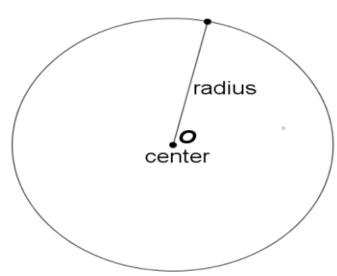
A. Circles

I. Activity

Draw a point somewhere in the middle of a sheet of paper. Now, using a ruler, mark 20 other points that are 5 cm from the first point. Compare your work with that of your seatmates. What shape do you recognize?

You can probably recognize circles even when you were young. When you hear the word circle, round shapes may come to your mind. Now, we will learn how circles are shaped this way. In the activity above, you saw that points that points that are the same distance from a fixed point yields a round shape.

Definitions: A **circle** is the set of all points that are the same distance from a fixed point. This fixed point is called the **center** of the circle. A segment drawn from any point on the circle to the center is called a **radius.**

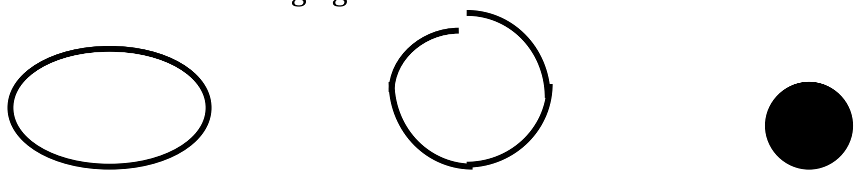


Note: A circle is named by its center. The circle at the right is called Circle *O*.

II. Questions to Ponder

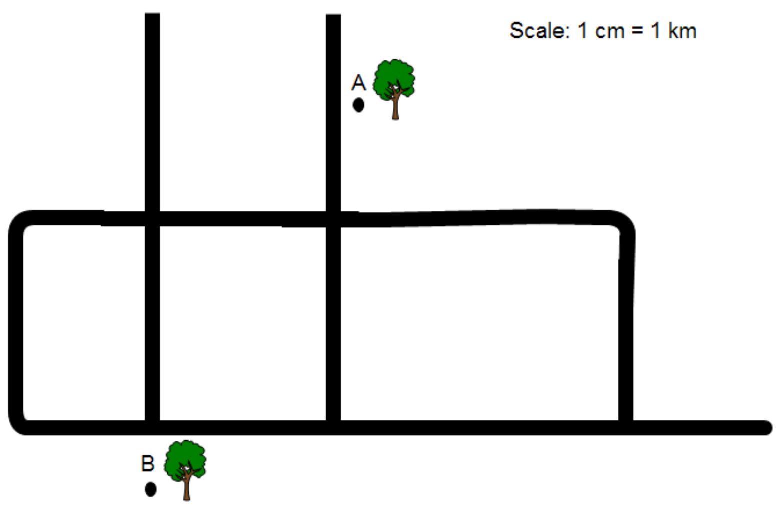
1. Why do all radii (plural of radius) of a circle have the same length? ***Because from the definition, all points on the circle is the same distance from its center. Thus, all radii have the same length.***

2. Which the following figures are circles?



None of the three are circles. The shape on the right may look like a circle, but by definition, only those points that are the same distance from the center are parts of the circle. Points inside the circle are closer to the center than points on the circle are, so they should not be part of the circle.

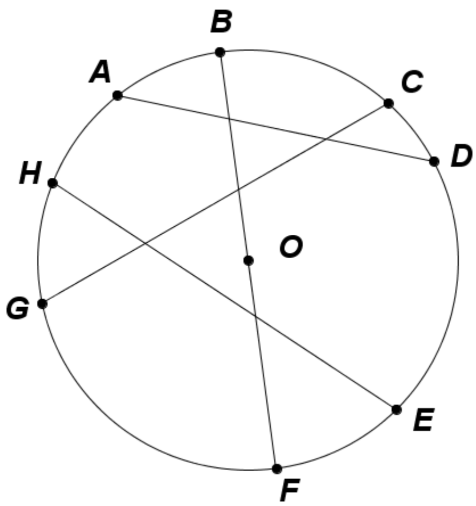
3. Your grandfather told you that when he was young, he and his playmates buried some old coins under the ground, thinking that these coins will be valuable after several years. He also remembered that these coins were buried exactly 4 kilometers from Tree A (see map) and 5 kilometers from Tree B. Where could the coins possibly be located? ***Construct a circle with radius 4 cm centered at A and another circle with radius 5 cm centered at B. The intersection points of these two circles are possible locations of the coins.***



B. Terms Related to Circles

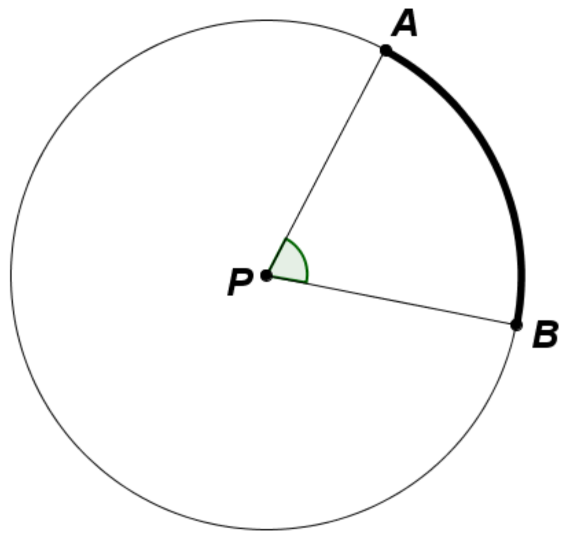
I. Activity

On Circle O, segments *AD*, *BF*, *CG*, and *HE* were constructed so that their endpoints are points on the circle.



Measure each segment, and determine which of these segments is the longest.

Find the measure of $\angle APB$ below.



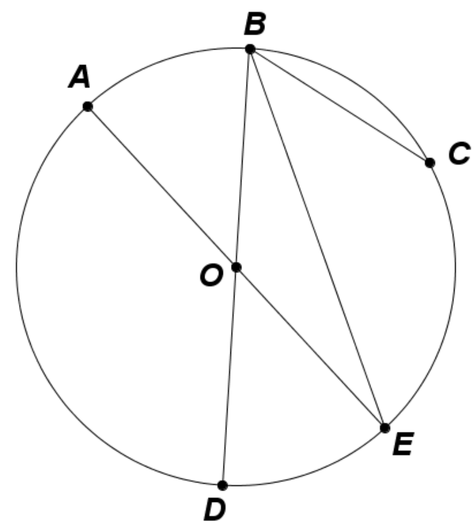
The activity above introduced you to other parts of a circle.

A **chord** is a segment that connects any two points of a circle. *AD*, *BF*, *CG*, and *HE* are chords of Circle *O*.

A **diameter** is a chord that passes through the center of a circle. *BF* is a diameter of Circle *O*. It is the longest chord of a circle and it is twice the length of a circle's radius.

A **central angle** is an angle whose vertex is on the circle's center, and whose sides intersect the circle at two points. $\angle APB$ is a central angle of Circle *P*.

An **arc** is a portion of a circle determined by a central angle. Arc AB is an arc of Circle P.



II. Points to Ponder

1. Determine whether each statement is true or false.
   a. Two radii always have the same length. ***True***

   b. Two chords always have the same length. ***False***
   c. All chords are diameters. ***False***
   d. All diameters are chords. ***True***
   e. All chords intersect at one point. ***False***
   f. A radius is not a chord. ***True***
   g. All diameters intersect at one point. ***True***

2. On Circle O,
   a. name each radius. ***AO, DO, EO, BO***
   b. name each diameter. ***AE, BD***
   c. name each chord. ***AE, BD, BE, BC***
   d. name each central angle. $\angle AOB$, $\angle BOE$, $\angle EOD$, $\angle DOA$
   e. name the arcs subtended by the central angles in (d).
   arc*AB*, arc*BE*, arc*ED*, arc*AD*

3. Using a compass, draw a circle whose radius is 5 cm. Then draw the following objects. Write "impossible" if the object cannot be drawn.
   a. One chord measuring 2 cm.
   b. One chord measuring 10 cm.
   c. One chord measuring 12 cm. ***This object is impossible to draw***
   d. Three radii measuring 5 cm.
   e. One central angle measuring 90°.
   f. One central angle measuring 135°.
   g. One arc subtended by an angle that measures 35°.
   h. Three adjacent central angles, each measuring 100°.

Lesson 37: Introduction to Statistics **Time: 1 hour**

Prerequisite Concepts: Measurement concepts, length measurements, measurement units

About the Lesson: This is an introductory lesson to basic concepts, uses, and importance of Statistics. The first lesson allows you to experience systematic gathering and organizing data. This makes use of your knowledge of arranging numbers according to some considerations, like arranging numbers in descending or ascending order.

Objective:

In this lesson, you are expected to:

2. Collect or gather statistical data and organize the data in a frequency table according to some systematic consideration;
3. Explain the basic concepts, uses and importance of Statistics; and
3. Pose questions and problems that may be answered using Statistics.

Lesson Proper:

I. Activity

Measuring the arm span: Stretch out both arms and measure the length from the tip of a middle finger to the tip of the other middle finger

http://www.gnbco.com/bow_length.php

You may present the following questions. Be sure to bring tape measures for use by your students. Ask the students the following and ask them to work in groups.

Do you think students in this class have different arm spans? How many in this class have the same arm spans? What is the most common measure of arm spans? To answer these questions, you will to do the following:

See to it that everybody in your class has both arms to avoid any embarrassment. Else, you may consider height or weight.

Measure your individual arm span using the centimeter (cm) unit of length. Round off the measures to the nearest cm. On the board, write your measures individually.

Possible Data: [Depending on the number of students]
120, 118, 123, 124, 138, 137, 130, 119, 120, 125, 118, 118, 123, 124, 132
125, 135, 119, 115, 120, 140, 123, 125, 119, 132, 130, 130, 130, 131, 132
132, 130, 118, 131, 130, 125, 125, 126, 128, 121, 140, 132, 119, 129, 108

Guide Questions:
What do these numbers represent?
These represent the measures of the arm span of 45 students in this class.

This is called raw data. Raw data are data which are not yet sorted or arranged according to some criteria or some systematic consideration.

Can we get clear and precise information immediately as we look at these numbers? Why?
No, because the numbers are not arranged from highest to lowest (or from lowest to highest) or sorted.

How can we make these numbers meaningful for anyone who does not know about the description of these numbers?

Students may have varied answers. Make them realize that this data must be sorted or arranged from highest to lowest. This will be their next task.

II. Activity
Sort out the raw data and present it in a way you think would be a good presentation.

A good presentation is according to a systematic consideration like the following:
1. *Arranging the data in either ascending or descending order.*
2. *Making a table to fit it the data.*
3. *Use graphs or frequency table.*

III. Discussion
In our daily activities, we encounter a lot of sorting and organizing objects, data, or things like what you just did. These are just few of the activities of doing Statistics.

What are some of the few activities that you just did?

Measuring the arm span, recording the measure, organizing the data, presenting the data in a systematic manner

> *Collecting data, organizing data, and presenting data according to some systematic considerations*

What is Statistics?
Statistics is the study of the collection, organization, analysis, and interpretation of data. It deals with all aspects of this, including the planning of data collection in terms of the design of surveys and experiments.

> Statistics is the science of collection, analysis, and presentation of data. Statisticians contribute to scientific enquiry by applying their knowledge to the design of surveys and experiments; the collection, processing, and analysis of data; and the interpretation of the results.

Give some examples of activities which you think Statistics is involved.
Census of population, registration of election voters.

> *Other examples would be:*
> - *46% of people polled trust the present government, while 14% doubt the government capacity, and the rest abstained*
> - *A school's drop-out rate has decreased by 2%.*
> - *88% of people questioned feel that it is better to exercise at least twice a week.*
>
> *You may tell the students that these are basic examples of statistics we see every day, and ask them whether they really understand what these mean. Remind them that with the study of statistics, these 'facts' that we hear every day can hopefully become a clearer.*

List down some problems or questions that can be answered using Statistics.
What fraction of Filipinos is living below poverty line?
What is the school's graduation rate?
A couple has 4 boys, and they are pregnant again: what is their chance of having another boy?

IV. Exercises
Make a survey in your community or in school to find out how far students are travelling in coming to school. Make the best estimate using kilometer.

V. Summary

In this lesson, you discussed different ways of presenting data in an organized manner. You were also introduced to a new area of mathematics called Statistics. You discussed the different activities involved in learning Statistics.

Furthermore,

Statistics is the science of collection, analysis, and presentation of data. Statisticians contribute to scientific enquiry by applying their knowledge to the design of surveys and experiments; the collection, processing, and analysis of data; and the interpretation of the results.

Raw data, also known as source data, is data that has not been processed in order to be displayed in any sort of presentable form.

The raw form may look very unrecognizable and be nearly meaningless without processing, but it may also be in a form that some can interpret, depending on the situation.

Some uses of Statistics.

Functions or Uses of Statistics

A. *Statistics helps in providing a better understanding and exact description of a phenomenon of nature.*
B. *Statistics helps in proper and efficient planning of a statistical inquiry in any field of study.*
C. *Statistics helps in collecting an appropriate quantitative data.*
D. *Statistics helps in presenting complex data in a suitable tabular and graphic form for an easy and clear comprehension of the data.*
E. *Statistics helps in drawing valid inferences, along with a measure of their reliability about the population parameters from the sample data.*

Lesson 38: Organizing and Presenting Data
Time: 1 hour
Prerequisite Concepts: Ratio and proportion, measurement concepts
About the Lesson: This lesson allows you to explore different ways of organizing and presenting data such as using tables, graphs or charts. Presenting data using graphs or charts such as frequency histogram, bar graphs, line graphs and pie charts or circle graphs will be studied. This will help you realize when to use such kind of graph and what information each of these types can provide.

Objective:

In this lesson, you are expected to:

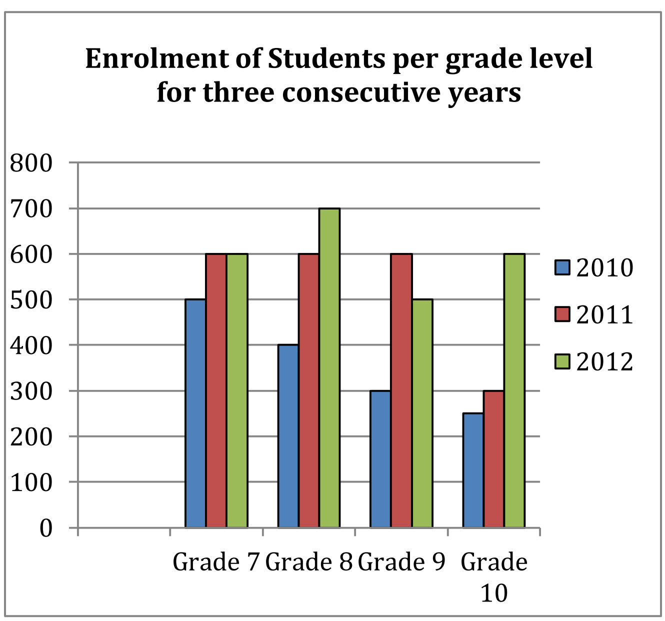
1. Organize data in a frequency table
2. Use appropriate graphs to represent data.

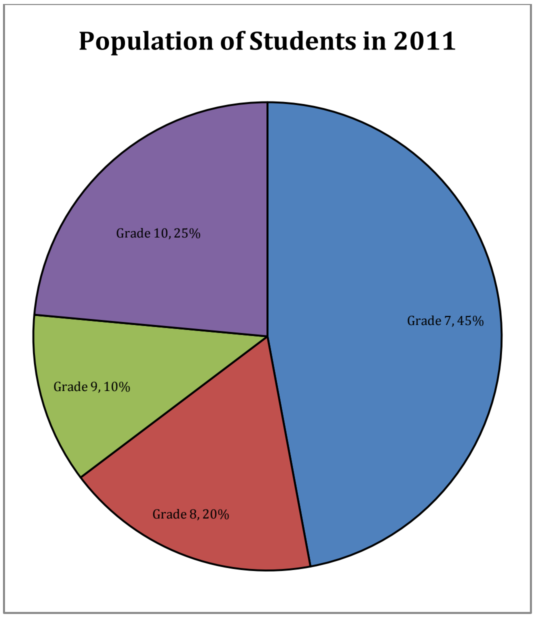
Lesson Proper:

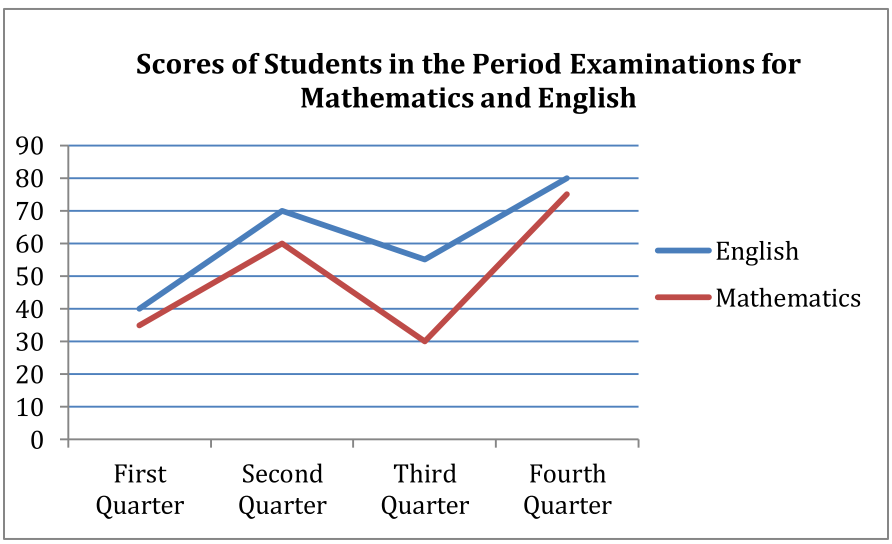
I. Activity

Present the following to your students and ask them to work in groups. Encourage them to write as many possible answers and explanations.

This is a good opportunity for students to explore different possible correct answers. Just remind them to provide justification. To enhance their skill on communicating mathematically, you may ask each group to write a story that each graph or chart describes.







1. What information can we get from each of the above charts or graphs? Discuss each chart or graph.

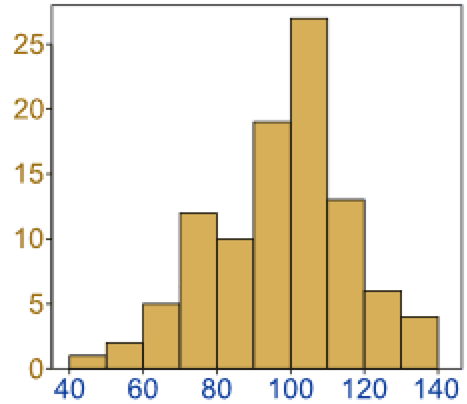
2. Do they present the same information?

3. Describe each of the charts/graphs. What do you think are some uses of each of the charts or graphs?

Ask each group to present their answer and their explanation.

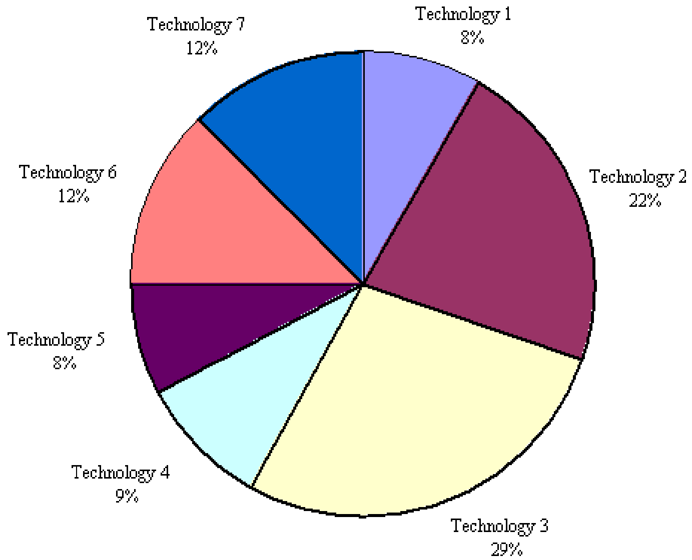
II. Discussion

A **histogram** is a graphical representation showing a visual impression of the distribution of data. A histogram consists of tabular frequencies, shown as adjacent rectangles, erected over intervals. The height of a rectangle is also equal to the frequency.



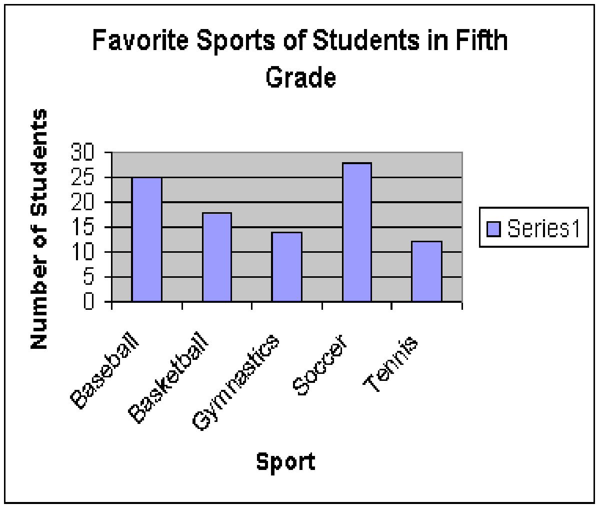
http://www.mathsisfun.com/data/histograms.html

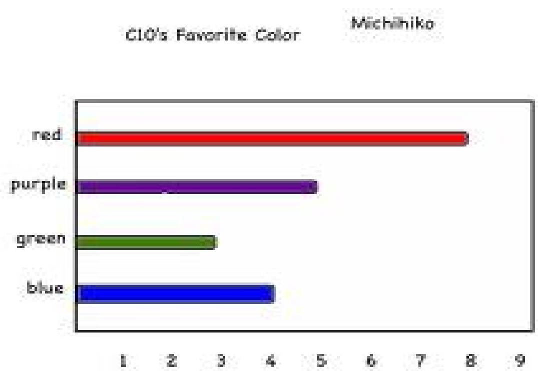
A pie chart is a circle divided into sectors proportional to the frequencies. It shows how a part of something relates to the whole. It is important to define what the whole represents.



http://hwachongilp.wikispaces.com/6.+How+to+interpret+pie+charts+

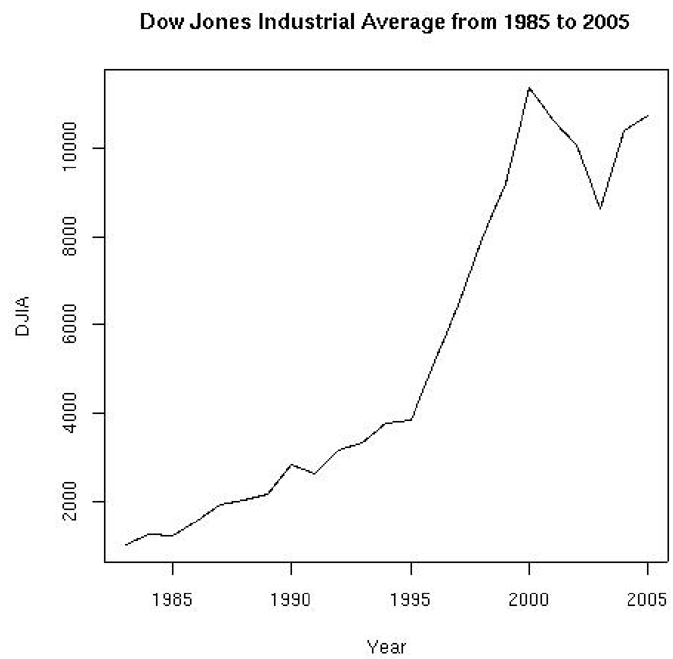
A bar graph is like a histogram except that its bars are separated. This uses parallel bars, either horizontal or vertical, to represent counts for several categories. One bar is used for each category with the length of the bar representing the count for that one category.





http://www.k12.hi.us/~gkolbeck/website/grade2.html

A line graph shows trends in data clearly. This displays data which are collected over a period of time to show how the data change at regular intervals.



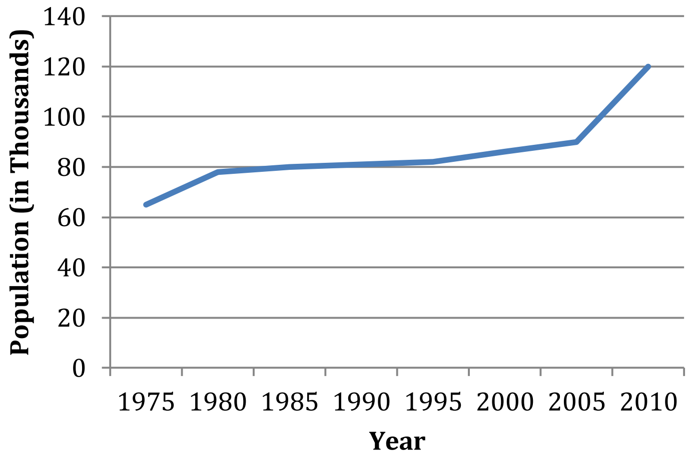
http://hwachongilp.wikispaces.com/7.+How+to+interpret+line+graphs

III. Activity

Directions:

1. Organize the following data and present using appropriate graph or chart. Explain why you are using such graph/chart in presenting your data.
   a. The data below shows the population [in thousands] of a certain city.

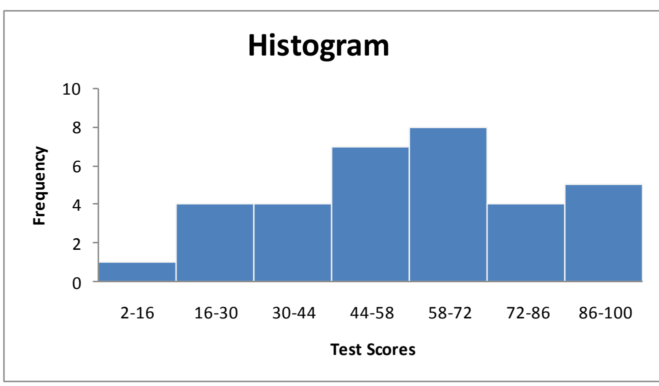
Year	1975	1980	1985	1990	1995	2000	2005	2010
Population in thousand	65	78	80	81	82	86	90	120



The line graph is the appropriate graph to use because the data shown are with respect to time.

b. The following data indicates the scores of 30 students who took the qualifying examination for mathematics challenge.

35, 60, 45, 56, 54, 20, 65, 80, 70, 34, 32, 90, 95, 70, 24, 11, 70, 65, 70, 45, 32, 70, 45, 55, 76, 77, 16, 92, 88, 86, 78, 54, 19

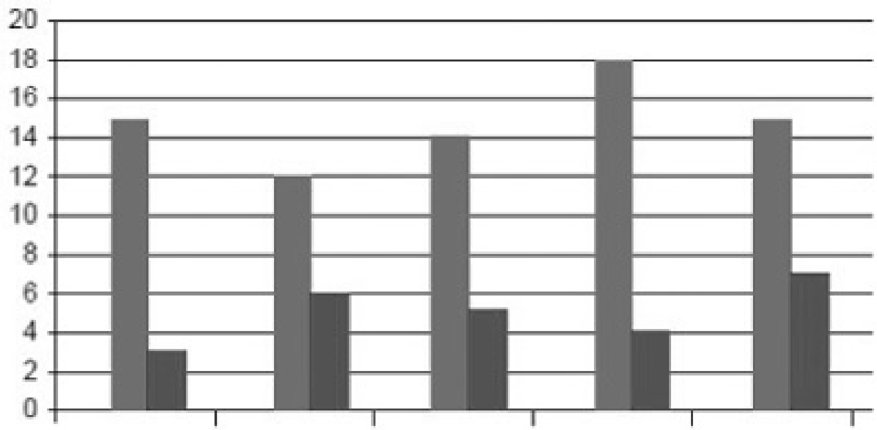


The histogram is used to group the test scores into intervals.

2. What is your story?

Work individually. After 10 minutes, share your answer to the person next to you.

The bar chart below does not have a title and other important information is missing:



Use your imagination and knowledge of charts to help make sense of the above chart. Think of a suitable title that explains what the bar chart is all about. Provide all the needed information and labels to complete the graph.

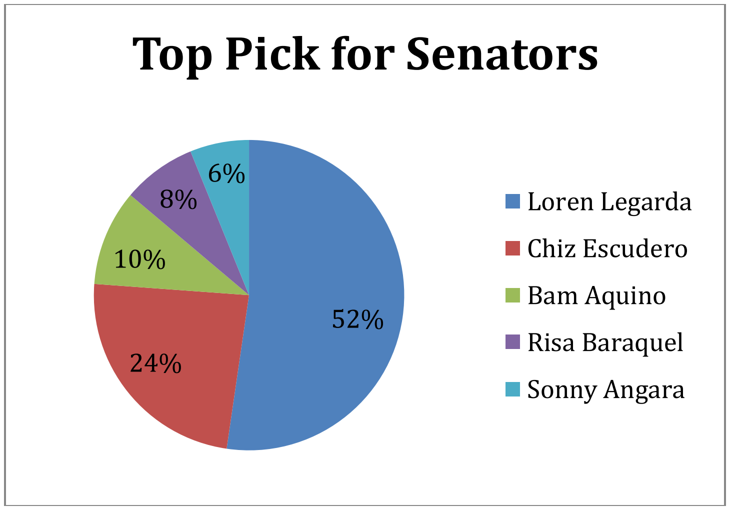
A suitable title is A Histogram of Networth of Lawmakers. The lighter-colored bars represent the Congressmen falling within the intervals of networth. The darker colored bars represent the Senators that fall within the intervals of networth.

IV. Questions to Ponder

1. What are some of the different ways of presenting data? *The different ways are a table, bar graph, and frequency table and histogram.*
2. Describe the information that can be obtained from a data presented using Bar Graph, Pie Chart, Line Graph, and Histogram.
   a. *Bar Graph – represents counts for several categories*
   b. *Pie Chart – shows how a part relates to a whole*
   c. *Line Graph – represents data over a period of time*
   d. *Histogram – represents counts of data that fall within intervals*

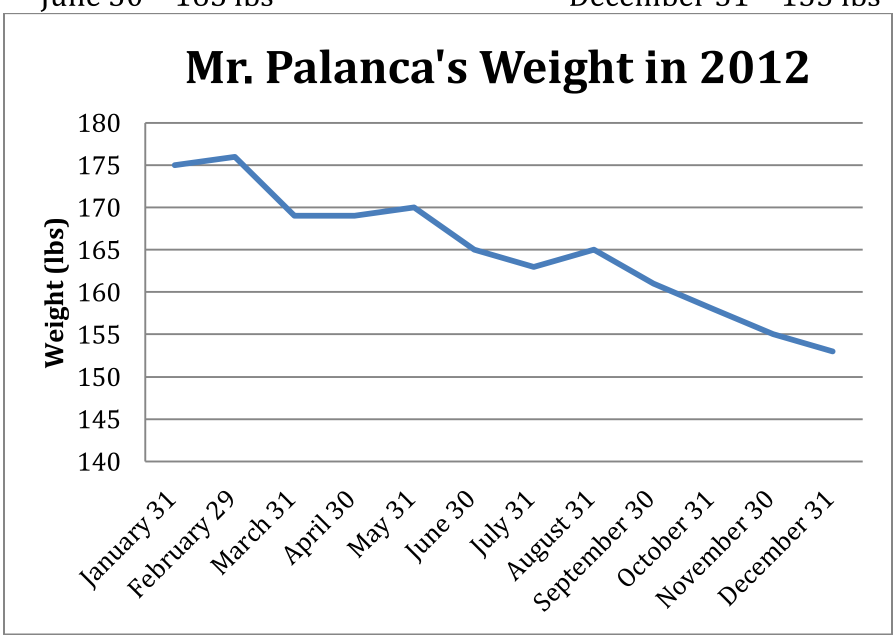
V. Exercises

1. Organize and present the following data using an appropriate chart or graph:
   a. An informal survey of 130 youth voters shows their top pick for senator:
      Loren Legarda – 68
      Chiz Escudero – 31
      Bam Aquino – 13
      Risa Baraquel – 10
      Sonny Angara – 8



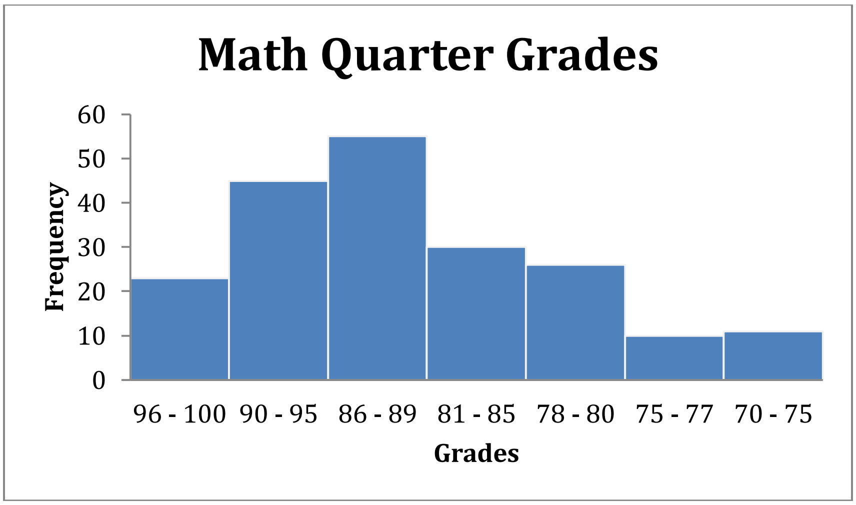
b. Mr. Palanca recorded his weight every end of the month in the year 2012.

January 31 – 175 lbs	July 31 – 163 lbs
February 29 – 176 lbs	August 31 – 165 lbs
March 31 – 169 lbs	September 30 – 161 lbs
April 30 – 169 lbs	October 31 – 158 lbs
May 31 – 170 lbs	November 30 – 155 lbs
June 30 – 165 lbs	December 31 – 153 lbs



c. Mrs. Bernardo tallied the Second Quarter Math grades of all 200 Grade 7 students in Lara's school as follows:

96 – 100	23 students
90 – 95	45 students
86 – 89	55 students
81 – 85	30 students
78 – 80	26 students
75 – 77	10 students
70 – 75	11 students



2. Go to your school EMIS (Educational Management Information System) office and ask permission from the personnel at the principal's office to guide you where the graphs/charts of the Performance Indicators of the school for the past 2 school years.

a. What are the performance indicators that you see?
b. What information can be obtained from the charts or graphs?
c. What is the significance of those charts or graphs?
d. Why is it important to put the data in graphs or in charts?

VI. Summary

In this lesson, you learned about the different ways of organizing and presenting data. These are histograms, pie charts, bar graphs and line graphs. You also learned which type of chart or graph suits a particular type of data to present.

Lesson 39: Organizing and Presenting Data Using Frequency Table and Histogram
Time: 1 hour
Prerequisite Concepts: Sorting and Presenting Data
About the Lesson: This lesson focuses on the use of frequency tables and histograms as a way of organizing and presenting data.

Objective:

In this lesson, you are expected to:

1. Sort and organize data using frequency table
2. Present data using a histogram.

Lesson Proper:

I. Activity

The following is a list of shoe sizes of 35 boys.

7	5	4	4	6	5	4
8	5	5	4	3	5	6
5	3	6	3	2	8	5
6	6	7	4	7	4	5
4	4	2	5	5	6	4

a. What information can you get from the above list?
b. What does the list tell us?
c. The list above does not tell us anything. So what do we do with the list?

II. Discussion

1. The Frequency Table

We construct a table with three columns as shown. The shoe sizes may be grouped as follows: 1-3, 3-5, 5-7, and 7-9. Then we refer to our list and go down each column and make a mark for each figure or number in the tally table. For boundary sizes, we can use the rule that we include the lower bound in the interval but not the upper bound. The first count is 2 (for sizes 1 and 2), so we put two strokes. The next count is 12 (for sizes 3 and 4), so we put 12 strokes, and so on.

Use the rule for determining the number of class intervals as follows: Let n be the number of data values. The number of intervals is approximately the $\sqrt{n}$. To determine the width of each interval, one suggestion is to subtract the lowest data value from the highest data value and divide it by the number of intervals. Below is a frequency table showing 4 intervals only.

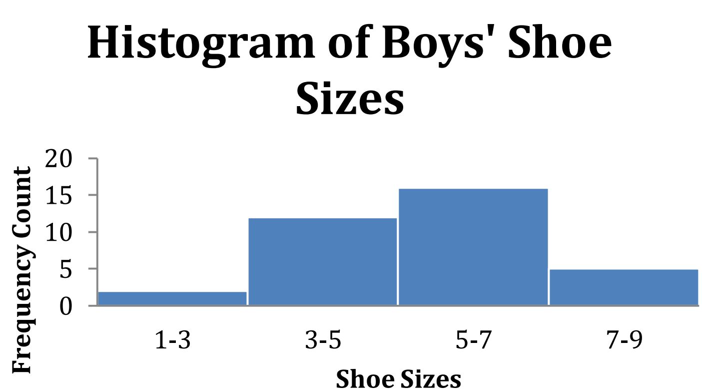
Shoe Size	Tally	Frequency
1-3	||	2
3-5	卌|||| ||	12
5-7	卌||卌|卌	16
7-9	|||| |	5
	卌||	
	Total:	35

Make sure that your students understand that the Histogram is useful for large quantities of numerical data.

After tallying, we count the tally marks in each row and put the number in the last column. This is now called the frequency table.

2. The Frequency Histogram

From the frequency table, we are going to construct a histogram. A *histogram* is "a representation of a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies."



a. What does the height of each of the rectangles tell us?
   The height of each rectangle corresponds to the frequency of the shoe sizes within an interval.
b. What is the total area of all the rectangles?
   The total area of the rectangles is equal to 35
c. What does the total area tell us?
   The total area of the rectangles is the total frequency.

III. Activity

Given the set of data, construct a frequency table and a frequency histogram using intervals of width 5.

> *It is better to specify either the width or the number of intervals to be used by students so that you don't have to check too many possible tables and graphs.*

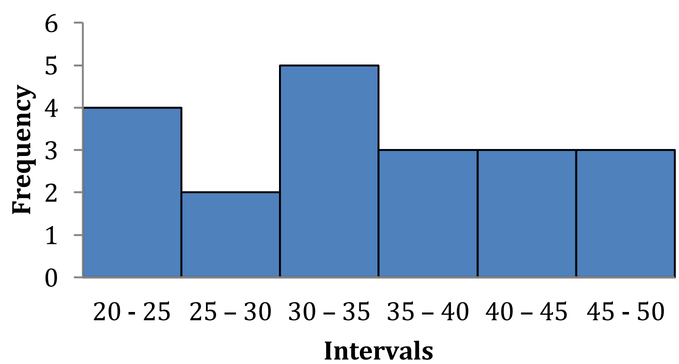
34	45	37	29	20
36	20	20	34	45
40	40	34	45	40
30	30	20	29	36

The lowest data value is 20 so we can use 6 intervals of width 5: 20 – 25, 25 – 30, 30 – 35, 35 – 40, 40 – 45, and 45 – 50.

Interval		Frequency
20 – 25		4
25 – 30		2
30 – 35		5
35 – 40		3
40 – 45		3
45 – 50		3
	Total	20

> *In counting boundary values, count the lower boundary value in the interval that it belongs. For example, the score 20 is counted in the first interval. The score of 45 is counted in the last interval.*

And here is the corresponding frequency histogram made by Excel:



> *Use Excel to get an accurate graph or Histogram.*

Can you come up with a different set of intervals and frequency count? Suppose you used 4 or 5 intervals only? How will the histogram differ?

IV. Questions/Points to Ponder

The **frequency** of a particular data value is the number of times the data value occurs.

What is a frequency table?
*A **frequency table** is a table that lists numerical data that have been grouped in intervals and the frequency of occurrence of the data.*

How do you construct a frequency table?
*A **frequency table** is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies. The data values are then grouped in intervals (e.g. 0-5,5-10). Following a rule for boundary values, frequency counts are noted for each interval.*

We use the following steps to construct a frequency table:

***Step 1:** Construct a table with three columns. In the first column, write down all of the data values grouped in intervals.*

***Step 2:** To complete the second column, go through the list of data values and place one tally mark at the appropriate place in the second column for every data value in the interval following a specified rule for counting boundary values. When the fifth tally is reached for a mark, draw a horizontal line through the first four tally marks as shown for 6 in the above frequency table. We continue this process until all data values in the list are tallied.*

***Step 3:** Count the number of tally marks for each interval and write the count in the third column.*

What is a frequency histogram?
*A **histogram** is a vertical bar graph of a frequency distribution of data values grouped into intervals.*

How do you construct a frequency histogram?

Step 1: Place the data intervals along the horizontal axis.

Step 2: Mark the frequency numbers on the vertical axis.

Step 3: Draw rectangles for each interval. The height of the rectangle is the frequency count fo that interval.

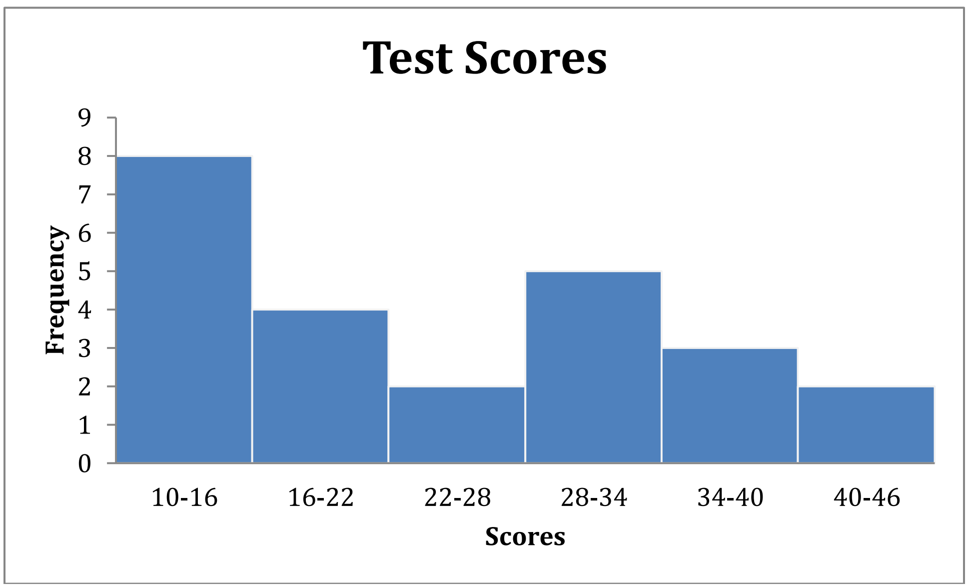
V. Exercises

The following are test scores of Section 1. Construct a suitable frequency table and a frequency histogram. Use intervals of width 6.

14 15 30 19 10 18 26 30 10 15
15 28

10 30 34 40 20 43 20 30 10 22
36 36

Interval	*Frequency*
10-16	*8*
16-22	*4*
22-28	*2*
28-34	*5*
34-40	*3*
40-46	*2*



VI. Summary

You learned how to construct frequency tables and a frequency histogram. A **frequency table** is a table that lists items and uses tally marks to record and show the number of times data values occur. A **histogram** is a vertical bar graph of a frequency distribution of data values grouped into intervals.

Lesson 40: Averages: Mean, Median, and Mode
Time: 1 hour
Prerequisite Concepts: Sorting and Presenting Data
About the Lesson: This is a lesson on computing for the values of Mean, Median, and Mode given a set of data. You will also describe data using the mean, median, and mode. Problem solving involving Mean, Median, and Mode is also included in this lesson.

Objective:

In this lesson, you are expected to:

1. Find the mean, median, and mode of a statistical data.
2. Solve problems involving mean, median, and mode.
3. Describe the data using information from the mean, median, and mode.

Lesson Proper:

I. Activity

The set of data shows a score of 35 students in their periodical test.

34	35	40	40	48	21	9
21	20	19	34	45	21	20
19	17	18	15	16	20	28
21	20	18	17	10	45	48
19	17	29	45	50	48	25

1. What score is typical to the group of students? Why?

The students may have a hard time figuring out what "typical" means. You may expound this further by using the word representative or maybe the number that can best describe the performance of the students taking the test. The answers may vary. Some may point out the median or the mean. Be sure to ask the students to justify their answers.

2. What score frequently appears? *20 and 21*

3. What score appears to be in the middle? How many students fall below this score? *21. 17 students are expected to fall below this score but there are two other scores in the bottom half that are the same as this middle score.*

II. Discussion

An average is a number that is typical for a set of data. There is more than one type of average. The type used most often is the mean value.
Measures of central tendency, or "location", attempt to quantify what we mean when we think of as the "typical" or "average" score in a data set. Statistics geared toward measuring central tendency all focus on this concept of "typical" or "average."

*By far the simplest, but also the least widely used, measure of central tendency is the **mode**. The mode in a distribution of data is simply the score that occurs most frequently.*

*Technically, the **median** of a distribution is the value that cuts the distribution exactly in half, such that an equal number of scores are larger than that value as there are smaller than that value.*

*The **mean**, or "average", is the most widely used measure of central tendency. The mean is defined technically as the sum of all the data scores divided by n (the number of scores in the distribution).*

> *It is important that the students will realize that there can be more than one mode and if no number occurs more than once in the set, then there is no mode for that set.*

The middlemost score is called the median. If there is an odd number of data values, the median is the middle value in the ordered list. If there is an even number of data values, the median is the mean of the two middle values in the ordered list.

III. Activity

1. The following sets of data show the height [in centimeters] of two groups of boys playing basketball.
   Group A: 135, 136, 140, 150, 134, 129, 126, 130
   Group B: 167, 136, 119, 136, 160, 178, 126, 140

   a. Compute for the mean.

      Group A - Mean is 135
      Group B – Mean is 145.25

   b. What information can you get from these two values?

      The boys in group B are generally taller than those in group A.

2. The following sets of data show the weekly income [in peso] of ten selected households living in two different barangays in the town of Kananga.

Brgy. Kawayan: *150, 1500, 1700, 1800, 3000, 2100, 1700, 1500, 1750, 1200*

Brgy. Montealegre: *1000, 1200, 1200, 1150, 1800, 1800, 1800, 2000, 1470, 8000*

a. Compute for the mean and the median for each Barangay.

Kawayan [Mean is 1345, Median is 1700]
Montealegre [Mean is 2142, Median is 1635]

b. What information can we get from these values?

Using the mean value, it seems that Barangay Montealegre's household income is far greater than that of Barangay Kawayan's. However, using the median, it seems that the two are almost the same.

Emphasize that the Median is more appropriate in this case because of extreme values. In the case of Barangay Kawayan, the mean is affected by the very low value which is 150 and in the case of Barangay Montealegre, the mean is affected by a very high value which is 8000. We are looking at a representative value (which is typical to the group) for each barangay that would more or less give an appropriate description of household income, therefore, the median is used because it is a practical estimate of each

c. Why do you think the median is more appropriate than the mean?

Mean and median are the two standard kinds of average. The Median is used when it's obvious that the mean would be misleading and this happens if there are extreme scores. Extreme scores are those are usually referred to as outliers. These are very high or very low scores. The mean is affected by extreme scores. In this example, Median household income is commonly considered, even though Gross Domestic Product per person is an equally

IV. Questions/Points to Ponder

The mean is the score obtained if all the scores are "evened out". For example, 5 boys have the following ages: 14, 12, 12, 15, and 12. If the ages are "evened out" (2 from 15 distributed to each of the 12 and 1 from 14 added to the other 12) then all 5 ages become 13. Thus, the mean age is 13. The mean is affected by extreme values.

The median is the middle score in the ordered list of the values. For example, in the case of the ages of the five boys, the ages may be arranged as: 15, 14, 12, 12, and 12. Hence, the median is 12, lower than the mean. The median is not affected by extreme values because its position in an ordered list stays the same.

The mode is the most common value. In the example, the mode is the age with the highest frequency count among the 5 boys and that is 12. The mode is useful if the interest is to know the most common value. For example, a company has give-away items for teens. To determine if the items are age-appropriate, the company might simply ask for the most common age in the group.

Mean and median are the two measures of central tendency. The Median is used when it's obvious that the mean would be misleading and this happens if there are extreme scores. Extreme scores are those are usually referred to as outliers. These are very high or very low scores. The mean is affected by extreme scores. In this example, Median household income is commonly considered, even though Gross Domestic Product per person is an equally accurately known as mean.

V. Exercises

1. Below are the mathematics grades of 30 Grade 7 students in the last quarter:

78	98	76	89	89	83
87	75	72	91	90	79
84	84	85	88	87	95
96	95	96	96	76	80
83	82	85	92	91	90

Compute for the mean, median and mode. *Answer: The mean is 86.4. The median is 87. The mode is 96.*

2. Mario took four examinations in a science class. His scores are 48, 65, 78, and 79. Which measure is more appropriate to use in order to determine how well Mario is performing in science? Answer: *The Bar graph is more appropriate.*

3. The National Housing Authority publishes data on resale prices of houses in Metro Manila. Which of mean, median and mode is more appropriate to use? Explain your answer. Answer: *The Median is more appropriate.*

4. Solve the following problems:
   a. The median for 10, 9, y, 12, and 6 is y. Find possible values of y, given that the values are whole numbers.
      Answer: 9 or 10

Solution: If the numbers are arranged, 6, 9, y, 10, 12. Since the median is also y, the value should either be 9 or 10 to satisfy the condition and since there is an odd number of data.

b. The mean of fifteen numbers is 30 and the mean of ten numbers is 25. What is the mean of all the twenty-five numbers?

Answer: 28

Solution: This is an example of the weighted mean where the overall mean will be computed based as follows:

$\frac{30(15)+25(10)}{25} = \frac{700}{25} = 28$

c. Given the set of numbers N = {7, 9, 10, 14, 8, 16, 13}. When a number *x* is added to the set, the new mean is 12. Calculate the value of *x*.

Answer: x = 19

Solution: $Mean = \frac{7+9+10+14+8+16+13+x}{8} = 12$. Solving for x:

$x = [12(8) - (7 + 9 + 10 + 14 + 8 + 6 + 13)]$. Hence *x* = 19.

VI. Summary

In this lesson, you learned about the three different "averages" of a set of numerical data: Mean, Median and Mode. The mean is the most commonly known average and is obtained by adding all the values and dividing the sum by the number of values. The median is the middle value in the ordered list of all values. The median is not affected by extreme values, unlike the mean. If there is an odd number of data values, the median is the middle value in the ordered list. If there is an even number of data values, the median is the mean of the two middle values in the ordered list. The mode is the value with the highest frequency count. It is useful in certain situations that simply ask for the most common value.

Lesson 41: Analyzing, Interpreting, and Drawing Conclusions from Graphics and Tabular Presentations
Time: 1 hour
Prerequisite Concepts: Organizing data using charts, tables and graphs; Mean, Median and Mode
About the Lesson: This lesson serves as a consolidation and practice of what you learned in the previous lessons. The problems will give you the opportunity to work in groups and discuss different solutions. You will also learn to justify your answers using data.

Objective:

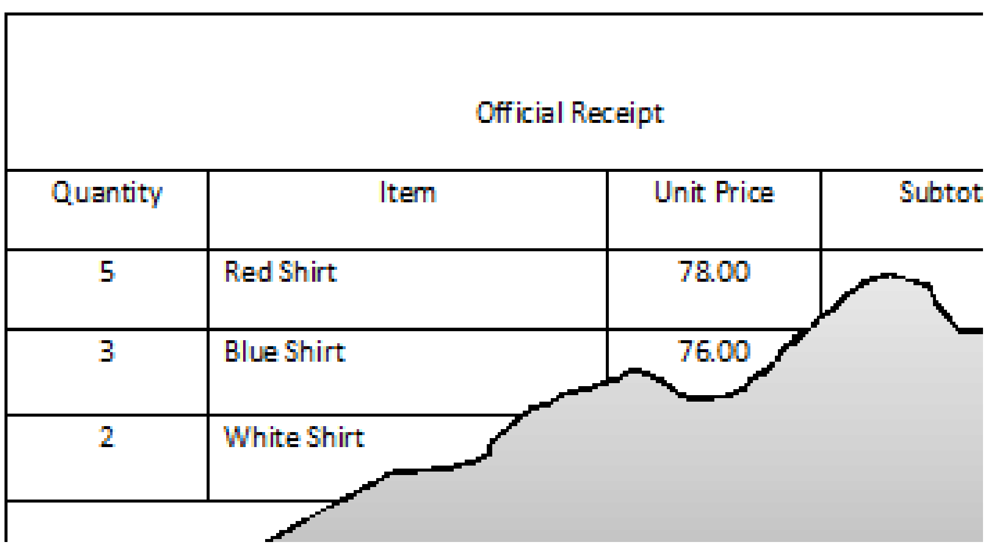
In this lesson, you are expected to analyze, interpret accurately and draw conclusions from graphics and tabular presentations of statistical data.

Lesson Proper:

I. Activity

Directions: Solve the following problems in groups or in pairs.

1. Daria bought 3 colors of T-shirts from a department store. She paid an average of PhP 74.00 per shirt. The receipt is shown below where part of it was torn.



   a. How much did she pay for each white shirt? *Answer: Php 61*
   b. How much did she pay in all? How did you determine this? *Answer: 740 since 10 x Php74 gives the total.*

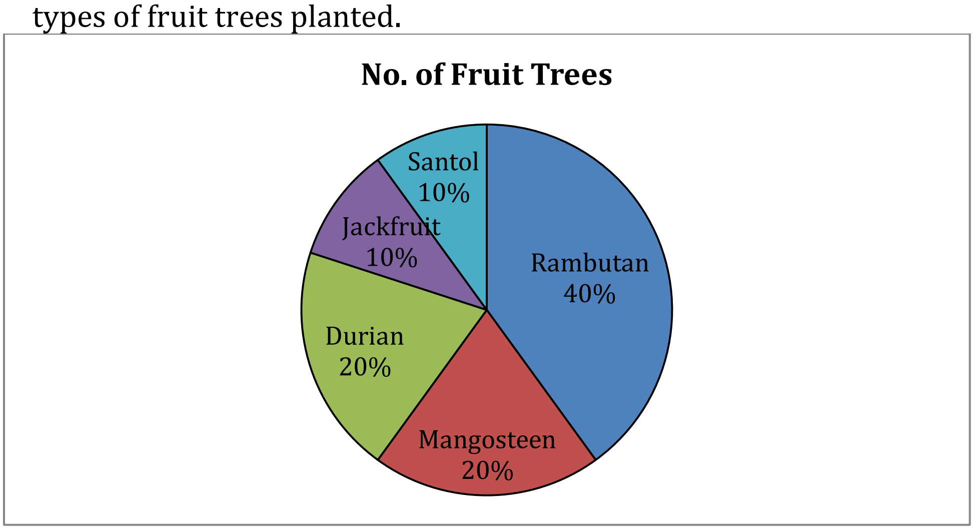
2. Samuel brought ten sachets of chocolate candies. He checked the sachets and found that they contain 12, 15, 16, 10, 15, 14, 12, 16, 15, and 12 candies.

   a. According to the data, what is the mean number of candies per sachet? *Answer: 13.7 candies*

AVERAGE CONTENT: 14

   b. The above information is written on each pack of candies. Why do you think this number is different from the answer to (a)? *Answer: You cannot have just a part of a candy. You should be counting whole candies. Hence, the mean was rounded off to the nearest whole number.*

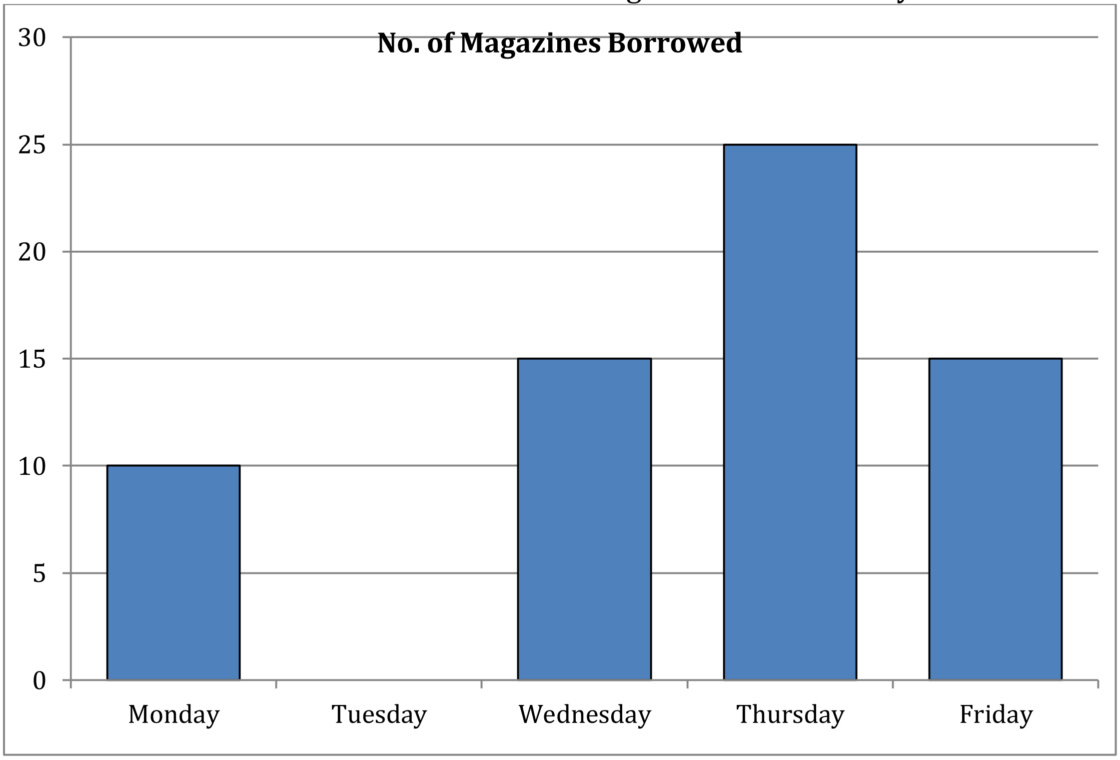
3. The Municipal Agriculturist of Kananga made a survey of fruit trees available in the orchard. He made a chart that shows the distribution of types of fruit trees planted.



   a. How do the number of Durian and Rambutan trees compare? *The number of Rambutan trees is twice the number of Durian trees.*
   b. What is the most common fruit tree? *Rambutan*
   c. What fraction of the fruit trees is Santol? *One-tenth*
   d. If there are 150 fruit trees altogether, how many are Mangosteen trees? *30 Mangosteen trees*

II. Activity

1. Mrs. Amodia, the School Librarian prepared a bar chart that shows the number of students who borrowed magazines in the library last week.



a. How many magazines were borrowed on Friday? *15 magazines* How many students borrowed magazines on this day? *It cannot be determined.*

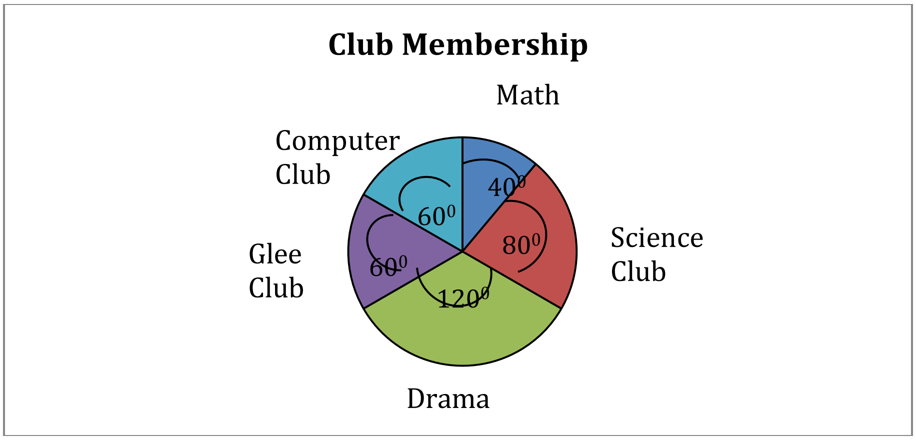
b. What is the mean number of magazines borrowed per day last week? *13 magazines were borrowed per day, on the average.* How many students borrowed magazines in all of the five days? *It cannot be determined.*

c. On what day had the most number of students borrow magazines? *It cannot be determined*.

d. How many students borrowed magazines on Tuesday? *None*

The number of magazines borrowed from the library does not necessarily equal to the number of students who borrowed the magazines. There is no mention about the some restrictions that only one magazine can be borrowed by a student. It is possible that a student could borrow more than one magazine.

2. The pie chart below shows the memberships of the different clubs in Ormoc City National High School.



a. What is the ratio of the number of students who are members of the Computer Club to that of the members of the Glee Club? *1:1*
b. What percentage of the students are members of the Drama Club? *One-third.*
c. If there are a total 240 students, how many are members of the Mathematics Club? *There are 96.*

The above chart is using degree measures of a circle instead of percentage. This is an opportunity for connection and integration of topics on geometry specifically on the degree measure of a circle.

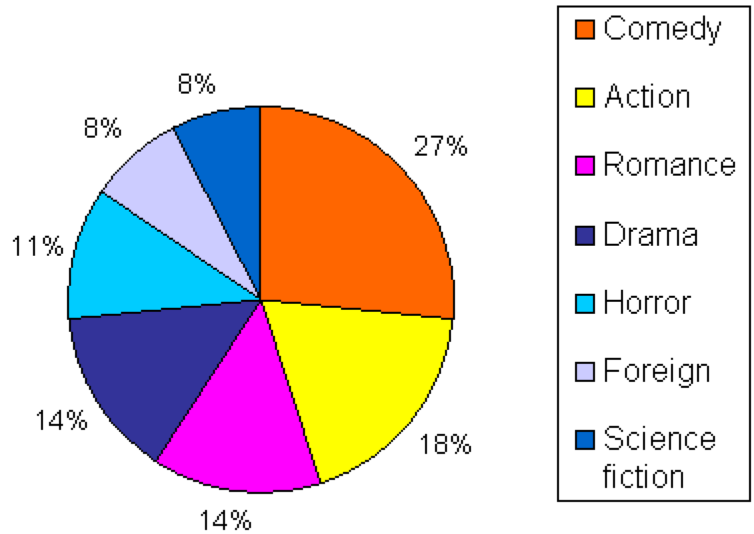
III. Questions/Points to Ponder

By analyzing data from graphs, charts and tables carefully, you could derive important information that lead to well-justified answers or conclusions. Hence it is important for students what each type of data presentation emphasizes. Combine with that the understanding of the measures mean, median and mode and the information gets richer. This is the most important part of this lesson.

IV. Exercise

1. From the local newspaper, cut out at least two different charts or graphs and write questions that will help your classmates interpret the graph or chart. Write down some implications that are important to you or to the community.

2. Study the pie chart below that shows the kinds of books owned by a local library:



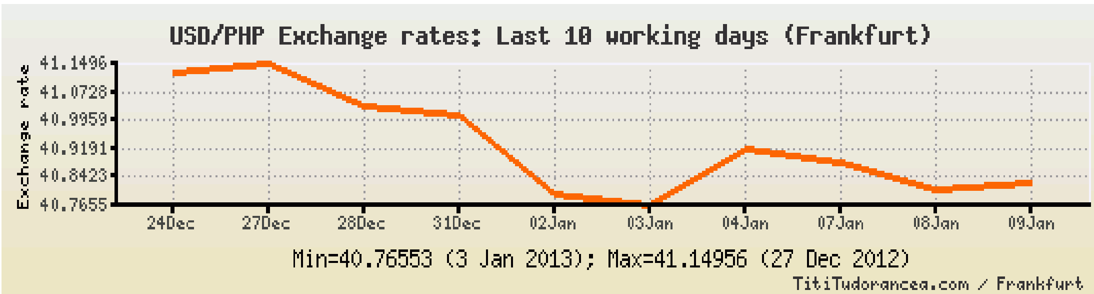
a. What type of book does the library have the most number of compared to any other type in its collection? *Comedy books*

b. If there are 850 books in the collection, how many books are there of each type? What is the "average" number of books per type? *68 books each of Foreign and Science Fiction; 94 books of Horror; 119 books of each of Drama and Romance; 153 books of Action; 230 books of Comedy*

c. How many more Science Fiction books should be added to the collection in order to match the Comedy Book collection? What percentage of the collection is each type when this happens*? There should be an additional 162 Science Fiction books. When this happens, each type is about 23% of the total collection.*

3. USD to PHP Exchange Rates: (US Dollar to Philippine Peso) Charts and historical data

Last 10 working days



a. On what day was the peso strongest against the US dollar? *Jan 3*

b. On what days did the peso appreciate against the US dollar? *Dec 28, Dec 31, Jan 2, Jan 3, Jan 7.*

c. Can you explain the scale on the vertical axis*? The higher the value, the more pesos equal a dollar.*

d. What is the “average” peso-dollar exchange rate from Dec 24 to Jan 9? Php 40.9 = US$1

V. Summary

In this lesson, you learned to interpret information gathered from tables, charts and graphs. You also learned to analyze data by obtaining the measures mean, median and mode.