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5 - FUNCTIONS - SOLUTIONS

5.1

(i) $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \frac{1}{1+x^2}$
f is not a function because not every arbitrary real number in the domain is assigned to a unique real number in the co-domain, i.e there exists real numbers in the domain which map to the same element in the co-domain, such as when $x = \pm 1$

(iii) $f : \mathbb{Z} \to \mathbb{Z}$ with $f(x) = \frac{1}{2}x$
f is not a function as each integer in the domain map doesn't map to an integer in the co-domain, rather it is assigned to a unique real number in the co-domain.

5.2

(i) $f_1(x) = 2x + 5$
f_1 is (1-1) as $\forall x, y \in A, (f_1(x) = f_1(y)) \Rightarrow (x = y)$ holds:

$$\begin{aligned} 2x + 5 &= 2y + 5 \\ 2x &= 2y \\ x &= y \end{aligned}$$

(iii) $f_3(x) = x^2 - 2x$
f_3 is onto, as it is not a 1-1 function.
1-1 contrapositive : $\exists x, y \in A, (x \neq y) \Rightarrow (f_3(x) \neq f_3(y))$
this is false, as there exists different x and y values which their image are equal.
so therefore, f_3 is onto

5.5

(i) If $g \circ f$ is (1-1), then f is (1-1)

Proof.

$$\begin{aligned} g \circ f(x) &= g \circ f(y) \\ g(f(x)) &= g(f(y)) \\ f(x) &= f(y) \\ x &= y \end{aligned}$$

Therefore If $g \circ f$ is 1-1, then f is 1-1 □

(ii) If $g \circ f$ is onto, then g is onto.

Proof.

$$\begin{aligned} g \circ f = g(f(x)) = g(y) &= z, \\ \exists x \in A, f(x) &= y \end{aligned}$$

Since $g \circ f$ is onto, then there exists an $x \in A$ such that $f(x) = y$ meaining it is also onto □

5.7 $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(i) $(124)(25)(58) = (12)(58)$