hw_es_bsm_knock_in

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Our goial is to apply exact sampling (ES) to Knock-In option associated to geometric Brownian motion.

Problem

- Asset follows $GBM(S_0, r, \sigma^2)$;
- Knock-in payoff is given as

$$F(S) = 1000 \cdot I(S_T > K) \cdot I\left(\min_{1 \le k \le m} S(t_k) < H \right).$$

- (**todo**) Find price

$$e^{-rT} \mathbb{E}[F(S)]$$

Parameters

- $r = 5\%, \sigma = 15\%, S(0) = 95$
- $T = 0.25, m = 50, H = 85, K = 96.$
- $k = 1000, n = 1000$

Algo

- Repeat k times of the following MC:

    - Simulate n paths for S_t with $m = 50$ steps;
    - Compute payoff for each path, and take average;

- Find mean, MSE, and confidence interval using k many MC outputs.

Code