1 - THE LANGUAGE OF MATHEMATICS - SOLUTIONS

1.1

(i) Proposition: *There exists a real number x such that $x^2 \neq x$*
This is a proposition as there is a specified mathematical structure (the set of real numbers) and there are no free variables(as x is bound by the existential quantifier).
Negation : *For all real numbers x, $x^2 = x$*

(ii) $x^2 < x$
This is a predicate as there is no specified mathematical structure and a free variable is present, namely x.

(iii) Proposition: *The integer $2^{10} - 1$ is a prime number*
This is a proposition as there is a specified mathematical structure(set of integers) and there are no free variables(as the variable has been assigned the value $2^{10} - 1$).
Negation: *The integer $2^{10} - 1$ is non-prime, i.e a composite number*

(iv) Proposition : *All natural numbers divisible by 2 are divisible by 4.*
This is a proposition as there is a specified mathematical structure(set of natural numbers) and there are no free variables(due to the universal quantifier).
Negation: *There exists a natural number such that it is divisible by 2 but not divisible by 4.*

(v) *If then x, x^2 and ≥ 0*
This is not a proposition, nor does it seem to be legible enough to be considered a predicate

1.2

(i) $(p \vee q) \vee (\neg p)$;

p	q	$p \vee q$	$\neg p$	$(p \vee q) \vee (\neg p)$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

(ii) $(p \wedge \neg p)$;

p	$\neg p$	$(p \wedge \neg p)$
T	F	F
F	T	F

(iii) $\neg[(p \wedge q) \vee (\neg p \wedge \neg q)]$;

p	q	$\neg p$	$\neg q$	$(p \wedge q)$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$\neg[(p \wedge q) \vee (\neg p \wedge \neg q)]$
T	T	F	F	T	F	T	F
T	F	F	T	F	F	F	T
F	T	T	F	F	F	F	T
F	F	T	T	F	T	T	F

(iv) $\neg[(p \vee q) \wedge (\neg p \vee \neg q)]$;

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$(\neg p \vee \neg q)$	$(p \vee q) \wedge (\neg p \vee \neg q)$	$\neg[(p \vee q) \wedge (\neg p \vee \neg q)]$
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	T	F	T

(v) $p \Rightarrow (p \vee q)$;

p	q	$(p \vee q)$	$p \Rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

From this, we can deduce that (i) and (v) are logically equivalent as they have the same truth values for each variation of statement variables.

(vi) $\neg p \Leftrightarrow (p \Rightarrow \neg q)$;

p	q	$\neg p$	$\neg q$	$(p \Rightarrow \neg q)$	$\neg p \Leftrightarrow (p \Rightarrow \neg q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	T

1.4

(i)

$$\begin{aligned}
& \forall a \in A, \exists b \in B, (p(a) \wedge q(b)) \\
& \neg[\forall a \in A, \exists b \in B, (p(a) \wedge q(b))] \\
& \exists a \in A, \forall b \in B, \neg(p(a) \wedge q(b)) \\
\Rightarrow \quad & \exists a \in A, \forall b \in B, (\neg p(a) \vee \neg q(b))
\end{aligned}$$

(iii)

$$\begin{aligned}
& \exists a \in A, \forall b \in B, p(a) \Rightarrow q(b) \\
& \exists a \in A, \forall b \in B, \neg p(a) \vee q(b) \\
& \neg[\exists a \in A, \forall b \in B, \neg p(a) \vee q(b)] \\
& \forall a \in A, \exists b \in B, \neg(\neg p(a) \vee q(b)) \\
\Rightarrow \quad & \forall a \in A, \exists b \in B, p(a) \wedge \neg q(b)
\end{aligned}$$