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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

E,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter}$$

there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2

and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar^2$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$E$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

E,

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.



That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{1}{2} \mu \omega_n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

$$E_n = \frac{1}{2} \mu \omega_n^2$$

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E = -V_0$$

$$E > -V_0$$

Figure 4

•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E + V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2mE}/\hbar \text{ outside.}$$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E = -V_0$$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$n^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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$V=0$

$V=V_0$

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Energy in units of  
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Energy in units of  
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Energy in units of  $V_0$

Figure 3

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Energy in units of

Figure 3

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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2

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E

Energy in units of  $V_0$

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$E$

Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$E$

Energy in units of

Figure 3

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Energy in units of  $V_0$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given  
by  

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string  
are also proportional to  $n^2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$   
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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E = V_0$$

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

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Energy in units of

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$V_0 (|x| > a)$ .

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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Energy in units of

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

$E = V_0$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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E

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of  $E_1$

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$V_0 \quad (|x| > a).$

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Energy in units of

Figure 3

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Energy in units of  $V_0$

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E < V_0$$

$$-a \leq x \leq a$$

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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to

to

E

Energy in units of

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Energy in units of  $V_0$

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of

Figure 3

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Energy in units of  $E_1$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4



•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Energy in units of

Figure 3

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E

Energy in units of

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Energy in units of

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Energy in units of

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$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))/\hbar}$  inside the well and  $p$

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$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$  Energy in units of

Figure 3

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

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$$\text{to } \psi(0) = 1$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

$$E_n \propto n^2$$

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$$V_0 \quad (|x| > a).$$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $E_0$

Figure 3

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string  
are also proportional to  $n^2$ ,  
because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$   
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both  
apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$\sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

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Energy in units of

Figure 3

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Energy in units of  $V_0$

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-a a

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Energy in units of

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to  $-a$   $a$

E

Energy in units of

Figure 3

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Energy in units of

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$\text{to -a a}$$

Energy in units of  $V_0$

Figure 3

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$$\text{to -a a}$$

$$\text{to -a a}$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$V=V_0$

-a a

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to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$^2$

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$V=0$

$V=V_0$

-a a

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(19)

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 $V=0$   
 $V=V_0$

-a a

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

-a a

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Energy in units of

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$E = V_0$

$E < V_0$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

$2m(E - V_0)/\hbar$

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Energy in units of

Figure 3

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

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$V=V_0$

$-a \quad a$

Figure 4

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \hbar k$   
 $k = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2mE}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

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Energy in units of

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Figure 5

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

$$2m(E - V_0)/\hbar$$

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because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

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-a a

Figure 4

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside}$$

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$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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to  $-a \leq x \leq a$

E

Energy in units of

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V=V_0$

$-a \ a$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \ a$

$E$

Energy in units of  
Figure 3

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$E$ ,

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  
 Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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Energy in units of  
 Figure 3

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√

E,

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( $x$ ) is given

by

$$V(x) = \begin{cases} \frac{1}{2} & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

$-a \leq x \leq a$

$x < -a$

$x > a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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are also proportional to  $n$

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$   
 $V_0 \quad (|x| > a).$   
 (19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

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Figure 4

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$2m(E - V_0)/\hbar$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$   
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 E  
 Energy in units of  
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$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0$  ( $|x| > a$ ).

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$-a$   $a$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

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Energy in units of  
 Figure 3  
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$$\begin{aligned}
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to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$V_0 \quad (|x| > a).$$

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Energy in units of  $V_0$

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to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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E  
Energy in units of  
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Energy in units of  
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of a string  
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Energy in units of  $E$   
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-a a

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

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2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$E = V_0$

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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E

Energy in units of

Figure 3

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$$E_n \propto n^2$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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E

Energy in units of

Figure 3

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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Energy in units of  $V_0$

Figure 3

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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E

Energy in units of

Figure 3

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$-a \quad a$

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$E < V_0$

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Figure 5

Energy in units of  $V_0$

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$E$

Energy in units of  $V_0$

Figure 3

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$E$

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$E$

Energy in units of  $E_1$

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V = V_0$

-a a

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.

$k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

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$E$

Energy in units of  $V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$V_0 \quad (|x| > a).$

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-a a

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E

Energy in units of

Figure 3

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Energy in units of  $E_1$

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(19)

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $V_0$

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-a a

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $V_0$

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$E$

Energy in units of

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Energy in units of  $V_0$

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Energy in units of

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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### 10.3.3 Finite square well

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by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
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Energy in units of  
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 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$   
 $V=V_0$   
 -a a  
 Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

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$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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E

Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$ . This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

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to  $-a$   $a$

E

Energy in units of

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

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Energy in units of

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Energy in units of

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$E$

Energy in units of

Figure 3

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Energy in units of

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$$0 \quad (|x| \leq a)$$

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$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

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Energy in units of  $V_0$

Figure 3

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$V=V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

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$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$E$

Energy in units of  $V_0$

Figure 3

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E

Energy in units of  $V_0$

Figure 3

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$$= \frac{1}{2} \mu c^2 k^2$$

$$= \frac{1}{2} \mu c^2 \left( \frac{n\pi}{L} \right)^2$$

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Energy in units of

Figure 3

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Figure 4

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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Figure 3

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Energy in units of  $V_0$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

$V = V_0$

$-a \quad a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

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Energy in units of

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E < V_0$

$E = V_0$

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to  $-a$   $a$

$E$

Energy in units of

Figure 3

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$E$

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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 E  
 Energy in units of  
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Energy in units of

Figure 3

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Energy in units of

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 \end{aligned}$$

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V_0 \quad (|x| \leq a).$$

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Energy in units of  $V_0$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

10.3.3 Finite square well

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

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E

Energy in units of

Figure 3

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$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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to  $-a$   $a$

$E$

Energy in units of

Figure 3

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Energy in units of

Figure 3

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$$E < -V_0$$

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$$\text{to -a a}$$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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10.3.3 Finite square well

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by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

$V_0$  ( $|x| \leq a$ ).

$0$  ( $|x| > a$ ).

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  
Figure 3  
gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of

Figure 3

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Energy in units of  $V_0$

Figure 3

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V=V_0$

$-a \ a$

Figure 4

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Energy in units of

Figure 3

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Figure 5

Energy in units of  $V_0$

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Energy in units of

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$E = V_0$

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Figure 4

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$n^2$

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Figure 5

Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $E_0$

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Energy in units of  $E_1$

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$

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Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p = \hbar k$ . This equals  $\sqrt{2m(E - V(x))}$  inside the well and  $\sqrt{2m(E - V_0)}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of  $E$

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

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2

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$-a \ a$

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Energy in units of

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Energy in units of  $V_0$

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$   
Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

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A possible wavefunction might look something like the one in Fig. 5. It is customary to

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Energy in units of  $E_0$

Figure 3

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Energy in units of

Figure 3

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$E$

Energy in units of  $V_0$

Figure 3

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Figure 5

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$E$

Energy in units of

Figure 3

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

$E = V_0$

$E < 0$

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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Energy in units of

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of  $V_0$

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Energy in units of  $E_0$

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Energy in units of  $V_0$

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$$-a \leq x \leq a$$

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$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

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E

Energy in units of  $V_0$

Figure 3

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$V(x) = \frac{1}{2}$

$0$  ( $|x| \leq a$ )

$V_0$  ( $|x| > a$ ).

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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 Energy in units of  
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Energy in units of  $V_0$

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Energy in units of  $V_0$

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Energy in units of

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(19)

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-a a

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E  
Energy in units of  
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Energy in units of  $E_1$   
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 V(x) &= V_0 & 0 \leq |x| \leq a \\
 &= 0 & |x| > a
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 \tag{19}$$

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E  
Energy in units of

Figure 3

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E

Energy in units of  $V_0$

Figure 3

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

$$E_n \propto n^2$$

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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Energy in units of  $V_0$

Figure 3

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Energy in units of  
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Energy in units of  
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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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Energy in units of

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Energy in units of  
Figure 3

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Energy in units of

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$E$

Energy in units of  $E_1$

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Energy in units of  $E$   
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Energy in units of

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Energy in units of  $E$   
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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
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$E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of  $V_0$

Figure 3

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E

Energy in units of

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$$E = \hbar^2 k^2 / 2m$$

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$E$

Energy in units of  $V_0$

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$

$V = V_0$

-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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E  
Energy in units of

Figure 3

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2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2



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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E < V_0$$

$$-a \leq x \leq a$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 (|x| \leq a)$$

$$0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

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$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2mE}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

$$\text{to } -a \leq x \leq a$$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$E$

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$V = V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a$$

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$E$

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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$E$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$E$ ,

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set } \psi(0) = 1$$

$$\text{to } \psi(0) = 1$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

$$E_n \propto n^2$$

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$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$-V_0 < E < 0$$

$$E > 0$$

$V=V_0$

-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$E$

Energy in units of  $E_1$

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$E$

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Energy in units of  $V_0$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case



of a string  
are also proportional to  $n^2$ ,  
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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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$$0 \quad (|x| \leq a)$$

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$$V_0 (|x| \leq a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E < V_0$

$E = V_0$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

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$E$

Energy in units of  $V_0$

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$E$

Energy in units of  $V_0$

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$E$

Energy in units of  $E_1$

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 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
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to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

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Energy in units of

Figure 3

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$2$

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE/\hbar} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$$E$$

Energy in units of

Figure 3

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are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}

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 (19)
 \end{aligned}$$

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$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
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Energy in units of  
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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

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$E$

Energy in units of

Figure 3

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$\psi(x)$  is given

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$2$$

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of

Figure 3

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Energy in units of

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E

Energy in units of  $V_0$

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$E$

Energy in units of

Figure 3

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$E_n \propto n^2$$

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$E$

Energy in units of  $E_1$

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$E$   
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Energy in units of  $E_0$

Figure 3

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Energy in units of  $V_0$

Figure 3

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$2mE/\hbar \text{ inside the well and } p$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to

-a a

E

Energy in units of

Figure 3



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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of  $V_0$

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Energy in units of

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$$\begin{aligned}
 V(x) &= V_0 & (|x| \leq a) \\
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$0 \quad (|x| \leq a)$   
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of

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Energy in units of  $V_0$

Figure 3

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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$$V_0 > 0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

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$$-a \leq x \leq a$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Figure 3 shows the energy levels for a string. The gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

E

Energy in units of  $V_0$

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Energy in units of

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$$V_0 \quad (|x| > a).$$

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Energy in units of  $V_0$

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$E$

Energy in units of

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Energy in units of

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Energy in units of  $V_0$

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

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$E$

Energy in units of

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$$0 \quad (|x| \leq a)$$

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$$p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

Figure 3

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Energy in units of  
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Energy in units of

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E  
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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p/\hbar = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

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-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

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$V(x) = \frac{1}{2}$

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$V_0 \quad (|x| > a).$

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to  $-a$   $a$

E

Energy in units of

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Energy in units of

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Energy in units of

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Energy in units of

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Energy in units of

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Energy in units of

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$$V_0 (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

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Energy in units of  $V_0$

Figure 3

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$-V_0 < E < 0$$

$$E > 0$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of  $V_0$

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Energy in units of  
Figure 3

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2

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2$$

$$2m(E - V_0)/\hbar^2$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future



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$V=0$

$V=V_0$

$-a \ a$

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A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize  $\psi(x)$  to 1.

Figure 3 shows the energy levels for a string and a quantum mechanical system. The gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ .

Figure 3

Energy in units of  $E_1$

Figure 3

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Energy in units of

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2

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apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

$E$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

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$$0 \quad (|x| \leq a)$$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{}$

$$2mE/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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Energy in units of

Figure 3

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are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto$

√

E,

where as the quantum mechanical system has  $\omega \propto E$ .

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2m(E - 0)}/\hbar$$

$$\sqrt{2m(E - V_0)}/\hbar \text{ outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$\sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$

E,

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 4

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p$$

$$2m(E - V_0)/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \frac{1}{2}$$

$0 \quad (|x| \leq a)$   
 $V_0 \quad (|x| > a).$   
 (19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.

$2m(E - V_0)/\hbar$

$2m(E - V_0)/\hbar$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

$E$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4



•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  $E$   
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  $E$   
 Figure 3  
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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & 0 \leq |x| \leq a \\ 0 & |x| > a \end{cases}$$

$V_0 (|x| > a)$ .

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \ a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 (|x| \leq a)$

$V_0 (|x| > a)$ .

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

$E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p$   
 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$0 \quad (|x| \leq a)$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This

$\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  $E$   
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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of  $V_0$

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Energy in units of  $E_0$

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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E  
Energy in units of

Figure 3

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

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E  
Energy in units of

Figure 3

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

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Energy in units of

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$E$  Energy in units of

Figure 3

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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to  $-a$   $a$

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Energy in units of

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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Energy in units of

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

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$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

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are also proportional to  $n^2$

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$$V_0 > 0$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V=V_0$

-a a

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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Energy in units of

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E

Energy in units of  $V_0$

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$E$   
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Energy in units of

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Energy in units of  $V_0$

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Energy in units of  
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E  
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 (19)
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### 10.3.3 Finite square well

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

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$$-a \leq x \leq a$$

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E

Energy in units of  $V_0$

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Energy in units of  $E_1$

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$$0 \quad (|x| \leq a)$$

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $V_0$

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Energy in units of

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$E = V_0$

$E < V_0$

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$E$

Energy in units of  $V_0$

Figure 3

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of  $E_1$   
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 V(x) &= 0 & 0 \leq |x| \leq a \\
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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

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E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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Energy in units of  $V_0$

Figure 3

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Energy in units of  $V_0$

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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Energy in units of  $V_0$

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$E$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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Energy in units of

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$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$E < V_0$$

$$-a \leq x \leq a$$

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$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

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E

Energy in units of  $V_0$

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$E$

Energy in units of

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Energy in units of  $V_0$

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

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$-a$   $a$

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 Energy in units of  
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Energy in units of  $E_1$

Figure 3

$$E_n$$

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Energy in units of

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Energy in units of  $V_0$

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\sqrt{\frac{2mE}{\hbar}} \text{ inside the well and } p = \sqrt{\frac{2m(E - V_0)}{\hbar}}$$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$-a \leq x \leq a$$

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of  
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Energy in units of

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E

Energy in units of

Figure 3

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Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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 E  
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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > 0$$

$$E < 0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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E

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

Figure 3

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Energy in units of  $V_0$

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll  
let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given  
by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string  
are also proportional to  $n^2$ ,  
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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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$$k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well.}$$

$$k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a}$$

$E$

Energy in units of

Figure 3

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Energy in units of

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Energy in units of

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Energy in units of

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 -a a

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E

Energy in units of

Figure 3

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$$V_0 (|x| \leq a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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E

Energy in units of  $V_0$

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$E$

Energy in units of  $V_0$

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$E$

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Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

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$V=V_0$

$-a$   $a$

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Energy in units of

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of  $E_0$

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$E$

Energy in units of

Figure 3

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$^2$

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$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

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Energy in units of  $V_0$

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Energy in units of

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E

Energy in units of  $V_0$

Figure 3

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Figure 5

Energy

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Energy in units of

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Energy in units of

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Energy in units of  $V_0$

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Energy in units of  $E_1$

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 -a a  
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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

$2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$   
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$2mE/\hbar^2 \text{ inside the well and } 2m(E - V_0)/\hbar^2 \text{ outside.}$$

$$2m(E - V_0)/\hbar^2$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3  
gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $\sqrt{2m(E - V(x))/\hbar}$ . This equals  $\sqrt{2m(E - V_0)/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 4

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$$2m(E - V_0)/\hbar$$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$E$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$E$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))/\hbar}$  inside the well and  $p$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p = \hbar k$ . This equals  $\sqrt{2m(E - V(x))}$  inside the well and  $\sqrt{2m(E - V_0)}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < -V_0$$

$$E > -V_0$$

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Figure 4

•  $E > -V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$\propto (E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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Energy in units of  $E_0$

Figure 3

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Energy in units of  $E_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$$-a \leq x \leq a$$

$$E$$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

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$$-a \quad a$$

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-a a

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$\sqrt{2m(E - V_0)}/\hbar$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of

Figure 3

E

Energy in units of

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$\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.

$\sqrt{2m(E - V_0)}/\hbar$

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$$2m(E - V_0)/\hbar^2$$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

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(19)

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize to  $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$ .

$E$

Energy in units of  $V_0$

Figure 3

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(19)

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-a a

E

Energy in units of

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2

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2

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-a a

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-a a

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Energy in units of  $E$

### Figure 3

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Energy in units of

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Energy in units of

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & |x| \leq a \\ 0 & |x| > a \end{cases}$$

$$V_0 \text{ for } |x| \leq a$$

$$0 \text{ for } |x| > a$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega = \hbar ck = \hbar c \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$

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$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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$E$

Energy in units of  $V_0$

Figure 3

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√

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Energy in units of

Figure 3

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Energy in units of

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Energy in units of

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Energy in units of  
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Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $\sqrt{2m(E - V(x))/\hbar}$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $\sqrt{2m(E - V_0)/\hbar}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a



Energy in units of  
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 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
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Energy in units of

Figure 3

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are also proportional to  $n^2$

$$E_n \propto n^2$$

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Energy in units of  $V_0$

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there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$E = V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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$$\begin{aligned}
 V(x) &= \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

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E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$0 \quad (|x| \leq a)$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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Energy in units of

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$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

to  $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$ . This

$$\text{equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

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$2$

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$2$

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$V_0 \quad (|x| > a).$

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$V=0$

$V=V_0$

$-a$   $a$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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to  $-a$   $a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$V=V_0$

$-a \ a$

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E

Energy in units of

Figure 3

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are also proportional to  $n$

2

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$$V_0 (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E = 0$

$E = V_0$

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Energy in units of

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$\psi(x)$  is given

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$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

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$E$

Energy in units of

Figure 3

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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Energy in units of

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Energy in units of  
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Energy in units of  $V_0$

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Energy in units of  $V_0$

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Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

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Figure 3

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Energy in units of  $V_0$

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Energy in units of  $V_0$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2  
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2  
(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

$$p = \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - V_0)}/\hbar$$

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E  
Energy in units of

Figure 3

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are also proportional to  $n$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

$$\text{inside}$$

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$-a \quad a$$

Figure 4

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$$2m(E - V_0)/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

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2

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$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

$$k = \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$-a \quad a$

Figure 4

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possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E$ :

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$V=V_0$

$-a$   $a$

Figure 4

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$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

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Energy in units of

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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$$E < -V_0$$

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Energy in units of

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Energy in units of

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$V_0 \quad (|x| > a).$

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Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3  
gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string  
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $V = 0$
- $V = V_0$

-a a

Figure 4

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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 $E < V_0$

-a a  
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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ inside the well and } p = \sqrt{2m(E - 0)}/\hbar \text{ outside.}$$

$$p = \sqrt{2m(E - 0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

to  $-a \leq x \leq a$

$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$0 \quad (|x| \leq a)$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 4

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Energy in units of

Figure 3

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2

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E,

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$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$E$

Energy in units of

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$E$   
Energy in units of  $V_0$

Figure 3

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Energy in units of

Figure 3

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$2$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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E:

$V=0$

$V=V_0$

-a a

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Figure 5

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$E$

Energy in units of

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Figure 5

Energy in units of  $V_0$

Figure 3

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Energy in units of

Figure 3

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(19)

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

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$E$

Energy in units of  $V_0$

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Energy in units of

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$E$

Energy in units of  $E_1$

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 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 $-a \leq x \leq a$

### Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$ 

$$\begin{aligned}
 &2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \\
 &\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \leq x \leq a
 \end{aligned}$$

Energy in units of  
 Figure 3  
 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p = \hbar k$ . This equals  $\sqrt{2m(E - V(x))}$  inside the well and  $\sqrt{2m(E - V_0)}$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$E_n \propto n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V=0$$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

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$E > V_0$

$-a \leq x \leq a$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$E$

Energy in units of  $V_0$

Figure 3

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by

$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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Energy in units of  $E_0$

Figure 3

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E

Energy in units of

Figure 3

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are also proportional to  $n$

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E  
Energy in units of

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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Energy in units of  $V_0$

Figure 3

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by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}.$$

(19)



This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \ a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$  outside.

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$V_0 \ (|x| > a).$

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Energy in units of

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize  $\psi$  to 1.

E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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$E$

Energy in units of  $V_0$

Figure 3

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E  
Energy in units of

Figure 3  
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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

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$E < V_0$

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Figure 4

Energy in units of  $E_0$

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Energy in units of  $V_0$

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Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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$V_0 \quad (|x| > a).$

(19)

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$-a$   $a$

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Energy in units of

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Energy in units of  $V_0$

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$E$

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Energy in units of

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$E$

Energy in units of

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Energy in units of

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Energy in units of  $E$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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$E$

Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

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Energy in units of  $V_0$

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Energy in units of

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$ . This

$$p = \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2mE}/\hbar \text{ outside.}$$

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Energy in units of

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$2$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy E:

$$E < V_0$$

$$E > V_0$$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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$E$

Energy in units of  $V_0$

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Energy in units of  $V_0$

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Energy in units of  $E_1$

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E

Energy in units of

Figure 3

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Figure 4  
Energy in units of  $V_0$  versus  $x/a$ . The well is centered at  $x = 0$  and has width  $2a$ .

Figure 5  
A possible wavefunction  $\psi(x)$  for a particle in a finite square well. The wavefunction is oscillatory inside the well and decays exponentially outside the well.

Figure 6  
Energy levels  $E_n$  versus well width  $2a$ . The energy levels decrease as the well width increases.

Figure 7  
Energy levels  $E_n$  versus well depth  $V_0$ . The energy levels increase as the well depth increases.

Figure 8  
Energy levels  $E_n$  versus well width  $2a$  and well depth  $V_0$ . The energy levels decrease as the well width increases and increase as the well depth increases.

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$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{}$$

$$2mE/\hbar \text{ inside the well and } p$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{}$

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$E$

Energy in units of  $V_0$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $E_1$

Figure 3

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### Figure 4

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$$V_0 \quad (|x| \leq a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

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are also proportional to  $n^2$

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$E$

Energy in units of

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of  $E_0$

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Energy in units of

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Energy in units of  
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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $p = \sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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E  
Energy in units of

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Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V$

$(x)$  is given

by

$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

E:

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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Figure 3

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$$E > V_0$$

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to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

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2

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E

Energy in units of  $V_0$

Figure 3

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are also proportional to  $n^2$

2

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$E$

Energy in units of  $E_1$

$$E$$

Figure 3

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{1}{2} \rho \omega^2 A^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \frac{1}{2}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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$$-a \quad a$$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$V=V_0$

$-a \ a$

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$E$

Energy in units of

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Energy in units of

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Energy in units of  $E$

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$E$

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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$V_0 \quad (|x| > a).$   
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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Figure 4



•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set  $E = 0$  as the zero of energy.

Energy in units of  $E_1$   
 Figure 3  
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 $E = V_0$   
 $E > V_0$

Figure 4  
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Energy in units of

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of  $V_0$

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Energy in units of

Figure 3

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Energy in units of  $V_0$

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by

$V(x) = \frac{1}{2}$

$0$  ( $|x| \leq a$ )

$V_0$  ( $|x| > a$ ).

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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$E$

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$$E > V_0$$

$$-a \leq x \leq a$$

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E

Energy in units of  $V_0$

Figure 3

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$$0 \quad (|x| \leq a)$$

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar^2. \text{ This equals } \sqrt{2m(E - V(x))/\hbar^2} \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar^2} \text{ outside.}$$

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$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$= \frac{1}{2} \mu c^2 k^2 \propto n^2, \text{ because although } \omega = ck \propto n, \text{ the energy is proportional to } \omega^2$$

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Energy in units of  $V_0$

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E  
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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

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Energy in units of  $V_0$

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 -a a

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Energy in units of  
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$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.

$2m(E - V_0)/\hbar$

$k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to set  $E = 0$  as the zero of energy.

Energy in units of  $E_1$   
 Figure 3  
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$$\begin{aligned}
 V(x) &= V_0 & 0 \leq |x| \leq a \\
 &= 0 & |x| > a
 \end{aligned}
 \tag{19}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$   
 $E = V_0$   
 $E > V_0$

Figure 4  
 •  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

E,

where as the quantum mechanical system has  $\omega \propto E$ .

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-a a

Figure 4

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Energy in units of

Figure 3

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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-a a

Figure 4

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to  $-a$   $a$

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Energy in units of

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$-a$   $a$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

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Energy in units of

Figure 3

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$$2mE/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)/\hbar}$$

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Energy in units of

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$$p = \sqrt{2m(E - V_0)}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\hbar\omega(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

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$$V_0 \quad (|x| \leq a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E < V_0$

$E > V_0$

$-a \leq x \leq a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

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Energy in units of  $V_0$

Figure 3

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$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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$E$

Energy in units of

Figure 3

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$k = \sqrt{2m(E - V(x))}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

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$\psi(x)$  is given

by

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$$V_0 > 0$$

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-a a

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E

Energy in units of

Figure 3

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Figure 3

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Energy in units of

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$2$

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Figure 3

E

Energy in units of  $V_0$

Figure 3

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2

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$E$

Energy in units of  $V_0$

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$E$

Energy in units of

Figure 3

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### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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Energy in units of

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$E$

Energy in units of  $V_0$

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-a a

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E

Energy in units of

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E

Energy in units of

Figure 3

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

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$-a$   $a$

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$E$

Energy in units of  $V_0$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

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$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

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-a a

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Energy in units of  $E_1$   
 Figure 3

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$$\begin{aligned}
 V(x) &= 0 & \text{for } |x| \leq a \\
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 \end{aligned}
 \tag{19}$$

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 $E = V_0$   
 $E > V_0$

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E  
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Figure 3

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > 0$$

$$E < 0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > 0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of  
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Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

2

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$   
 $V=V_0$   
 $-a \quad a$

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$   
 Energy in units of  $V_0$

Figure 3

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$$0 \quad (|x| \leq a)$$

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$$E > V_0$$

$$-a \leq x \leq a$$

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$
(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $V = 0$
- $V = V_0$

-a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string  
are also proportional to  $n^2$   
, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$   
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lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

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$$0 \quad (|x| \leq a)$$

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$$V=0$$

$$V=V_0$$

$$-a \quad a$$

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E

Energy in units of

Figure 3

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$$2$$

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$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V(x))}/\hbar$  inside the well and  $p$

$2m(E - V_0)/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

$\omega$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$$2m(E - V_0)/\hbar^2$$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of

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$$-a \leq x \leq a$$

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{inside the well and } p = \sqrt{2mE}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

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$-a \ a$

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Energy in units of  
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Energy in units of

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Energy in units of

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Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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 Energy in units of  
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$$E$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ .

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of

Figure 3

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Energy in units of  
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E  
Energy in units of

Figure 3

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E

Energy in units of  $V_0$

Figure 3

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Energy in units of

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$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$V_0 > 0 \quad (|x| > a).$$

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Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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$E$

Energy in units of

Figure 3

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Energy in units of  $V_0$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

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Energy in units of  $V_0$

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a \quad a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form

of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE/\hbar}$  inside the well and  $p$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| \leq a).$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$E > V_0$

$E < V_0$

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Energy in units of  $V_0$

Figure 3

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Energy in units of  
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Energy in units of

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$E$

Energy in units of

Figure 3

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$E$

Energy in units of  $E_1$

Figure 3

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Energy in units of  
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E

Energy in units of

Figure 3

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$^2$

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Energy in units of

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2

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2

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Energy in units of  
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Energy in units of

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Energy in units of  $V_0$

Figure 3

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Energy in units of

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$E$  Energy in units of

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$$V_0 \quad (|x| > a).$$

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$-a \leq x \leq a$

Figure 4

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Energy in units of  $V_0$

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar \text{ outside.}$$

$$2m(E - V_0)/\hbar$$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$$2$$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E,

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by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$E$

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$E_n$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

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$E$

Energy in units of  $E_1$

Figure 3

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt$

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$V(x) = \frac{1}{2}$

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normalize  $\psi$  to 1.

Figure 3

E

Energy in units of

Figure 3

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inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  ( $E = \hbar \omega$ ). This equals  $\hbar^2 k^2 / 2m$  inside the well and  $p$

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-a a

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Figure 5

Energy in units of  $V_0$

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < 0$$

$$E > 0$$

$$-a \leq x \leq a$$

Figure 4

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$

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$$E$$

Energy in units of

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E

Energy in units of  $V_0$

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gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$V = V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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E

Energy in units of



Figure 3

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$$E < V_0$$

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E

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

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$n^2$

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Energy in units of  $V_0$

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Energy in units of

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 gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$\begin{aligned}
 V(x) &= \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \\
 (19)
 \end{aligned}$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
 -a a

Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$   
 $\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{\frac{2mE}{\hbar}}$  inside the well and  $\sqrt{\frac{2m(E - V_0)}{\hbar}}$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a$   $a$

E

Energy in units of

Figure 3

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V=0$$

$$V=V_0$$

$$-a \quad a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both inside and outside the well. } k \text{ is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to } -a \quad a$$

$$E$$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of  $V_0$

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E

Energy in units of

Figure 3

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2

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Energy in units of

Figure 3

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Energy in units of

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10.3.3 Finite square well



to  $-a$   $a$

E

Energy in units of

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E

Energy in units of

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to -a a

E

Energy in units of

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$E$

Energy in units of  $E_1$

Figure 3

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$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and p

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2 m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and p

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### 10.3.3 Finite square well

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

$$(19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E < V_0$$

$$E > V_0$$

$$-a \leq x \leq a$$

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$E$

Energy in units of

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Energy in units of  $V_0$

Figure 3

gap

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$V=V_0$

-a a

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Energy in units of

Figure 3

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$V_0 \quad (|x| > a).$

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$V=V_0$

-a a

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E  
Energy in units of  
Figure 3  
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$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

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Energy in units of

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Energy in units of  $V_0$

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to  $-a \leq x \leq a$

E

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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That wouldn't be much of a state, because the probability would be zero everywhere. The

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

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plot  $\psi(x)$  versus  $x$ .

$E$

Energy in units of  $V_0$

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Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

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Energy in units of  
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Energy in units of

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$V_0 \quad (|x| > a).$

(19)

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Energy in units of

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E

Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $E_1$

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-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\frac{2m(E - V(x))}{\hbar}$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$

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Energy in units of  $V_0$

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$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2m(E - V(x))}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$2m(E - V_0)/\hbar$$

$$2m(E - V_0)/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \quad a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

$E$ ,

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

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The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section

10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll

let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} \frac{1}{2} & 0 \leq |x| \leq a \\ V_0 & |x| > a \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

$-a$   $a$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$\sqrt{2m(E - V_0)}/\hbar$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of

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$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$\psi(x)$  is given

by

$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 > 0$$

$$V_0 > 0$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$E = V_0$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar$$

$$k = \sqrt{2mE}/\hbar \text{ inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$k = \sqrt{2m(E - V_0)}/\hbar$$

outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside

and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a$   $a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

$$E_n \propto n^2$$

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$$V_0 > 0$$

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This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$



$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to -a a

E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

$n^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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by

$V(x) = \frac{1}{2}$

$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$\sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

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outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
Figure 3  
gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n^2$ , because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$  (because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ , where as the quantum mechanical system has  $\omega \propto E$ . There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero. That wouldn't be much of a state, because the probability would be zero everywhere. The lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below), because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.

### 10.3.3 Finite square well

Things get more complicated if we have a finite potential well. For future convenience, we'll let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

E  
Energy in units of  
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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V = 0$   
 $V = V_0$   
-a a

Figure 4

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Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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$$V_0 \quad (|x| > a).$$

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•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$2m(E - V(x))/\hbar$ . This equals  $\sqrt{2m(E - V_0)}/\hbar$  inside the well and  $\sqrt{2m(E - 0)}/\hbar$  outside.

$2m(E - V_0)/\hbar$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$$V(x) = \begin{cases} \frac{1}{2}V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

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Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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$2$

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Energy in units of

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$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar$$

$$\sqrt{2mE}/\hbar \text{ inside the well and } \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary

to  $-a \leq x \leq a$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case

of a string

are also proportional to  $n$

2

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

2

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

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E,

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### 10.3.3 Finite square well

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$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E > V_0$$

$$E < V_0$$

$$-a \leq x \leq a$$

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Figure 4

- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$p = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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$E$

Energy in units of

Figure 3

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$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of

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Energy in units of

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$k$  is larger inside the well, so the wavelength is shorter there.

A possible wavefunction might look something like the one in Fig. 5. It is customary to

normalize  $\psi(x)$  to 1.

Figure 3

E

Energy in units of  $E_1$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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E

Energy in units of

Figure 3

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Energy in units of

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$$\text{inside the well and } p = \sqrt{2m(E - V_0)}/\hbar$$

$$\text{outside. } k \text{ is therefore real everywhere, so } \psi(x) \text{ is an oscillatory function both}$$

inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n^2$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

(because the time derivative in Eq. (4.50) brings down a factor of  $\omega$ ). So Figs. 2 and 3 both

apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$

where as the quantum mechanical system has  $\omega \propto E$ .

There is no  $n = 0$  state, because from Eq. (18) this would make  $\psi$  be identically zero.

That wouldn't be much of a state, because the probability would be zero everywhere. The

lack of a  $n = 0$  state is consistent with the uncertainty principle (see Section 10.4 below),

because such a state would have  $\Delta x \Delta p = 0$  (since  $\Delta x < L$ , and  $\Delta p = 0$  because  $n = 0 \Rightarrow k = 0 \Rightarrow p = \hbar k = 0$ ), which would violate the principle.



√

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$0 \quad (|x| \leq a)$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$V = 0$$

$$V = V_0$$

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Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$2m(E - V(x))/\hbar. \text{ This equals } \sqrt{2mE}/\hbar \text{ inside the well and } p$$

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Energy in units of

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 $V = V_0$   
 $-a \leq x \leq a$

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Energy in units of

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Energy in units of

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Energy in units of  $V_0$

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Energy in units of

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Energy in units of  $V_0$

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apply to both systems. The difference between the systems is that a string has  $\omega \propto \sqrt{E}$ ,

$E$ ,

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let  $x = 0$  be located at the center of the well. If we label the ends as  $\pm a$ , then  $V(x)$  is given

by

$$V(x) = \begin{cases} 0 & (|x| \leq a) \\ V_0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$$E = 0$$

$$E = V_0$$

$$-a \leq x \leq a$$

Figure 4

•  $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p$

$$k = \sqrt{2m(E - V(x))}/\hbar. \text{ This equals } \sqrt{2m(E - V_0)}/\hbar \text{ outside the well and } \sqrt{2m(E - 0)}/\hbar \text{ inside the well.}$$

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possible wavefunction might look something like the one in Fig. 5. It is customary to  $-a \leq x \leq a$

$E$

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

$$E_n \propto n^2$$

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$-a \quad a$

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E

Energy in units of

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

are also proportional to  $n$

2

, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega m(E - V(x))/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $p$

$2m(E - V_0)/\hbar$

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$$0 \quad (|x| \leq a)$$

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

$V=0$

$V=V_0$

-a a

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(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy

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$V=0$

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-a a

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases} \quad (19)$$

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$E < V_0$

$E > V_0$

$E = V_0$

Figure 4

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to  $-a$   $a$

$E$

Energy in units of  $V_0$

Figure 3

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Energy in units of

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E

Energy in units of  $V_0$

Figure 3

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$E$

Energy in units of

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$E$

Energy in units of  $V_0$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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Energy in units of

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Energy in units of  $E$

Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string

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Energy in units of

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$E$

Energy in units of  $E_1$

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and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A

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$E$

Energy in units of

Figure 3

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega^2$

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$$V(x) = \begin{cases} V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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$E$

Energy in units of  $V_0$

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$\psi(x)$  is given

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$$V(x) = \begin{cases} -V_0 & (|x| \leq a) \\ 0 & (|x| > a) \end{cases}$$

$$V_0 \quad (|x| > a).$$

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$$E < 0$$

$$E > 0$$

$$-a \leq x \leq a$$

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Energy in units of

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$0 \quad (|x| \leq a)$

$V_0 \quad (|x| > a).$

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Energy in units of

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- $E > V_0$  (unbound state): From Eq. (11), the wavenumber  $k$  takes the general form of  $p = \sqrt{2m(E - V(x))}/\hbar$ . This equals  $\sqrt{2mE}/\hbar$  inside the well and  $\sqrt{2m(E - V_0)}/\hbar$  outside.  $k$  is therefore real everywhere, so  $\psi(x)$  is an oscillatory function both inside and outside the well.  $k$  is larger inside the well, so the wavelength is shorter there. A possible wavefunction might look something like the one in Fig. 5. It is customary to -a a

Energy in units of  
Figure 3

gap between the energies grows as  $n$  increases. Note that the energies in the case of a string are also proportional to  $n$

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, because although  $\omega = ck \propto n$ , the energy is proportional to  $\omega$

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E

Energy in units of  $V_0$

Figure 3

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$E$

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Energy in units of  $E_1$

Figure 3

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E

Energy in units of

Figure 3

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$E$

Energy in units of  $V_0$

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$0 \ (|x| \leq a)$

$V_0 \ (|x| > a).$

(19)

This is shown in Fig. 4. Given  $V_0$ , there are two basic possibilities for the energy  $E$ :

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$-a \ a$

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Energy in units of  $V_0$

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i am number 1

i am number 2

i am number 3

i am number 4

i am number 5

i am number 6

i am number 7

i am number 8

i am number 9

i am number 10

i am number 11  
i am number 12  
i am number 13  
i am number 14  
i am number 15  
i am number 16  
i am number 17  
i am number 18  
i am number 19  
i am number 20  
i am number 21  
i am number 22  
i am number 23  
i am number 24  
i am number 25  
i am number 26  
i am number 27  
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i am number 32