

Arbitrage-Free Valuation of a Gas-Fired Tolling Agreement

Model Specification

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1 Objective

A gas-fired tolling agreement grants the holder the right, but not the obligation, to convert natural gas into electricity at a specified power plant. Economically, it represents a **strip of American-style call options on the spark spread**, with path dependency induced by start-up costs and unit-level dispatch constraints.

2 Market Characteristics

- **Natural Gas** (G_t): Storable commodity; forward curve derived from traded instruments.
- **Electricity** (P_t): Non-storable commodity; forward curve is primitive.

The power spot price is defined consistently as:

$$S_t := F_P(t, t).$$

3 Risk-Neutral Measure and No-Arbitrage

All valuation is performed under the risk-neutral measure \mathbb{Q} . No-arbitrage requires that traded forward prices are martingales.

3.1 Natural Gas Dynamics under \mathbb{Q}

$$dG_t = \mu_{\text{fwd}}(t)G_t dt + \sigma_g G_t dW_t^{\mathbb{Q},g}, \quad \mu_{\text{fwd}}(t) = \frac{\partial}{\partial t} \ln F_G(0, t) \quad (1)$$

Ensuring:

$$\mathbb{E}^{\mathbb{Q}}[G_t] = F_G(0, t).$$

This gives us:

$$G_t = F(0, t) \exp\left(-\frac{1}{2}\sigma_g^2 t + \sigma_g W_t^{\mathbb{Q},g}\right) \quad (2)$$

3.2 Power Dynamics under \mathbb{Q} (MRJD with Compensation)

The power price is modeled as:

$$P_t = F_P(0, t) \exp(X_t), \quad X_0 = 0 \quad (3)$$

where the stochastic factor X_t follows:

$$dX_t = -\kappa X_t dt + \sigma_p dW_t^{\mathbb{Q},p} + J dN_t - \lambda (\mathbb{E}[e^J] - 1) dt \quad (4)$$

with:

- N_t : Poisson process with intensity λ
- $J \sim \mathcal{N}(\mu_J, \sigma_J^2)$
- $\mathbb{E}[e^J] = \exp(\mu_J + \frac{1}{2}\sigma_J^2)$

Since:

$$\mathbb{E}[e^J] = \exp\left(\mu_J + \frac{1}{2}\sigma_J^2\right),$$

the compensator ensures:

$$\mathbb{E}^{\mathbb{Q}}[P_t] = F_P(0, t),$$

thus preserving no-arbitrage.

3.3 Correlation Structure

$$dW_t^{\mathbb{Q},g} dW_t^{\mathbb{Q},p} = \rho dt \quad (5)$$

4 Physical Measure (\mathbb{P})

The physical measure describes real-world price evolution and is used for risk management.

4.1 Gas

$$dG_t = \mu_{\text{real}} G_t dt + \sigma_g G_t dW_t^{\mathbb{P},g} \quad (6)$$

4.2 Power

$$d(\ln P_t) = \kappa(\theta_{\text{LRMC}}(t) - \ln P_t)dt + \sigma_p dW_t^{\mathbb{P},p} + J dN_t^{\mathbb{P}} \quad (7)$$

Jump intensities and diffusion drifts may differ between \mathbb{P} and \mathbb{Q} .

5 Plant Representation: Multiple Units

The plant consists of N_{units} independent units:

Unit	Heat Rate	Capacity	Start Cost
i	HR_i	Cap_i	$K_{\text{start},i}$

6 Spark Spread Payoff

For unit i at hour h :

$$\pi_{h,i} = (P_{t_h} - \text{HR}_i G_{t_h}) \cdot \text{Cap}_i \quad (8)$$

7 Optimal Dispatch Problem

Daily dispatch is obtained by:

$$\text{Daily Value}_D = \sum_{i=1}^{N_{\text{units}}} \max \left\{ \sum_{h=1}^{24} \pi_{h,i} - K_{\text{start},i}, 0 \right\} \quad (9)$$

A merit-order heuristic is applied for tractability.

8 Tolling Agreement Valuation

$$V_0 = \sum_{D=1}^{365} e^{-rT_D} \mathbb{E}^{\mathbb{Q}} [\text{Daily Value}_D] \quad (10)$$

9 Monte Carlo Valuation Algorithm

1. Simulate correlated (G_t, X_t) paths under \mathbb{Q} .
2. Construct $P_t = F_P(0, t)e^{X_t}$.
3. Compute hourly spark spreads for each unit.
4. Apply dispatch and start logic.
5. Average discounted cashflows.