CSE 431/531 Analysis of Algorithms
Problem Set 5

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Due Date: August 13, 2023 23:59 PM EST.

Problem 1. (40%)

1. In HW2, we have the joint-match problem for graphs. What is the certificate for this problem? Write the verifier algorithm.
2. In HW2, we have the diagonal submatrix finding problem. What is the certificate for this problem? Write the verifier algorithm.
3. In HW4, we have the Bloxorz game problem. What is the certificate for this problem? Write the verifier algorithm.
4. On our last slides page 14 we have the k-IS problem. What is the certificate for the problem? Write the verifier algorithm.

If you answer the above problems on certificates more than 20 words each. You get 0 points.

Problem 2. (20%)

The 3-SAT problem is defined on page 9 of our last slide. The SUBSET-SUM problem is defined as given a set of integers S and a target value t does there exist a subset T of S that sums up precisely to t?

Here is a polynomial reduction from 3-SAT problem to SUBSET-SUM problem:

(1) Let $x_1, x_2, ..., x_n$ be the variables of the Boolean formula.

Let $c_1, c_2, ..., c_m$ be the clauses.

(2) Construct a bunch of numbers for every variable x_i below:

$$a_i = 10^{m+i} + \sum_{j: x_i \text{ is in } c_j} 10^j$$

$$b_i = 10^{m+i} + \sum_{j: \overline{x_i} \text{ is in } c_j} 10^j$$

We get $2n$ numbers.

(3) Construct a bunch of numbers for every clause c_j below:

$$d_j = h_j = 10^j$$

We get $2m$ numbers.

(4) Merge all numbers in the step (2) and (3) to form a set S of $2m + 2n$ numbers.

(5) Construct a number

$$t = \sum_{i=1}^{n} 10^{m+i} + 3 \times \sum_{j=1}^{m} 10^j$$

(6) The SUBSET-SUM instance we get is $\{S, t\}$.

1. Given a 3-SAT instance of $\phi = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$, what is the SUBSET-SUM instance we get using the above reduction? Is it a yes-instance or a no-instance?

2. Given a 3-SAT instance of $\phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2})$, what is the SUBSET-SUM instance we get using the above reduction? Is it a yes-instance or a no-instance?

Problem 3. (40%)

You and your teammate were in a co-op game and won n loot boxes. Each loot box i has a value v_i. You want to decide which loot boxes belong to you or your teammate so that both get **exact** equal amount of total value. If the total value is different either you or your teammate becomes mad. The problem of whether one of you will be mad seems very hard.

1. Formulate this decision problem mathematically as what we did in the lecture slides. Give it a name.

2. Show that this problem is in NP. State what is the certificate and write the verifier algorithm.

3. Is this problem in P? If so, give a polynomial algorithm to solve it. If not, show that this problem is NP-hard by constructing a polynomial reduction from a known NP-complete problem to it.