Project 1 for “Algorithms for Big-Data Analysis”

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1 Submission Requirement

1. Pack all of your codes named as ”proj1-name-ID.zip” and send it to both me and TA:

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2. If you get significant help from others on one routine, write down the source of references at the beginning of this routine.

2 Algorithms for ℓ_1 minimization

Consider the Basis Pursuit (BP) problem

$$\min_x \quad \|x\|_1, \text{ s.t. } Ax = b, \tag{2.1}$$

where $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$ are given. Test matrices:

```
n = 1024;
m = 512;
A = randn(m,n);
u = sprandn(n,1,0.1);
b = A*u;
```

See `http://bicmr.pku.edu.cn/~wenzw/bigdata/Test_BP.m`

1. Solve (2.1) using CVX by calling different solvers mosek or gurobi.
   CVX, Mosek and Gurobi are available free at:
   CVX: `http://cvxr.com/cvx/`
   Mosek: `http://www.mosek.com/`
   Gurobi: `http://www.gurobi.com/`

2. Solve (2.1) by calling mosek or gurobi directly.

3. Write down and implement one of the following algorithms in Matlab:

(a) Classical Augmented Lagrangian method (or Bregman method), where each augmented Lagrangian function is minimized by using the proximal gradient method
Reference: Wotao Yin, Stanley Osher, Donald Goldfarb, Jerome Darbon, *Bregman Iterative Algorithms for l1-Minimization with Applications to Compressed Sensing*

(b) Classical Augmented Lagrangian method (or Bregman method), where each augmented Lagrangian function is minimized by using the accelerated proximal gradient method (FISTA or Nesterov's method)
Reference on FISTA: Amir Beck and Marc Teboulle, *A fast iterative shrinkage thresholding algorithm for linear inverse problems*

4. Write down and implement one of the following algorithms in Matlab:

(a) Alternating direction method of multipliers (ADMM) for the dual problem
Reference: Junfeng Yang, Yin Zhang, *Alternating direction algorithms for l1-problems in Compressed Sensing*

(b) Alternating direction method of multipliers with linearization for the dual problem
Reference: Junfeng Yang, Yin Zhang, *Alternating direction algorithms for l1-problems in Compressed Sensing*

5. Requirement:

(a) The interface of each method should be written in the following format

```
[x, out] = method_name(x0, A, b, opts);
```

Here, x0 is a given input initial solution, A and b are given data, opts is a struct which stores the options of the algorithm, out is a struct which saves all other output information.

(b) Compare the efficiency (cpu time) and accuracy (checking optimality condition) in the format as
`http://bicmr.pku.edu.cn/~wenzw/bigdata/Test_BP.m`

3 Algorithms For Sparse Inverse Covariance Estimation

Let $S^n = \{X \in \mathbb{R}^{n\times n} \mid X^\top = X\}$. Let $S \in S^n$ be a given observation of covariance matrix.

1. Consider the model

$$\max_{X\succeq 0} \log\det X - \mathrm{Tr}(SX) - \rho\|X\|_1, \tag{3.1}$$

where $\|X\|_1 = \sum_{ij} |X_{ij}|$.

(a) data sets: set $n = 30$, generate models 1 and 2 in section 5.1 of page 599 at
`http://www-stat.wharton.upenn.edu/~tcai/paper/Precision-Matrix.pdf`

(b) Derive the dual problem of (3.1).

(c) Solve (3.1) using CVX using a few ρ, for example, $10, 0.1, 0.001$.

(d) Write down and implement a first-order type algorithm for solving (3.1) with the same ρ in (b).

2. Consider the model

$$\min_{X \succeq 0} \|X\|_1, \text{ s.t. } \|SX - I\|_F \leq \sigma, \tag{3.2}$$

where I is the identity matrix and $\|\cdot\|_F$ is the Frobenius norm. The test data is the same as 1(a).

   (a) Derive the dual problem of (3.2).

   (b) Solve (3.2) using CVX using a few σ, for example, $10, 0.1, 0.001$.

   (c) Solve (3.2) with the same σ in (b) by calling SDPT3 directly.
   SDPT3: `http://www.math.nus.edu.sg/~mattohkc/sdpt3.html`

   (d) Write down and implement a first-order type algorithm for solving (3.2) with the same σ in (b).

Note that there is a constraint $X \succeq 0$ in both (3.1) and (3.2).