An Introduction to Time Series Forecasting with Python

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An Introduction to Time Series Forecasting with Python

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What is a Time Series?

A sequential set of data points
measured over time.

$$x(t), t = 0, 1, 2, 3, \ldots$$

t - a variable that represents the elapsed time.

Examples

- **hourly** number of page views for a website
- **daily** air temperature in a city
- **monthly** average number of reported bugs
- **quarterly** income for a company
- **annual** population data in a country

A Time Series may contain information
about general tendency in data,
seasonal effects, occasional events, and so on.

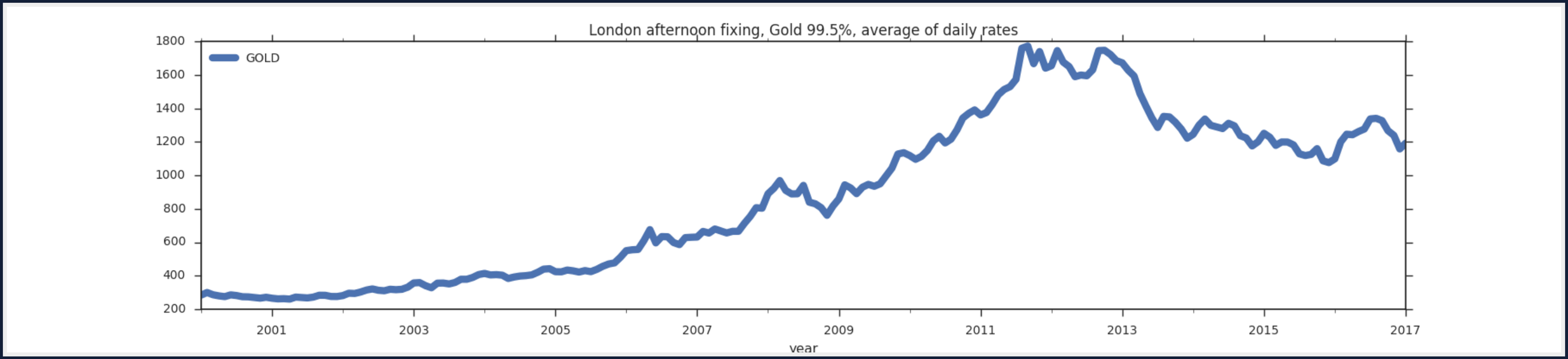
Time Series Decomposition

Trend

The general (long-term, non-periodic) tendency of a time series

⚠ Often modeled using the logistic growth model, or can be extracted from data using Moving Average technique

Example



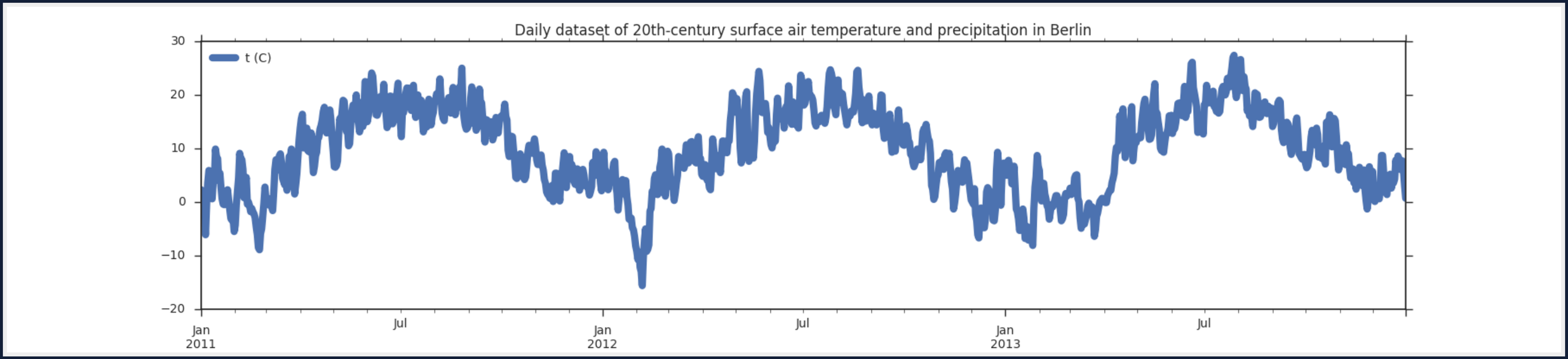
The WoldBank Dataset, London afternoon fixing, Gold 99.5%, average of daily rates

Seasonal

The periodic fluctuations caused by regular influences

⚠ Can be modeled using Fourier series

Example

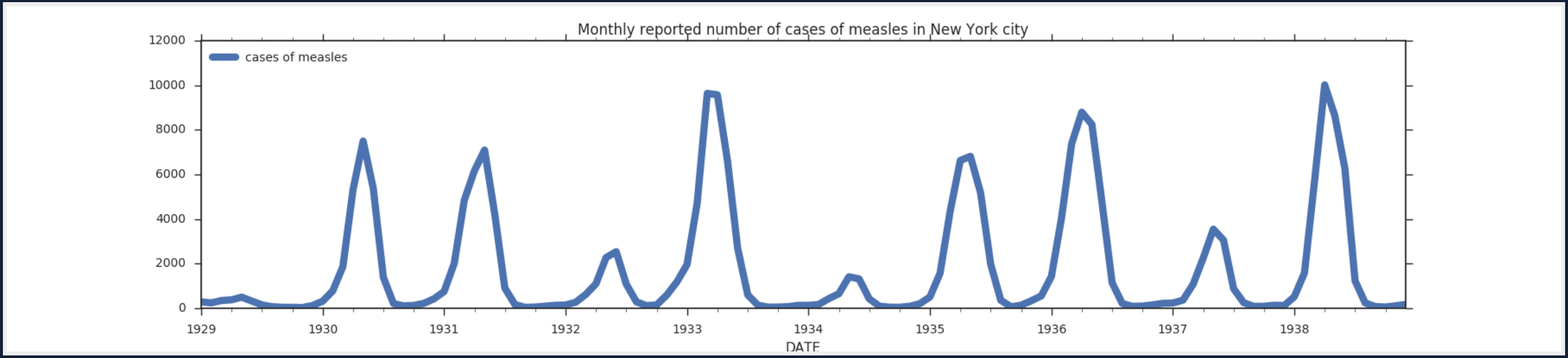


European Climate Assessment & Dataset, Air temperature in Berlin

Cyclical

The medium-term fluctuations caused
by cyclicaly occured non-periodic influences

Example

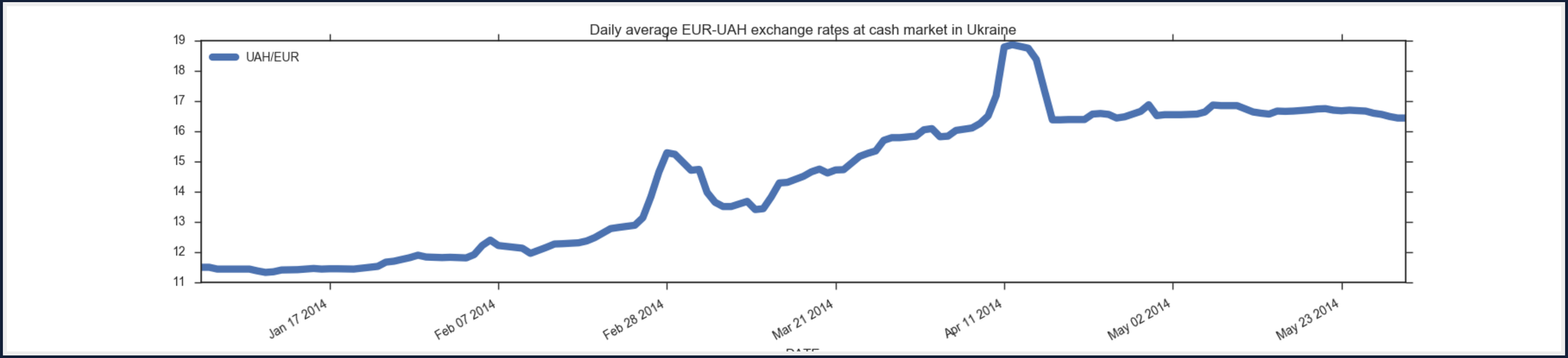


London W., Yorke J.A., The monthly reported number of cases of measles, New York city

Irregular

Random variations caused
by unpredictable influences

Example



valuta.today, Daily average EUR-UAH exchange rates in the cash market, Ukraine

Time Series Decomposition Model

There are 2 main models that generally used to model the effects of these components

⚠ The choice of the decomposition model is called **test for log-level specification**.

Additive model

Assumes that the four components are ***independent***

$$Y(t) = T(t) + S(t) + C(t) + I(t)$$

When to choose the Additive model

- The behaviors of the components are independent,
  i.e. an increase in the trend-cycle will not cause an increase in the magnitude of seasonal

- The difference of the trend and the raw data
  is roughly constant in similar periods of time
  irrespectively of the tendency of the trend

- The pattern of seasonal variation
  is roughly stable over the year
  i.e. the seasonal movements are the approximately same from year to year

https://ec.europa.eu/eurostat/sa-elearning/additive-decomposition-0

Multiplicative model

Assumes that the components are ***dependent***

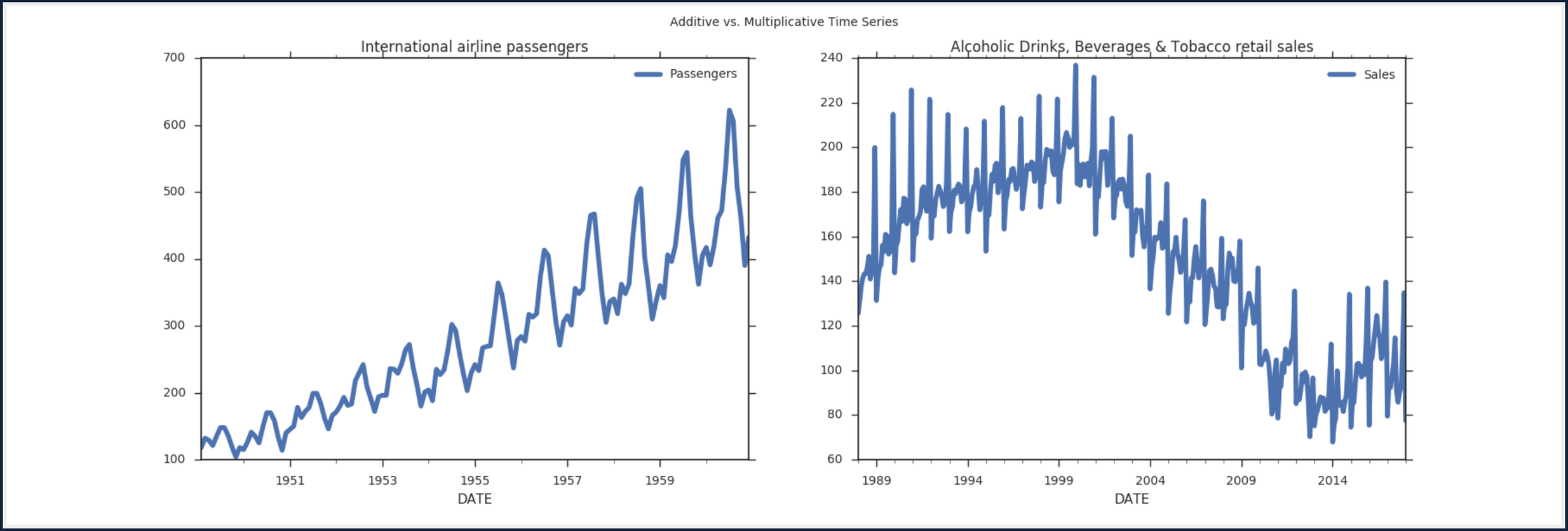
$$Y(t) = T(t) \cdot S(t) \cdot C(t) \cdot I(t)$$

By taking *log*s it is possible to present the multiplicative model in an additive form. This is called *log-additive* model.

When to choose the Multiplicative model

- Components on the graph jump in magnitude over time
- The seasonal and irregular fluctuations change in a specific manner after the trend
  i.e. the amplitude of the seasonality increase (decrease) with an increasing (decreasing) trend

Additive vs. Multiplicative



Additive: Alcoholic Drinks, Beverages & Tobacco retail sales, UK; **Multiplicative**: International airline passengers

Demo

Meet the dataset
Total number of visits to the UK by overseas residents
Source: Office for National Statistics

https://github.com/gakhov/pycon-ua-2018/blob/master/look-into-the-data.ipynb

The Roadmap

Data – Decomposition Model –
– Forecasting Model – Training – Prediction

How To Measure The Forecast Performance

The Mean Forecast Error (MFE)

$$\frac{1}{n}\sum_{t=1}^{n}(x_t - f_t)$$

A good forecast has the MFE close to zero

- Provides the direction of the error
- $MFE = 0$ doesn't guarantee that the forecast contains no errors (positives can cancel negative errors)
- Sensitive to observations scale and data transformations
- Doesn't penalize extreme errors

The Mean Squared Error (MSE)

$$\frac{1}{n}\sum_{t=1}^{n}(x_t - f_t)^2$$

```
from sklearn.metrics import mean_squared_error
```

A good forecast has the MSE close to zero

- Shows the overall idea of the error
- Emphasizes the fact that the total forecast error is much affected by large individual errors
- Sensitive to observations scale and data transformations
- Penalizes extreme errors

The Normalized Mean Squared Error (NMSE)

$$\frac{1}{\sigma^2 n}\sum_{t=1}^{n}(x_t - f_t)^2$$

The smaller the NMSE, the better the forecast

- It's balanced error measure
- Very effective in judging forecast accuracy of the model

The Coefficient of Determination (R^2)

$$R^2 = 1 - \frac{\sum (x_t - f_t)^2}{\sum (x_t - \bar{x})^2}$$

```
from sklearn.metrics import r2_score
```

A good forecast has the R^2 close to one

- Estimates the variability of the forecast around its mean
- $R^2 \leq 1$
- Doesn't indicate whether the model is adequate

Theil's U

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(x_t - f_t)^2}}{\sqrt{\frac{1}{n}\sum f_t^2}\sqrt{\frac{1}{n}\sum x_t^2}}$$

A good forecast has the U close to zero

- A normalized measure of total forecast error
- $0 \leq U \leq 1$
- $U = 0$ means the perfect fit

Models

There are many different approaches,
but here we focus on the top 3 families.

Stochastic Models

with Python

ARMA Model

Assumes that data are
a realization of a stationary process

Augmented Dickey-Fuller (ADF) unit root test,
Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test

AR – autoregression process (long memory)

Have a relatively long memory, because the current observation
is correlated with all previous ones, although with decreasing coefficients

MA – moving average process (short memory)

Have a short-term memory, since it's just a function of a finite,
and generally small, number of its past observations.

ARMA is to combine these properties
and represent model in a simplified form
whose first q coefficients can be any,
whereas the following ones decay according to simple rules.

$$ARMA(p, q)$$

- p - the lag order (AR)
  the number of lag observations to include

- q - the order of moving average (MA)
  the size of the moving average window

ARIMA

Autoregressive Integrated Moving Average

A generalization of an ARMA for
Integrated (Difference Stationary) Time Series

Non-stationary time series whose differences of some order are stationary.

- Non-stationary time series are differenced until stationarity is achieved (Box–Jenkins method)
- Doesn't support seasonal component modeling

$ARIMA(p, d, q)$

- p – the lag order (AR)
  the number of lag observations to include

- d – the degree of differencing (I)
  the number of times that the raw observations were differenced

- q – the order of moving average (MA)
  the size of the moving average window

The data has to be prepared by a degree of differencing to make time series stationary,
i.e. by removing trend and seasonal structures that negatively affect the regression model.
ARMA corresponds to d=0.

SARIMA

Seasonal Autoregressive Integrated Moving Average

A modification of ARIMA to support a seasonal time series.

$$ARIMA(p, d, q) \times (P, D, Q)_s$$

- (p,d,q) are the non-seasonal parameters
- (P, D, Q) the seasonal component of the time series.
- s is the seasonal periodicity
  (4 for quarterly periods, 12 for yearly periods, etc.)

- Check whether there is any evidence of a trend or seasonal effects
- Stationarization by taking differences
- Seasonal component research

Python Tools

Statsmodels

A high-level module to work with statistical models,
conducting statistical tests and data exploration.
https://www.statsmodels.org/

```
from statsmodels.tsa.statespace import sarimax

model = sarimax.SARIMAX(
    X_train,
    trend='n',
    order=(1,1,1),
    seasonal_order=(1,1,0,12),
    enforce_stationarity=True,
    enforce_invertibility=True)
results = model.fit()

steps = X_train.shape[0]

forecast = results.get_forecast(steps=steps)
forecast_ci = forecast.conf_int()

yhat_test = forecast.predicted_mean.values
```

Demo

Build a Seasonal AutoRegressive Integrated Moving Average model
to predict the number of visits to the UK by overseas residents

https://github.com/gakhov/pycon-ua-2018/blob/master/stochastic-models.ipynb

Python Tools

Prophet

A tool for producing forecasts for time series data
that has multiple seasonality with linear or non-linear growth.
https://github.com/facebook/prophet

Prophet Forecasting Model

Based on ARIMA and utilizes a Bayesian-based curve fitting method

$$X(t) = T(t) + S(t) + H(t) + \varepsilon_t$$

$T(t)$ - trend, $S(t)$ - seasonal effects, $H(t)$ - holidays (potentially irregular), ε_t - unpredicted effects.

The seasonality is modeled by the Fourier Series.

```
from fbprophet import Prophet

m = Prophet()
m.fit(df)

forecast = m.predict(df)
m.plot(forecast)
```

```
m.plot_components(forecast);
```

Demo

Build a Prophet Forecasting Model
to predict the number of visits to the UK by overseas residents

https://github.com/gakhov/pycon-ua-2018/blob/master/prophet.ipynb

Artificial Neural Networks

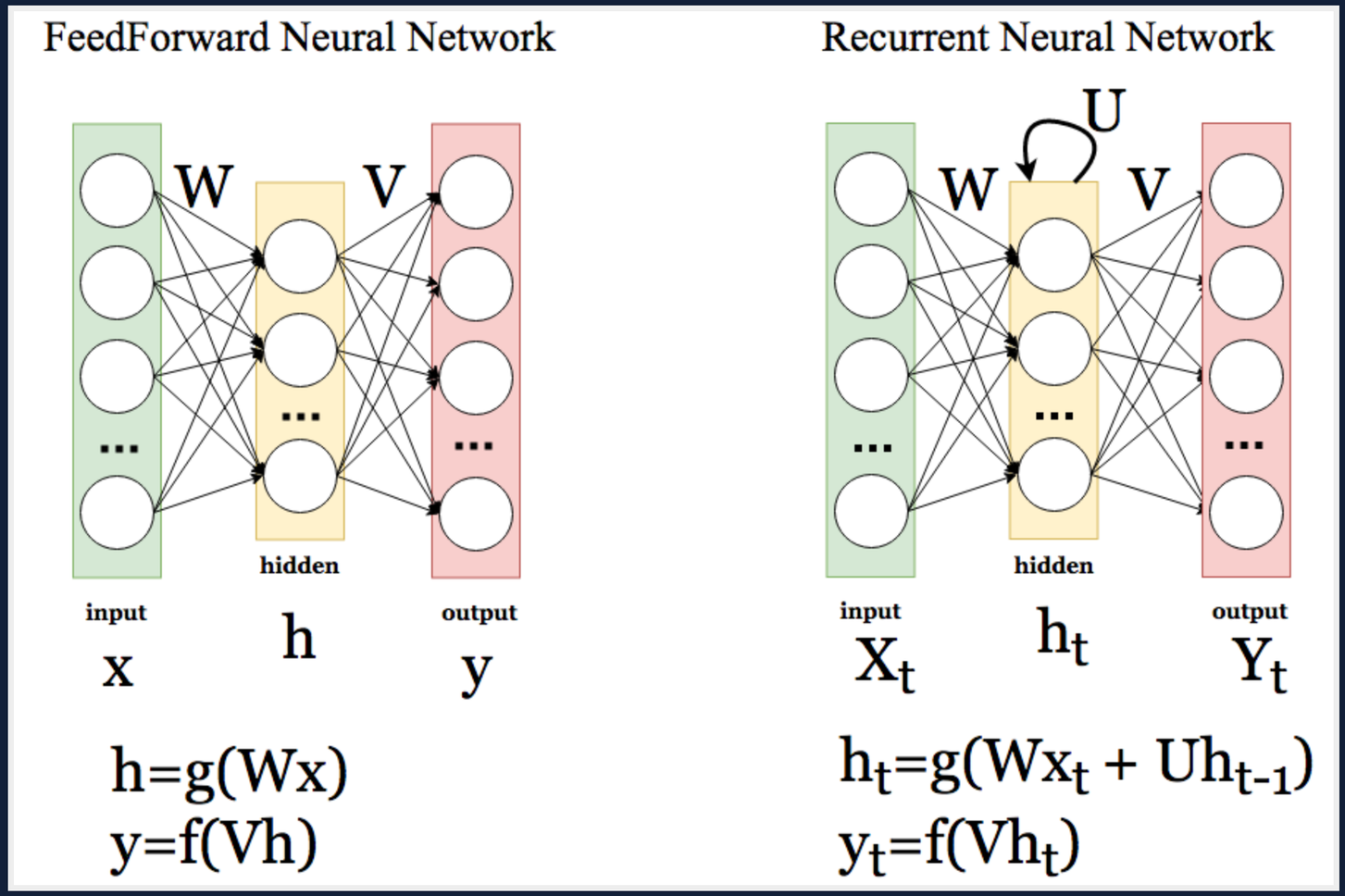
with Python

ANN can be seen as a computational graph

- **nodes** are computing units
- **directed edges** transmit numeric information

Each computing unit (neuron) is capable of evaluating
a single primitive function (activation function) of its input.
At practive, units are organized in layers.

Popular ANNs types



Recurrent Neural Network

Similar to Feedforward networks, but allows a **recurrent** hidden state
with activation **depended on the previous cycles**.

Long Short-Term Memory (LSTM) architecture

Unlike the traditional recurrent unit,
which overwrites its content each timestep,
the LSTM unit can decide whether to keep the existing memory
via introduced additional convolution steps (gates)

Read More: https://www.slideshare.net/gakhov/recurrent-neural-networks-part-1-theory

The main problem while working with ANNs
is the choice of the network's parameters,
such as the number of layers and number of neurons.

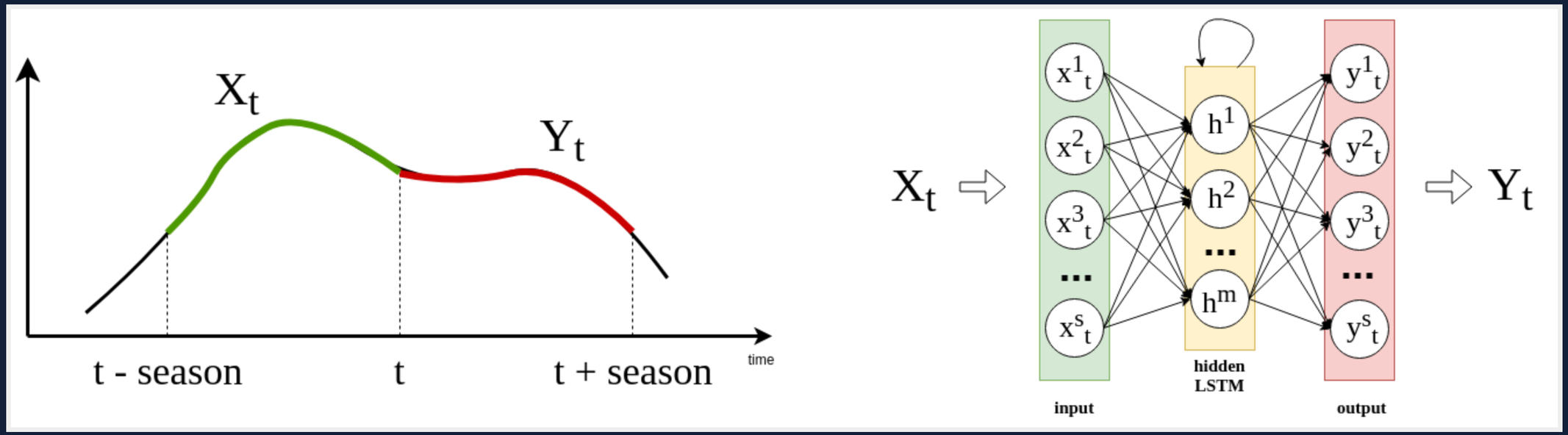
SANN

Seasonal Artificial Neural Networks

- only 1 hidden layer
- **number of input and output neurons**
  can be taken as number of observations in a season
- **number of hidden nodes**
  should be detemined experimentally (a few)

  * as recommended by C. Hamzaçebi.

How to build a model



We want to teach SANN to learn next season value from the previous season values

Using sliding-window technique transform time series to a number of s-dimensional vectors

Python Tools

Keras

A high-level neural networks API,
capable of running on top of TensorFlow, CNTK, or Theano.
https://keras.io/

```
from keras.layers import InputLayer, Dense, LSTM
from keras.models import Sequential
from keras.optimizers import SGD

model = Sequential()
model.add(InputLayer(input_shape=(1, seasons), name="input"))
model.add(
    LSTM(4, name="hidden", activation='sigmoid',
         use_bias = True, bias_initializer='ones')
)
model.add(
    Dense(seasons, name="output", activation='linear',
          use_bias = True, bias_initializer='ones')
)

model.compile(loss='mean_squared_error',
              optimizer=SGD(lr=0.05, decay=1e-6, momentum=0.9),
              metrics=["mae", "mse"])
history = model.fit(X_train, y_train, epochs=100, batch_size=1,
                    validation_data=(X_val, y_val))

yhat_test = model.predict(X_test[::seasons])
```

Demo

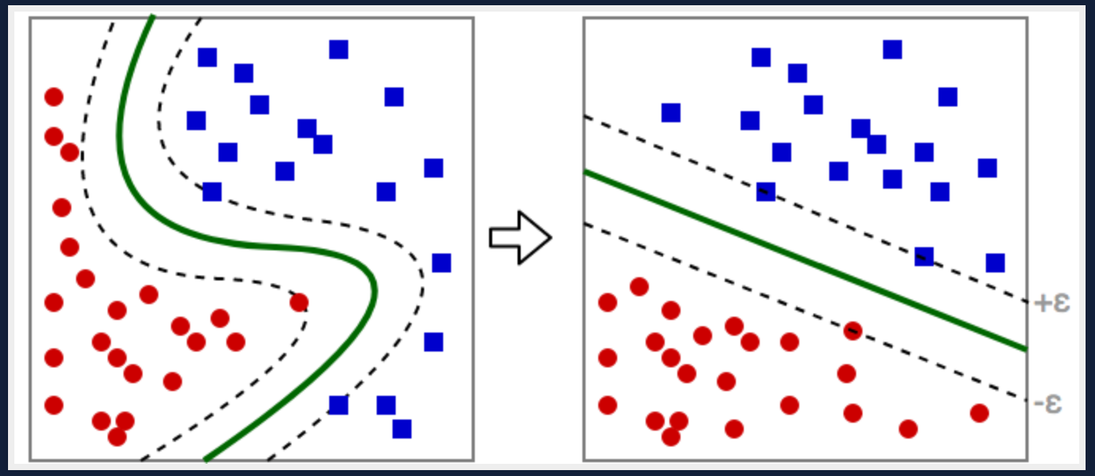
Build a Seasonal Artificial Neural Network with LSTM layer
to predict the number of visits to the UK by overseas residents

https://github.com/gakhov/pycon-ua-2018/blob/master/artificial-neural-networks.ipynb

Support Vector Machines

with Python

A technique that maps data into a high-dimensional feature space where it can be linearly separable in the best way.



Support vectors are "critical" observations that fully specify the decision function

Support Vector Machine Regression

Performs a linear regression
in the high-dimension feature space
with a special ε-intensive loss function.

Main SVM meta-parameters

ε-tube for no penalty

Controls the width of the ε-insensitive zone, used to fit the training data.

Bigger ε-values result in more 'flat' estimates (fewer support vectors).

Penalty parameter C

Controls the amount up to which deviations larger that ε is tolerated.

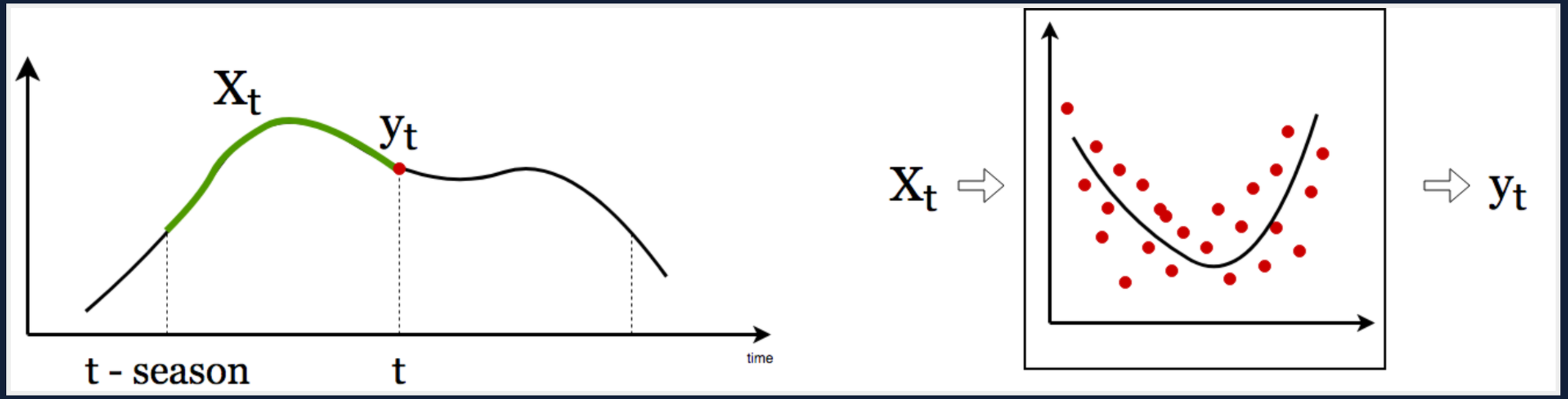
Prevents overfitting and in a trade-off with the model complexity.

Kernel function type

Induces the high-dimentinal feature space.

Most popular are linear, polynomial and Gaussian Radial Basis Function (RBF) kernels.

How to build a model



We want to train SVM Regressor to learn value at time t from the previous season values

Using sliding-window technique transform time series to a number of s-dimensional vectors

Python Tools

Scikit-learn

A simple and efficient tool for machine learning,
data mining and data analysis.
http://scikit-learn.org/

```
from sklearn import svm
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import ShuffleSplit

C_range = np.logspace(-2, 10, 5)
gamma_range = np.logspace(-9, 3, 5)
param_grid = {
    "gamma": gamma_range,
    "C": C_range,
    "epsilon": 0.1
}

cv = ShuffleSplit(n_splits=3, test_size=0.25, random_state=73)

grid = GridSearchCV(
    svm.SVR(kernel="rbf", max_iter=100),
    param_grid=param_grid,
    cv=cv)

grid.fit(X_train, y_train)
model = grid.best_estimator_

yhat_test = model.predict(X_test)
```

Demo

Build a Seasonal Support Vector Regressor
to predict the number of visits to the UK by overseas residents

https://github.com/gakhov/pycon-ua-2018/blob/master/support-vector-machines.ipynb

Final Notes

- Always visualize the data
- Think which decomposition model better describes you data
- Select a proper forecasting model
- Divide the raw data into 2 parts (training / test sets). Small part of the training set reserve for validation

Final Notes

- The observations only from the training set shell be used for the model construction
- Use the test set to verify how accurate the constructed model performs
- Many models expect normalized, scaled data as input
  (check Box-Cox Transformation, log-scaling, etc.)

Thank you

These slides can be found at
https://github.com/gakhov/pycon-ua-2018

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