classmate

Date

Page

LINEAR ALGEBRA

UE18MA251

Jayant Sipani

PES1201800173

CSE, 4th SEM

ASSIGNMENT - 5

1) $y = A + Bx + Cx^2$

$$1 = A + B + C$$
$$-1 = A + 2B + 4C$$
$$1 = A + 3B + 9C$$

$$Ax = b$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$\Rightarrow$ $\underline{C = 2}$ $B = -8$

$A = 7$

$\boxed{\therefore \text{Eqn. of Parabola} = 2x^2 - 8x + 7}$

Scanned with CamScanner

2)
$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $\qquad l_{21} = 2$

$R_3 \to R_3 + 5R_1$ $\qquad l_{31} = -5$

$R_4 \to R_4 - 5R_1$ $\qquad l_{41} = 5$

$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 12 \\ 0 & -4 & 11 & -31 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$ $\qquad l_{32} = -2$

$R_4 \to R_4 + 2R_2$ $\qquad l_{42} = -2$

$$\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$l_{43} = 3$

$$A \sim \underbrace{\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}}_{} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$$A = LU$$

3) $T(x, y, z) = [x+2y-z, y+z, x+y-2z]$

$T: R^3 \to R^3$

(i) std. basis for $R^3 = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T = (1,0,0) = (1,0,1)$
$T = (0,1,0) = (2,1,1)$
$T = (0,0,1) = (-1,1,-2)$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

(ii) $$T = \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & -2 & b_3 \end{bmatrix} \quad R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & -1 & -1 & b_3 - b_1 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_2 + b_3 - b_1 \end{bmatrix}$$

$\rho(T) = 2$

Basis of $c(T) = \{(1,0,1), (2,1,1)\}$

Scanned with CamScanner

Date
Page

$R_1 \to R_1 - 2R_2$

$$T \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Free variables

Pivot variables

Expressing Free variables in terms of pivot variables

$$z = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Basis of $N(T) = \{(3, -1, 1)\}$
Basis of $C(T^T) = \{(1, 2, -1), (0, 1, 1)\}$

Left Null Space: $\rho(A) = \rho[A \; b]$
if $b_2 + b_3 - b_1 = 0$

$A^T y = 0$

Basis of $N(T^T) = \{(-1, 1, 1)\}$

(iii) Eigen values & Eigen vectors of T

$$|T - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 = \pm\sqrt{3}$$

$\Rightarrow \lambda = 0$ or $\lambda = \sqrt{3}$ or $\lambda = -\sqrt{3}$

Scanned with CamScanner

When $\lambda = 0$

$$T - \lambda T = T$$

Eigen vector is nullspace of $T = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ for $\lambda = 0$

When $\lambda = \sqrt{3}$

$$T - \lambda T = \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix}$$

$$R_3 \to R_3 - \left(\frac{1}{1-\sqrt{3}}\right) R_1$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1 - \frac{2}{1-\sqrt{3}} & -2-\sqrt{3} + \frac{1}{1-\sqrt{3}} \end{bmatrix}$$

$$R_2 \to R_3 - \left(\frac{1 - 2/_{1-\sqrt{3}}}{1-\sqrt{3}}\right) R_2$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & -2-\sqrt{3} + \frac{1}{1-\sqrt{3}} - \frac{1-\sqrt{3}-2}{(1-\sqrt{3})^2} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to R_1 - \left(\frac{2}{1-\sqrt{3}}\right) R_2$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 0 & -1 - \frac{2}{1-\sqrt{3}} \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-1+\sqrt{3}-2}{(1-\sqrt{3})^2} \\ 0 & 1 & \frac{1}{1-\sqrt{3}} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{-1+\sqrt{3}-2}{(1-\sqrt{3})^2}$$

$$\Rightarrow \frac{-3-\sqrt{3}}{2}$$

$$\frac{1}{1-\sqrt{3}} \Rightarrow \frac{-1-\sqrt{3}}{2}$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-3-\sqrt{3}}{2} \\ 0 & 1 & \frac{-1-\sqrt{3}}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Eigen vector when $\lambda = \sqrt{3}$ is $\begin{bmatrix} \frac{3+\sqrt{3}}{2} \\ \frac{1+\sqrt{3}}{2} \\ 1 \end{bmatrix}$

Scanned with CamScanner

When $\lambda = -\sqrt{3}$

$$T - \lambda I = \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix}$$

$$R_3 \to R_3 - \left(\frac{1}{1+\sqrt{3}}\right) R_1$$

$$\sim \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & \frac{1-2}{1+\sqrt{3}} & -2+\sqrt{3}+\frac{1}{1+\sqrt{3}} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & -2+\sqrt{3}+\frac{1}{1+\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{-1-\frac{2}{1+\sqrt{3}}}{1+\sqrt{3}} \end{bmatrix}$$

$$R_1 \to R_1 - \left(\frac{2}{1+\sqrt{3}}\right) R_2$$

$$\sim \begin{bmatrix} 1+\sqrt{3} & 0 & -1-\frac{2}{1+\sqrt{3}} \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-1}{1+\sqrt{3}} - \frac{2}{(1+\sqrt{3})^2} \\ 0 & 1 & \frac{1}{1+\sqrt{3}} \\ 0 & 0 & 0 \end{bmatrix}$$

Scanned with CamScanner

$$\frac{-1}{1+\sqrt{3}} - \frac{2}{(1+\sqrt{3})^2} \Rightarrow \frac{-3+\sqrt{3}}{2}$$

$$\frac{1}{1+\sqrt{3}} \Rightarrow \frac{1-\sqrt{3}}{-2}$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-3+\sqrt{3}}{2} \\ 0 & 1 & \frac{1-\sqrt{3}}{-2} \\ 0 & 0 & 0 \end{bmatrix}$$

Eigen vector for $\lambda = -\sqrt{3}$

$$\left(\frac{3-\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}, 1\right)$$

(iv) $T = QR$ decomposition

$$q_1 = \frac{a}{\|a\|} = \left[\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right]$$

$$q_2 = \frac{b}{\|B\|} = \left[\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}, \frac{-1}{\sqrt{6}}\right]$$

$\|B\| = \sqrt{\frac{3}{2}}$

$$q_3 = \frac{c}{\|c\|} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = [0, 0, 0]$$

q_3

$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix}$$

$q_1^T a = \sqrt{2}$

$q_1^T b = 3/\sqrt{2}$

$q_1^T c = -3/\sqrt{2}$

$q_2^T b = 3/\sqrt{6}$

$q_2^T c = 3/\sqrt{6}$

$q_3^T c = 0$

$$R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = QR$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2/3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{3} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}$$

4)

x	-4	1	2	3
y	4	6	10	8

$$y = c + dx$$

$$4 = c - 4d$$

$$6 = c + d$$

$$10 = c + 2d$$

$$8 = c + 3d$$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

To find $\hat{x}$

$$A^T A \hat{x} = A^T b$$

$$A^T A = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}$$

$$(A^T A)^{-1} A^T = \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & 24 \\ -18 & 2 & 6 & 10 \end{bmatrix}$$

$$(A^T A)^{-1} A^T b = \frac{1}{116} \begin{bmatrix} 772 \\ 80 \end{bmatrix}$$

$$= \begin{bmatrix} 193/29 \\ 20/29 \end{bmatrix} = \begin{bmatrix} c \\ d \end{bmatrix}$$

$\therefore$ Best fit line $\Rightarrow$ $y = cx + d$

$$\boxed{y = \frac{193}{29} + \frac{20}{29}x}$$

5) $Q = A(A^T A)^{-1} A^T$

$P = I - Q$

Eqn. of plane $x_1 + x_2 + 3x_3 + 4x_4 = 0$

$$A = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

$$A^TA = [27]$$

$$A^TA^{-1} = \left[\frac{1}{27}\right]$$

$$Q = \frac{1}{27} \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 4 \end{bmatrix}$$

$$= \frac{1}{27} \begin{bmatrix} 1 & 1 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 12 & 16 \end{bmatrix}$$

$$P = I - Q$$

$$= \frac{1}{27} \begin{bmatrix} 27 & 0 & 0 & 0 \\ 0 & 27 & 0 & 0 \\ 0 & 0 & 27 & 0 \\ 0 & 0 & 0 & 27 \end{bmatrix} - \frac{1}{27} \begin{bmatrix} 1 & 1 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 12 & 16 \end{bmatrix}$$

$$P = \frac{1}{27} \begin{bmatrix} 26 & -1 & -3 & -4 \\ -1 & 26 & -3 & -4 \\ -3 & -3 & 18 & -12 \\ -4 & -4 & -12 & 11 \end{bmatrix}$$

Classmate
Date
Page

6) $A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$

$R_2 \rightarrow R_2 - \left(\frac{2}{a}\right) R_1$

$R_3 \rightarrow R_3 - \left(\frac{2}{a}\right) R_1$

$$A = \begin{bmatrix} a & 2 & 2 \\ 0 & \frac{a^2-4}{a} & \frac{2a-4}{a} \\ 0 & \frac{2a-4}{a} & \frac{a^2-4}{a} \end{bmatrix}$$

$R_3 \rightarrow R_3 - \left(\frac{2a-4}{a^2-4}\right) R_2$

$$\sim \begin{bmatrix} a & 2 & 2 \\ a & \frac{a^2-4}{a} & \frac{2a-4}{a} \\ 0 & 0 & \frac{a^2-4}{a} - \left(\frac{2a-4}{a^2-4}\right)\left(\frac{2a-4}{a}\right) \end{bmatrix}$$

$\therefore\ a > 0 \quad ; \quad \frac{a^2-4}{a} > 0$

$\Rightarrow a > 2$

$$\frac{a^2-4}{a} - \frac{(2a-4)^2}{a(a^2-4)} > 0$$

$\Rightarrow a > 2 \ \& \ a > -4 \quad (\because a > 0)$

$\Rightarrow a \in (2, \infty)$

In this interval, A is positive definite

$$f = x^T A x = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$$

$$= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2 + 2a_{23}x_2x_3 + a_{33}x_3^2 + 2a_{13}x_1x_3$$

Comparing the co-efficients,

$a_{11} = 2$ $\quad a_{12} = -1$

$a_{22} = 2$ $\quad a_{23} = -1$

$a_{33} = 2$ $\quad a_{13} = 0$

$\therefore$ Required Matrix is

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Scanned with CamScanner

1)

$$A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

To find Eigen values $|A^TA - \lambda I| = 0$

$$\begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 90\lambda = 0$$

$$\lambda = 0 \quad \text{or} \quad \lambda = 90$$

Eigen vectors, for $\lambda = 0$

$$\begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 81 & -27 \\ 0 & 0 \end{bmatrix}$$

Eigen vector is $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

for $\lambda = 90$

$$A - 90I = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}$$

Eigen vector is $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Scanned with CamScanner

$$V_1 = \frac{x_1}{\|x_1\|} = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

$$V_2 = \frac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

$\because$ x_1 & x_2 are orthogonal

$$V = [v_1 \quad v_2] = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \\ 0 & 0 \end{bmatrix} \qquad \sqrt{\lambda_1} > \sqrt{\lambda_2}$$

$$= \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\sigma_1 = \sqrt{\lambda_1}$

$\sigma_2 = \sqrt{\lambda_2}$

σ_1 & σ_2 are singular values of A

$\sigma_1 = \sqrt{90} \qquad \sigma_2 = 0$

$$u_1 = \frac{Av_1}{\sigma_1} \quad \text{iff } \sigma_1 \neq 0$$

Eigen values of AA^T are 90, 0, 0

For $\sigma_1 = \sqrt{90}$, $u_1 = \dfrac{Av_1}{\sigma_1} = \dfrac{1}{\sqrt{90}} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$

$$u_1 = \begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix}$$

For $\sigma_2 = 0$, Previous formula can't be used

wkt u_2 & u_3 are orthogonal vectors associated with eigen values of AA^T.

$\therefore \lambda = 0 \quad (AA^T - 0I)x = 0$

$Ax = 0$

$$\begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

To find Null space,

$$\begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$y = \begin{bmatrix} 20 \\ 1 \\ 0 \end{bmatrix} \qquad z = \begin{bmatrix} -20 \\ 0 \\ 1 \end{bmatrix}$

or $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

Scanned with CamScanner

classmate

Date

Page

$\therefore$ $x_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ & $x_3 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ are eigen vectors of AA^T when $\lambda = 0$

x_2 & x_3 are orthogonal to V_1

To find u_2 & u_3, apply Gram Schmidt process on x_2 & x_3

To find u_2, $\mu_2 = \frac{x_2}{\|x_2\|} = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix}$

To find μ_3, find vector $\perp$ to both μ_1 & μ_2

$$c = x_3 - (\mu_1 - x_3)\mu_1 - (\mu_2 + x_3)\mu_2$$

$$\Rightarrow \mu_3 = \frac{c}{\|c\|} = \begin{bmatrix} 2/3\sqrt{5} \\ -4/3\sqrt{5} \\ \sqrt{5}/3 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & \sqrt{5}/3 \end{bmatrix}$$

$$A = u \Sigma V^T = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & \sqrt{5}/3 \end{bmatrix} \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$