Abstract

- Goal
  - Apply foward time, centered space (FTCS) to heat equation, and investigate stability.
- Ref
  - FDM heat, [Hir13]

Problem

- Heat equation:
$$u_t = \alpha u_{xx}, \quad a \le x \le b, t \ge 0$$
- Boundary condition:
$$u(a,t) = g(t); \quad u(b,t) = h(t).$$
- Initial condition:
$$u(x,0) = f(x).$$

Anal

Grid with Δx and Δt will be
$$x_i = a + i\Delta x;\ \Delta x = \frac{b-a}{N},\ i = 0, 1, \ldots, N$$
and
$$t_k = k\Delta t;\ \Delta t = T/M,\ k = 0, 1, \ldots, M.$$
Use forward difference in time, central difference in space, one obtains iterative scheme inside the domain:
$$u_{i,k} = \rho u_{i-1,k-1} + (1-2\rho)u_{i,k-1} + \rho u_{i+1,k-1}, \quad 1 \le i \le N-1, 1 \le k \le M$$
where
$$\rho = \frac{\alpha \Delta t}{(\Delta x)^2}.$$
It is shown that $\rho < 1/2$ is needed for its stability.

Para

- $\alpha = 1, a = 0, b = 1;$
- $f(x) = \sin(\pi x)$
- $g(t) = h(t) = 0.$
- Exact solution:
$$u(x,t) = e^{-\pi^2 t}\sin(\pi x).$$

Implementation - 01

We first choose $\rho = 0.4$ other parameters given below.

```
al = 1 #alpha
dx = .2 #space mesh size
rho = .4 #conditinal number, to be less than .5 for the stability
dt = rho*(dx**2)/al #time step size
```

- (todo) find L^∞ error between exact solution and ftcs solution
- (todo) plot three error curves in one figure corresponding to $t = 1.95, 1.97, 1.98.$
- (todo) plot a surface of eact solution.

Implementation - 02

We change $\rho = 1$ with others unchanged.

- find L^∞ error between exact solution and ftcs solution
- plot three error curves corresponding to $t = 1.88, 1.92, 1.96$
- observe intersections of three curves, if any. identify erros at each intersections.
- (**A-bonus**) can you reveal why three error curves have common intersections?