Homework 4

This homework is to implement classes for solving the sound wave equation and the complex Ginzburg-Landau equation. These classes are called `SoundWaves` and `CGLEquation` in `equations.py`. Please make use of the `spectral.py` file we have been discussing in class; you can also make changes to this file if that makes things easier for you. The `SoundWaves` class is initialized with a domain, fields `u` and `p` which contain the initial velocity and pressure perturbations, and the background pressure `p0`. The `CGLEquation` class is initialized with a domain and a field `u`. Then the `evolve` method should take `num_steps` timesteps of size `dt` using the `timestepper`. Be sure your code:

- Generates sparse `M` and `L` matrices for efficient implicit timstepping
- Does not include any aliasing errors
- The sound wave equations work with both real and complex variables

I have included the tests associated with the sound wave equation.

Linearized sound waves in an ideal gas satisfy the equations

$$\partial_t u + \partial_x p' = 0,$$

$$\partial_t p' + \gamma p_0 \partial_x u = 0.$$

Here u and p' are the velocity and pressure perturbation, p_0 is the background pressure, and γ is the ratio of specific heats. For simplicity, we will take $\gamma = 1$. You will solve this equation on $x \in [0, L]$, with boundary conditions $u(0) = u(L) = 0$. *Hint:* It might be advantageous to rewrite the pressure equation as

$$\partial_t p' + \partial_x u = (1 - p_0)\partial_x u.$$

The complex Ginzburg-Landau equation is

$$\partial_t u = u + (1 + ib)\partial_x^2 u - (1 + ic)|u|^2 u.$$

It is used to model weakly nonlinear phenomena. You will use the parameters $b = 0.5, c = -1.76$. You will solve the equation on $x \in [0, L]$ with homogeneous Dirichlet boundary conditions $u(0) = u(L) = 0$.