Optimisation and Decision Models Homework 3

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Question 3

(a) Decision Variables

x_1= Units of product line 1 to produce
x_2 = Units of product line 2 to produce
x_3 = Units of product line 3 to produce
x_4 = Units of product line 4 to produce
y_1 = Binary variable: 1 if product line 1 is produced, 0 otherwise
y_2 = Binary variable: 1 if product line 2 is produced, 0 otherwise
y_3 = Binary variable: 1 if product line 3 is produced, 0 otherwise
y_4 = Binary variable: 1 if product line 4 is produced, 0 otherwise

Objective Function

Maximise total profit,

$$70x_1 - 50000y_1 + 60x_2 - 40000y_2 + 90x_3 - 70000y_3 + 80x_4 - 60000y_4$$

Constraints

subject to $y_1 + y_2 + y_3 + y_4 \leq 2$

$$y_3 \leq y_1 + y_2$$
$$x_1 + x_2 \leq 20000 + Mz \qquad z \in \{0,1\}$$
$$x_3 + x_4 \leq 20000 + M(1-z) \qquad z \in \{0,1\}$$
$$x_1 \leq My_1$$
$$x_2 \leq My_2$$
$$x_3 \leq My_3$$
$$x_4 \leq My_4$$
$$x_1 \leq 10000$$
$$x_2 \leq 15000$$
$$x_3 \leq 12500$$
$$x_4 \leq 9000$$
$$x_1, x_2, x_3, x_4 \geq 0$$
$$x_1, x_2, x_3, x_4 \in Z$$
$$y_1, y_2, y_3, y_4, z \in \{0,1\}$$
$$M \text{ is a large number}$$

(b) The Excel solution is in "Question 3" tab of the Excel file "Ex3_Qn3b.xlsx".
The optimal solution is: $x_1 = 0, x_2 = 15000, x_3 = 12500, x_4 = 0$
The optimal objective value is £1915000

The profit-maximising production mix is thus: 15000 units of Product Line 2 and 12500 units of Product Line 3.

Question 4

(a) Decision Variables

x_C = Number of furniture sets to purchase from Caroline Woodworks
x_N = Number of furniture sets to purchase from Nashawtuc Millworks
x_A = Number of furniture sets to purchase from Adirondack Furnishing Designs
x_L = Number of furniture sets to purchase from Lancaster Artisan Company
y_C = Binary variable: 1 if there is purchase from Caroline Woodworks, 0 otherwise
y_N = Binary variable: 1 if there is purchase from Nashawtuc Millworks, 0 otherwise
y_L = Binary variable: 1 if there is purchase from Lancaster Artisan Company, 0 otherwise

Objective Function

Minimise overall cost,

$$10000y_C + 2500x_C + 20000y_N + 2450x_N + 2510x_A + 13000y_L + 2470x_L$$

Constraints

subject to $x_C \leq My_C$
$x_N \leq My_N$
$x_L \leq My_L$
$x_C \leq 1000$
$x_N \leq 1200$
$x_A \leq 800$
$x_L \leq 1100$
$x_C + x_N + x_A + x_L = 2000$
$x_C, x_N, x_A, x_L \geq 0$
$x_C, x_N, x_A, x_L \in Z$
$y_C, y_N, y_L \in \{0, 1\}$
$M\ is\ a\ large\ number$

(b) The AMPL model file is attached as "Ex3_Qn4b.mod" and run file is "Ex3_Qn4b.run".

The optimal solution is: $x_C = 0, x_N = 1200, x_A = 0, x_L = 800$
The optimal objective value is £4949000

(c) Decision Variables

x_C= Number of furniture sets to purchase from Caroline Woodworks
x_N = Number of furniture sets to purchase from Nashawtuc Millworks
x_A = Number of furniture sets to purchase from Adirondack Furnishing Designs
x_L = Number of furniture sets to purchase from Lancaster Artisan Company
x_D = Number of furniture sets (up to the first 1000) to purchase from Delaware Mills
x_{D2} = Number of furniture sets (beyond the first 1000) to purchase from Delaware Mills
y_C = Binary variable: 1 if there is purchase from Caroline Woodworks, 0 otherwise
y_N = Binary variable: 1 if there is purchase from Nashawtuc Millworks, 0 otherwise
y_L = Binary variable: 1 if there is purchase from Lancaster Artisan Company, 0 otherwise
y_D = Binary variable: 1 if there is purchase (up to the first 1000 furniture sets) from Delaware Mills, 0 otherwise

y_{D2} = Binary variable: 1 if there is purchase (beyond the first 1000 furniture sets) from Delaware Mills, 0 otherwise

Objective Function

Minimise overall cost,

$$10000y_C + 2500x_C + 20000y_N + 2450x_N + 2510x_A + 13000y_L + 2470x_L + 9000y_D + 2530x_D + 7000y_{D2} + 2430x_{D2}$$

Constraints

subject to $x_C \leq My_C$

$x_N \leq My_N$

$x_L \leq My_L$

$x_D \leq My_D$

$x_{D2} \leq My_{D2}$

$1000y_{D2} \leq x_D$

$x_C \leq 1000$

$x_N \leq 1200$

$x_A \leq 800$

$x_L \leq 1100$

$x_D \leq 1000$

$x_{D2} \leq 500$

$x_C + x_N + x_A + x_L + x_D + x_{D2} = 2000$

$x_C, x_N, x_A, x_L, x_D, x_{D2} \geq 0$

$x_C, x_N, x_A, x_L, x_D, x_{D2} \in Z$

$y_C, y_N, y_L, y_D, y_{D2} \in \{0, 1\}$

$M\ is\ a\ large\ number$

The AMPL model file is attached as "Ex3_Qn4c.mod" and run file is "Ex3_Qn4c.run".

The optimal solution is: $x_C = 0, x_N = 1200, x_A = 0, x_L = 800, x_D = 0, x_{D2} = 0$

The optimal objective value is £4949000