note_fourier_carr_madan

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1 Fourier transform by Carr-Madan

1.1 Abstract

Our goal is

- Use alternative Fourier transform method proposed by Carr and Madan for european call pricing whenever characteristic function is available for its log price.

1.2 Analysis

Propostion

Let interest rate be r and the characteristic function of $\ln(S_T)$ be ϕ. Then, the price $C(K)$ of Call(T, K) satisfies, for any $\alpha > 0$,

$$C(K) = \frac{e^{-\alpha \ln K}}{\pi} \int_0^\infty e^{-iv \ln K} \psi(v) dv,$$

where

$$\psi(v) = \frac{e^{-rT} \phi(v - (\alpha + 1)i)}{\alpha^2 + \alpha - v^2 + i(2\alpha + 1)v}.$$

Q. Does the above proposition still hold if $\alpha = 0$?

1.2.1 Proof

step1

Let $k = \ln K$ be the log strike, and $c(k)$ be

$$c(k) = e^{\alpha k} C(e^k).$$

Then, Fourier transform of $c(K)$

$$\psi(v) = \int_{-\infty}^\infty e^{ivk} c(k) dk$$

exists, and the inverse transform of $\psi(v)$ recovers $c(k)$, i.e.

$$c(k) = \frac{1}{2\pi} \int_{-\infty}^\infty e^{-ivk} \psi(v) dv = \frac{1}{\pi} \int_0^\infty e^{-ivk} \psi(v) dv$$

The second equality is valid since ψ is odd in its imaginary part and even in its real part (why?).

step2

Therefore, it's enough to show the representation of ψ in terms of ϕ. This can be done Fubini's theorem.