Chapter 2 - The Basic Theory of Interest

Internal Rate of Return

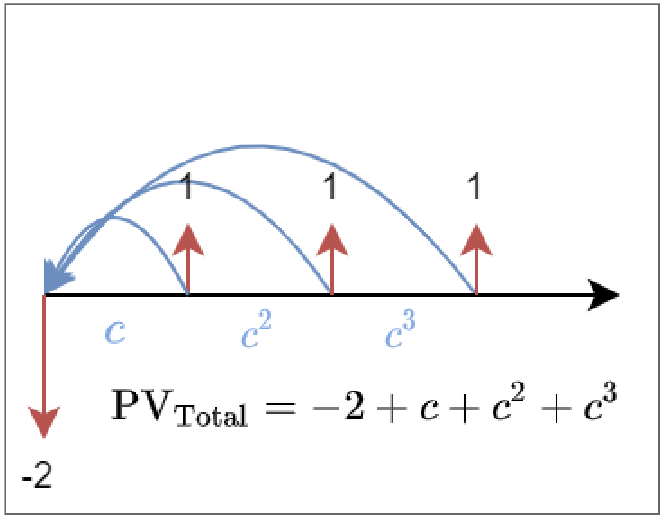
- The internal rate of return is another way of summarizing or describing a series of cash flows.
- It is the per period interest rate that would make the PV of the cash flow stream zero.

Internal Rate of Return: Let $(x_0, x_1, \ldots, x_n)$ be a cash flow stream. Then the internal rate of return, r, of this stream satisfies

$$0 = x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + \ldots + \frac{x_n}{(1+r)^n}.$$

Equivalently, is the r such that the discount factor, $c = \frac{1}{1+r}$, satisfies

$$0 = x_0 + x_1 c + x_2 c^2 + \ldots + x_n c^n.$$



In [3]:

```
import numpy as np
from numpy.polynomial import Polynomial
irr_poly = Polynomial([-2, 1, 1, 1])
real_roots = np.real(irr_poly.roots())
discount_factor = np.max(real_roots)
print(f"IRR: {1/discount_factor - 1:.2f}")
```

```
IRR: 0.23
```

Evaluation Criteria

Net Present Value

- To evaluate alternatives using NPV, we simply rank them by their PV.
- This allows us to compare different cash flow streams, with inflows and outflows at different times, as long as we use the appropriate interest rate.
- Widely regarded as the single best measure of an investment's measure.
- Heck, you can even add two PVs together to evaluate a combined investment.

Internal Rate of Return

- Alternatively, we can rank alternatives according to their IRR.
- Usually, an investment is not worth considering unless its IRR is greater than the prevailing, accessible interest rate. Can you see why?

Discussion of the Criteria

- Theorists tend to use NPV.
- IRR can be valuable when investment decisions can be repeated, i.e., the returns can be compounded.
- However, NPV could still be used if you consider *all* the future cash flows.
- NPV can be very tricky in practice: What interest rate goes where?
- NPV doesn't tell you about *return*: you may have to invest a lot to get a positive NPV.

Applications and Extensions

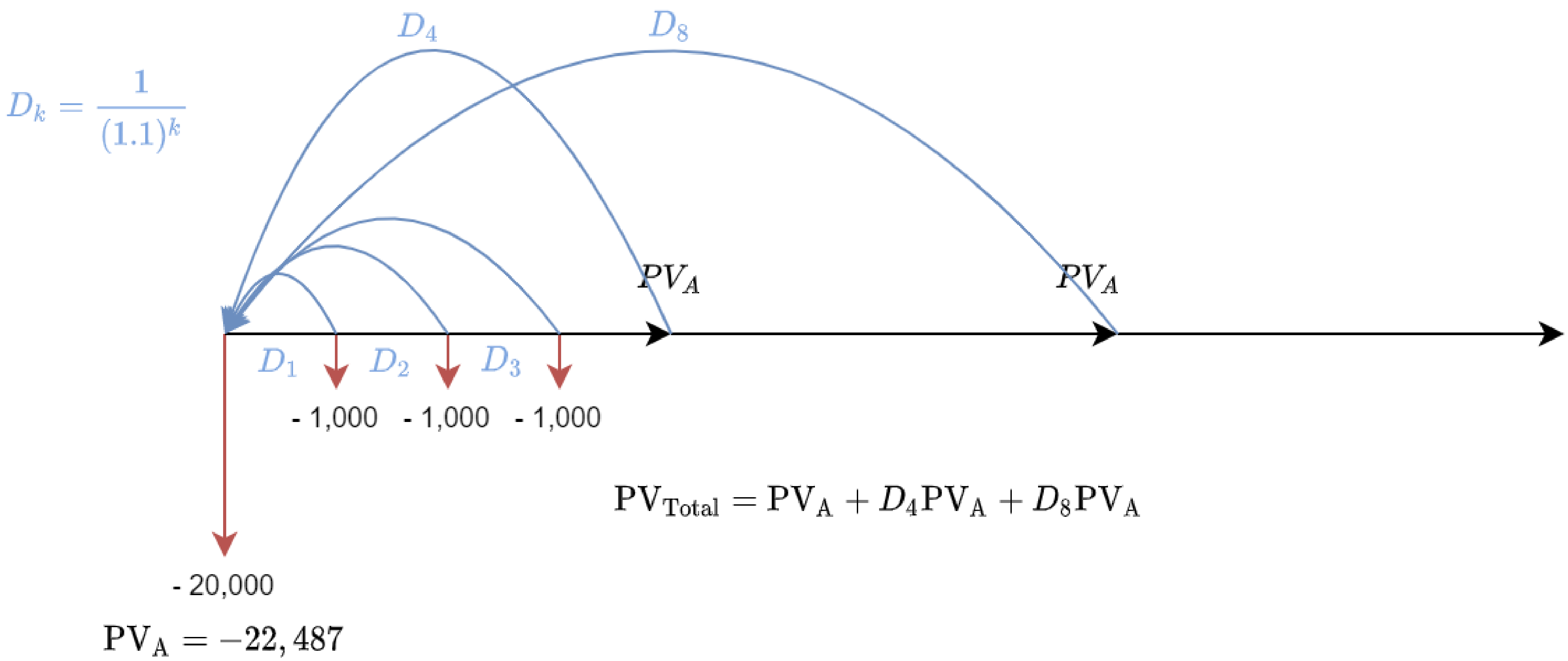
Cycle Problems

- What about NPV comparisons for cash flows with different lengths?
    - We can line up them up by repeating them an appropriate number of times OR we can repeat them indefinitely.

Example 2.7: (Automobile Purchase)

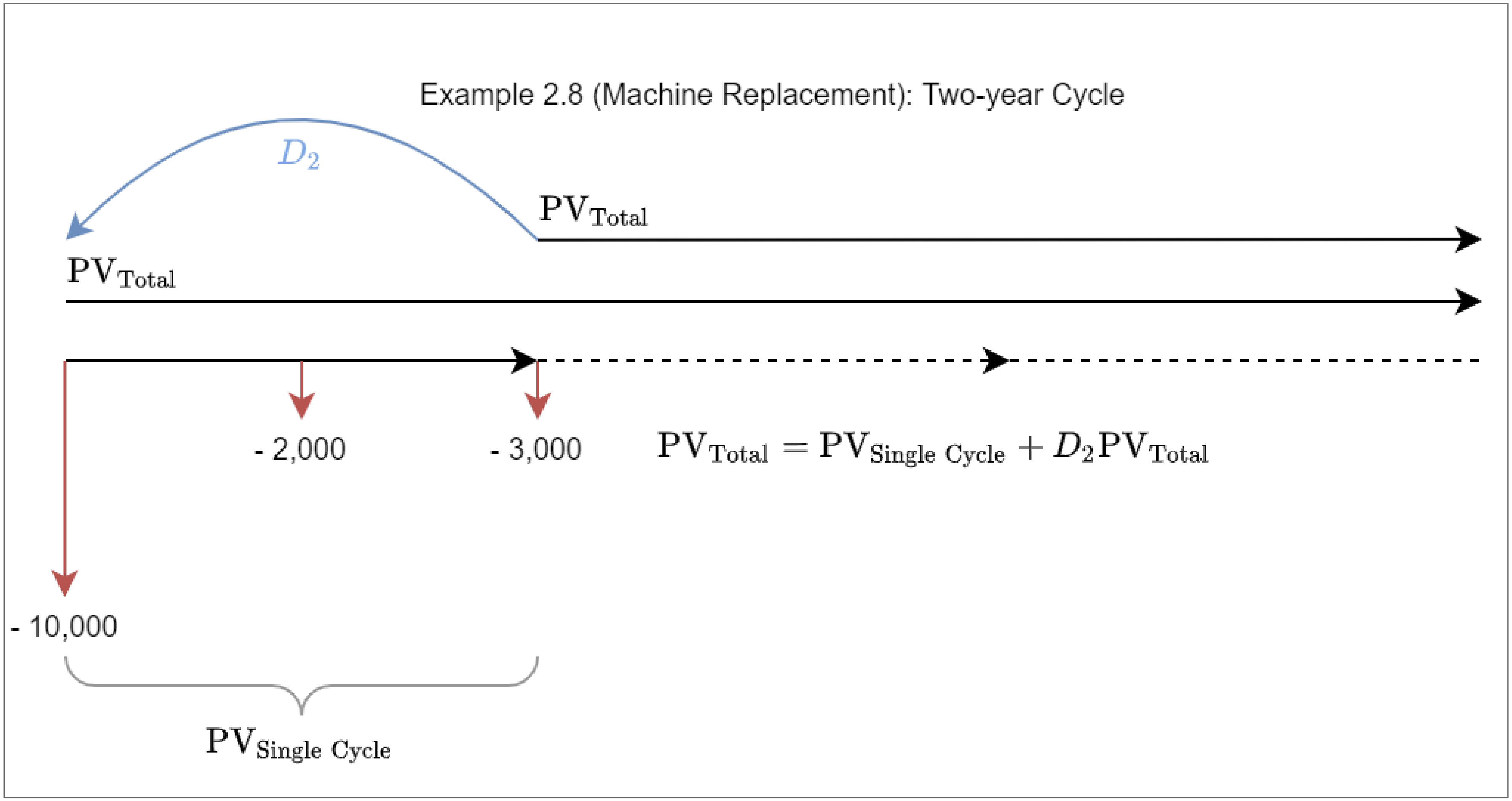
- The prevailing interest rate is 10% annually.
- Car A:
  - Initial cost of 20,000 USD.
  - Annual cost of 1000 USD (payable at the beginning of each year after the first year).
  - Lasts 4 years.
- Car B:
  - Initial cost of 30,000 USD.
  - Annual cost of 2000 USD (payable at the beginning of each year after the first year).
  - Last 6 years.
- To compare them, we repeat the Car A lifecycle three times and the Car B lifecycle twice.

Example 2.7 (Automobile Purchase): Car A Lifecycle



Example 2.8: (Machine Replacement)

- The prevailing interest rate is 10% annually.
- Specialized Machine:
  - Initial cost of 10 000 USD.
  - Operating cost of 2000 USD during the first year, with payment at the end of the year.
  - Operating costs increase by 1000 USD each year after the first.
- How often should the machine be replaced?
  - We use the same "trick" from the previous example: we identify when the cash flow cycles repeat.
  - With an added twist: we pretend these cycles go on forever...



Example 2.8 (Machine Replacement): Two-year Cycle
D_2
$\mathrm{PV}_{\mathrm{Total}}$
$\mathrm{PV}_{\mathrm{Total}}$
- 2,000
- 3,000
$\mathrm{PV}_{\mathrm{Total}} = \mathrm{PV}_{\mathrm{Single\ Cycle}} + D_2 \mathrm{PV}_{\mathrm{Total}}$
- 10,000
$\mathrm{PV}_{\mathrm{Single\ Cycle}}$

In [2]:

```python
# Example: Machine Replacement.
def pv_cycle(k):
    current_sum = 0
    for i in range(0, k):
        current_sum += (-2000 - 1000 * i) / (1.1 ** (i + 1))
    return -10000 + current_sum


def pv_total(k):
    return pv_cycle(k) / (1 - 1 / (1.1 ** k))


print(f"Cycle Length \t Total PV")
for n in range(1, 7):
    print(f"{n:8} \t {pv_total(n):8.0f}")
```

```
Cycle Length     Total PV
       1          -130000
       2           -82381
       3           -69577
       4           -65359
       5           -64481
       6           -65196
```

Inflation

- *Nominal* value is measured in terms of money, whereas *real* value is measured against goods and services.
- The *real* value of something is the nominal value (the "not-really" value) adjusted for inflation:
    - The nominal FV of 100 USD deposited at 10% for 1 year is 110 USD. But that's not its real value: one year from now, 110 USD will be able to buy less than what 110 USD can buy today. If we expect a 2% rise in prices, then we are only *really* achieving an 8% interest rate (approximately).
- Nominal dollars grow at the nominal interest rate.
- Real dollars grow at the nominal interest rate, r, but then deflate (discount) at the inflation rate, f, such that the real rate of interest, r_0 is:

$$1 + r_0 = \frac{1 + r}{1 + f}$$

Summary

- The time value of money is concretely captured by an interest rate.
- Compounded at any frequency, hence "nominal" vs "effective".
- Future Value and Present Value and the Ideal Bank.
- Net Present Value and Internal Rate of Return.
- Complications of inflows and outflows, cycle lengths, taxes and inflation.