hw_char_fun_normal

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Our goal is to derive the characteristic function for a normal distribution.

- (todo) Let
$$X_t = e^{iuW_t}$$
for a brownian motion W. Find
$$\phi(t) = \mathbb{E}[X_t].$$

- (todo) Let
$$Y \sim \mathcal{N}(\mu, \sigma^2)$$
be a random variable. Prove that its characteristic function
$$\phi(u) = \mathbb{E}[e^{iuY}]$$
is
$$\phi(u) = e^{iu\mu - \frac{1}{2}u^2\sigma^2}.$$