# A Lyapunov Optimization Approach to the Quality of Service for Electric Vehicle Fast Charging Stations


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***Abstract*—In this paper, an optimal power allocation strategy of a fast charging station (FCS) is presented to maximize the quality of service (QoS) of electric vehicle (EV) charging when the charging demand is greater than the available power at the station. In this regard, the problem is formulated in the form of a Lyapunov optimization problem that enables solving the problem online while eliminating the need for future information. The QoS is modeled through Jain's fairness index to account for the charging time increase for each customer due to the power limit. Moreover, each EV's demand is characterized by the amount of charging power. The power is calculated with the help of a second-order equivalent circuit model of the battery pack of the EV. Lastly, the problem is evaluated in a case study.**

***Index Terms*—EV fast charging, fast charging station, Lyapunov optimization, quality of service**

## I. Introduction

Electric vehicles (EVs) powered by renewable generations have marked advantages of mitigating our climate impact, diminishing operational costs, and providing better torque profile [1]. Moreover, EVs can be leveraged as electricity network storage [2], time-shiftable loads which can be exploited to handle the inherent intermittency of the renewable power plants [3], and flexible loads that help the demand-side management in the paradigm of electricity markets [4], [5]. On the other hand, EV development is challenging. The main bottleneck of EV technology advancement is the speed of recharging EVs battery compared to that of refueling conventional vehicles. This multifaceted problem boils down to the lithium-ion battery chemistry and the limitations pertaining to fast-charging stations (FCSs).

In order to address the fast charging impediments that stem from the battery chemistry, researchers are, on the one hand, trying to discover/invent new materials to be used in lithium-ion batteries; on the other hand, they are working on sophisticated battery models, such as electrochemical models [6], and state-of-the-art optimization strategies [7]. Additionally, drawing immense power from the electricity network disturbs the power grid by inducing voltage deviation, thermal loading, and feeder overloading, which are more pronounced at the distribution level [8]. Consequently, there is ongoing research on reducing the amount of power FCSs absorb from the network. In this respect, having renewable generation on the site of FCS and/or utilizing energy storage systems are proposed in the literature [1]. Furthermore, efficient and optimal siting and sizing of the FCSs are helpful in that the power limitation imposed on an FCS follows the topology of the surrounding distribution network. The complexity of the optimization problem comes from the need to consider the configuration of both distribution and transportation networks to find the optimal location and size of the station [9]. In addition, resolving the issues of FCSs for the electricity network should be dealt with alongside enhancing the quality of service (QoS) for the customers, i.e., shortening the charging time whilst satisfying the charging demand. QoS is the measurement of the overall performance of the system seen by the user. In the scope of FCSs, QoS can be defined as delivering the required energy to the customers, waiting times in the queue at the station, charging time, variations of charging power, etc. [10]. The studies on QoS optimization mostly focus on the problems of pricing and queuing frameworks [11].

In the interaction between power limitation on the FCS imposed by the electricity grid and improving the QoS, the priority is on the stability and health of the distribution network. Therefore, sometimes the maximum allowed power limit is less than the total charging demand at the station. In this circumstance, a predetermined power allocation policy is applied to reduce the power delivered to each EV being charged [1]. However, the division of power between charging piles (CPs) can be formulated and solved as an optimization problem. To the best of our knowledge, there has been no study on optimizing the power allocation to the CPs in the fast charging station when demand exceeds the station's power cap.

In this paper, we propose an optimization problem to find the optimal power allocation for each CP where an EV is connected under limited power situations. The problem is defined in the form of a Lyapunov optimization problem. Lyapunov optimization approach relaxes the need for future information [12] and decomposes an optimization problem into several sub-problems that are solved at each time step [13]. The advantage of this method is that the lack of historical data that disables us from accurately predicting the future is not an issue. Additionally, decomposing the problem boosts the speed of problem-solving. It decreases the processing time, while at the same time, the Lyapunov optimization guarantees the result is near the global optimal solution.

The remaining sections of the paper are as follows. The problem is formulated in Section II, and an online algorithm is introduced to implement Lyapunov optimization. Section III presents the simulation results and Section IV concludes the paper.

## II. Problem Formulation

The optimal power allocation problem is formulated using Lyapunov optimization in this section. In the following, Jain's fairness method [1], Lyapunov function, Lyapunov drift equation, and the iterative process of solving the problem are presented.

### A. Optimization Problem

In this work, we consider a case where the total demand exceeds the available power at the station. Therefore, a simple power allocation between different CPs is to reduce the power at each CP evenly. However, uniform power reduction at each CP may not be fair because the EVs are at a different stage of the charging process. For example, if one EV has just been connected to be charged and another vehicle requires a couple of minutes to finish the charge, then identically reducing the available power for them may not be the optimal choice seeing that they may experience different percentage increases in charging time. Accordingly, the system time ratio (STR) is used as a metric to quantify QoS as follows [1]

$$x_i(t) = \frac{T_r^l(t)}{T_r(t)}, \tag{1}$$

where $x_i(t)$ denotes STR at the $i$th CP, $T_r^l(t)$ indicates the required remaining charging time considering the limited available power, and $T_r(t)$ represents the required remaining charging time without power limit.

In the following, Eq. (2) computes $x_i(t)$ based on the remaining charging request and charging power as

$$x_i(t) = \frac{E_i^r(t)/P_i(t)}{E_i^r(t)/P_i^b(t)} = \frac{P_i^b(t)}{P_i(t)}, \tag{2}$$

where $E_i^r(t)$ is the remaining energy required for charging the EV at the $i$th CP, $P_i^b(t)$ shows the charging power if the EV was allowed to draw as much power as needed from the station, and $P_i(t)$ indicates the charging power under limited available power at FCS. Therefore, $E_i^r(t)/P_i(t)$ and $E_i^r(t)/P_i^b(t)$ represent charging time. It should be noted that $P_i(t) \leq P_i^b(t)$.

Optimal QoS is obtained through maximizing Jain's fairness score for STR of vehicles. The maximum score, i.e., one, is achieved when all EVs have equal STRs. Jain's fairness score is defined as the following [1],

$$S(t) = \frac{\left[\sum_{i=1}^{N} x_i(t)\right]^2}{N \sum_{i=1}^{N} x_i^2(t)}, \tag{3}$$

where $S(t)$ represents QoS and $N$ is the number of CPs at the station. The value of $S(t)$ is between zero and one, i.e., $0 \leq S(t) \leq 1$. The maximum $S(t)$ is obtained if all $x_i(t)$ are equal.

Consequently, the problem of optimal power allocation is as the following,

$$\min_{P_i(t)} \quad \left[-\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{S(t)\}\right], \tag{4.a}$$

subject to

$$\bar{Q} < \infty, \tag{4.b}$$

$$\sum_{i=1}^{N} P_i(t) \leq P_s(t), \ \forall t \tag{4.c}$$

$$P_i(t) \leq P_i^b(t), \ \forall t \tag{4.d}$$

$$P_i(t) \geq 0, \ \forall t \tag{4.e}$$

where $\bar{Q} = \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\{Q(t)\}$ is the time average of $Q(t)$ in (5) that should be finite, and $P_s(t)$ shows the power limit at the FCS. $Q(t)$ is the total unmet demand of EVs at the station, excluding the vehicles in the queue. In other words, $Q(t)$ only includes the total remaining requested energy by EVs that are being charged and is formulated as

$$Q(t+1) = \max[Q(t) - e(t), 0] + a(t), \tag{5}$$

where $e(t)$ expresses the total amount of delivered energy to EVs at time slot $t$ and $a(t)$ is the total demand of EVs that begin their charging process at $t$. Hence, the following hold,

$$e(t) = \Delta t \sum_{i=1}^{N} P_i(t), \ \forall t \tag{6}$$

$$Q(1) = \sum_{i=1}^{N} E_i^r(1), \tag{7}$$

where $\Delta t$ is the length of each time slot. Equation (5) is a virtual queue expression that replaces the constraint on $\bar{Q}$ in the optimization problem.

Next, the Lyapunov function is defined as the following positive-definite quadratic function,

$$L[Q(t)] \triangleq \frac{1}{2}Q^2(t). \tag{8}$$

This function is positive if there is unsatisfied demand and is zero when all EVs are completely charged.

Then, the Lyapunov drift is defined as [12]

$$\Delta[Q(t)] \triangleq \mathbb{E}\{L[Q(t+1)] - L[Q(t)] | Q(t)\}. \tag{9}$$

Lyapunov drift shows the expected change of Lyapunov function over time.

Lastly, the drift-plus-penalty term is [12]

$$\Delta[Q(t)] - V\mathbb{E}\{S(t)|Q(t)\}, \tag{10}$$

where $V$ is a non-negative constant that provides a trade-off between the QoS and satisfied demand. According to Lyapunov optimization, if a constant upper bound exists for the Lyapunov drift function $\Delta[Q(t)]$ at each time slot, then the stability constraint, i.e., $\bar{Q} < \infty$, is met. Furthermore, optimizing the penalty term minimizes the time-average of QoS, which is equivalent to the minimization of the objective function in (4.a). We derive the upper bound expression for the Lyapunov drift in the following.

**Theorem 1.** *At any time slot t, the Lyapunov drift-plus-penalty function has the following upper bound:*

$$\begin{aligned}\Delta[Q(t)] - V\mathbb{E}\{S(t)|Q(t)\} \leq B +\\ Q(t)\mathbb{E}\{a(t)-e(t)|Q(t)\} - V\mathbb{E}\{S(t)|Q(t)\},\end{aligned} \tag{11}$$

*where $B$ is a positive constant defined as follows*

$$B = \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right). \tag{12}$$

*Proof.* We start with taking the square of $Q(t+1)$ as follows

$$Q^2(t+1) = \left(\max[Q(t)-e(t),0] + a(t)\right)^2.$$

With the help of the following inequality [14],

$$(\max[b-c,0]+a)^2 \leq a^2 + b^2 + c^2 + 2b(a-c),$$

we have

$$Q^2(t+1) \leq Q^2(t) + e^2(t) + a^2(t) + 2Q(t)\left(a(t)-e(t)\right).$$

Note that $e(t)$ is the amount of energy delivered to EVs at time slot $t$. Assume that each of the $N$ CPs have a power rating of $P_{\text{CP}}^{\max}$, then the maximum possible value for $e(t)$ is $e_{\max} = NP_{\text{CP}}^{\max}\Delta t$. On the other hand, $a(t)$ indicates the total energy added to the demand at time slot $t$. As there are $N$ CPs, at most, $N$ newly arrived EVs can be connected to the station and start charging. If the maximum battery capacity of the EVs in the system is $C_b^{\max}$, then the upper bound of $a(t)$ is $a_{\max} = NC_b^{\max}$. Thus, by substituting $e_{\max}$ and $a_{\max}$ in the inequality, the following is obtained,

$$\begin{aligned}\frac{1}{2}\left[Q^2(t+1) - Q^2(t)\right] \leq \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right)\\ + Q(t)\left(a(t)-e(t)\right).\end{aligned}$$

The left-hand side of the inequality is the difference between Lyapunov functions at time slots $t+1$ and $t$,

$$L[Q(t{+}1)] - L[Q(t)] \leq \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right) + Q(t)\left(a(t)-e(t)\right).$$

By taking conditional expectations with respect to $Q(t)$ from both hand sides, we obtain

$$\begin{aligned}\mathbb{E}\{L[Q(t+1)] - L[Q(t)]|Q(t)\} \leq \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right)\\ + Q(t)\mathbb{E}\{a(t)-e(t)|Q(t)\}.\end{aligned}$$

**Algorithm 1** Minimizing Upper Bound

**Inputs:** $V$, $P_s(t)$, $\Delta t$, $a(t)$, $E_i^r(1)$
**Outputs:** $P_i(t)$
**Initialization:** $Q(1) \leftarrow \sum_{i=1}^N E_i^r(1)$

```
for t ∈ [1, T] do
    find P_i(t) by solving problem (13)
    e(t) = Δt Σ_{i=1}^N P_i(t)
    update a(t)
    Q(t) ← max[Q(t) − e(t), 0] + a(t)
end for
```

The left-hand side of the expression is identical to Eq. (9); therefore, the upper bound of the Lyapunov drift is

$$\Delta[Q(t)] \leq \frac{1}{2}\left(e_{\max}^2 + a_{\max}^2\right) + Q(t)\mathbb{E}\{a(t)-e(t)|Q(t)\}.$$

By adding the penalty term to both hand sides of the inequality, we obtain the upper bound of the drift-plus-penalty function as follows

$$\begin{aligned}\Delta[Q(t)] - V\mathbb{E}\{S(t)|Q(t)\} \leq B +\\ Q(t)\mathbb{E}\{a(t)-e(t)|Q(t)\} - V\mathbb{E}\{S(t)|Q(t)\}.\end{aligned}$$

□

Subsequently, an iterative algorithm is proposed to minimize the upper bound of (11) for each time slot through solving the following problems,

$$\min_{P_i(t)} \quad Q(t)\left(a(t)-e(t)\right) - V\cdot S(t), \tag{13.a}$$

subject to

$$\sum_{i=1}^N P_i(t) \leq P_s(t), \tag{13.b}$$

$$P_i(t) \leq P_i^b(t), \tag{13.c}$$

$$P_i(t) \geq 0, \tag{13.d}$$

We exploit the Algorithm 1 to derive the upper limit of the objective function in Eq. (4.a).

**Theorem 2.** *For all $T > 0$, the objective function in* (4.a) *has the following upper limit,*

$$-\frac{1}{T}\sum_{t=1}^T \mathbb{E}\{S(t)\} \leq \frac{B}{V} - p^* + \frac{1}{TV}\mathbb{E}\{L[Q(1)]\}, \tag{14}$$

*where $p^*$ is the maximum value of $\frac{1}{T}\sum_{t=1}^T \mathbb{E}\{S(t)\}$.*

*Proof.* Firstly, we show that, given enough time, the time-average of $a(t) - e(t)$ is zero. Assume that all EVs at each CP come and finish charging during the considered time period $[1, T]$, i.e., no EV will come before $t = 1$ or finish charging after $t = T$. Since the total amount of energy charged to the EVs equal to the total amount of energy required by the EVs at each CP, then for any $k$th CP, with $k \in \{1, 2, \cdots, N\}$, $\sum_{t=1}^T [a_k(t) - e_k(t)] = 0$.

Next, by taking expectations from both sides of the inequality (11), and summing over all time slots, we have

$$\mathbb{E}\{L[Q(T)]\}-\mathbb{E}\{L[Q(1)]\} - V\sum_{t=1}^{T}\mathbb{E}\{S(t)\} \leq$$
$$TB - VTp^* + \sum_{t=1}^{T} Q(t)\mathbb{E}\{a(t) - e(t)\}.$$

The condition with respect to $Q(t)$ is removed through the law of iterated expectations. Since the time average of $a(t) - e(t)$ is zero; the inequality becomes as the following,

$$\mathbb{E}\{L[Q(T)]\} - \mathbb{E}\{L[Q(1)]\} - V\sum_{t=1}^{T}\mathbb{E}\{S(t)\} \leq TB - VTp^*.$$

Moreover, $L[Q(T)]$ is a positive-definite function; therefore, $\mathbb{E}\{L[Q(T)]\} \geq 0$. Thus, we can omit this part from the left-hand side of the inequality and obtain

$$-\mathbb{E}\{L[Q(1)]\} - V\sum_{t=1}^{T}\mathbb{E}\{S(t)\} \leq TB - VTp^*.$$

By rearranging and dividing by $VT$, we get

$$-\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{S(t)\} \leq \frac{B}{V} - p^* + \frac{1}{TV}\mathbb{E}\{L[Q(1)]\}.$$

□

Theorem 1 holds for every time slot and only requires the information of the current time slot. Thereby, based on Theorem 2, minimizing the upper bound of each time slot leads to minimization of the time-average penalty term with at most $\mathcal{O}(1/V)$ deviation from the true optimal solution. Additionally, the problem is solved iteratively for every time slot as it is realized online. This helps decrease the computational burden of solving the problem seeing that the problem is effectively broken into $T$ sub-problems. Further, solving the problem does not require future forecasting, which involves uncertainty. The uncertainty in the future forecast leads to a sub-optimal solution without guaranteeing how far the sub-optimal solution will be from the true optimal solution. In contrast, in Lyapunov optimization, the drift from the optimal solution is restricted to $\mathcal{O}(1/V)$ and can be controlled through constant $V$ as it is user input. Finally, this optimization approach is implemented by executing the Algorithm 1.

## III. Simulation Results

This section presents simulation results to assess the efficacy of the Lyapunov optimization approach. A fast-charging station with 20 CPs is considered, where each CP has a power rating of 350 kW. In addition, we consider a case where the power limit of the station is 2 MW, i.e., $P_s(t) = 2$ (MW), while the regular power cap of the station is $20 \times 350 = 7$ (MW). The power limit may stem from a temporary voltage drop, the station's hardware failure, or the decision of the station owner to purchase less electricity due to high prices at peak hours. As such, the proposed methodology considers the QoS based on the percentage increase of charging time for each customer. The objective of the optimization problem does not factor in the cost or profit of the station because the maximum power is drawn from the station. Thus, as long as the EVs' demand exceeds the power limit of the station, the station will constantly sell 2 MW of power to the EVs.

This study assumes two types of vehicles with different battery packs. Both types have a battery pack with 16 modules. Each module comprises six groups of 74 parallel cells [2]. The difference between battery packs is that one type uses A123 ANR26650M1 cells with 2.1 Ah capacity [2], while the other utilizes Samsung ICR18650-26F lithium-ion cells with the capacity of 2.6 Ah [15]. The battery packs are characterized through a second-order equivalent circuit model. The packs' terminal voltage and charging current are used to calculate the charging power for each vehicle. This paper assumes that EVs are charged through the constant current-constant voltage (CC-CV) charging method. It is worth noting that the cell model is extended into the pack model following the proposed approach in [16]. The problem is solved using Bonmin solver in CasADi environment on a computer with 3.00 GHz CPU and 16 GB RAM [17]. Each iteration of the Lyapunov approach took less than 25 ms to process.

A span of 2000s is considered when the power limit of the station is limited to 2 MW. The simulation results are illustrated in Fig. 1. In the beginning, most of the CPs are occupied except CPs number 9, 10, 11, 12, and 13; the EV at CP 14 requires around 40 seconds to finish the charging process. The red dashed line curves depict how EVs would continue to be charged if the power limit of 2 MW had not been imposed. On the other hand, the solid blue lines represent the obtained results of the optimization problem. As it is seen, the total demand for EVs is not greater than 2 MW during the entire 2000s period. Based on the red dashed lines (without considering station power limit), EVs' required charging power surpasses the 2 MW limit in the first 138 seconds; then it drops below the limit until $t = 200$; it goes beyond 2 MW at $t = 200$ until $t = 240$ when it falls again and remains smaller than 2 MW for roughly a minute. Afterward, it violates the power limit for 37 seconds and drops below it for the rest of the period. Note that the optimization problem is unaware of these demand fluctuations. Lyapunov optimization solved the problem without future information or prediction.

The optimization problem results demonstrate that, during the first 151 seconds, the maximum power, i.e., 2 MW, is drawn from the station when the power limit is activated. In the next 49 seconds, the total demand is smaller than 2 MW until a new EV is connected to CP 9. Only for this case, a third type of EV is assumed, with twice as much battery and charge capacity. Therefore, CP 9 consumes over 300 kW power after $t = 500$, which is possible because other EVs have either fully charged and stopped absorbing energy from the station or reached the constant-voltage stage of the charging process when the charging power reduces. Subsequently, the total charging power of EVs rises to the limit for another 54 seconds, followed by 46 seconds of reduced

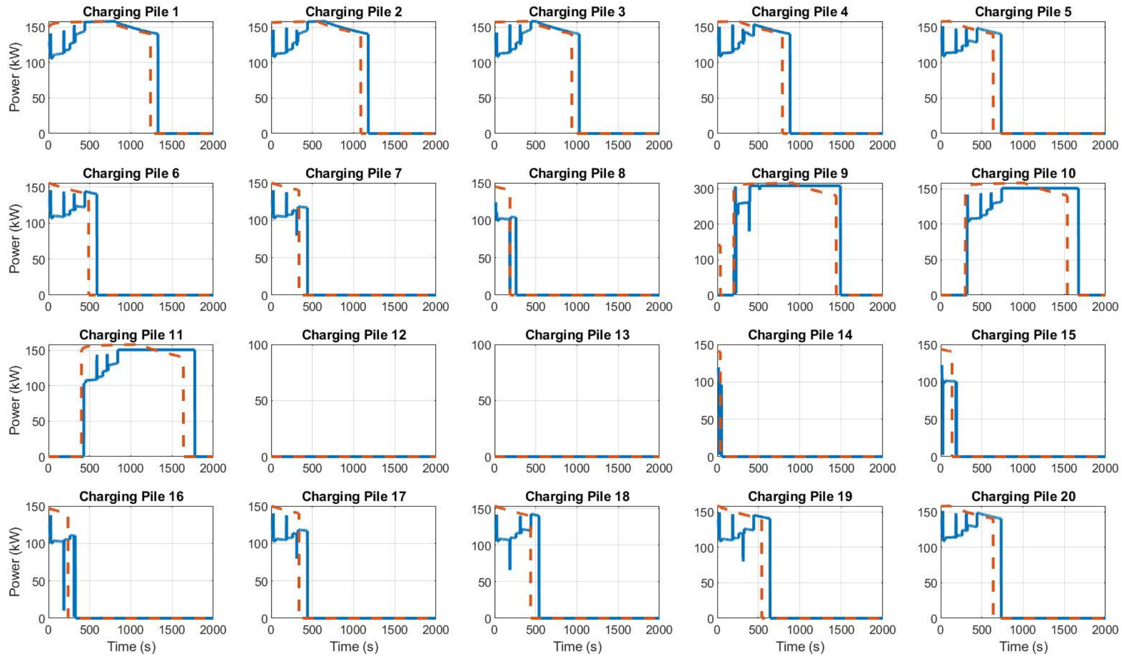


Fig. 1. Charging profile of all 20 CPs

total demand until a new EV connects to the CP 10 and starts to charge. Again, the total demand becomes 2 MW for 57 seconds before dropping below the limit. Finally, since a new EV does not come to the station, all vehicles complete their charging process in the next 22 minutes.

It can be seen that the optimized solutions, i.e., solid blue lines in Fig. 1, altered the CC-CV profile of the charging process when the power limit of 2 MW is imposed. In other words, mainly in the first 500 seconds, the charging profiles have shifted from CC-CV to a pulse-charge-like format. The reason lies in the fact that the requested power exceeds 2 MW; thus, the optimization problem has to allocate the power accordingly. Since the Lyapunov approach optimizes every time step, the power division between EVs is performed at each time slot in such a way that if the EVs were to continue the charging process with the allocated power, the percentage increase of the charging time for the customers was as close as possible. However, the power division in the next time slot may change because one EV may finish the charging, or some EVs may decrease their power demand as they approach the end of the charge. Consequently, when the objective is recalculated in the next time slot, the power allocation may be modified, leading to an overall profile analogous to a pulse-charge method while keeping the total delivered power at a constant value of 2 MW. For example, the EV at CP 15 completes the charging process at near $t = 200$, which frees up around 100 kW of power. Simultaneously, EVs at CPs 1-7, 17, 19, and 20 see a power surge, while those at CPs 8, 16, and 18 experience an abrupt power fall. These three EVs are going to finish their charge before the rest of the customers. The power drop extends the charging duration for customers at CPs 8, 16, and 18. Still, the ultimate percentage increase of their charging time is comparably similar to others, which is the goal of the optimization problem in this paper. On the other hand, the power alteration does not last long as in $t = 200$, an EV connects to the station at CP 9 and starts to charge.

In Fig. 1, in the time slots when the total demand is short of 2 MW, the charging profiles are mainly similar to that of the CC-CV method. In fact, the solid blue lines follow the red dotted ones with a delay that is caused due to the power limitation.

The obtained average QoS is 0.8. As mentioned earlier, the maximum possible value for the QoS is one. The fact that the QoS is smaller than one means that the percentage increase in charging time is slightly different for different customers. There are two reasons for this smaller QoS: i) In Lyapunov optimization, there is a trade-off between optimizing the objective function and minimizing the backlogged queue, which in our case is the total energy demand from the station. The trade-off is controlled by coefficient $V$, which is a user input value. Increasing the value of $V$ leads to a better optimized objective function at the cost of deteriorating the queue minimization. It means that an attempt to make the QoS tend to one will lead to an increased charging time for every CP. Put another way, QoS can be improved by extending the charging time of each CP because this is the only possible way to have identical STRs, as the power limit does not allow identical STRs without considerable extension of charging

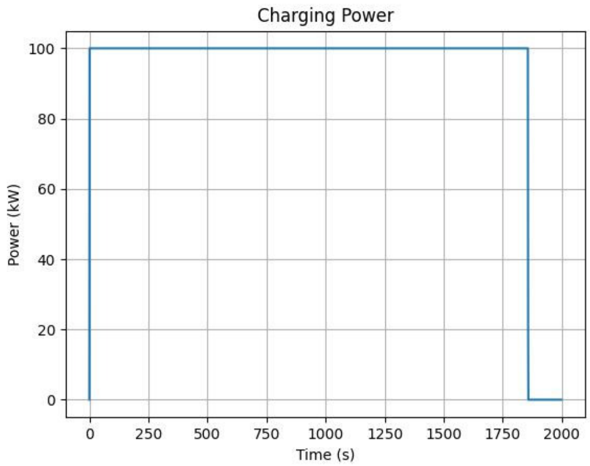


Fig. 2. Charging profile of the EV connected to CP 1 while the available power is uniformly allocated

time. ii) The EVs that finish their charging process sooner than others experience more charging time increases since their remaining charge process coincides with the time spans when the total demand is greater than 2 MW.

The results of Lyapunov optimization are compared with a case where, due to power limit, a uniform policy is adopted for power allocation [1]. In this regard, the 2 MW available power at the station is uniformly divided between 20 CPs, restricting each CP's power to a 100 kW limit. Fig. 2 shows the case for CP 1 where the EV is fully charged with a 100 kW power limit. The charging profile changed from CC-CV to a constant-power profile, and the charging time increased by 620 seconds. In contrast, the proposed approach extends the charging time by 92 seconds. Suppose the power limit is uniformly divided between those EVs being charged (not divided by the number of CPs, seeing that CPs 12 and 13 are unoccupied or some EVs finish the charging process leaving a CP unoccupied). In that case, the charging time for CP 1 will have a 207 seconds increase which is more than 100% higher than 92 seconds. Likewise, the EV at CP 7 will experience 143 seconds of increased charging time under uniform power allocation. In comparison, the proposed algorithm prolongs the charging time for CP 7 by only 103 seconds (38% improvement). Other customers experience similar enhancements, but the detail is not mentioned for brevity.

## IV. Conclusion

In this paper, the problem of optimal power allocation between CPs under limited available power at the FCS is formulated and solved using the Lyapunov optimization approach. The Lyapunov approach finds the best power allocation for every time step to maximize the QoS. Jain's fairness score is considered for the QoS metric, which tries to attain an identical percentage increase in EV charging time due to power limitation. The problem is decomposed into $T$ subproblems that are solved as each time slot realizes. The results demonstrate that the proposed methodology outperforms the power allocation policies commonly used in the literature. When power limitation is present, the CC-CV charging profiles are changed into a pulse-like charge; when the total demand is smaller than the power limit, the results follow the CC-CV profile.

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