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Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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Energy in units of

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

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Energy in units of

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$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

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$$V_0 \quad (|x| > a).$$

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Energy in units of

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

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Energy in units of
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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$$V_0 \quad (|x| > a).$$

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$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

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Energy in units of

Figure 3

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Energy in units of

Figure 3

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Energy in units of V_0

Figure 3

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

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Energy in units of V_0

Figure 3

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Energy in units of

Figure 3

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

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$E < V_0$

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$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V_0 \quad (|x| > a).$$

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-a a

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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$$0 \quad (|x| \leq a)$$

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$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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√

E,

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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 $V = V_0$
 $-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \hbar k$
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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$2m(E - V_0)/\hbar$$

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E

Energy in units of

Figure 3

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Energy in units of

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Energy in units of

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E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

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10.3.3 Finite square well

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convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

-a a

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Energy in units of

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E

Energy in units of V_0

Figure 3

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Energy in units of V_0

Figure 3

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of V_0

Figure 3

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Energy in units of

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Figure 5

Energy in units of V_0

Figure 3

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

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Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of V_0

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Energy in units of

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E

Energy in units of E_1

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

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That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

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$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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E

Energy in units of

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

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Energy in units of

Figure 3

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

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$-a \leq x \leq a$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

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Energy in units of V_0

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Energy in units of

Figure 3

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n = \frac{1}{2} \hbar c k_n = \frac{1}{2} \hbar c \frac{n\pi}{L}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside.

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Energy in units of V_0

Figure 3

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 . This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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$$V_0 \quad (|x| > a).$$

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$$E = 0$$

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$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a$$

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$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$E = 0$$

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$$-a \leq x \leq a$$

Figure 4

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$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$V=V_0$

$-a \ a$

Figure 4

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$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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E

Energy in units of

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

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$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

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inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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Energy in units of
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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of
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Energy in units of
Figure 3

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$$\sqrt{2m(E - V_0)}/\hbar$$

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E

Energy in units of V_0

Figure 3

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E

Energy in units of

Figure 3

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are also proportional to n^2

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$-a \quad a$

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Energy in units of

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plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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Energy in units of

Figure 3

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

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$E > V_0$

$-a \leq x \leq a$

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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Energy in units of

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$$-a \leq x \leq a$$

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Figure 5

Energy in units of V_0

Figure 3

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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 V_0 & \quad (|x| > a).
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$-a \quad a$
 E
 Energy in units of

Figure 3

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E

Energy in units of

Figure 3

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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 E
 Energy in units of
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Energy in units of

Figure 3

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 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

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$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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Energy in units of

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

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Figure 4

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$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$0 (|x| > a).$$

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of

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Energy in units of

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$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } V_0 = 1 \text{ in units of } E.$$

$$V_0 = 1$$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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Energy in units of E

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by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$V=V_0$

-a a

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$E_n \propto n^2$$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

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2

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E

Energy in units of

Figure 3

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

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$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

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Energy in units of V_0

Figure 3

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$$-a \quad a$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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-a a

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Energy in units of V_0

Figure 3

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E Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

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Energy in units of V_0

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Energy in units of E_1

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$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

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Energy in units of

Figure 3

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are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

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Energy in units of

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

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$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E
 Energy in units of
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 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned} V(x) &= V_0 & (|x| \leq a) \\ V(x) &= 0 & (|x| > a). \end{aligned} \quad (19)$$

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$E < V_0$

$E > V_0$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

$$2m(E - V_0)/\hbar$$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

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Energy in units of V_0

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Figure 5

Energy in units of V_0

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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10.3.3 Finite square well
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$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

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$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

Figure 3

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$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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Energy in units of
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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
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Energy in units of

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$$-a \leq x \leq a$$

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

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-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))/\hbar}$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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Energy in units of V_0

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Energy in units of V_0

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Energy in units of V_0

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$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $p = \sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

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Figure 4

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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 $V = V_0$
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Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

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2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$-a \leq x \leq a$$

Figure 4

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

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$$E < V_0$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

Energy in units of V_0

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$2m(E - V_0)/\hbar^2 \text{ outside the well and } 2m(E - 0)/\hbar^2 \text{ inside the well.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

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$V=0$

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Figure 4

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

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E

Energy in units of

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2

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possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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n^2

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$V_0 \ (|x| > a)$.

(19)

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$-a \ a$

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Figure 3
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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$0 \quad (|x| \leq a)$

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Energy in units of V_0

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Energy in units of

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Energy in units of V_0

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Energy in units of

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$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

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$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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 (19)
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Energy in units of

Figure 3

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Energy in units of

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Figure 3

Energy in units of E_1

Figure 3

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$$V_0 (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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Energy in units of

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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Energy in units of

Figure 3

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k is larger inside the well, so the wavelength is shorter there.

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Energy in units of V_0

Figure 3

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n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of

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$$V_0 \quad (|x| > a).$$

$$(19)$$

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$V=V_0$

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E
Energy in units of

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$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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2
 2

2
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Energy in units of

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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E
 Energy in units of
 Figure 3
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Energy in units of E

Figure 3

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E

Energy in units of

Figure 3

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are also proportional to n

$$2$$

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

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inside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

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-a a

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Energy in units of

Figure 3

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

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, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto$

√

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

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Energy in units of

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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Energy in units of V_0

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

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E_n

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \omega_n^2}{2m c^2} \propto n^2$$

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$$V_0 \quad (|x| > a).$$

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$E > V_0$

$E < V_0$

$E = V_0$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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E

Energy in units of V_0

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

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Energy in units of V_0

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are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

$$\text{inside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = c^2 k^2 \propto n^2$.

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

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$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$E = V_0$$

$$-a \leq x \leq a$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

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$$\text{to } -a \quad a$$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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E

Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \text{ } a$$

$$\text{to } -a \text{ } a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \text{ } a$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

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-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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E
 Energy in units of
 Figure 3
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Energy in units of E
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2
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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

$2m(E - 0)/\hbar$

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Energy in units of

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$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Figure 4

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

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Energy in units of V_0

Figure 3

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Energy in units of

Figure 3

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$$E_n = \frac{1}{2} \rho \omega^2 L^2$$

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a)$$

$$0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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Energy in units of V_0

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Energy in units of

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Energy in units of V_0

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$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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10.3.3 Finite square well
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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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Figure 5

Energy in units of V_0

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Energy in units of E

Figure 3

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Energy in units of

Figure 3

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Energy in units of E_1

Figure 3

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gap between the energies grows as n increases. Note that the energies in the case of a string

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

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Energy in units of
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

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Energy in units of

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Energy in units of

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
 and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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$E > V_0$

$E = V_0$

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$E < V_0$

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Energy in units of

Figure 3

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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n^2

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E

Energy in units of

Figure 3

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

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Energy in units of

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$0 \quad (|x| \leq a)$

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(19)

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E:

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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Energy in units of V_0

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Figure 5

Energy in units of V_0

Figure 3

E

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Energy in units of E_1

Figure 3

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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 -a a
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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$2m(E - V_0)/\hbar$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

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Energy in units of

Figure 3

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$E = 0$

$E = V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

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Energy in units of

Figure 3

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Figure 3

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apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - 0)/\hbar}$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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Things get more complicated if we have a finite potential well. For future convenience, we'll

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by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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$E > V_0$

-a a

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Energy in units of

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Energy in units of

Figure 3

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are also proportional to n^2

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-a a

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of V_0

Figure 3

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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10.3.3 Finite square well

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-a a

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Energy in units of V_0

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of V_0

Figure 3

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Figure 5

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Energy in units of V_0

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to plot } \psi(x) \text{ versus } x \text{ from } -a \text{ to } a.$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$-a \ a$

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Energy in units of V_0

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Energy in units of V_0

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Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Figure 4
Energy in units of V_0 versus x/a . The well is centered at $x = 0$ and has width $2a$.

Figure 5
A possible wavefunction $\psi(x)$ for a particle in a finite square well. The wavefunction is oscillatory inside the well and decays exponentially outside the well.

Figure 6
Energy levels E_n versus well width $2a$. The energy levels decrease as the well width increases.

Figure 7
Energy levels E_n versus well depth V_0 . The energy levels increase as the well depth increases.

Figure 8
Energy levels E_n versus well width $2a$ and well depth V_0 . The energy levels decrease as the well width increases and increase as the well depth increases.

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Figure 11
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Energy levels E_n versus well width $2a$ and well depth V_0 . The energy levels decrease as the well width increases and increase as the well depth increases.

Figure 33
Energy levels E_n versus well width $2a$ and well depth V_0 . The energy levels decrease as the well width increases and increase as the well depth increases.

Figure 34
Energy levels E_n versus well width $2a$ and well depth V_0 . The energy levels decrease as the well width increases and increase as the well depth increases.

Figure 35
Energy levels E_n versus well width $2a$ and well depth V_0 . The energy levels decrease as the well width increases and increase as the well depth increases.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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-a a

Figure 4

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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Energy in units of V_0

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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Energy in units of V_0

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Energy in units of V_0

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$2m(E - V_0)/\hbar$

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Energy in units of

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$$-a \leq x \leq a$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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n^2

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

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$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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to $-a \quad a$

E

Energy in units of

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of

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$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

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$V=V_0$

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Energy in units of

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of E_0

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of V_0

Figure 3

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Energy in units of

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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 Energy in units of
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$\begin{aligned}
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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

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E

Energy in units of

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \mu \omega_n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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E
Energy in units of

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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 -a a

Figure 4

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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to $-a$ a

E

Energy in units of

Figure 3

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E

Energy in units of

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$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

Figure 3

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are also proportional to n^2

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of

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are also proportional to n

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ω

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Energy in units of

Figure 3

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are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$V_0 \quad (|x| > a).$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \rho \omega^2 L^2$$

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

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Energy in units of

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Energy in units of V_0

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$V=V_0$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of E_1

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

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$$-a \quad a$$

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E

Energy in units of

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Energy in units of

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$

$$\text{inside the well and } \sqrt{2m(E - 0)}/\hbar$$

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$$\text{inside the well and } p = \hbar k = \sqrt{2m(E - 0)} \quad \text{outside.}$$

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Energy in units of

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Energy in units of V_0

Figure 3

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Energy in units of

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of V_0

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of

$$V_0$$

Figure 3

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$$E_n = \frac{1}{2} \hbar \omega_n$$

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))}/\hbar.$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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-a a

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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Energy in units of V_0

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Energy in units of

Figure 3

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are also proportional to n

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$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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E

Energy in units of V_0

Figure 3

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E

Energy in units of

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Energy in units of

Figure 3

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Energy in units of V_0

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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E

Energy in units of V_0

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$$\text{Energy in units of } E$$

Figure 3

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Energy in units of V_0

Figure 3

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E

Energy in units of

Figure 3

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Energy in units of V_0

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Energy in units of
Figure 3

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Energy in units of E_0

Figure 3

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Energy in units of
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 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar}}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

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(19)

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Energy in units of V_0

Figure 3

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Energy in units of V_0

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$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

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Energy in units of
Figure 3
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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

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Energy in units of

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E

Energy in units of

Figure 3

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$$V_0 \quad (|x| > a).$$

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E

Energy in units of V_0

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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Energy in units of

Figure 3

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of
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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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Energy in units of V_0

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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 . So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 & \quad (|x| \leq a) \\
 V_0 & \quad (|x| > a).
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of V_0

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Energy in units of
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Energy in units of E

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$$V_0 \quad (|x| > a).$$

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Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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Energy in units of

Figure 3

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Energy in units of
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$-a$ a

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Energy in units of

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$V(x) = \frac{1}{2}$

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$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

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$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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Energy in units of

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E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

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are also proportional to n^2
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$V = 0$
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-a a

Figure 4

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

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-a a

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E

Energy in units of

Figure 3

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

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$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2m(E - 0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

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$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

E

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

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where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$p = \sqrt{2m(E - V(x))}/\hbar$. This equals

$\sqrt{2mE}/\hbar$ inside the well and

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

$-a \quad a$

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$2m(E - V_0)/\hbar^2$$

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-a a

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E

Energy in units of

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$$0 \quad (|x| \leq a)$$

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$E > V_0$

-a a

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Energy in units of V_0

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E Energy in units of

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Figure 5

Energy in units of V_0

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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n^2

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

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Energy in units of E_1

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$2m(E - V_0)/\hbar^2$$

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E

Energy in units of V_0

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n^2

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Energy in units of E

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Energy in units of

Figure 3

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of
Figure 3

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$$E = \frac{1}{2} \int_{-a}^a \psi^2 dx = \frac{1}{2} \int_{-a}^a \psi^2 dx$$

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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E

Energy in units of

Figure 3

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of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar$. This equals $\sqrt{\frac{2mE}{\hbar}}$

$\sqrt{\frac{2m(E - V_0)}{\hbar}}$

inside and outside the well. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$2m(E - V_0)/\hbar$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

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$2m(E - V_0)/\hbar$

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Energy in units of

Figure 3

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Energy in units of
Figure 3
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$$\begin{aligned} V(x) &= V_0 & (|x| \leq a) \\ V(x) &= 0 & (|x| > a). \end{aligned} \quad (19)$$

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
 n^2 equals $\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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-a a

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Figure 5

Energy

in units of E_1

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Energy in units of

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

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Figure 4

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

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E

Energy in units of

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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Energy in units of

Figure 3

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Energy in units of
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Energy in units of V_0

Figure 3

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Energy in units of E_1

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$$E_n = \frac{1}{2} \hbar \omega_n$$

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Energy in units of V_0

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

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That wouldn't be much of a state, because the probability would be zero everywhere. The

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$V(x) = \frac{1}{2}$

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$V=V_0$

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Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

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Figure 5

Energy in units of V_0

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Energy in units of V_0

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$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of

Figure 3

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Energy in units of V_0

Figure 3

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E

Energy in units of E_1

Figure 3

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 $\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and $\sqrt{\frac{2m(E - V_0)}{\hbar^2}}$ outside.

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Figure 4

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\sqrt{2mE}/\hbar$$

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Energy in units of

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$$-a \leq x \leq a$$

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$-a \quad a$

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Energy in units of

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Energy in units of

Figure 3

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Energy in units of

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E

Energy in units of

Figure 3

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 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

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Figure 4

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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Energy in units of
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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

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$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$\sqrt{2m(E - V_0)}/\hbar$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (since $E = \hbar\omega$). This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$V=V_0$

$-a$ a

Figure 4

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$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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2

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$$V_0 > 0 \quad (|x| > a).$$

(19)

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Figure 4

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of
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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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Energy in units of V_0

Figure 3

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Energy in units of E_1

Figure 3

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Things get more complicated if we have a finite potential well. For future

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

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$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar^2 \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar^2$$

$$\sqrt{2m(E - V_0)}/\hbar^2 \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

$$\text{there. A}$$

$$\text{possible wavefunction might look something like the one in Fig. 5. It is customary}$$

$$\text{to -a a}$$

$$\text{E}$$

$$\text{Energy in units of}$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Energy in units of

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$$\sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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Figure 5

Energy in units of V_0

Figure 3

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$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of V_0

Figure 3

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $2m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and p

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That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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$V(x) = \frac{1}{2}$

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V_0 ($|x| > a$).

(19)

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

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are also proportional to n

2

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$$V_0 (|x| \leq a).$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

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Energy in units of

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

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to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

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Energy in units of

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Energy in units of

Figure 3

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$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
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Energy in units of V_0

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

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-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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Energy in units of
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Energy in units of V_0

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10.3.3 Finite square well

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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E
Energy in units of
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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$E > V_0$

$E = V_0$

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Energy in units of

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of V_0

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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That wouldn't be much of a state, because the probability would be zero everywhere. The

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E < V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

ω^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Figure 4

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$\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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That wouldn't be much of a state, because the probability would be zero everywhere. The

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

$$V_0$$

Figure 3

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gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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10.3.3 Finite square well

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \leq x \leq a$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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Energy in units of

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$$E_n \propto n^2$$

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to $-a$ a

E

Energy in units of

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Energy in units of

Figure 3

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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Energy in units of V_0

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$p = \hbar k = \sqrt{2m(E - V(x))} \quad \text{This equals } \sqrt{2m(E - V_0)} \text{ inside the well and } \sqrt{2mE} \text{ outside.}$$

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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Energy in units of

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E

Energy in units of V_0

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$$\frac{2mE}{\hbar^2} \text{ inside the well and } p = \frac{2m(E - V_0)}{\hbar^2}$$

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E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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Energy in units of V_0

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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Energy in units of
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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Energy in units of

Figure 3

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Energy in units of

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 3
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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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10.3.3 Finite square well
Things get more complicated if we have a finite potential well. For future

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of V_0

Figure 3

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Energy in units of
Figure 3

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Energy in units of

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Energy in units of

Figure 3

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E

Energy in units of

Figure 3

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Energy in units of E_1

Figure 3

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of

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2

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$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of

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$$-a \leq x \leq a$$

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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Energy in units of

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E_n

E

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

2

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$$-a \quad a$$

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

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Energy in units of

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$$V_0 > 0 \quad (|x| > a).$$

(19)

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$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

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$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

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$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

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$$E > 0$$

$V=V_0$

-a a

Figure 4

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to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$V=V_0$

-a a

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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Figure 5

Energy in units of V_0

Figure 3

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$-a \quad a$

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possible wavefunction might look something like the one in Fig. 5. It is customary

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E

Energy in units of

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

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V_0 ($|x| > a$).

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E

Energy in units of

Figure 3

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

$E > V_0$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Figure 5

Figure 6

Figure 7

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

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Energy in units of

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Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \ a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$k \text{ is larger inside the well, so the wavelength is shorter there.}$$

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Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar^2$. This equals $\sqrt{2m(E - V(x))/\hbar^2}$ inside the well and $p = \sqrt{2m(E - V_0)/\hbar^2}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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by

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Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

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are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

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apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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10.3.3 Finite square well

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

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Energy in units of

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 Energy in units of
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$E > V_0$

$E = V_0$

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Energy in units of V_0

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Energy in units of

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$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well.}$$

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Energy in units of

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

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$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

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Energy in units of V_0

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Energy in units of E_1

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 (19)
 \end{aligned}$$

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$V=0$
 $V=V_0$
 -a a
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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\sqrt{2m(E - V(x))/\hbar}.$$

$$\sqrt{2m(E - V_0)/\hbar}$$

$$\sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

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$$-a \quad a$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$$p = \sqrt{2m(E - V_0)}/\hbar$$

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Energy in units of

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, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to

to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n \propto n^2$$

Energy in units of V_0

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E_n \propto n^2$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

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$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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$$0 \quad (|x| \leq a)$$

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Energy in units of

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Energy in units of

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$V_0 \quad (|x| > a).$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Figure 4

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

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Energy in units of
Figure 3

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E

Energy in units of V_0

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$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
, because although $\omega = ck \propto n$, the energy is proportional to ω

2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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$$V_0 \quad (|x| \leq a)$$

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-a a

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$$\sqrt{2m(E - V_0)}/\hbar$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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E

Energy in units of V_0

Figure 3

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E

Energy in units of

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Figure 3

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$V(x) = \frac{1}{2}$

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Figure 5

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$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

Energy in units of E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

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E

Energy in units of V_0

Figure 3

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Energy in units of

Figure 3

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Energy in units of

Figure 3

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$$V_0 \quad (|x| > a).$$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

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$$E < -V_0$$

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Energy in units of V_0

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apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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Energy in units of
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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :
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-a a

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

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$V(x) = \frac{1}{2}$

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V_0 ($|x| > a$).

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$V=V_0$

-a a

Figure 4

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$2m(E - V_0)/\hbar$ outside.

$2m(E - V_0)/\hbar$

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Energy in units of

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$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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E

Energy in units of

Figure 3

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$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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$V = 0$
 $V = V_0$
 -a a

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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E,

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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10.3.3 Finite square well

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10.3.3 Finite square well

Figure 3

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$E > V_0$$

$$-a \leq x \leq a$$

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$V=V_0$

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Energy in units of V_0

Figure 3

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 Energy in units of
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$$E > V_0$$

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$$-a \leq x \leq a$$

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Energy in units of

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Figure 5

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E:

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$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

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10.3.3 Finite square well

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Figure 3

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Energy in units of E_1

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

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Energy in units of V_0

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Energy in units of V_0

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E_n

E_n

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = (ck)^2 = \hbar^2 k^2 / 2m$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

$$\text{inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

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E_n

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 = (ck)^2 = \hbar^2 k^2 / 2m$

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$k = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$V_0 \quad (|x| > a).$$

(19)

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$E > V_0$

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-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

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k is larger inside the well, so the wavelength is shorter there. A

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Energy in units of V_0

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of V_0

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gap between the energies grows as n increases. Note that the energies in the case of a string

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$V(x) = \begin{cases} \frac{1}{2} & -a \leq x \leq a \\ 0 & |x| > a \end{cases}$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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E

Energy in units of

Figure 3

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E

Energy in units of V_0

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Energy in units of

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$2mE/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

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$E > V_0$

$E < V_0$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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n^2

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Energy in units of

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$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$
 $V=V_0$
 -a a
 Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

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Energy in units of
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 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of E_0

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Energy in units of

Figure 3

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Energy in units of V_0

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10.3.3 Finite square well

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \frac{1}{2}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

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Energy in units of

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

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Energy in units of

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Energy in units of V_0

Figure 3

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Energy in units of

Figure 3

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Energy in units of E_1

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$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \mu \omega_n^2$$

$$= \frac{1}{2} \mu c^2 k^2 = \frac{1}{2} \mu c^2 \left(\frac{n\pi}{L} \right)^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of

Figure 3

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E

Energy in units of E_1

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Energy in units of V_0

Figure 3

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$$V_0 \quad (|x| > a).$$

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Energy in units of

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Energy in units of V_0

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Energy in units of V_0

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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Energy in units of

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$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

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$V=0$

$V=V_0$

$-a \ a$

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normalize ψ to 1. The energy E is in units of V_0 .

Figure 3

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside.

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Energy in units of

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$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\frac{2m(E - V(x))}{\hbar}$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and p

$\frac{2m(E - V_0)}{\hbar^2}$ outside.

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))/\hbar$. This equals $\sqrt{\frac{2mE}{\hbar^2}}$ inside the well and p

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Energy in units of V_0

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$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

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Energy in units of

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Energy in units of

Figure 3

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Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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E

Energy in units of

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Energy in units of V_0

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

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$$E_n \propto n^2$$

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of V_0

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Energy in units of

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a). \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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Figure 3

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where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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10.3.3 Finite square well

$2mE/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)/\hbar}$

$2mE/\hbar$ inside the well and p

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

$$2m(E - V_0)/\hbar$$

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$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

Figure 5

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega^2 m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$\begin{aligned}
 V(x) &= \frac{1}{2} \\
 0 &(|x| \leq a) \\
 V_0 &(|x| > a).
 \end{aligned}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$$\sqrt{E}$$

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$V(x) = \frac{1}{2}$

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 (19)

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$V=0$

$V=V_0$

$-a \quad a$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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 E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2
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Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

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Energy in units of V_0

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \ a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of

Figure 3

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Energy in units of
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Energy in units of V_0

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, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > 0$$

$$E < 0$$

$$-a \leq x \leq a$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

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That wouldn't be much of a state, because the probability would be zero everywhere. The

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case

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Energy in units of E
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

because although $\omega = ck \propto n$, the energy is proportional to ω (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

to $-a$ a

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$2$$

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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where as the quantum mechanical system has $\omega \propto E$.

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

Figure 3

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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$V=0$

$V=V_0$

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Energy in units of

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$$V_0 \quad (|x| > a).$$

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$E < V_0$

-a a

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega \hbar (E - V(x)) / \hbar$. This equals $\sqrt{2m(E - V(x)) / \hbar}$ inside the well and $\sqrt{2m(E - V_0) / \hbar}$ outside.

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$E > V_0$

$E < V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

$E > V_0$

$E = V_0$

$E < V_0$

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$E = V_0$

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E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$\sqrt{2m(E - V_0)}/\hbar$

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Energy in units of

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Energy in units of V_0

Figure 3

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

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Energy in units of V_0

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 (19)
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E
 Energy in units of V_0

Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

E

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow$

$k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

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E Energy in units of

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E
 Energy in units of
 Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

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Energy in units of

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E

Energy in units of V_0

Figure 3

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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Energy in units of E

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Energy in units of

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E

Energy in units of E_1

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Energy in units of V_0

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That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

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Energy in units of

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gap between the energies grows as n increases. Note that the energies in the case of a string

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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$E > V_0$

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by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

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Energy in units of

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Energy in units of

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

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-a a

Figure 4

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$0 \quad (|x| \leq a)$

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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Energy in units of
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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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0 ($|x| \leq a$)

V_0 ($|x| > a$).

(19)

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Energy in units of

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Energy in units of

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$$= \frac{1}{2} \mu c^2 k^2$$

$$= \frac{1}{2} \mu c^2 \left(\frac{n\pi}{L} \right)^2$$

$$= \frac{1}{2} \mu c^2 \frac{n^2 \pi^2}{L^2}$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

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$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter

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possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

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$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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to $-a \leq x \leq a$

Energy in units of E_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{to -a a}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of

Figure 3

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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Energy in units of V_0

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Energy in units of
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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

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Energy in units of V_0

Figure 3

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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Energy in units of

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-a a

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Energy in units of V_0

Figure 3

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Energy in units of

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Energy in units of

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Energy in units of

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because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$2m(E - V_0)/\hbar^2$$

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Energy in units of

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Energy in units of V_0

Figure 3

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$E > V_0$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{Energy in units of } E$$

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E

Energy in units of

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Energy in units of V_0

Figure 3

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$$E_n$$

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Energy in units of

Figure 3

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k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$V_0 (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$E_n \propto n^2$

E_n

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

E_n

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E_n

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

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by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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-a a

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$\sqrt{2m(E - V_0)}/\hbar$ outside.

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Energy in units of

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Energy in units of
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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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$E < V_0$

$E > V_0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

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Energy in units of

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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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$$2m(E - V_0)/\hbar^2$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a). \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

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Energy in units of

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Energy in units of

Figure 3

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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E
Energy in units of
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$$V = V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

$$n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

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$$\begin{aligned}
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 \end{aligned}
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Energy in units of V_0

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$$\text{Energy in units of } V_0$$

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Energy in units of

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10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

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E

Energy in units of

Figure 3

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E

Energy in units of E_1

Figure 3

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Figure 5

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where

as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

by

$$V(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

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Energy in units of

Figure 3

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2

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Energy in units of

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$$E$$

Energy in units of V_0

Figure 3

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

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2

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$0 \quad (|x| \leq a)$

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Energy in units of
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Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $p = \sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar} \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \rho \omega_n^2 L$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$(because the time derivative in Eq. (4.50) brings down a factor of ω).$$

So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

$$E,$$

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a$ a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a$ a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

$\psi(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{to -a a}$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$\text{to -a a}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

V_0 ($|x| \leq a$).

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

inside
 and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
 Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case

of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to $\omega(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string
are also proportional to n^2
, because although $\omega = ck \propto n$, the energy is proportional to ω^2
(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both
apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,
where as the quantum mechanical system has $\omega \propto E$.
There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.
That wouldn't be much of a state, because the probability would be zero everywhere. The
lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),
because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
 $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V(x))/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c k$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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, because although $\omega = ck \propto n$, the energy is proportional to ω

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

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Energy in units of V_0

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$$\sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

$$\text{and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A}$$

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of a string
are also proportional to n^2
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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll
let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given
by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p
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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string
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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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Energy in units of

Figure 3

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are also proportional to n^2

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$$0 \quad (|x| \leq a)$$

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$2m(E - V_0)/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and $\sqrt{2m(E - V_0)/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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Energy in units of
 Figure 3
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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

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Energy in units of
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$E < V_0$

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside.

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Energy in units of V_0

Figure 3

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E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$\sqrt{2m(E - 0)}/\hbar \text{ outside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well.}$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

$$\omega^2 \propto k^2 \propto n^2$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$$E_n \propto n^2$$

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$\frac{2m(E - V(x))}{\hbar}. \text{ This equals } \sqrt{\frac{2mE}{\hbar}}$$

$$\text{inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar}}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case

of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

$\omega^2(E - V(x))/\hbar$. This equals $\sqrt{\frac{2mE}{\hbar}}$

$\sqrt{\frac{2m(E - V_0)}{\hbar}}$

inside and outside the well. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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$$k = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

to $-a \leq x \leq a$

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

$\sqrt{2m(E - V_0)/\hbar}$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

plot $\psi(x)$ versus x .

E

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$

E ,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$

$0 \ (|x| \leq a)$

$V_0 \ (|x| > a)$.

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of

$\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE/\hbar}$ inside the well and

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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Energy in units of

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(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

E,

where as the quantum mechanical system has $\omega \propto E$.

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω

n^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically

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The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then V

(x) is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside.

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside
and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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Things get more complicated if we have a finite potential well. For future convenience, we'll let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - 0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E
Energy in units of
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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given by

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

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E

Energy in units of V_0

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E
Energy in units of

Figure 3
gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2
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2
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Energy in units of V_0

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√

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E

Energy in units of

Figure 3

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Figure 5

Energy in units of V_0

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Energy in units of

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$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$
 $V_0 \quad (|x| > a).$
 (19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V_0)}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \quad a$

E

Energy in units of E_1

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
 Figure 3
 gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$. There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a $n = 0$ state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have $\Delta x \Delta p = 0$ (since $\Delta x < L$, and $\Delta p = 0$ because $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A

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E
Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

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$2m(E - V(x))/\hbar$. This equals $\sqrt{2mE/\hbar}$ inside the well and p

$2m(E - V_0)/\hbar$

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Energy in units of

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Energy in units of
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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$V = 0$
 $V = V_0$
-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $\sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2m(E - V_0)/\hbar}$ inside the well and $\sqrt{2m(E - V(x))/\hbar}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

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Energy in units of

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$$E$$

Energy in units of

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E = 0$$

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$$-a \leq x \leq a$$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

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There is no $n = 0$ state, because from Eq. (18) this would make ψ be identically zero.

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form

of $p = \hbar k$

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2mE}$ outside.

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and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to $-a \leq x \leq a$

E

Energy in units of

Figure 3

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let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E = 0$

$E = V_0$

$-a \leq x \leq a$

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > 0$$

$$E < 0$$

$$-a \leq x \leq a$$

Figure 4

• $E > 0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

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E

Energy in units of

Figure 3

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$$E_n \propto n^2$$

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let $x = 0$ be located at the center of the well. If we label the ends as $\pm a$, then $V(x)$ is given

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > 0$$

$V=V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to $\omega m(E - V(x))/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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, because although $\omega = ck \propto n$, the energy is proportional to ω

(because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$,

where as the quantum mechanical system has $\omega \propto E$.

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

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$$V=V_0$$

$$-a \quad a$$

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E < V_0$

$E > V_0$

-a a

Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and p

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$2m(E - V(x))/\hbar$. This equals $\sqrt{2m(E - V(x))}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside.

$2m(E - V_0)/\hbar$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

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, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega(E - V(x))$. This equals $\sqrt{2m(E - V(x))}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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(19)

This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

$\sqrt{2m(E - V(x))}$. This equals $\sqrt{2m(E - V_0)}$ inside the well and $\sqrt{2m(E - V_0)}$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter

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there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2 , because although $\omega = ck \propto n$, the energy is proportional to ω^2 (because the time derivative in Eq. (4.50) brings down a factor of ω). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has $\omega \propto \sqrt{E}$, where as the quantum mechanical system has $\omega \propto E$.

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$V = 0$
 $V = V_0$
 -a a

Figure 4

- $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of $p = \sqrt{2m(E - V(x))}/\hbar$. This equals $\sqrt{2mE}/\hbar$ inside the well and $\sqrt{2m(E - V_0)}/\hbar$ outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside and outside the well. k is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of
Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω

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$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given V_0 , there are two basic possibilities for the energy E :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

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Energy in units of V_0

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string are also proportional to n^2

$$E_n = \frac{1}{2} \hbar \omega_n$$

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside. k is therefore real everywhere, so $\psi(x)$ is an oscillatory function both inside

and outside the well. k is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to $-a \leq x \leq a$

E

Energy in units of V_0

Figure 3

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$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

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Energy in units of

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Figure 4

• $E > V_0$ (unbound state): From Eq. (11), the wavenumber k takes the general form of p

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Energy in units of

Figure 3

gap between the energies grows as n increases. Note that the energies in the case of a string

are also proportional to n

2

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

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, because although $\omega = ck \propto n$, the energy is proportional to $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))/\hbar}$. This equals $\sqrt{2mE\hbar c^2}$ inside the well and $\sqrt{2m(E - V_0)\hbar c^2}$ outside.

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$E > V_0$

-a a

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(19)

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Figure 4

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E

Energy in units of

Figure 3

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Energy in units of V_0

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Energy in units of

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ω

, because although $\omega = ck \propto n$, the energy is proportional to ω^2

i am number 1

i am number 2

i am number 3

i am number 4

i am number 5

i am number 6

i am number 7

i am number 8

i am number 9

i am number 10

i am number 11
i am number 12
i am number 13
i am number 14
i am number 15
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