SMACOF Algorithm

a) Basic notations: Stress function $\sigma(X) = \sum_{r<s}(d_{rs} - \delta_{rs})^2$

$\Delta = (\delta_{rs})$ is dissimilarity matrix of original data set Y

X is a configuration in lower dimension R^d and $d_{rs}(X)$ = Minkowski distance between obervations r, s of X

ε is the tolerence, and k = max number of iterations

b) Guttman transformation $X = \frac{1}{n} B(Z)\, Z$, where the matrix B(Z) is defined as (for $r \neq s$):

$$b_{rs} = 0, \text{ if } d_{rs}(Z) = 0; \quad b_{rs} = -\frac{\delta_{rs}}{d_{rs}(Z)}, \text{ if } d_{rs}(Z) \neq 0;$$

$$(\text{for } r = s): \quad b_{rr} = \sum_{s=1, s\neq r}^{n} b_{rs} = -\sum_{s=1, s\neq r}^{n} \frac{\delta_{rs}}{d_{rs}(Z)} .$$

c) Stop criterion: either | diff | < ε, or max number of iterations k has been reached.

d) SMACOF Algorithm (see the chart of flow on the right)

i) Find an initial configuration of points in R^d. Call this $\mathbf{X}^0$.

ii) Compute the raw stress $\sigma(\mathbf{X}^0)$

iii) Set $Z = \mathbf{X}^0$ then Using Guttman transformation to find $\mathbf{X}^1$.

iv) Compute the stress for $\mathbf{X}^1$, $\sigma(\mathbf{X}^1)$

v) Compute the diff of stress values diff $= \sigma(\mathbf{X}^1) - \sigma(\mathbf{X}^0)$

vi) If stop criterion is satisfied, then stop and set $X = \mathbf{X}^1$.

vii) Otherwise go to step iii) and set $Z = \mathbf{X}^1$ and find $\mathbf{X}^2$

viii) Go to steps iv), v) and iterate until stop criterion is satisfied,

set $X = \mathbf{X}^k$