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Energy in units of  
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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$   
 $E > V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$$\begin{aligned}
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$$p = \hbar k = \sqrt{2m(E - V(x))} \quad \text{This equals } \sqrt{2m(E - V_0)} \text{ inside the well and } \sqrt{2mE} \text{ outside.}$$

$$p = \hbar k = \sqrt{2m(E - V_0)} \quad \text{outside.}$$

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$k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Figure 5

Energy in units of  $V_0$

Figure 3

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Energy in units of  $E_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

$E$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$ ,

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$$V_0 > 0$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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Energy in units of

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

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$$E < V_0$$

$$E = V_0$$

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E

Energy in units of

Figure 3

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $E_1$

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 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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 (19)
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

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$V=V_0$

$-a$   $a$

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E

Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x)) / \hbar^2$ . This equals  $\sqrt{2mE / \hbar^2}$  inside the well and  $p = \sqrt{2m(E - V_0) / \hbar^2}$  outside.

$$k = \frac{\sqrt{2mE}}{\hbar} \text{ inside the well and } p = \frac{\sqrt{2m(E - V_0)}}{\hbar} \text{ outside.}$$

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Energy in units of

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Energy in units of  $V_0$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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Energy in units of  
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$E$

Energy in units of

Figure 3

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

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-a a

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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√

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to plot } \psi(x) \text{ versus } x \text{ from } -a \text{ to } a.$$

Figure 5

Energy in units of  $V_0$

Figure 3

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 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V=V_0$

-a a

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Energy in units of  $E_1$

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 Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by  

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
 (19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :  
 $V=0$   
 $V=V_0$   
 -a a  
 Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

$2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$   
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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by  
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$   
(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

$V = V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.

$2mE/\hbar$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))/\hbar}$  inside the well and  $p = \sqrt{2m(E - V_0)/\hbar}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))/\hbar}$ . This equals  $\sqrt{2m(E - V_0)/\hbar}$  inside the well and  $p = \sqrt{2m(E - V_0)/\hbar}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E,

where as the quantum mechanical system has  $\omega \propto E$ .

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

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apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p = \hbar k$ . This equals  $\sqrt{2m(E - V(x))}$  inside the well and  $\sqrt{2m(E - V_0)}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

$\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$



$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

$$E_n \propto n^2$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$$E_n \propto n^2$$

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$$E_n \propto n^2$$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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Energy in units of

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Energy in units of  $E_0$

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$E$

Energy in units of

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$E$

Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-a \leq x \leq a$$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of

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Energy in units of  $V_0$

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
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$E = 0$   
 $E > V_0$   
 -a a

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 
 $2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a



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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p/\hbar = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$   
 $E = V_0$   
 $-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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2

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Energy in units of

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to  $-a$   $a$

E

Energy in units of

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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2

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$E$

Energy in units of

Figure 3

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E

Energy in units of

Figure 3

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$$E > V_0$$

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$E$

Energy in units of

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$$-V_0 < E < 0$$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

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E

Energy in units of

Figure 3

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$^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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Energy in units of  
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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E  
Energy in units of  
Figure 3  
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$

2

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2

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E

Energy in units of

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$E$

Energy in units of

Figure 3

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Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both



apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$2$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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are also proportional to  $n^2$

$\omega$

$\omega$

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$E$

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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 E  
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Energy in units of

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to  $-a$   $a$

E

Energy in units of

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Energy in units of

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 -a a  
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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form  
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Energy in units of  
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 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of  $E_0$

Figure 3

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$   
 $V=V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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E  
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Figure 3

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$2$   
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$2$   
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well.}$$

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E  
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Figure 3

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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$E$

Energy in units of

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Energy in units of

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2

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$$2m(E - V_0)/\hbar$$

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$E$

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

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Energy in units of

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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Energy in units of

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-V_0 < E < 0$$

$$E > 0$$

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-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$V_0$  ( $|x| \leq a$ ).

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$E$

Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-a \quad a$$

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$E$

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Energy in units of

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$E > V_0$

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Figure 5

Energy in units of  $V_0$

Figure 3

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Energy in units of  
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$E$

Energy in units of  $E_1$

Figure 3

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of  
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Energy in units of

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$E$

Energy in units of  $V_0$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

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2

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E

Energy in units of

Figure 3

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Energy in units of

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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$E$

Energy in units of

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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$V_0$

$(|x| > a)$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

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Energy in units of

Figure 3

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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(19)

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Energy in units of

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$E$

Energy in units of  $V_0$

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$   
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-a a  
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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - V(x))}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

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$V_0 \quad (|x| > a).$

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

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Figure 5

Energy in units of

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Energy in units of  $E_0$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E > V_0$

-a a

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Energy in units of  $V_0$

Figure 3

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Energy in units of  $V_0$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \ a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of

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normalize  $\psi$  to 1. The energy  $E$  is in units of  $V_0$ .

Figure 3 shows the energy levels for a string. The gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

$E_n = \frac{1}{2} \hbar \omega_n$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar \omega$ .

$E_n$

$E_n = \frac{1}{2} \hbar \omega_n = \frac{1}{2} \hbar ck_n = \frac{1}{2} \hbar c \frac{n\pi}{L} = \frac{\hbar^2 c^2}{8mL^2} n^2$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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Figure 5

Energy in units of  $V_0$

Figure 3

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to  $-a$   $a$

E

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Energy in units of

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E

Energy in units of

Figure 3

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

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$E$

Energy in units of  $V_0$

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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Energy in units of  $V_0$

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Energy in units of  $E_1$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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 $V = V_0$   
 $-a \leq x \leq a$

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Energy in units of  $E$

Figure 3

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$E$   
 Energy in units of  $V_0$   
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Energy in units of

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$$2m(E - V_0)/\hbar^2$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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E

Energy in units of

Figure 3

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$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

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Figure 5

Energy in units of  $V_0$

Figure 3

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$-a \ a$

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Energy in units of  
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Energy in units of

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Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of

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 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

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 $(x)$  is given  
 by  

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
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$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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E

Energy in units of

Figure 3

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$E_n \propto n^2$$

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of  $E_1$

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$E$

Energy in units of  $V_0$

Figure 3

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Figure 5

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

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$E$

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Energy in units of  $V_0$

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$E$

Energy in units of  $E_1$

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Energy in units of  $E$   
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 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= V_0 & 0 \leq |x| \leq a \\
 V(x) &= 0 & |x| > a
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

$2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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2

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E  
Energy in units of  
Figure 3  
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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E  
Energy in units of  
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$\omega^2 \propto k^2 \propto n^2$$

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E

Energy in units of

Figure 3

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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$E$

Energy in units of

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to  $-a$   $a$

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of  $V_0$

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$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p/\hbar$

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Energy in units of

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Energy in units of

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Energy in units of

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Energy in units of

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Energy in units of  $E$

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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Figure 3

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$2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$   
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $E$   
Figure 3  
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to  $-a$   $a$

E

Energy in units of

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

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Energy in units of

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$$2m(E - V_0)/\hbar$$

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of  $E_0$

Figure 3

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Energy in units of  $V_0$

Figure 3

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Energy in units of  $E_0$

Figure 3

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Energy in units of  $E_1$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$

$E$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \ a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E:

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.

$k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot  $\psi(x)$  versus  $x$ .

Figure 3

Energy in units of  $E_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

consider a finite square well of width  $L$  and depth  $V_0$ .

Figure 4

Energy in units of  $E_0$

Figure 4

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

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Energy in units of

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E  
Energy in units of

Figure 3  
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$^2$   
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Figure 4

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$^2$   
 $^2$

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Energy in units of Figure 3

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Energy in units of  $E$

### Figure 3

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Energy in units of

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E < V_0$$

$$E = V_0$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 = c^2 k^2 \propto n^2$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2mE}/\hbar$  outside.

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$E$

Energy in units of  $V_0$

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Energy in units of

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(19)

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-a a

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to  $-a$   $a$

E

Energy in units of

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Energy in units of  $E_0$

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$0 \quad (|x| \leq a)$$

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$$E > V_0$$

$$-a \leq x \leq a$$

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Energy in units of  $E$

### Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$

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 The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),  
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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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Energy in units of

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are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar^2 \text{ inside the well and } 2mE/\hbar^2 \text{ outside.}$$

$$2m(E - 0)/\hbar^2 \text{ outside.}$$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of

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Energy in units of

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$\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  $E$   
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
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$$0 \quad (|x| \leq a)$$

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$E$

Energy in units of

Figure 3

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Energy in units of  $E_0$

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$E$

Energy in units of  $V_0$

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Energy in units of  $V_0$

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$   
 $V=V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of  
Figure 3  
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$2$   
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$2$   
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Energy in units of

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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to -a a

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Energy in units of

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to  $-a \ a$

$E$

Energy in units of

Figure 3

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E < V_0$

$-a \leq x \leq a$

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of  $V_0$

Figure 3

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are also proportional to  $n^2$

$\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$E$

Energy in units of

Figure 3

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$E$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Figure 5

Energy in units of  $V_0$

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Energy in units of

Figure 3

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$V=V_0$$

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $V_0$

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Energy in units of  $V_0$

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$-a \quad a$

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plot  $E$

Energy in units of

Figure 3

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Energy in units of

Figure 3

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2

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$E$

Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $V_0$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

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$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to plot } \psi(x) \text{ versus } x \text{ from } -a \text{ to } a.$$

Figure 5

Energy in units of  $V_0$

Figure 3

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Energy in units of

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E

Energy in units of

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$E$

Energy in units of  $E_1$

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$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

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Energy in units of  $E$   
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10.3.3 Finite square well  
Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
  
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :  
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$E$

Energy in units of

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar \omega$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$ . This

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.

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Energy in units of

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

( $x$ ) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

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Energy in units of

Figure 3

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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Energy in units of

Figure 3

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$2$

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-a a

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of  
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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

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$$-a \leq x \leq a$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$ . This

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to  $-a$   $a$

E

Energy in units of

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Energy in units of

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Energy in units of

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$E$

Energy in units of

Figure 3

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Energy in units of

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inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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-a a

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

$V = V_0$

-a a

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Energy in units of

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of  $E_1$

Figure 3

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Energy in units of

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of  $V_0$

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Energy in units of

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Energy in units of

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$2m(E - V_0)/\hbar$

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of  $V_0$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$E_n = \frac{1}{2} \hbar \omega_n$

$n$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar \omega$  ( $E = \frac{1}{2} \hbar \omega$ ). This equals  $\frac{1}{2} \hbar \omega$

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$E$

Energy in units of  $V_0$

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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Energy in units of

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$n^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega$

2

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Energy in units of

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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$E$

Energy in units of

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$\sqrt{2m(E - V_0)/\hbar}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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$$\text{to -a a}$$

Energy in units of  $V_0$

Figure 3

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Energy in units of  $V_0$

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$V=V_0$

-a a

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $V = 0$
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-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))/\hbar}$ . This equals  $\sqrt{2m(E - V(x))/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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Energy in units of

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$0 \quad (|x| \leq a)$

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(19)

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to  $-a$   $a$

E

Energy in units of  $V_0$

Figure 3

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E

Energy in units of  $V_0$

Figure 3

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$$E$$

Energy in units of  $E_1$

Figure 3

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Energy in units of  $V_0$

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$E$

Energy in units of

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$V_0 (|x| > a)$ .

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 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$



(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p = \hbar k$ . This equals  $\sqrt{2m(E - V(x))}$  inside the well and  $\sqrt{2m(E - V_0)}$  outside.

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

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$E$

Energy in units of

Figure 3

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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Energy in units of

Figure 3

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$$n^2$$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$2$$

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E

Energy in units of

Figure 3

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E  
Energy in units of  
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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E  
Energy in units of  
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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$E > V_0$

$E = V_0$

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Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

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$V(x) = \frac{1}{2}$

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$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

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$V=0$

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$$2m(E - V_0)/\hbar^2$$

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$E$

Energy in units of  $V_0$

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$E$  Energy in units of  $V_0$

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

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$$V(x) = \frac{1}{2}$$

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$$E$$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of  $V_0$

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

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E  
Energy in units of

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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### 10.3.3 Finite square well

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of

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Energy in units of

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Energy in units of  $E_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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Energy in units of  $V_0$

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$E > V_0$

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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$$E_n = \frac{1}{2} \hbar \omega_n$$

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$E$

Energy in units of

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

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Energy in units of

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E

Energy in units of

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Figure 5

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

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$E$

Energy in units of

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Energy in units of  $V_0$

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 V(x) &= V_0 & (|x| \leq a) \\
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 \tag{19}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
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$E$

Energy in units of

Figure 3

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Energy in units of  $V_0$

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$E$

Energy in units of

Figure 3

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Energy in units of  $E_1$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $V_0$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

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$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

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Energy in units of

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$$V_0 > 0 \quad (|x| > a).$$

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$$\text{to } \psi(0) = 1$$

Energy in units of  $V_0$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E < V_0$$

$$-a \leq x \leq a$$

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Figure 5

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
 Energy in units of  
 Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Energy in units of

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)/\hbar}$  inside the well and  $p$

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E

Energy in units of  $V_0$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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$E$

Energy in units of  $V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-a \leq x \leq a$$

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$E$

Energy in units of

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$E$

Energy in units of  $E_1$

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Energy in units of

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E

Energy in units of

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$$V_0 \quad (|x| > a).$$

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Figure 5

Energy in units of  $V_0$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

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$E$

Energy in units of  $V_0$

Figure 3

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E

Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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E  
Energy in units of

Figure 3

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$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p = \hbar k$

$\sqrt{2m(E - V(x))}$ . This equals  $\sqrt{2m(E - V_0)}$  inside the well and  $\sqrt{2mE}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$\psi(x)$  is given

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

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•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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Energy in units of

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$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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Energy in units of

Figure 3

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

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2

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$V=0$

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Energy in units of

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of  $V_0$

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Energy in units of

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$0$  ( $|x| \leq a$ )

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Energy in units of  $V_0$

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Energy in units of

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to  $-a$   $a$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

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$E$

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$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-a \leq x \leq a$$

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$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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E

Energy in units of  $V_0$

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Energy in units of

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$-a$   $a$

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Energy in units of  $V_0$

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$-a \quad a$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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Energy in units of

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

$E$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$$E_n \propto \omega_n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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Figure 3

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Energy in units of  
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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well.}$$

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Energy in units of

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to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$E_n = \frac{1}{2} \mu \omega_n^2$$

$$= \frac{1}{2} \mu c^2 k^2$$

$$= \frac{1}{2} \mu c^2 \left( \frac{n\pi}{L} \right)^2$$

$$= \frac{1}{2} \mu c^2 \frac{\pi^2 n^2}{L^2}$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$V = V_0$

$-a \quad a$

Figure 4

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Energy in units of  $V_0$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

( $x$ ) is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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-a a

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Energy in units of  $V_0$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2



and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

$E = V_0$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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to -a a

E

Energy in units of

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Energy in units of

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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$$-a \leq x \leq a$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of  
Figure 3

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Energy in units of

Figure 3

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

$E$

Energy in units of  $V_0$

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Energy in units of  $V_0$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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Energy in units of  
 Figure 3  
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$$\begin{aligned}
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 (19)
 \end{aligned}$$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

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$E = 0$

$E = V_0$

$-a \leq x \leq a$

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

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$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$E$ ,

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### 10.3.3 Finite square well

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$-a$   $a$

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possible wavefunction might look something like the one in Fig. 5. It is customary to

plot  $\psi(x)$  versus  $x$ . The energy  $E$  is shown in units of  $V_0$ .

$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$0$  ( $|x| \leq a$ )

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E

Energy in units of

Figure 3

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2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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Energy in units of  
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$E$

Energy in units of  $V_0$

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Energy in units of

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$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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Figure 5

Energy in units of  $V_0$

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Energy in units of  $V_0$

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Figure 5

Energy in units of  $V_0$

Figure 3

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E

Energy in units of

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$0 \quad (|x| \leq a)$   
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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of  $E_1$

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Energy in units of  $V_0$

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

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$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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$E$

Energy in units of

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Figure 3 shows the energy levels for a string and a particle in a potential well. The gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

E

Energy in units of  $\hbar^2/(2ma^2)$

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E

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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Energy in units of

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Energy in units of  $V_0$

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to  $-a$   $a$

E

E

Energy in units of

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap

between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$V=V_0$

-a a

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to -a a

E

Energy in units of

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$V=0$

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Energy in units of

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E  
Energy in units of  
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$2$   
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Energy in units of

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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2

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Energy in units of

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$E$

Energy in units of

Figure 3

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E  
Energy in units of  
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Energy in units of  
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2

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E

Energy in units of  $V_0$

Figure 3

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$E$  Energy in units of  $V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3  
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are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

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Energy in units of  $E$

### Figure 3

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

-a a

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Energy in units of

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$E$

Energy in units of  $E_1$

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to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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2

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Energy in units of  
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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E

Energy in units of  $V_0$

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Energy in units of

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$$V_0 \quad (|x| > a).$$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of

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Energy in units of

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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- $V = V_0$

-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$$V_0 (|x| \leq a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

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$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

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Energy in units of  $E_0$

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E

Energy in units of

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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E

Energy in units of  $V_0$

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$-a$   $a$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of

Figure 3

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Energy in units of

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$E$

Energy in units of  $E_1$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

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$E$

Energy in units of

Figure 3

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$(|x| > a)$

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Energy in units of

Figure 3

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Energy in units of  $V_0$

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to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Energy in units of  $V_0$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E < V_0$

$E > V_0$

$-a \leq x \leq a$

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$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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Energy in units of

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Energy in units of  $E_0$

Figure 3

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of  
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$   
 $E < V_0$   
-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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Energy in units of  $E_0$

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$$V_0 \quad (|x| > a).$$

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$$V=V_0$$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.

$k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot  $\psi(x)$  versus  $x$  in units of  $a$ .

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

$E$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.

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$$V_0 \quad (|x| > a).$$

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$$V=V_0$$

$$-a \quad a$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E,

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$$0 \quad (|x| \leq a)$$

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$E$

Energy in units of

Figure 3

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$V_0 \ (|x| > a).$

(19)

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$V=V_0$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a)$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

-a a

Figure 4

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$V_0 \quad (|x| > a).$$

(19)

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$E < V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$\sqrt{2m(E - V(x))}/\hbar.$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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E  
Energy in units of  
Figure 3  
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- $V = 0$
- $V = V_0$

-a a

Figure 4

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E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string  
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(19)

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$$V=V_0$$

$$-a \quad a$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

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$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

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$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

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Energy in units of

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Energy in units of

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Energy in units of  $V_0$

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$$\text{Energy in units of } E$$

$$\text{Figure 3}$$

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of

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### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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Figure 5

Energy in units of  $V_0$

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Energy in units of

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 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
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Energy in units of  
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E

Energy in units of  $V_0$

Figure 3

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$E$

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That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by  

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$E = V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a  
 E  
 Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by  

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
 (19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :  
 $V=0$   
 $V=V_0$   
 -a a  
 Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

$2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$   
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$



2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

E,

where as the quantum mechanical system has  $\omega \propto E$ .

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$$E_n \propto n^2$$

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$E$ ,

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of p

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and p

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and p

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E,

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$^2$

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$E$

Energy in units of

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$$V_0 \quad (|x| > a).$$

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$-a \quad a$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of

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Energy in units of

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Energy in units of  $V_0$

Figure 3

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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E

Energy in units of

Figure 3

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E > V_0$$

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$$E < V_0$$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$-a \ a$

Figure 4

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$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

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$E$

Energy in units of

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

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$$E_n \propto n^2$$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

$V=V_0$

-a a

Figure 4

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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to -a a

E

Energy in units of

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$V=0$

$V=V_0$

-a a

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$\sqrt{2m(E - V_0)}/\hbar$  outside.

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E  
Energy in units of  
Figure 3  
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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of

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Energy in units of

Figure 3

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

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2

2

10.3.3 Finite square well

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$V(x) = \frac{1}{2}$

$0$  ( $|x| \leq a$ )

$V_0$  ( $|x| > a$ ).

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Energy in units of

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

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Energy in units of

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega \hbar (E - V(x)) / \hbar$ . This equals  $\sqrt{2m(E - V(x)) / \hbar}$  inside the well and  $\sqrt{2m(E - V_0) / \hbar}$  outside.

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Figure 4

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

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$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

$E > V_0$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

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$$0 \quad (|x| \leq a)$$

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$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of

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Figure 4

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Energy in units of  $E$

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$E$$

Energy in units of

Figure 3

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
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Energy in units of  $E_0$

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Energy in units of

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  
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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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E

Energy in units of

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Energy in units of  $E_0$

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(19)

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$V_0$

$(|x| > a)$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $E$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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normalize  $\psi(x)$  to 1.

Figure 3

E

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Energy in units of

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Energy in units of  $V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of

Figure 3

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Figure 5

Energy in units of  $V_0$

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$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-a \leq x \leq a$$

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$$\text{Energy in units of } V_0$$

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are also proportional to  $n^2$

$$E_n = \frac{1}{2} \mu \omega_n^2$$

$$= \frac{1}{2} \mu c^2 k_n^2$$

$$= \frac{1}{2} \mu c^2 \left( \frac{n\pi}{L} \right)^2$$

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of  
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E  
Energy in units of

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to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V = 0$

$V = V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

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E

Energy in units of

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$E$

Energy in units of

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$E > V_0$

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Figure 4

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$$\text{inside the well and } p = \sqrt{2m(E - 0)}/\hbar$$

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$E$

Energy in units of

Figure 3

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inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

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$E$

Energy in units of

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-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string



are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))$ . This equals  $\sqrt{2m(E - V(x))}$  inside the well and  $p = \sqrt{2m(E - V_0)}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  $V_0$

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Energy in units of  
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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

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2

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Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

$$V_0$$

Figure 3

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$$E_n = \frac{1}{2} \hbar \omega_n$$

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Figure 3

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of

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$E$

Energy in units of

Figure 3

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

Figure 3

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

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Energy in units of  $V_0$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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### 10.3.3 Finite square well

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

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$n^2$

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to  $-a$   $a$

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Energy in units of

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$-a$   $a$

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Energy in units of

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inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

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Energy in units of  $E_0$

Figure 3

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E

Energy in units of

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Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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to  $-a$   $a$

$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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E  
Energy in units of  
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Energy in units of

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Energy in units of

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $V_0$

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Energy in units of

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$E$

Energy in units of  $E_1$

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Energy in units of

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$$V_0 \quad (|x| > a).$$

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

$2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$   
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$\frac{2m(E - V(x))}{\hbar^2}$$

$$\frac{2mE}{\hbar^2} \text{ inside the well and } p = \frac{2m(E - V_0)}{\hbar^2}$$

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$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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Energy in units of

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$E$

Energy in units of

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Energy in units of

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$-a$   $a$

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Energy in units of

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize  $\psi$  to 1. The energy levels for a particle in a potential well are also proportional to  $n^2$ .

Figure 3

Energy in units of  $V_0$

Figure 3

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$V_0 \quad (|x| > a).$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

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Energy in units of  $V_0$

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Energy in units of  $E_0$

Figure 3

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Energy in units of  $V_0$

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$E$

Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of  $E_1$

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$$\text{, because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2 m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

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Energy in units of  
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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
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$$V_0 \quad (|x| \leq a).$$

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$$E$$

Energy in units of

Figure 3

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$$E_n \propto n^2$$

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Energy in units of  $E_0$

Figure 3

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$E$

Energy in units of

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$E$

Energy in units of  $V_0$

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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Energy in units of

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$$E = \frac{1}{2} \hbar \omega^2 = \frac{1}{2} \hbar c^2 k^2 = \frac{1}{2} \hbar c^2 \left( \frac{n\pi}{L} \right)^2$$

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E

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

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Energy in units of  $V_0$

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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Energy in units of

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

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Energy in units of

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$V_0 \ (|x| > a)$ .

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

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Energy in units of

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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Energy in units of  
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Energy in units of

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Energy in units of  $E_0$

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$E$

Energy in units of  $E_1$

Figure 3

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E  
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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to  $-a$   $a$

E

Energy in units of

Figure 3

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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$E > V_0$

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Energy in units of  $V_0$

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Energy in units of  $E_0$

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

Figure 3

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Energy in units of  $V_0$

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Figure 3

Energy in units of  $E_0$

Figure 3

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Figure 3

Energy in units of  $E_0$

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$E$

Energy in units of

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Energy in units of  $V_0$

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$i$  am number 1

$i$  am number 2

$i$  am number 3

$i$  am number 4

$i$  am number 5

$i$  am number 6

$i$  am number 7

$i$  am number 8

$i$  am number 9

$i$  am number 10

i am number 11  
i am number 12  
i am number 13  
i am number 14  
i am number 15  
i am number 16  
i am number 17  
i am number 18  
i am number 19  
i am number 20  
i am number 21  
i am number 22  
i am number 23  
i am number 24  
i am number 25  
i am number 26  
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