Vectorising log. reg.

~~Instead of looping over m examples:~~

$z^{(1)} = w^T x^{(1)} + b \qquad z^{(2)} = w^T x^{(2)} + b \qquad z^{(3)} = w^T x^{(3)} + b$

$a^{(1)} = \sigma(z^{(1)}) \qquad a^{(2)} = \sigma(z^{(2)}) \qquad a^{(3)} = \sigma(z^{(3)})$

$$X = \begin{bmatrix} \vdots & \vdots & & \vdots \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ \vdots & \vdots & & \vdots \end{bmatrix} \qquad \mathbb{R}^{n_x \times m}$$

We want to comp. everything in 1 line:

$$[z^{(1)}\ z^{(2)} \dots z^{(m)}] = w^T X + \underbrace{[b \dots b]}_{\mathbb{R}^{1 \times m}} \qquad w^T \begin{bmatrix} \vdots & \vdots & & \vdots \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

$$\cancel{[w^T x^{(1)} \dots w^T x^{(m)}]}$$

$$Z = [z^{(1)}\ z^{(2)} \dots z^{(m)}] = \underbrace{[\underbrace{w^T x^{(1)} + b}_{z^{(1)}}\ \ \underbrace{w^T x^{(2)} + b}_{z^{(2)}} \dots w^T x^{(m)} + b]}_{1 \times m}$$

$$\boxed{Z = \underbrace{np.dot(w.T, X)}\ + \underbrace{b}_{\mathbb{R}^{1 \times 1}}}$$

Python automatically expands this result to a row vector (broadcasting)

$$\boxed{A = [a^{(1)}\ a^{(2)} \dots a^{(m)}] = \sigma(Z)}$$

Gradient output

$dz^{(1)} = a^{(1)} - y^{(1)}, \quad dz^{(2)} = a^{(2)} - y^{(2)}$

$dZ = \underbrace{[dz^{(1)}\ dz^{(2)} \dots dz^{(m)}]}_{1 \times m}$

$A = [a^{(1)} \dots a^{(m)}], \quad Y = [y^{(1)} \dots y^{(m)}]$

$dZ = A - Y$

$\quad = [a^{(1)} - y^{(1)}\ \ a^{(2)} - y^{(2)} \dots]$

$dw = 0$	$db = 0$
$dw \mathrel{+}= x^{(1)} dz^{(1)}$	$db \mathrel{+}= dz^{(1)}$
$dw \mathrel{+}= x^{(2)} dz^{(2)}$	$db \mathrel{+}= dz^{(2)}$
$\vdots$	$\vdots$
$dw \mathrel{/}= m$	$db \mathrel{+}= dz^{(m)}$
	$db \mathrel{/}= m$