Optimisation and Decision Models Homework 1

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<u>Question 3</u>

(a) <u>Decision Variables</u>:

s = number of special risk insurance to sell

m = number of mortgages to sell

<u>Objective Function</u>:

Maximise total expected profit, $5s + 2m$

<u>Constraints</u>:

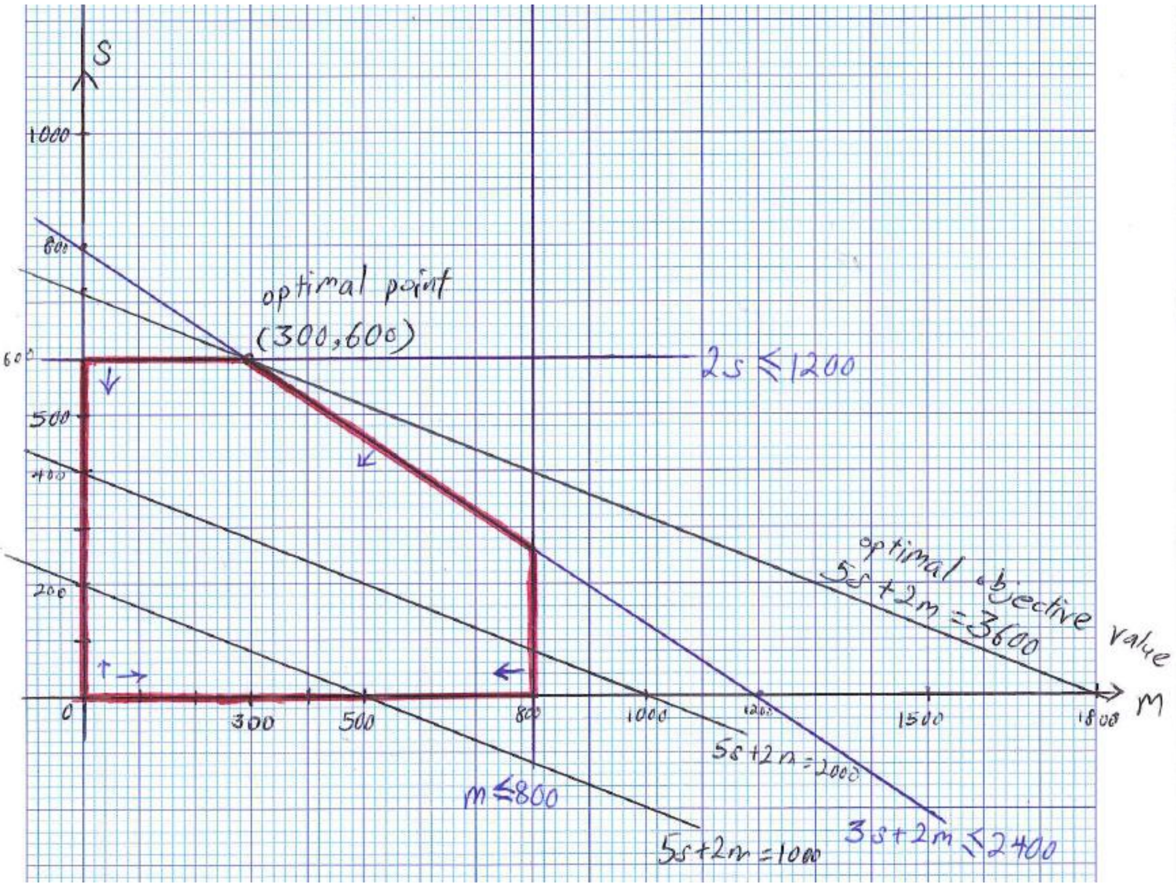
subject to $3s + 2m \le 2400$

$m \le 800$

$2s \le 1200$

$s, m \ge 0$

(b) The graphical solution is as below.



The optimal solution is $m = 300, s = 600$

The optimal objective value is £3600

From the graph above, we can see that there are two binding constraints (where they are satisfied as equalities at optimal solution):

$$2s \le 1200$$
$$3s + 2m \le 2400$$

(c) At the optimal solution, the binding constraints must be satisfied as equalities. Hence, we will need to solve the below linear equations:

$$2s = 1200$$
$$3s + 2m = 2400$$

From the two equations,

$$s = 600$$
$$3(600) + 2m = 2400 \quad \Rightarrow \quad 2m = 600 \quad \Rightarrow \quad m = 300$$

Hence we have $m = 300, s = 600$ as the optimal solution, the optimal objective value is therefore $£5(600) + £2(300) = £3600$

(d) The Excel solution is in Q3 tab of the Excel file "Homework1.xlsx".

<u>Question 4</u>

(a) <u>Decision Variables</u>:

s = number of "Stir Fry" mixes to produce
b = number of "Barbecue" mixes to produce
h = number of "Hearty Mushrooms" mixes to produce
v = number of "Veggie Crunch" mixes to produce

<u>Objective Function</u>:

Maximise total earnings, $0.22s + 0.20b + 0.18h + 0.18v$

<u>Constraints</u>:

subject to $0.0625s + 0.050b + 0.0625v \le 3750$

$$0.075s + 0.100h \le 2000$$
$$0.0625s + 0.050b + 0.075h + 0.0625v \le 3375$$
$$0.050s + 0.075b + 0.075h + 0.0625v \le 3500$$
$$0.075b + 0.0625v \le 3750$$
$$s, b, h, v \ge 0$$

(b) The AMPL model file is attached as "Q4b.mod" and run file is "Q4b.run".

The optimal solution is: $s = 26666.67, b = 18333.33, h = 0, v = 12666.67$
The optimal objective value is £11813.33

At the optimal solution, the binding constraints must be satisfied as equalities. From the AMPL output, we can see that the following three constraints are binding constraints based on the definition:

$$Mushrooms\ Constraint\text{:} \quad 0.075s + 0.100h \le 2000$$
$$Green\ Peppers\ Constraint\text{:} \quad 0.0625s + 0.050b + 0.075h + 0.0625v \le 3375$$
$$Broccoli\ Constraint\text{:} \quad 0.050s + 0.075b + 0.075h + 0.0625v \le 3500$$

CORRECTION:

$$NonNegative\ Mushrooms\ Constraint:\ \ h\ \ \geq 0$$

(c) If there is an extra 100kg of green peppers, the following green peppers constraint will be relaxed to:

$$Green\ Peppers\ Constraint: 0.0625s + 0.050b + 0.075h + 0.0625v\ \ \leq 3475$$

The AMPL model file is attached as “Q4c.mod” and run file is “Q4c.run”. The new optimal objective value is £11877.33

For an extra 100kg of green peppers, the increase in earnings is:

$$£11877.33 - \ £11813.33 = £64$$

$$\text{In other words, the value of an extra 100kg of green peppers is £64}$$

Nature’s Best Frozen Foods company should be willing to pay up to $£64\ \div 100 = £0.64$ per extra kg of green peppers.