hw_vasicek_calibration

February 14, 2019

1 Vasicek model calibration

Ref Chapter 7 of [Hir13]

Our goal is to design pricing engine and calirate vasicek model from data

1.1 Pricing formula for Vasicek model

Notations: - $P(t, T)$: zero coupon bond price - $L(t, T)$: LIBOR rate - $s(t, T, N)$: swap rate with N terms on $[t, T]$

The relation to the pricing formulas are

$$L(t, T) = \frac{100}{T - t}\left(\frac{1}{P(t, T)} - 1\right)$$

and

$$s(t, T, N) = 100\frac{1 - P(t, T)}{\Delta \sum_{j=1}^{N} P(t, t + j\Delta)}, \text{ where } \Delta = \frac{T - t}{N}.$$

If we further assume Vasicek model for the short rates with parameter $\theta = (\kappa, \mu, \sigma, r_0)$, i.e.

$$dr_t = \kappa(\mu - r_t)dt + \sigma dW_t, \text{ with } r_0,$$

the rate has its explicit form given by

$$r_t = r_0 e^{-\kappa t} + \mu(1 - e^{-\kappa t}) + \sigma e^{-\kappa t} \int_0^t e^{-\kappa s} dW_s.$$

Todo

Verify the above explicit form r_t as the solution to the original vasicek model.

Todo

Design pricing engine of ZCB $P(0, T)$ using the above explicit formula.

(Hint) ZCB value is determined by

$$P(0, T) = \mathbb{E}[e^{-\int_0^T r_t dt}] = e^{A(0,T) - B(0,T) r_0}$$

with

$$B(t, T) = \frac{1 - e^{-\kappa(T-t)}}{\kappa}, \quad A(t, T) = \left(\mu - \frac{\sigma^2}{2\kappa^2}\right)[B(t, T) - (T - t)] - \frac{\sigma^2}{4\kappa} B^2(t, T).$$

Todo

Design alternative pricing engine of ZCB $P(0, T)$ using exact sampling.

(hint) $R(T) = \int_0^T r_t dt$ follows a normal distribution with

$$\mathbb{E}R_T = \mu T + (r_0 - \mu)\frac{1 - e^{-\kappa T}}{\kappa}$$

and

$$Var(R_T) = \frac{\sigma^2}{2\kappa^3}(2\kappa T - 3 + 4e^{-\kappa T} - e^{-2\kappa T}).$$

Todo

Compute ZCB $P(0,1)$ Libor $L(0,1)$ with the following parameters using above two different pricing engines.

In [1]:
```
'''======paras======='''
theta = [.1, .05, .003, .03]
kappa, mu, sigma, r0 = theta
```

Todo

find 10 term swap rates with term length 1/2 year, i.e $s(t = 0, T = 5, N = 10)$.

Todo

Pick a date, and using Libor market data of that data, callibrate Vasicek model. - Then compare market rate and calibrated rate in a plot. - You may use SSRE - You may do calibration twice using two pricing engines, and see which one is better. - You may use the following provided market data too.

In [3]:
```
import pandas as pd
dfLiborRate = pd.DataFrame({'maturity (months)': [1, 2, 3, 6, 12],
                            '20081029 rate(%)': [3.1175, 3.2738, 3.4200, 3.4275, 3.4213]
                            '20110214 rate(%)': [0.2647, 0.2890, 0.3140, 0.4657, 0.7975]
                          })
```

In [4]:
```
dfLiborRate
```

Out[4]:

	20081029 rate(%)	20110214 rate(%)	maturity (months)
0	3.1175	0.2647	1
1	3.2738	0.2890	2
2	3.4200	0.3140	3
3	3.4275	0.4657	6
4	3.4213	0.7975	12

Todo

Pick a date, and using swap market data of that data, callibrate Vasicek model. - Then compare market rate and calibrated rate in a plot. - You may use SSRE - You may do calibration twice using two pricing engines, and see which one is better. - You may use the following provided market data too.

In [5]:
```
dfSwapRate = pd.DataFrame({'term (year)': [2, 3, 5, 7, 10, 15, 30],
                            '20081029 rate(%)': [2.6967, 3.1557, 3.8111, 4.1497, 4.3638,
                            '20110214 rate(%)': [1.0481, 1.5577, 2.5569, 3.1850, 3.7225,
                          })
```

```
In [6]: dfSwapRate
```

```
Out[6]:    20081029 rate(%)  20110214 rate(%)  term (year)
        0            2.6967            1.0481            2
        1            3.1557            1.5577            3
        2            3.8111            2.5569            5
        3            4.1497            3.1850            7
        4            4.3638            3.7225           10
        5            4.3753            4.1683           15
        6            4.2772            4.4407           30
```