Computational Frameworks

MapReduce

Big data challenges

Large computational costs:

- Any processing requiring a *superlinear number of operations* may easily turn out *unfeasible on large inputs*.
- If input size is really huge, just *touching all data items* is already quite time consuming. (E.g., *an index for* $\simeq 50 \cdot 10^9$ *web pages of about 20KB each* $\rightarrow$ *1000TB of data*.)
- For computation-intensive (e.g., optimiziation) algorithms, exact solutions may be too costly. Need to resort to *accuracy-efficiency tradeoffs*.

Need of high-performance (e.g., parallel/distributed) platforms:

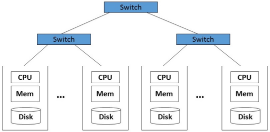
- Specialized hw is costly and becomes rapidly obsolete
- When using parallel/distributed platforms:
  - Fault-tolerance becomes serious issue: *low Mean-Time Between Failures* (MTBF).
  - Effective programming requires *high skills*

MapReduce

- Introduced by Google in 2004 (see [DG08])
- Programming framework for big data processing on distributed platforms.
- Employed in many application scenarios on clusters of commodity processors and cloud infrastructures
- Main features:
  - Data centric view
  - Inspired by functional programming (map, reduce functions)
  - Ease of programming. Messy details (e.g., task allocation; data distribution; fault-tolerance; load-balancing) are hidden to the programmer
- Main implementation: Apache Hadoop
- Hadoop ecosystem: several variants and extensions aimed at improving Hadoop's performance (e.g., Apache Spark)

Typical cluster architecture

- Racks of 16-64 compute nodes (commodity hardware), connected (within each rack and among racks) by fast switches (e.g., 10 Gbps Ethernet)



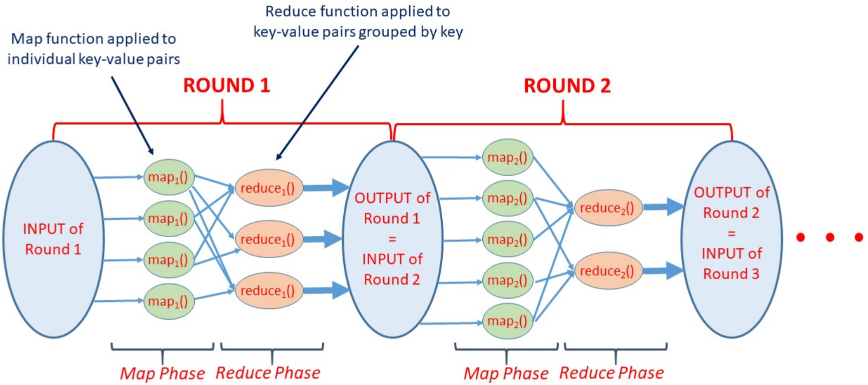
- Distributed File System
  - Files divided into *chunks* (e.g., 64MB per chunk)
  - Each chunk replicated (e.g., 2x o 3x) with replicas in different nodes and, possibly, in different racks
  - The distribution of the chunks of a file is represented into a *master node* file which is also replicated. A directory (also replicated) records where all master nodes are.
  - Examples: Google File System (GFS); Hadoop Distributed File System (HDFS)

MapReduce computation

- Computation viewed as a sequence of rounds.
  (*The original formulation considered only one round*.)
- Each round transforms a set of key-value pairs into another set of key-value pairs (*data centric view*!), through the following two phases
  - Map phase: a user-specified map function is applied separately to each input key-value pair and produces ≥ 0 other key-value pairs, referred to as intermediate key-value pairs.
  - Reduce phase: the intermediate key-value pairs are grouped by key and a user-specified reduce function is applied separately to each group of key-value pairs with the same key, producing ≥ 0 other key-value pairs, which is the output of the round.
- The output of a round is the input of the next round.

Remark: over the course of the algorithm , the domains of key and values may change.

MapReduce computation (cont'd)



Implementation of a round

- Input file is split into X chunks and each chunk forms the input of a map task.
- Each map task is assigned to a worker (a compute node) which applies the map function to each key-value pair of the corresponding chunk, buffering the intermediate key-value pairs it produces in its local disk
- The intermediate key-values pairs, while residing in the local disks, are partitioned into Y buckets through a hash function h:

$$(k, v) \quad \rightarrow \quad \text{Bucket } \; i = h(k) \bmod Y.$$

- Each bucket forms the input of a different reduce task which is assigned to a worker (a compute node).

  Obs.: *hashing helps balancing the load of the workers*
- The worker applies the reduce function to each group of key-value pairs with the same key, writing the output on the DFS. The application of the reduce function to a group is referred to as a reducer

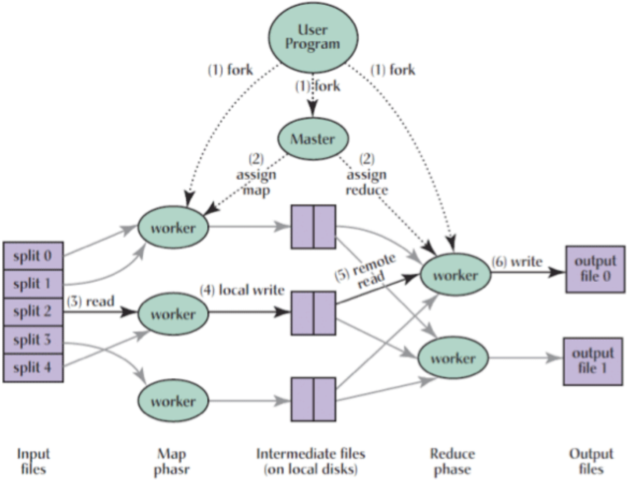
Implementation of a round (cont'd)

- The user program is forked into a master process and several worker processes. The master is in charge of assigning map and reduce tasks to the various workers, and to monitor their status (idle, in-progress, completed).
- Input and output files reside on a Distributed File System, while intermediate data are stored on the workers' local disks.
- The round involves a data shuffle for moving the intermediate key-value pairs from the compute nodes where they were produced (by map tasks) to the compute nodes where they must be processed (by reduce tasks).

  Obs.: *shuffle is often the most expensive operation of the round*
- The values X and Y are design parameters.

Implementation of a round (cont'd)

From the original paper (split $i \equiv$ chunk i):



Dealing with faults

- The Distributed File System is fault-tolerant
- Master pings workers periodically to detect failures
- Worker failure:
  - Map tasks completed or in-progress at failed worker are reset to idle and will be rescheduled. Note that even if a map task is completed, the failure of the worker makes its output unavailable to reduce tasks, hence it must be rescheduled.
  - Reduce tasks in-progress at failed worker are reset to idle and will be rescheduled.
- Master failure: the whole MapReduce task is aborted

Specification of a MapReduce algorithm

A *MapReduce (MR) algorithm* should be specified so that

- The input and output of the algorithm is clearly defined
- The sequence of rounds executed for any given input instance is unambiguously implied by the specification
- For each round the following aspects are clear
  - input, intermediate and output sets of key-value pairs
  - functions applied in the map and reduce phases.
- Meaningful values (or asymptotic) bounds for the key performance indicators (defined later) can be derived.

Specification of a MapReduce algorithm (cont'd)

For example, an MR algorithm with a fixed number of rounds R, we can use the following style:

Input: description of the input as set of key-value pairs
Output: description of the output as set of key-value pairs

Round 1:

- *Map phase*: description of the function applied to each key-value pair
- *Reduce phase*: description of the function applied to each group of key-value pairs with the same key

Round 2: as before ..

. . .

Round R: as before ...

Obs.: output key-value pairs may be returned at any time during the algorithm's execution.

Analysis of a MapReduce algorithm

The analysis of an MR algorithm aims at estimating the following key performance indicators (see [P+12]):

- Number of rounds R.
- Local space M_L: maximum amount of space required by *any* application of a map or reduce function during the course of the algorithm for storing the input of the function and any temporary data (output pairs emitted without being saved do not count).
- Aggregate space M_A: max amount of space which, at any time during the execution of the algorithm, is required to store all data that the algorithm is using or will use at future times.

Observations:

- The indicators are usually estimated through asymptotic analysis (either worst-case or probabilistic) as functions of the instance size.
- M_L bounds the amount of main memory required at each worker, while M_A bounds the overall amount of memory space that the executing platform must provide.

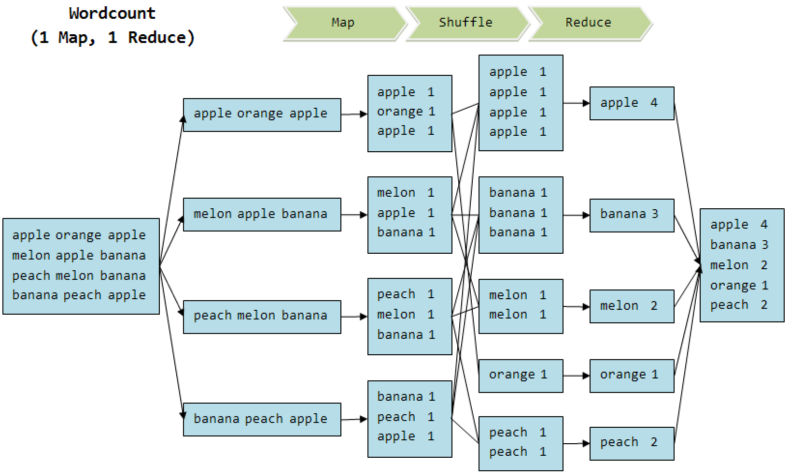
Example: Word count

Input: collection of text documents $D_1, D_2, \ldots, D_k$ containing N words occurrences (counting repetitions). Each document is a key-value pair, whose key is the document's name and the value is its content.
Output: The set of pairs $(w, c(w))$ where w is a word occurring in the documents, and $c(w)$ is the number of occurrences of w in the documents.

Round 1:

- *Map phase*: for each document D_i, produce the set of intermediate pairs $(w, 1)$, one for each occurrence of a word $w \in D_i$.
  N.B.: the word is the key of the pair.
- *Reduce phase*: For each word w, gather all intermediate pairs $(w, 1)$ and return the output pair $(w, c(w))$ where $c(w)$ is the sum of all values (1's) of these pairs.

Example: Word count (cont'd)



Analysis of Word count

Worst-case analysis with respect to input size N (= aggregate number of word occurrences)

- $R = 1$
- $M_L = O(N)$. Bad case: only one word occurs repeated N times over all documents.
- $M_A = O(N)$

Design goals for MapReduce algorithms

Here is a simple yet important observation.

Theorem

For every computational problem solvable by a sequential algorithm in space $S(|input|)$ *there exists a 1-round MapReduce algorithm with* $M_L = M_A = \Theta(S(|input|))$

Proof.

Run sequential algorithm on whole input with one reducer. $\square$

Remark: the trivial solution implied by the above theorem is impractical for very large inputs for the following reasons:

- A platform with very large main memory is needed.
- No parallelism is exploited.

Design goals for MapReduce algorithms (cont'd)

In general, to process efficiently very large input instances, an algorithm should aim at: *breaking the computation into a (hopefully small) number of rounds that execute several tasks in parallel, each task working efficiently on a (hopefully small) subset of the data.*

In MapReduce terms, the design goals are the following:

Design Goals

- Few rounds (e.g., $R = O(1)$)
- Sublinear local space (e.g., $M_L = O(|\text{input}|^{\epsilon})$, for some constant $\epsilon \in (0, 1)$)
- Linear aggregate space (i.e., $M_A = O(|\text{input}|)$), or only slightly superlinear
- Polynomial complexity of each map or reduce function .

Obs.: algorithm design aims at exhibiting interesting tradeoffs between performance indicators. Often, small M_L enables high parallelism but may incur large R.

Improved Word count 1

Obs.: the Word count algorithm described before does not meet the sublinear local space goal.

As a simple optimization we require the map function to produce only one pair for each word in a document. The improved algorithm is the following (same input-output as before):

Round 1:

- *Map phase*: for each document D_i, produce the set of intermediate pairs $(w, c_i(w))$, one for each word $w \in D_i$, where $c_i(w)$ is the number of occurrences of w in D_i.
- *Reduce phase*: For each word w, gather all intermediate pairs $(w, c_i(w))$ and return the output pair $(w, c(w))$ where $c(w)$ is the sum of the $c_i(w)$'s.

Analysis of improved Word count 1

Let N_i be the number of words in D_i ($\Rightarrow N = \sum_{i=1}^{k} N_i$). The optimization yields:

- $R = 1$
- $M_L = O\left(\max_{i=1,k} N_i + k\right)$.
- $M_A = O\left(N\right)$

Observation: The sublinear local space requirement is satisfied as long as $N_i = o(N)$, for each i, and $k = o(N)$

Observations

The MapReduce framework features some clear advantages:

- ease of use: the programmer/algorithm designer must only focus on data transformations aiming at the aforementioned general design goals. Allocation of tasks to the workers, data management, handling of failures are ensured in a totally transparent way.
- portability/adaptability: applications can run on different platforms and the underlying implementation of the framework will do best effort to exploit parallelism and minimize data movement costs.
- data-centric view, ideal for big-data processing.

However, some important caveats must be taken into account:

- round complexity metric is somewhat weak: it ignores the runtimes of map and reduce functions and the actual volume of data shuffled at each round. More sophisticated (yet, less usable) metrics exist.
- curse of the last reducer: in some cases, one, or a few reducers may be much slower than the other ones, thus delaying the end of the round. When designing MapReduce algorithms one should try to ensure some load balancing (if at all possible) among reducers.

Basic techniques and primitives

Partitioning technique

When some aggregation functions (e.g., sum of the $c_i(w)$'s for every word i) may potentially receive large inputs (e.g., large k) or skewed ones, it is advisable to partition the input, either deterministically or randomly, and perform the aggregation in stages.

We will see two examples:

- An improved version of Word count
- A Class count primitive

Improved Word count 2

Consider the word count problem, k documents and N word occurrences overall, in a realistic scenario where $k = \Theta(N)$ and N is huge (e.g., *huge number of small documents*). The following algorithm reduces local space requirements at the expense of an extra round.

Idea: partition intermediate pairs randomly in $o(N)$ groups and compute counts in two stages

Round 1:

- *Map phase*: for each document D_i, produce the intermediate pairs $(x, (w, c_i(w)))$, one for every word $w \in D_i$, where x (the key of the pair) is a random integer in $[0, \sqrt{N})$ and $c_i(w)$ is the number of occurrences of w in D_i.
- *Reduce phase*: For each key x gather all pairs $(x, (w, c_i(w)))$, and for each word w occurring in these pairs produce the pair $(w, c(x, w))$ where $c(x, w) = \sum_{(x,(w,c_i(w)))} c_i(w)$. Now, w is the key for $(w, c(x, w))$.

Improved Word count 2 (cont'd)

Round 2:

- *Map phase*: identity function
- *Reduce phase*: for each word w, gather the at most $\sqrt{N}$ pairs $(w, c(x, w))$ resulting at the end of the previous round, and return the output pair $(w, \sum_x c(x, w))$.

Analysis. Let m_x be the number of intermediate pairs with key x produced by the Map phase of Round 1, and let $m = \max_x m_x$. As before, let N_i be the number of words in D_i. We have

- $R = 2$
- $M_L = O\left(\max_{i=1,k} N_i + m + \sqrt{N}\right)$.
- $M_A = O(N)$

How large can m be? We need a *very useful* technical tool.

Technical tool: Chernoff bound

Chernoff bound

Let $X_1, X_2, \ldots, X_n$ be n i.i.d. Bernoulli random variables, with $\Pr(X_i = 1) = p$, for each $1 \leq i \leq n$. Thus, $X = \sum_{i=1}^{n} X_i$ is a Binomial(n, p) random variable. Let $\mu = E[X] = n \cdot p$. For every $\delta_1 \geq 5$ and $\delta_2 \in (0, 1)$ we have that

$$\begin{aligned} \Pr(X \geq (1+\delta_1)\mu) &\leq 2^{-(1+\delta_1)\mu} \\ \Pr(X \leq (1-\delta_2)\mu) &\leq 2^{-\mu\delta_2^2/2} \end{aligned}$$

The proof can be found in [MU05].

Estimate of m for word count 2

Theorem

Suppose that the keys assigned to the intermediate pairs in Round 1 are i.i.d. random variables with uniform distribution in $[0, \sqrt{N})$. Then, with probability at least $1 - 1/N^5$

$$m = O\left(\sqrt{N}\right).$$

Therefore, from the theorem and the preceding analysis we get

$$M_L = O\left(\max_{i=1,k} N_i + \sqrt{N}\right),$$

with probability at least $1 - 1/N^5$. In fact, for large N the probability becomes *very close to 1*.

N.B.: This is an example of probabilistic analysis, as opposed to more traditional worst-case analysis.

Estimate of m word count 2 (cont'd)

Proof of theorem.

Let $N' \le N$ be the number of intermediate pairs produced by the Map phase of Round 1, and consider an arbitrary key $x \in [0, \sqrt{N})$.

Crucial observation: m_x is a $\text{Binomial}(N', 1/\sqrt{N})$ random variable with expecation $\mu = N'/\sqrt{N} \le \sqrt{N}$.

By the Chernoff bound we have

$$\Pr(m_x \ge 6\sqrt{N}) \le \frac{1}{2^{6\sqrt{N}}} \le \frac{1}{N^6}.$$

Now, by union bound we have that

$$\begin{aligned}\Pr(m \ge 6\sqrt{N}) &\le \sum_{x \in [0,\sqrt{N})} \Pr(m_x \ge 6\sqrt{N}) \\ &\le \sqrt{N}\,\Pr(m_x \ge 6\sqrt{N}) \le \frac{1}{N^5}\end{aligned}$$

Therefore, with probability at least $1 - 1/N^5$ we have $m \le 6\sqrt{N}$, i.e., $m = O\left(\sqrt{N}\right)$. □

Observations

The choice of partitioning the intermediate pairs into groups of size $O\left(\sqrt{N}\right)$, in the first round of improved word count 2 is somewhat arbitrary and convenient to provide an example of reduction of M_L to a sublinear value.

In the rest of the course we will often make this choice.

However, it is important to understand that the approach can be generalized to attain a given target bound on M_L (e.g., $M_L = O\left(N^{\epsilon}\right)$ for some $\epsilon \in (0, 1)$) exercising a finer tradeoff between M_L and number of rounds. An exercise will explore this issue.

Class count

Suppose that we are given a set S of N objects, each labeled with a class from a given domain, and we want to count how many objects belong to each class.

More precisely:

Input: Set S of N objects represented by pairs $(i, (o_i, \gamma_i))$, for $0 \leq i < N$, where o_i is the i-th object, and γ_i its class.
Output: The set of pairs $(\gamma, c(\gamma))$ where γ is a class labeling some object of S and $c(\gamma)$ is the number of objects of S labeled with γ.

Observation: the straightforward 1-round algorithm may require $O(N)$ local space, in case a large fraction of objects belong to the same class. In the next slide we will see a more efficient algorithm.

Class count (cont'd)

Round 1:

- *Map phase*: map each pair $(i, (o_i, \gamma_i))$ into the intermediate pair $(i \bmod \sqrt{N}, (o_i, \gamma_i))$ ($\bmod =$ reminder of integer division)
- *Reduce phase*: For each key $j \in [0, \sqrt{N})$ gather the set (say $S^{(j)}$) of all intermediate pairs with key j and, for each class γ labeling some object in $S^{(j)}$, produce the pair $(\gamma, c_j(\gamma))$, where $c_j(\gamma)$ is the number of objects of $S^{(j)}$ labeled with γ.

Round 2:

- *Map phase*: identity function
- *Reduce phase*: for each class γ, gather the at most $\sqrt{N}$ pairs $(\gamma, c_j(\gamma))$ resulting at the end of the previous round, and return the output pair $(\gamma, \sum_j c_j(\gamma))$.

Exercise

Analyze the above algorithm.

Trading accuracy for efficiency

There are problems for which exact MR algorithms may be too costly, namely they may require a large number of rounds, or large (i.e., close to linear) local space, or large (i.e., superlinear) aggregate space. These algorithms become impractical for very large inputs.

In these scenarios, giving up exact solutions (if acceptable for the application) may greatly improve efficiency.

We'll now see an example through an important primitive for the processing of pointsets from metric spaces (e.g., spatial datasets).

Maximum pairwise distance

Suppose that we are given a set S of N points from some metric space (e.g., $\Re^3$) and we want to determine the maximum distance between two points, for a given distance function $d(\cdot,\cdot)$.

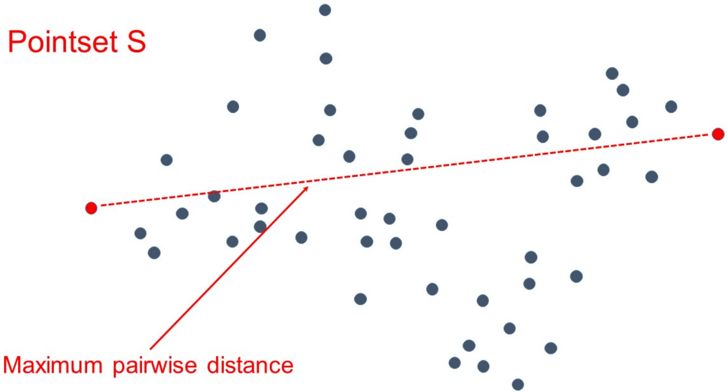
More precisely:

Input: Set S of N points represented by pairs (i, x_i), for $0 \leq i < N$, where x_i is the i-th point.
Output: $(0, \max_{0 \leq i,j < N} d(x_i, x_j))$.

Exercise

Design an MR algorithm for the problem which requires $R = O(1)$, $M_L = O\left(\sqrt{N}\right)$ and $M_A = O\left(N^2\right)$.

Maximum pairwise distance (cont'd)



Maximum pairwise distance (cont'd)

We can substantially reduce the aggregate space requirements if we tolerate a factor 2 error in the estimate of $d_{\max}$. For an arbitrary point x_i define

$$d_{\max}(i) = max_{0 \leq j < N} d(x_i, x_j).$$

Lemma

For any $0 \leq i < N$ we have $d_{\max} \in [d_{\max}(i), 2d_{\max}(i)]$.

Proof.

It is immediate to see that $d_{\max} \geq d_{\max}(i)$. Suppose that $d_{\max} = d(x_r, x_t)$ for some r, t. By the triangle inequality we have

$$d(x_r, x_t) \leq d(x_r, x_i) + d(x_i, x_t) \leq 2d_{\max}(i).$$

□

Maximum pairwise distance (cont'd)

Round 1:

- *Map phase*: map each pair (i, x_i) into the intermediate pair $(i \bmod \sqrt{N}, (i, x_i))$. Also, create the $\sqrt{N} - 1$ pairs $(j, (0, x_0))$, for $1 \leq j < \sqrt{N}$.
- *Reduce phase*: For each key $j \in [0, \sqrt{N})$ gather the set $S^{(j)}$ of all intermediate pairs with key j (which include $(j, (0, x_0))$) and produce the pair $(0, d_{\max}(0, j))$ where $d_{\max}(0, j)$ is the maximum distance between x_0 and the points associated with pairs in $S^{(j)}$

Round 2:

- *Map phase*: identity.
- *Reduce phase*: gather all pairs $(0, d_{\max}(0, j))$ with $j \in [0, \sqrt{N})$ and return the output pair $(0, \max_{0 \leq j < \sqrt{N}} d_{\max}(0, j))$.

Analysis: $R = 2$, $M_L = O\left(\sqrt{N}\right)$, $M_A = O\left(N\right)$.

Exploiting samples

When facing a big-data processing task, we should ask ourselves the following question:

Can we profitably process a small sample of the data?

In many cases, the answer is YES! Specifically, a small sample, *suitably extracted*, could be exploited for the following purposes.

- To subdivide the dataset in smaller subsets to be analyzed separately.
- To provide a succint yet accurate representation of the whole dataset, which contains a good solution to the problem and filters out noise and outliers, thus allowing the execution of the task on the sample.

In what follows, we will see an example of the first type through the sorting primitive, while an example of the second type will be given later.

Sorting

Input: Set $S = \{s_i \; : \; 0 \le i < N\}$ of N distinct sortable objects (each s_i represented as a pair (i, s_i))

Output: Sorted set $\{(i, s_{\pi(i)}) \; : \; 0 \le i < N\}$, where π is a permutation such that $s_{\pi(1)} \le s_{\pi(2)} \le \cdots \le s_{\pi(N)}$.

The MR SampleSort algorithm is based on the following idea:

- Fix a suitable integral design parameter K.
- Randomly select some objects (K on average) as splitters.
- Partition the objects into subsets based on the ordered sequence of splitters.
- Sort each subset separately and compute the final ranks.

Obs.: SampleSort is at the base of TeraSort, a popular benchmark for measuring the performance of computing platforms, as well as most winners of the big-data *Sort Benchmark competition*.

MR SampleSort

Round 1:

- *Map phase*: for each pair (i, s_i) do the following:
  - transform the pair into $(i \bmod K, (0, s_i))$ (call it *regular pair*);
  - with probability $p = K/N$, independently of other objects, select s_i as a splitter and, if selected, create K *splitter pairs* $(j, (1, s_i))$, with $0 \le j < K$. (Note that a binary flag is used to distinguish between splitter from regular pairs).

  Obs. Let t be the number of splitters selected in the Map phase. At the end of the phase, the splitter pairs represent K copies of the splitters, one for each key j.

- *Reduce phase*: for $0 \le j < K$ do the following: gather all regular and splitter pairs with key j; sort the t splitters and let $x_1 \le x_2 \le \cdots x_t$ be the splitters in sorted order. Transform each regular pair $(j, (0, s))$ into the pair (ℓ, s), where $\ell \in [0, t]$ is such that $x_\ell < s \le x_{\ell+1}$ (assume $x_0 = -\infty$ and $x_{t+1} = +\infty$).

MR SampleSort (cont'd)

- Round 2:
  - *Map phase*: identity
  - *Reduce phase*: for every $0 \leq \ell \leq t$ gather, from the output of the previous round, the set of pairs $S^{(\ell)} = \{(\ell, s)\}$, compute $N_\ell = |S^{(\ell)}|$, and create $t+1$ replicas of N_ℓ (use suitable pairs for these replicas).
- Round 3:
  - *Map phase*: identity
  - *Reduce phase*: for every $0 \leq \ell \leq t$ do the following: gather $S^{(\ell)}$ and the values $N_0, N_1, \ldots, N_t$; sort $S^{(\ell)}$; and compute the final output pairs for the objects in $S^{(\ell)}$ whose ranks start from $1 + \sum_{h=0}^{\ell-1} N_h$.

Example

$$N = 32,\ K = 4$$

$$\begin{aligned} S \;=\; & 16, 32, 1, 15, 14, 7, 28, 20, 12, 3, 29, 17, 11, 10, 8, 2, \\ & 25, 21, 13, 5, 19, 23, 30, 26, 31, 22, 9, 6, 27, 24, 4, 18 \end{aligned}$$

Round 1. Call S_j the set of intermediate pairs $(j, (0, s))$ after the Map phase. We have (objects only):

$$\begin{aligned} S_0 &= \mathbf{16}, 32, 1, 15, 14, 7, 28, 20 \\ S_1 &= 12, 3, \mathbf{29}, 17, 11, 10, 8, 2 \\ S_2 &= 25, \mathbf{21}, 13, 5, 19, 23, 30, 26 \\ S_3 &= 31, 22, \mathbf{9}, 6, 27, 24, \mathbf{4}, 18 \end{aligned}$$

The $t = 5$ splitters are highlighted in blue. In sorted order:

$$x_1 = 4, x_2 = 9, x_3 = 16, x_4 = 21, x_5 = 29.$$

Example (cont'd)

Round 1 (cont'd). Call $S_j^{(\ell)}$ the set of intermediate pairs $(j,(0,s))$ with $x_\ell < s \leq x_{\ell+1}$. We have (objects only):

j	$S_j^{(0)}$	$S_j^{(1)}$	$S_j^{(2)}$	$S_j^{(3)}$	$S_j^{(4)}$	$S_j^{(5)}$
0	1	7	16,15,14	20	28	32
1	3,2	8	12,11,10	17	29	
2		5	13	21,19	25,23,26	30
3	4	9,6		18	22,27,24	31

Round 2 The $S^{(\ell)}$'s (objects only) and the N_ℓ's values are

$$
\begin{array}{lcll}
S^{(0)} & = & 1,3,2,4 & N_0 = 4 \\
S^{(1)} & = & 7,8,5,9,6 & N_1 = 5 \\
S^{(2)} & = & 16,15,14,12,11,10,13 & N_2 = 7 \\
S^{(3)} & = & 20,17,21,19,18 & N_3 = 5 \\
S^{(4)} & = & 28,29,25,23,26,22,27,24 & N_4 = 8 \\
S^{(5)} & = & 32,30,31 & N_5 = 3
\end{array}
$$

Example (cont'd)

Round 3. Final output

$$
\begin{array}{ll}
(1,1),\cdots,(4,4) & \text{from rank } 1 \\
(5,5),\cdots,(9,9) & \text{from rank } N_0+1=5 \\
(10,10),\cdots,(16,16) & \text{from rank } N_0+N_1+1=10 \\
(17,17),\cdots,(21,21) & \text{from rank } N_0+N_1+N_2+1=17 \\
(22,22),\cdots,(29,29) & \text{from rank } N_0+N_1+N_2+N_3+1=22 \\
(30,30),\cdots,(32,32) & \text{from rank } N_0+\cdots+N_4+1=30
\end{array}
$$

Analysis of MR SampleSort

- Number of rounds: $R = 3$
- Local Space M_L:
  - Round 1: $O\left(t + N/K\right)$, since each reducer must store all splitter pairs and a subset of N/K intermediate pairs.
  - Round 2: $O\left(\max\{N_\ell\ ;\ 0 \leq \ell \leq t\}\right)$ since each reducer must gather one $S^{(\ell)}$.
  - Round 3: $O\left(t + \max\{N_\ell\ ;\ 0 \leq \ell \leq t\}\right)$, since each reducer must store all N_ℓ's and one $S^{(\ell)}$.

  $\Rightarrow$ overall $M_L = O\left(t + N/K + \max\{N_\ell\ ;\ 0 \leq \ell \leq t\}\right)$
- Aggregate Space M_A: $O\left(N + t \cdot K + t^2\right)$, since in Round 1 each splitter is replicated K times, and in Round 3 each N_ℓ is replicated $t + 1$ times. The objects are never replicated.

Analysis of MR SampleSort (cont'd)

Lemma

With reference to the MR SampleSort algorithm, for any $K \in (2 \ln N, N)$ the following two inequalities hold with high probability (i.e., probability at least $1 - 1/N$):

1. $t \leq 6K$, *and*
2. $\max\{N_\ell \,;\, 0 \leq \ell \leq t\} \leq 4(N/K) \ln N$.

Proof.

Deferred. □

Theorem

By setting $K = \sqrt{N}$, MR SampleSort runs in 3 rounds, and, with high probability, it requires local space $M_L = O\left(\sqrt{N} \ln N\right)$ and aggregate space $M_A = O(N)$.

Proof.

Immediate from the lemma and the previous analysis. □

Analysis of MR SampleSort (cont'd)

Proof of Lemma

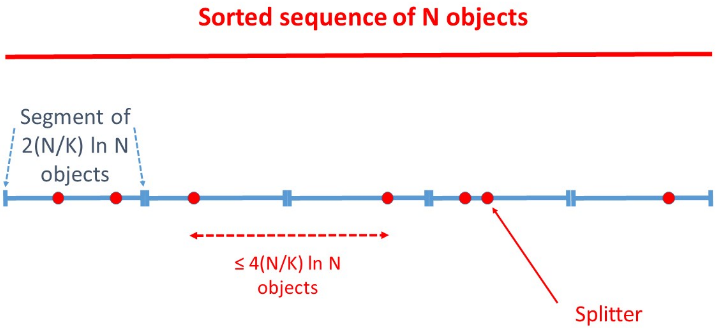
We show that each inequality holds with probability at least $1 - 1/(2N)$.

1. t is a Binomial$(N, K/N)$ random variable with $E[t] = K > 2 \ln N$. By the Chernoff bound (first inequality with $\delta_1 = 5$) we have that $t > 6K$ with probability at most $1/(2N)$.
2. View the *sorted* sequence of objects as divided into $K/(2 \ln N)$ contiguous segments of length $N' = 2(N/K) \ln N$ each, and consider one arbitrary such segment. The probability that no splitter is chosen among the objects of the segment is

$$\left(1 - \frac{K}{N}\right)^{N'} = \left(1 - \frac{1}{(N/K)}\right)^{(N/K) 2 \ln N} \leq \left(\frac{1}{e^{\ln N}}\right)^2 = \frac{1}{N^2}.$$

(We used the well-known inequality $(1 - 1/x)^x \leq 1/e, \quad \forall\, x \geq 0$.)

Analysis of MR SampleSort (cont'd)



Analysis of MR SampleSort (cont'd)

Proof of Lemma.

2. (cont'd) So, there are $K/(2\ln N)$ segments and we know that, for each segment, the event "*no splitter falls in the segment*" occurs with probability $\leq 1/N^2$. Hence, the probability that any of these $K/(2\ln N)$ events occurs is $\leq K/(N^2 2\ln N) \leq 1/(2N)$ (union bound!). Therefore, with probability at least $(1-1/(2N))$, at least 1 splitter falls in each segment, which implies that each N_ℓ cannot be larger than $4(N/K)\ln N$. Hence, we have that the second inequality stated in the lemma holds with probability at least $(1-1/(2N))$.

In conclusion, we have that the probability that at least one of the two inequalities does not hold, is at most $2 \cdot 1/(2N) = 1/N$, and the lemma follows. □

Exercises

Exercise 1

For each of the 4 problems listed in Exercise 2.3.1 of [LRU14], design an $O(1)$-round MR algorithm which uses $O\left(\sqrt{N}\right)$ local space and linear aggregate space, where N is the input size.

Exercise 2

Suppose that an instance of the Class count problem is given consisting only of object-class pairs (o, γ) (i.e., without the initial integer keys in $[0, N)$). Consider a modification of the algorithm described in the slides, where in the Map phase of Round 1 each pair (o, γ) is mapped into a pair $(j, (o, \gamma))$, where j is a random integer in $[0, \sqrt{N})$. The rest of the algorithm is unchanged. Analyze probabilistically the local space requirements of the modified algorithm.

Exercises (cont'd)

Exercise 3

Design and analyze an efficient MR algorithm for approximating the maximum pairwise distance, which uses local space $M_L = O\left(N^{1/4}\right)$. Can you generalize the algorithm to attain $M_L = O\left(N^{\epsilon}\right)$, for any $\epsilon \in (0, 1/2)$?

Exercise 4

Design an $O(1)$-round MR algorithm for computing a matrix-vector product $W = A \cdot V$, where A is an $m \times n$ matrix and V is an n-vector. Your algorithm must use $o(n)$ local space and linear (i.e., $O(mn)$) aggregate space.

Summary

- Big Data Challenges
- MapReduce Framework
    - Main Features
    - MapReduce Computation: high-level structure, implementation, algorithm specification, analysis (key peformance indicators)
    - Algorithm design goals
- Basic Techniques and Primitives
    - Chernoff bound
    - Partitioning: Word count and Class count.
    - Efficiency-accuracy tradeoffs: Maximum pariwise distance
    - Sampling: Sorting

References

LRU14 J. Leskovec, A. Rajaraman and J. Ullman. Mining Massive Datasets. Cambridge University Press, 2014. Chapter 2 and Section 6.4

DG08 J. Dean and A. Ghemawat. MapReduce: simplified data processing on large clusters. OSDI'04 and CACM 51,1:107113, 2008

MU05 M. Mitzenmacher and E. Upfal. Proability and Computing: Randomized Algorithms and Probabilistic Analysis. Cambridge University Press, 2005. (Chernoff bounds: Theorems 4.4 and 4.5)

P+12 A. Pietracaprina, G. Pucci, M. Riondato, F. Silvestri, E. Upfal: Space-round tradeoffs for MapReduce computations. ACM ICS'112.

Errata

Changes w.r.t. first draft:

- Slide 50: “You algorithms” $\rightarrow$ “Your algorithm”