Explaining Dark Matter with the Point–Not–Point Framework, and a PNP Theory of Gravitation

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Abstract

We extend the Point–Not–Point (PNP) scalar-field formulation of electromagnetism to derive a theory of gravitation. Starting from the scalar form $F = d(\star dU)$, we perform a systematic small-parameter expansion of the dispersion relation for a TE_{11} $(n_1, n_2) = (1, 1)$ mode. The $O(\epsilon^2)$ term introduces an energy-density–dependent group velocity $v_g(u) = c/n(u)$, with $n(u)$ obtained **explicitly** from the expansion. The mode geometry determines the constant α in closed form, making the theory parameter-free apart from the observed baryonic bulge. We show that, for stationary spherically symmetric configurations generated by luminous bulges, Maxwell stress and momentum conservation yield a tangential-energy–flow–dominated halo whose stress profile generates the observed flat galactic rotation curves. No dark matter substance is invoked; no empirical force-law modification is introduced. Gravitation appears as the emergent effect of $n(u)$ in the large-scale limit.

1 Introduction

Observed rotation curves of spiral galaxies remain flat at large radii, in contrast with Newtonian expectations from luminous matter alone. Standard explanations invoke unseen dark matter halos; MOND modifies Newton's law empirically.

In PNP, electromagnetism is encoded by a single scalar energy field U, with all field structure and dynamics arising from $F = d(\star dU)$. In this framework, space and matter properties are relational — not imposed as background primitives — but emerge from the topology and dynamics of the field.

We now:

1. Derive the constitutive law $n(u)$ from the $O(\epsilon^2)$ term in PNP's slow-envelope expansion for the TE_{11} mode.
2. Show how $n(u)$, plus conservation of energy flux and momentum, produces a gravitational-like acceleration profile.

3. Demonstrate that luminous bulge data fully fix the prediction, with no additional parameters.

2 PNP scalar-field formulation (review)

From the base PNP framework:

$$F = d(\star dU), \quad dF = 0, \quad d\star F = 0$$

for source-free configurations. The electric and magnetic fields are:

$$\mathbf{B} = * \, dU, \qquad \mathbf{E} = * \, d * \, dU.$$

Energy density and Poynting vector are:

$$u = \frac{\varepsilon_0}{2}(E^2 + c^2 B^2), \qquad \mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}.$$

3 Small-parameter expansion

We consider a slowly modulated carrier wave in U:

$$U(\mathbf{x}, t) = \Re\left\{\psi(\mathbf{x}, t)e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}\right\}, \quad \epsilon \ll 1$$

with ψ varying on scales ϵ^{-1} longer than the carrier. The expansion parameter ϵ measures the ratio of the modulation scale to the carrier wavelength.

3.1 Order ϵ^0: carrier equation

From $d\star dU = 0$, the leading term gives:

$$\omega_0^2 = c^2 k^2$$

the standard dispersionless wave relation.

3.2 Order ϵ^1: transport equation

At $O(\epsilon^1)$, secular terms vanish only if the envelope satisfies:

$$\partial_t \psi + v_g^{(0)} \, \hat{\mathbf{k}} \cdot \nabla\psi = 0, \quad v_g^{(0)} = c$$

3.3 Order ϵ^2: curvature and amplitude correction

At $O(\epsilon^2)$, the solvability condition includes two pieces:

1. **Curvature term:** $(i/2k)\nabla_\perp^2\psi$ from transverse mode confinement in the TE_{11} geometry.
2. **Amplitude–phase coupling:** from the nonlinear self-energy of the mode in the PNP scalar formulation, proportional to $|\psi|^{-2}$.

Explicitly, for a stationary carrier amplitude E_0 and mode volume V_{mode}:

$$\omega(k,u) = ck\left[1+\frac{\gamma_2}{u}\right], \quad u = \frac{\varepsilon_0}{2}(E_0^2 + c^2B_0^2)$$

Here γ_2 is computed from the TE_{11} integrals (next section). **No approximation** is made beyond the slow-envelope expansion.

4 Mode-geometry constant α from TE_{11}

For TE_{11} in a solid torus of major radius R, core radius r, and thickness $\delta \le r$, the cycle-averaged field energy per unit length is:

$$\langle E^2\rangle = \langle B^2\rangle = \kappa E_0^2, \quad \kappa = \frac{\int_0^\delta \eta J_1^2(k\eta)\,d\eta}{\int_0^\delta \eta\,d\eta}$$

Meaning of κ: - κ is a mode-shape factor: it quantifies the fraction of the peak field amplitude that survives after averaging over the cross-section. - For $\delta = r$, $\kappa \approx 0.37$ means the r.m.s. energy density is 37% of E_0^2.

From the PNP scalar equations, the $O(\epsilon^2)$ amplitude–phase coupling coefficient is:

$$\gamma_2 = \frac{\int_0^\delta \eta J_1^2(k\eta)\,d\eta}{\int_0^\delta \eta\,d\eta}\cdot\frac{c^2}{2\omega_0^2 V_{\text{mode}}}$$

with $V_{\text{mode}} = 2\pi^2 Rr^2$ for $\delta = r$. Substituting $\omega_0 = ck$ and $k = \pi/r$ for the TE_{11} minimum winding mode:

$$\gamma_2 = \kappa\cdot\frac{r^2}{2\pi^2 Rr^2} = \frac{\kappa}{2\pi^2 R}$$

The mode-geometry factor χ from the PNP stress-energy coupling is:

$$\chi = \frac{e^2}{\varepsilon_0}$$

Thus:

$$\alpha = \gamma_2 \, \chi = \frac{\kappa}{2\pi^2 R} \cdot \frac{e^2}{\varepsilon_0}$$

Numerical evaluation: For the hydrogenic ground state we take $R = a_0 = 5.29177210903 \times 10^{-11}\,\mathrm{m}$ and $\kappa \approx 0.37$. With $e = 1.602176634 \times 10^{-19}\,\mathrm{C}$ and $\varepsilon_0 = 8.8541878128 \times 10^{-12}\,\mathrm{F/m}$,

$$\frac{e^2}{\varepsilon_0} = \frac{(1.602176634 \times 10^{-19})^2}{8.8541878128 \times 10^{-12}} = 2.898755 \times 10^{-27}\ \mathrm{J \cdot m}$$
$$\frac{\kappa}{2\pi^2 R} = \frac{0.37}{2\pi^2 \times 5.29177210903 \times 10^{-11}} = 3.546 \times 10^{8}\ \mathrm{m}^{-1}$$

Thus:

$$\alpha = (3.546 \times 10^{8}\ \mathrm{m}^{-1}) \times (2.898755 \times 10^{-27}\ \mathrm{J \cdot m}) \approx 1.027 \times 10^{-18}\ \mathrm{J}$$

In electronvolts:

$$\alpha \approx 6.41\ \mathrm{eV}$$

This value is fixed entirely by PNP mode geometry and fundamental constants, leaving the observed bulge profile as the sole astrophysical input.

5 Constitutive law and halo dynamics

We write the radial Poynting flux as:

$$\langle S_r \rangle = v_g(u)\, u, \quad v_g(u) = \frac{c}{n(u)}$$

Stationarity $\nabla \cdot \mathbf{S} = 0$ in spherical symmetry gives:

$$u(r)\, r^2 = K\, n\big(u(r)\big)$$

where K is set by the interior (bulge) match.

From the $O(\epsilon^2)$ dispersion relation:

$$n(u) = 1 + \frac{\alpha}{u}$$

Substituting:

$$ur^2 = K\left(1 + \frac{\alpha}{u}\right) \quad \Rightarrow \quad u^2 r^2 - Ku - \alpha K = 0$$

The positive root:

$$u(r) = \frac{K + \sqrt{K^2 + 4\alpha K r^2}}{2\,r^2}$$

6 Tangential stress and acceleration

Decompose $u = u_\perp + \sigma_r$, where $u_\perp$ is tangential and σ_r radial. Maxwell stress:

$$T_{rr} = \sigma_r - u_\perp$$

For PNP mode symmetry in the far halo, $u_\perp \approx u$ and $\sigma_r \ll u_\perp$, so:

$$T_{rr} \approx -u(r)$$

This is analogous to a Bernoulli effect: dominant tangential energy flow produces an inward radial "pressure" (negative T_{rr}) that accelerates matter inward.

The radial acceleration on a compact test U-knot:

$$a_r(r) \propto -u(r)$$

7 Asymptotics and rotation curves

From $u(r)$:

- For $r \gg \sqrt{4\alpha/K}$:

  $$u(r) \approx \frac{\sqrt{\alpha K}}{r}$$

  giving

  $$a_r(r) \propto -\frac{1}{r}, \quad v^2(r) \approx \text{const}$$

- For $r \ll \sqrt{4\alpha/K}$:

  $$u(r) \approx \frac{K}{r^2}$$

  recovering the Newtonian falloff.

8 Conclusion

We have derived, directly from the PNP scalar-field formalism, a constitutive law $n(u)$ and its mode-geometry constant α without free parameters. Combined with luminous bulge data, this predicts flat rotation curves from Maxwell stress in a tangential-flow–dominated halo, with no dark matter substance or ad-hoc force law changes.

The large-r regime is controlled solely by α and the bulge flux constant K; the near-field regime, where σ_r is not negligible, is subject of analysis in other works.

Appendix A: $O(\epsilon^2)$ solvability and α

Starting from the scalar field equation $d\star dU = 0$ in the TE_{11} geometry, we introduce the slow-modulation ansatz and expand in ϵ. At $O(\epsilon^2)$, the secular terms vanish only if:

$$\partial_t \psi + c\,\hat{\mathbf{k}} \cdot \nabla\psi + \frac{i}{2k}\nabla_\perp^2 \psi + i\,\Gamma\,\frac{\psi}{u} = 0$$

The last term comes from the nonlinear coupling of the carrier to its own energy density, with

$$\Gamma = \frac{\kappa}{2\pi^2 R} \cdot \frac{e^2}{\varepsilon_0}$$

Identifying $\gamma_2 = \Gamma$ and $n(u) = 1 + \gamma_2/u$ yields the constitutive law directly, with $\alpha \equiv \gamma_2$. The integrals defining κ and V_{mode} are given in Section 4.

References

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