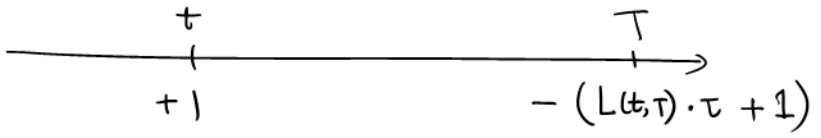
Introduction to Rate

Notation

① $P(t,T)$: zero coupon bond (ZCB)

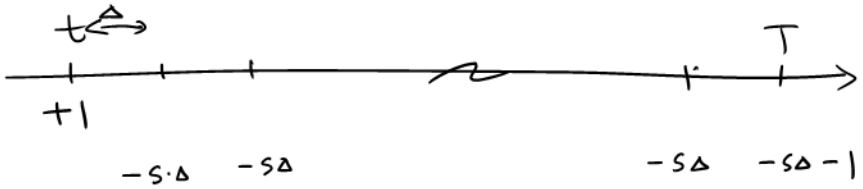
② $L(t,T)$: Libor rate, $\tau = T-t$



$$+1-(L(t,T)\tau+1)P(t,T)=0$$

$$L(t,T)=\frac{1-P(t,T)}{\tau P(t,T)}\cdot 100\%$$

③ $S(t,T,N)$: swap rate, $\Delta=\frac{\tau}{N}=\frac{T-t}{N}$



$$+1-S\cdot\Delta\cdot P(t,t+\Delta)-S\cdot\Delta\cdot P(t+2\Delta)-\cdots-S\Delta P(t,t+(N-1)\Delta)-S\Delta P(t,T)-1\cdot P(t,T)=0$$

$$1-P(t,T)=S\Delta\sum_{i=1}^{N}P(t,t+i\Delta)$$

$$S=\frac{1-P(t,T)}{\Delta\sum_{i=1}^{N}P(t,t+i\Delta)}\cdot 100\%$$

Relation between short rate & ZCB

Given

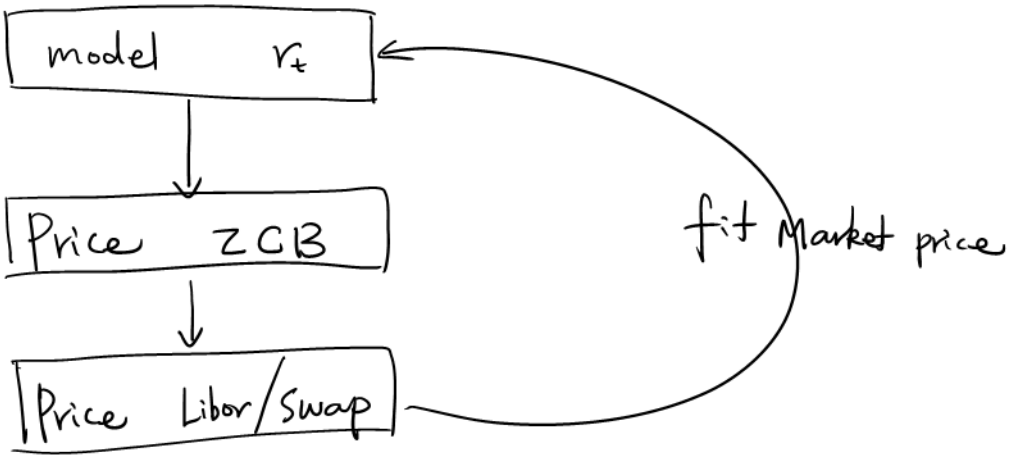
① short rate $r_t \sim$ SDE (stoch. differ. eqn).

② ZCB $P(t,T)$

FTAP (Fundamental thm of Asset Pricing) implies

$$P(t,T) = \mathbb{E}\left[e^{-\int_t^T r_s ds} \cdot 1 \,\middle|\, \mathcal{F}_t \right]$$

Calibration



$$C = \mathbb{E}\left[e^{-rT}(S_T - K)^+\right]$$

$$= \mathbb{E}\left[e^{-rT}\left((S_T - K)\,I(S_T \geq K) + 0 \cdot I(S_T < K)\right)\right]$$

$$= \underbrace{\mathbb{E}\left[e^{-rT} S_T\, I(S_T \geq K)\right]}_{\overset{?}{=}\ S_0\, I_1} - K e^{-rT} \underbrace{\mathbb{E}\left[I(S_T \geq K)\right]}_{\mathbb{P}(S_T \geq K)\ =\ I_2^?}$$

$$(S_T - K)^+ = \begin{cases} S_T - K & S_T \geq K \\ 0 & S_T < K \end{cases}$$

$$I(S_T \geq K) = \begin{cases} 1, & S_T \geq K \\ 0, & S_T < K \end{cases}$$

① $\displaystyle\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$

cont (i) $\mathbb{P}(S_T \geq K) = \mathbb{P}(\underbrace{\ln S_T}_{X} \geq \underbrace{\ln K}_{H})$

$$= I_2(\phi_{\ln S_T}, \ln K)$$

(ii) $\mathbb{E}\left[e^{-rT} S_T\, I(S_T \geq K)\right] \overset{?}{=} S_0 \cdot I_1$

$$= S_0 \cdot \frac{\mathbb{E}\left[e^{-rT} S_T\, I(\ln S_T \geq \ln K)\right]}{\mathbb{E}\left[e^{-rT} S_T\right]}$$

b/c Discounted stk price is Mtgl w.r.t. $\mathbb{Q}$

$S_0 = \mathbb{E}\left[e^{-rT} S_T\right]$

$$= S_0 \cdot \frac{\mathbb{E}\left[S_T\, I(\ln S_T \geq \ln K)\right]}{\mathbb{E}[S_T]}$$

($\ln S_T = X$, $\mathbb{E}[S_T] \to e^x$, $\ln K = H$)

= Lemma with $\ln S_T = X$, $\ln K = H$

$= S_0 \cdot I_1(\phi, H)$