

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$V_0 \quad (|x| > a)$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

$$\text{of } p$$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < 0$$

$$E > 0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < 0$$

$$E > 0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c k_n = \hbar c \frac{n\pi}{L}$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$2mE/\hbar^2 \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to

to

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x)) / \hbar}$ inside the well and $\sqrt{2m(E - V_0) / \hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x)) / \hbar}$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and $\sqrt{2m(E - V(x)) / \hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x)) / \hbar}$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and $\sqrt{2m(E - V(x)) / \hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x)) / \hbar}$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and $\sqrt{2m(E - V(x)) / \hbar}$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x)) / \hbar}$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and $\sqrt{2m(E - V(x)) / \hbar}$ outside.

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

,

because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)/\hbar} \text{ inside the well and } p$$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar^2$. This equals $\sqrt{2mE/\hbar^2}$ inside the well and $\sqrt{2m(E - V_0)/\hbar^2}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$$\text{to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

$$\hbar\omega$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$\omega \propto E$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$E \propto \omega^2 \propto k^2 \propto n^2$$

$$E \propto \omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar.$$

This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

$$\text{gap between the energies grows as } n \text{ increases. Note that the energies in the case of a string are also proportional to } n^2$$

$$\text{are also proportional to } n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Figure 3

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\text{apply to both systems. The difference between the systems is that a string has } \omega \propto \sqrt{E},$$

$$\text{where as the quantum mechanical system has } \omega \propto E.$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

$$V_0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

ω , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p
 $\sqrt{2m(E - V_0)}/\hbar$
 outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both
 inside
 and outside the well. k is larger inside the well, so the wavelength is shorter
 there. A
 possible wavefunction might look something like the one in Fig. 5. It is customary
 to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case
 of a string
 are also proportional to n
 2
 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This
 equals $\sqrt{2mE}/\hbar$ inside the well and p
 $\sqrt{2m(E - V_0)}/\hbar$
 outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both
 inside
 and outside the well. k is larger inside the well, so the wavelength is shorter
 there. A
 possible wavefunction might look something like the one in Fig. 5. It is customary
 to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case
 of a string
 are also proportional to n
 2
 , because although $\omega = ck \propto n$, the energy is proportional to ω
 2
 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2
 and 3 both
 apply to both systems. The difference between the systems is that a string has $\omega \propto$
 \sqrt{E} ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically
 zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The
 lack of a $n = 0$ state is consistent with the uncertainty principle (see Section
 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$
 $k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future
 convenience, we'll
 let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V
 (x) is given
 by
 $V(x) = \frac{1}{2}$
 $0 \ (|x| \leq a)$
 $V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &\sqrt{2m(E - V(x))/\hbar}. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap

between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < -V_0$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega$ $\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E < V_0$$

$$E > V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$-a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$\omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$\omega \propto \sqrt{E}$$

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of $\hbar^2/(2ma^2)$.

Figure 3 shows the energy levels for a finite square well. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

E

Energy in units of $\hbar^2/(2ma^2)$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 .

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$. This equals $\hbar\omega$ inside the well and $\hbar\omega$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of $\hbar^2/(2ma^2)$.

Figure 3 shows the energy levels for a finite square well. The gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 .

E

Energy in units of $\hbar^2/(2ma^2)$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 .

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$.

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

\sqrt{E} ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

2
 2
outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

ω ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x)) / \hbar}.$$

$$\sqrt{2mE / \hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \text{ } (|x| \leq a)$
 $V_0 \text{ } (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E Energy in units of E_1 Figure 3 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E Energy in units of E_1 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n = \frac{1}{2} \hbar \omega_n$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :
 $V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{1}{2} \hbar \omega_n$

$E_n = \frac{1}{2} \hbar \omega_n$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$E_n = \frac{1}{2} \hbar \omega_n$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$E_n = \frac{1}{2} \hbar \omega_n$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar^2}. \text{ This equals } \sqrt{\frac{2m(E - V(x))}{\hbar^2}}$$

$$\sqrt{\frac{2mE}{\hbar^2}} \text{ inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2m(E - V_0)/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

2

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{to -a a}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < -V_0$

$-V_0 < E < 0$

$E > 0$

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{1}{2} \hbar \omega_n = \frac{1}{2} \hbar c k_n = \frac{1}{2} \hbar c \frac{n\pi}{L}$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega$ (not $\hbar \omega^2$). This equals $\sqrt{2m(E - V(x))}/\hbar$. This

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{1}{2} \hbar \omega_n = \frac{1}{2} \hbar c k_n = \frac{1}{2} \hbar c \frac{n\pi}{L}$

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to

to

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to plot } \psi(x) \text{ versus } x \text{ from } -a \text{ to } a.$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

to -a a}

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2 \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$-a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$\omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$\omega \propto \sqrt{E}$$

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$\frac{1}{2}$

ω

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$\frac{1}{2}$

ω

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

\sqrt{E} ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \frac{1}{2} \\ 0 &(|x| \leq a) \\ V_0 &(|x| > a). \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

$$\text{gap between the energies grows as } n \text{ increases. Note that the energies in the case of a string are also proportional to } n^2$$

$$\text{are also proportional to } n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

2

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

2

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Figure 3 shows the energy levels for a string and a quantum mechanical system. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 .

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 .

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2} V_0$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2m(E - V(x))}{\hbar^2}}$$

$$\frac{2mE}{\hbar^2} \text{ inside the well and } p = \frac{2m(E - V_0)}{\hbar^2}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\text{apply to both systems. The difference between the systems is that a string has } \omega \propto \sqrt{E},$$

$$\text{where as the quantum mechanical system has } \omega \propto E.$$

$$\text{There is no } n = 0 \text{ state, because from Eq. (18) this would make } \psi \text{ be identically zero.}$$

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$

$$p = \hbar k = \sqrt{2m(E - V(x))} \quad \text{This equals } \sqrt{2m(E - V_0)} \text{ inside the well and } \sqrt{2mE} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 4
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega (E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E < V_0$$

$$E > V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize to $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$.

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

\sqrt{E} ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

2
 2
outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}} \text{ outside.}$$

$$\frac{2mE}{\hbar^2} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\frac{2m(E - V_0)}{\hbar^2}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside.

$$\frac{2mE}{\hbar^2} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\frac{2m(E - V_0)}{\hbar^2}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x)) / \hbar}.$$

$$\sqrt{2mE / \hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$0 \ (|x| \leq a)$
 $V_0 \ (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$E = \hbar \omega$

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$E = \hbar \omega$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$\sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < 0$$

$$E > 0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :
 $V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Figure 3

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \hbar k$
 $k = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2mE}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

ω

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2 \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck \propto \hbar k^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$\text{inside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - 0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar^2$. So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 & \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 & \text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto \omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$\frac{1}{2}$

ω

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$\frac{1}{2}$

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$E_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar.$$

This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{\hbar^2 k_n^2}{2m}$

E_n

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{\hbar^2 k_n^2}{2m}$

E_n

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E_n

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \rho \omega^2 L^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \rho \omega^2 L^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

ω ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Energy in units of E_0

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Energy in units of E_0

Figure 3

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & -a \leq x \leq a \\ 0 & |x| > a \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p
 $\sqrt{2m(E - V_0)}/\hbar$
 outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both
 inside
 and outside the well. k is larger inside the well, so the wavelength is shorter
 there. A
 possible wavefunction might look something like the one in Fig. 5. It is customary
 to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case
 of a string
 are also proportional to n
 2
 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This
 equals $\sqrt{2mE}/\hbar$ inside the well and p
 $\sqrt{2m(E - V_0)}/\hbar$
 outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both
 inside
 and outside the well. k is larger inside the well, so the wavelength is shorter
 there. A
 possible wavefunction might look something like the one in Fig. 5. It is customary
 to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case
 of a string
 are also proportional to n
 2
 , because although $\omega = ck \propto n$, the energy is proportional to ω
 2
 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2
 and 3 both
 apply to both systems. The difference between the systems is that a string has $\omega \propto$
 \sqrt{E} ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically
 zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The
 lack of a $n = 0$ state is consistent with the uncertainty principle (see Section
 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$
 $k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future
 convenience, we'll
 let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V
 (x) is given
 by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E = V_0$
 $E < V_0$

-a a
Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)/\hbar}$ outside.

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

$$A \text{ possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

Figure 5

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

2
 2
outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \hbar k$
 $k = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x)) / \hbar}.$$

$$\sqrt{2mE / \hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{Energy in units of } E$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{Energy in units of } E$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \text{ } (|x| \leq a)$
 $V_0 \text{ } (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p/\hbar outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set $E = 0$ as the zero of energy.

Energy in units of E_0
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & 0 \leq |x| \leq a \\
 &= 0 & |x| > a
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$
 $E = V_0$
 $E > V_0$

Figure 4
 • $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{to -a a}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

V_0 ($|x| \leq a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to

to

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto \omega_n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

Figure 3

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

2
 2

2
outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.
 The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V(x))/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto \omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c k$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

$\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

$\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

$\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

to $-a \leq x \leq a$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar^2$. This equals $\sqrt{2mE/\hbar^2}$ inside the well and $\sqrt{2m(E - V_0)/\hbar^2}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$$\text{to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Figure 3

$$\text{gap between the energies grows as } n \text{ increases. Note that the energies in the case of a string are also proportional to } n^2$$

$$\text{, because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2 m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$-a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$\omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$\omega \propto \sqrt{E}$$

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \mu \omega_n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n = \frac{1}{2} \mu \omega_n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

Figure 3

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \frac{1}{2} \\ 0 &(|x| \leq a) \\ V_0 &(|x| > a). \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n \propto n^2$

$E_n \propto n^2$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$E_n \propto n^2$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \mu \omega_n^2$$

$$= \frac{1}{2} \mu c^2 k^2 \propto n^2, \text{ because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2$$

$$(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both$$

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

$$The$$

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

$$because \text{ such a state would have } \Delta x \Delta p = 0 \text{ (since } \Delta x < L, \text{ and } \Delta p = 0 \text{ because } n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0), \text{ which would violate the principle.}$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

$$let \ x = 0 \text{ be located at the center of the well. If we label the ends as } \pm a, \text{ then } V(x) \text{ is given}$$

$$by$$

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a)$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Figure 5

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega$ $\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E < V_0$$

$$E > V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

Figure 3

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

2
 2
outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x)) / \hbar}.$$

$$\sqrt{2mE / \hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar^2}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar^2$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \text{ } (|x| \leq a)$
 $V_0 \text{ } (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to plot } \psi(x) \text{ versus } x \text{ from } -a \text{ to } a.$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 & \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 & \text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p/\hbar outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set $E = 0$ as the zero of energy.

Energy in units of E_1
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & 0 \leq |x| \leq a \\
 &= 0 & |x| > a
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$
 $E = V_0$
 $E > V_0$

Figure 4
 • $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)
 V_0 ($|x| > a$).

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside.

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{1}{2} \hbar \omega_n$

$E_n = \frac{1}{2} \hbar \omega_n$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$E_n = \frac{1}{2} \hbar \omega_n$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$E_n = \frac{1}{2} \hbar \omega_n$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 $-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1.

Figure 3

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.
 The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 &(|x| \leq a) \\
 V_0 &(|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

$$\text{inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c k$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$ (19)

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$p = \sqrt{2m(E - V(x))}/\hbar$. This equals

$\sqrt{2mE}/\hbar$ inside the well and

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

$-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$\sqrt{2m(E - V_0)}/\hbar$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

\sqrt{E}

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

\sqrt{E}

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$E = \frac{1}{2} \int_{-a}^a \psi^2 dx = \frac{1}{2} \int_{-a}^a \psi^2 dx$$

$$E = \frac{1}{2} \int_{-a}^a \psi^2 dx$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

$\frac{1}{2}$

ω , because although $\omega = ck \propto n$, the energy is proportional to $\frac{1}{2}m\omega^2(E - V(x))$. This

equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

$\frac{1}{2}$

ω , because although $\omega = ck \propto n$, the energy is proportional to $\frac{1}{2}m\omega^2$

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

V_0 ($|x| \leq a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \frac{1}{2} \\ 0 &(|x| \leq a) \\ V_0 &(|x| > a). \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

$$\text{gap between the energies grows as } n \text{ increases. Note that the energies in the case of a string are also proportional to } n^2$$

$$\text{are also proportional to } n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of V_0

Figure 3

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$p = \sqrt{2m(E - V(x))}/\hbar$. This equals

$\sqrt{2mE}/\hbar$ inside the well and

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

$-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

$$V_0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - 0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\text{apply to both systems. The difference between the systems is that a string has } \omega \propto \sqrt{E}$$

$$\text{inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega (E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E < V_0$$

$$E > V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of V_0 .

Figure 3 shows the energy levels for a string. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$.

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of V_0 .

Figure 3 shows the energy levels for a string. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$.

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of V_0 .

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$.

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of V_0 .

Figure 3 shows the energy levels for a string. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$.

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize to $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$.

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

$$V_0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

2

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

2

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2m(E - V_0)/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$ outside the well.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \ (|x| \leq a)$
 $V_0 \ (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside.

$\frac{2m(E - V_0)}{\hbar}$

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside.

$\frac{2m(E - V_0)}{\hbar}$

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar
 \end{aligned}$$
 outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V(x))/\hbar}. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| > a) \\ 0 & (|x| \leq a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p/\hbar \text{ outside.}$$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2} V_0$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2m(E - V_0)}{\hbar}}$$

$$\text{inside the well and } p = \sqrt{\frac{2mE}{\hbar}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, where as the quantum mechanical system has $\omega \propto E$.

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E
 Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside the well.}$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$2m(E - V_0)/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

$$\sqrt{E},$$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x)) / \hbar. \text{ This equals } \sqrt{2mE} / \hbar \text{ inside the well and } \sqrt{2m(E - V_0)} / \hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{\hbar^2 k_n^2}{2m}$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n = \frac{\hbar^2 k_n^2}{2m}$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

$$\text{there. A possible wavefunction might look something like the one in Fig. 5. It is customary}$$

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

$$\text{of } p$$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

$$\text{there. A possible wavefunction might look something like the one in Fig. 5. It is customary}$$

to $-a \leq x \leq a$

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c k$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

$$The$$

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

$$because \text{ such a state would have } \Delta x \Delta p = 0 \text{ (since } \Delta x < L, \text{ and } \Delta p = 0 \text{ because } n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0), \text{ which would violate the principle.}$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

$$let \ x = 0 \text{ be located at the center of the well. If we label the ends as } \pm a, \text{ then } V(x) \text{ is given}$$

$$by$$

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x))$. This equals $\sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ outside the well and k inside the well.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar^2$. This equals $\sqrt{2mE/\hbar^2}$ inside the well and $\sqrt{2m(E - V_0)/\hbar^2}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$$\text{to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

$$\hbar\omega$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$\omega \propto E$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\frac{2m(E - V_0)}{\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 &(|x| \leq a) \\
 V_0 &(|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\frac{2m(E - V_0)}{\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Figure 3

$$\text{gap between the energies grows as } n \text{ increases. Note that the energies in the case of a string are also proportional to } n^2$$

$$\text{, because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2 m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E \propto \omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

$$\text{there. A possible wavefunction might look something like the one in Fig. 5. It is customary}$$

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \frac{1}{2} \\ 0 &(|x| \leq a) \\ V_0 &(|x| > a). \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

2

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$E > V_0$

$E < V_0$

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E = V_0$
 $E < V_0$

-a a
Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside the well and $p = \sqrt{2m(E - V_0)}$ inside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 4
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside the well and $p = \sqrt{2m(E - V_0)}$ inside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 4
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

to $-a \quad a$

to $-a \quad a$

to $-a \quad a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar.$$

This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$, \text{ because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x)) / \hbar}.$$

$$\sqrt{2mE / \hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$0 \ (|x| \leq a)$
 $V_0 \ (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 & \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 & \text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside.

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

ω

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar^2 \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$E_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - 0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < 0$$

$$E > 0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto \omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :
 $V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \mu \omega_n^2$$

$$= \frac{1}{2} \mu c^2 k^2 \propto n^2, \text{ because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2$$

$$(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both$$

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Figure 3 shows the energy levels for a string and a quantum mechanical system. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 .

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 .

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2} V_0$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

Figure 3

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

2
 2
outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar^2}. \text{ This equals } \sqrt{\frac{2m(E - V(x))}{\hbar^2}}$$

$$\sqrt{\frac{2mE}{\hbar^2}} \text{ inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \hbar k$
 $\sqrt{2m(E - V(x))}$. This equals $\sqrt{2mE}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Figure 5

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$\omega \propto \sqrt{E}$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 &(|x| \leq a) \\
 V_0 &(|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $k = \sqrt{2m(E - V(x))}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

A

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $k = \sqrt{2m(E - V(x))}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $k = \sqrt{2m(E - V(x))}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

$$\text{there. A possible wavefunction might look something like the one in Fig. 5. It is customary}$$

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

$$\text{there. A possible wavefunction might look something like the one in Fig. 5. It is customary}$$

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c k$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

Figure 3 shows the energy levels for a string and a quantum mechanical system. The gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 .

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 .

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 .

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar.$$

This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

\sqrt{E} ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\text{apply to both systems. The difference between the systems is that a string has } \omega \propto \sqrt{E},$$

$$\text{where as the quantum mechanical system has } \omega \propto E.$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Things get more complicated if we have a finite potential well. For future convenience, we'll

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.
 The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E \propto \omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$
 $E > V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

$\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

$\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

$\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega$

$$E_n = \frac{1}{2} \hbar \omega_n$$

$$\text{inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V(x))/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 3

Energy in units of E_1

Figure 3

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 3

Energy in units of E_1

Figure 3

Energy in units of E_1

Figure 3

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \text{ } (|x| \leq a)$
 $V_0 \text{ } (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto \omega_n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to

to

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 $-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
 Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E
 Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto \omega_n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar.$$

This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$E_n = \frac{1}{2} \hbar \omega_n$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$p = \sqrt{2m(E - V(x))}/\hbar$. This equals

$\sqrt{2mE}/\hbar$ inside the well and

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

$-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

$$\text{possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{E}$$

$$\text{Energy in units of}$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ outside the well and k inside the well.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar^2$. This equals $\sqrt{2mE/\hbar^2}$ inside the well and $\sqrt{2m(E - V_0)/\hbar^2}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

$$\sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \frac{1}{2} \\ 0 &(|x| \leq a) \\ V_0 &(|x| > a). \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
 Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

$$The$$

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

$$because \text{ such a state would have } \Delta x \Delta p = 0 \text{ (since } \Delta x < L, \text{ and } \Delta p = 0 \text{ because } n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0), \text{ which would violate the principle.}$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

$$let \ x = 0 \text{ be located at the center of the well. If we label the ends as } \pm a, \text{ then } V(x) \text{ is given}$$

$$by$$

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\text{apply to both systems. The difference between the systems is that a string has } \omega \propto \sqrt{E},$$

$$\text{where as the quantum mechanical system has } \omega \propto E.$$

$$\text{There is no } n = 0 \text{ state, because from Eq. (18) this would make } \psi \text{ be identically zero.}$$

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &\sqrt{2m(E - V(x))/\hbar}. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar^2$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$$V=V_0$$

$$-a \quad a$$

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega_n(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$-a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$\omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$\omega \propto \sqrt{E}$$

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of V_0 .

Figure 3

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of V_0 .

Figure 3

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$$k = \frac{\sqrt{2m(E - V(x))}}{\hbar}.$$

This equals $\frac{\sqrt{2mE}}{\hbar}$ inside the well and

$$\frac{\sqrt{2m(E - V_0)}}{\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$
- $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\frac{2mE}{\hbar^2} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\frac{2m(E - V_0)}{\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

$$\frac{2mE}{\hbar^2} \text{ inside the well and } \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\frac{2m(E - V_0)}{\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

to $-a$ a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$\omega \propto \sqrt{E}$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

E

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0) / \hbar}$ inside the well and p

$2m(E - V_0) / \hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \ (|x| \leq a)$
 $V_0 \ (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 &(|x| \leq a) \\
 V_0 &(|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
 , because although $\omega = ck \propto n$, the energy is proportional to ω

2
 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \ a$$

$$\text{to } -a \ a$$

$$\text{to } -a \ a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E = -V_0$$

$$E > -V_0$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2mE}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E = -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :
 $V=0$
 $V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

$\omega^2(E - V(x))/\hbar^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

$\omega^2(E - V(x))/\hbar^2$. So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
 That wouldn't be much of a state, because the probability would be zero everywhere.
 The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.
 The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

\sqrt{E} ,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

$x < -a$

$x > a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

to $-a \quad a$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E,
 where as the quantum mechanical system has $\omega \propto E$.
 There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$2m(E - V_0)/\hbar^2$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V_0)}/\hbar$

inside the well and p

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E > V_0$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E < V_0$$

$$-a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{to -a a}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

Figure 3

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x))$. This equals $\sqrt{2m(E - V(x))\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))\hbar}$. This equals $\sqrt{2m(E - V_0)\hbar}$ inside the well and $p = \sqrt{2m(E - V_0)\hbar}$ outside.

Figure 4

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

$E > V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$

$2m(E - V_0)/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

to -a a}

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

V_0

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

$$\omega^2 \propto k^2 \propto n^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

$$-a \leq x \leq a$$

to

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$\omega^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$

$0 \leq |x| \leq a$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize $\psi(x)$ to 1.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

\sqrt{E} ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \frac{1}{2} \\ 0 & \quad (|x| \leq a) \\ V_0 & \quad (|x| > a). \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$

$V_0 (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\text{apply to both systems. The difference between the systems is that a string has } \omega \propto \sqrt{E},$$

$$\text{where as the quantum mechanical system has } \omega \propto E.$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega = \hbar ck = \hbar c k$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E$$

$$E$$

$$E$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of

Figure 3

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E$$

$$E$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of $\hbar^2/(2ma^2)$.

Figure 3

E

Energy in units of $\hbar^2/(2ma^2)$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE\hbar^2/(2m)} = \hbar^2 k^2/(2m)$ inside the well and $\sqrt{2m(E - V_0)\hbar^2/(2m)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize ψ to 1. The energy E is in units of $\hbar^2/(2ma^2)$.

Figure 3

E

Energy in units of $\hbar^2/(2ma^2)$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))/\hbar}$.

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, while a quantum mechanical system has $\omega \propto E$.

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar.$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
 are also proportional to n^2
 n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 n^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2
 n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 n^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

2

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

2

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \ (|x| \leq a)$
 $V_0 \ (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Figure 3

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

Figure 3

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 & \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 & \text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E = V_0$
-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$E > -V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > -V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \frac{1}{2}$
 $0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n = \frac{1}{2} \hbar \omega_n$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

E

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2 ,
because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$\sqrt{2m(E - V_0)/\hbar} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$2m(E - V(x))/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2}$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 &(|x| \leq a) \\
 V_0 &(|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),
 because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This

$\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of E
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

$$2mE/\hbar^2 \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2mE}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\propto (ck)^2 \propto k^2 \propto n^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\propto (ck)^2 \propto k^2 \propto n^2$. So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n = \frac{1}{2} \omega_n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n = \frac{1}{2} \hbar \omega_n$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$

(19)

$V(x) = \frac{1}{2}$

$0 \leq |x| \leq a$

$V_0 \quad |x| > a$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

$\sqrt{2m(E - V_0)}/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))} / \hbar$. This equals $\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

$\sqrt{2m(E - V(x))} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

$\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

$\sqrt{2m(E - V_0)} / \hbar$ inside the well and $\sqrt{2m(E - V_0)} / \hbar$ outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

$$\text{to } -a \quad a$$

$$\text{to } -a \quad a$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$
 $E > V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$

$-a$ a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ outside the well and $\sqrt{2m(E - V_0)}/\hbar$ inside the well.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4
• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$x < -a$

$x > a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar^2$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$x < -a$

$x > a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

E

E

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

E

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

E

E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

E

E

E

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

to -a a}

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$\psi(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$\sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$\sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$\sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$\sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$E = \frac{1}{2} \hbar \omega^2 = \frac{1}{2} \hbar c^2 k^2 = \frac{1}{2} \hbar c^2 \left(\frac{n\pi}{L} \right)^2$$

$$E = \frac{1}{2} \hbar c^2 \left(\frac{n\pi}{L} \right)^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$E = V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2 , because although $\omega = ck \propto n$, the energy is proportional to ω

2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a
Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V(x))/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2m(E - V(x))}/\hbar$. This

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals \sqrt

$\frac{2mE}{\hbar}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

2

2

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n

ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar^2} \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p = \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p
 $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a
 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \end{aligned} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
 V(x) &= 0 & (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$
 $E < V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V(x))}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero. That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

V_0 ($|x| > a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$E = 0$

$E = V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and p

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n , because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\propto k^2 \propto n^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\propto k^2 \propto n^2$. So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside the well and $p = \sqrt{2m(E - V_0)}$ inside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 4
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)
This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside the well and $p = \sqrt{2m(E - V_0)}$ inside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 4
gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar \omega$ $\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{Energy in units of } E$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\text{Energy in units of } E$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well.

k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar$. So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by
 $V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

$\omega \propto \sqrt{E}$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$p = \sqrt{2m(E - V(x))}/\hbar$. This equals

$\sqrt{2mE}/\hbar$ inside the well and

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2mE/\hbar$ outside.

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

$$\text{possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{E}$$

$$\text{Energy in units of}$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar.$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

i am number 1

i am number 2

i am number 3

i am number 4

i am number 5

i am number 6

i am number 7

i am number 8

i am number 9

i am number 10

i am number 11
i am number 12
i am number 13
i am number 14
i am number 15
i am number 16
i am number 17
i am number 18
i am number 19