Optimisation Prize 2 Attempt

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Part a

The robust constraint can be rewritten as:

$$z \le b$$

Where z is the optimal objective value from the below linear program:

$$maximise \quad x^T a$$

$$s.t. \quad Fa \le g$$

$$x, a \in R^n, F \in R^{k \times n}, g \in R^k$$

The dual problem of this linear program is:

$$minimise\ g^T y$$

$$F^T y = x$$

$$x, a \in R^n, F \in R^{k \times n}, g \in R^k, y \in R_+^k$$

If the value of any $g^T y$ is $\le b$, then the minimum of $g^T y$ will also be $\le b$. Hence we can rewrite the robust constraint into:

$$g^T y \le b$$

$$F^T y = x$$

$$x, a \in R^n, F \in R^{k \times n}, g \in R^k, y \in R_+^k$$

Part b

Since there is uncertainty in steel requirements, the steel requirement could be rewritten as:

$$a_1W + a_2P \le 27000$$

$$a_1 \le 2$$

$$-a_1 \le -1$$

$$a_2 \le 1.5$$

$$-a_2 \le -0.5$$

Using the results from part a, this can be rewritten as $z \le 27000$

Where z is the optimal objective value of the below linear program:

$$maximise\ Wa_1 + Pa_2$$

$$s.t.\quad \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \le \begin{pmatrix} 2 \\ -1 \\ 1.5 \\ -0.5 \end{pmatrix}$$

The dual of this linear program is:

$$minimise\ 2y_1 - y_2 + 1.5y_3 - 0.5y_4$$

$$s.t.\quad \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} W \\ P \end{pmatrix} \quad \Rightarrow \quad W = y_1 - y_2 \quad \& \quad P = y_3 - y_4$$

$$y_1, y_2, y_3, y_4 \ge 0$$

Rewriting this robust constraint in the original Winnipeg Tool Company's problem:

$$maximise\ 0.13W + 0.10P$$

$$s.t.\quad 2y_1 - y_2 + 1.5y_3 - 0.5y_4 \le 27000$$

$$1.0W + 1.0P \le 21000$$

$$0.3W + 0.5P \le 9000$$

$$W \le 15000, P \le 16000$$

$$W \ge 0, P \ge 0$$

$$W = y_1 - y_2 \quad \& \quad P = y_3 - y_4$$

$$y_1, y_2, y_3, y_4 \ge 0$$

Rewrite the whole problem in terms of y:

$$\textit{maximise}\ 0.13y_1 - 0.13y_2 + 0.10y_3 - 0.10y_4$$

$$s.t. \quad 2y_1 - y_2 + 1.5y_3 - 0.5y_4 \le 27000$$

$$y_1 - y_2 + y_3 - y_4 \le 21000$$

$$0.3y_1 - 0.3y_2 + 0.5y_3 - 0.5y_4 \le 9000$$

$$y_1 - y_2 \le 15000, y_3 - y_4 \le 16000$$

$$y_1 - y_2 \ge 0, y_3 - y_4 \ge 0$$

$$y_1, y_2, y_3, y_4 \ge 0$$

We now use AMPL to solve the above linear program; the optimal value is £1795 and the optimal solution is:

$$y_1 = 1500$$

$$y_2 = 0$$

$$y_3 = 16000$$

$$y_4 = 0$$

Substitute into the W and P equations:

$$W = 1500$$

$$P = 16000$$