Dimension and Space as Emergent Properties of Distance in a Cause-Effect Model of the Emergence of Time

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Abstract

We build on a previously introduced framework [1], in which time and distance emerge from internal cause-effect relationships within a single, stable Node. In that model, "time" arises from a partial ordering of subnodes' interactions, while "distance" is the minimal causal-chain length between subnodes. Here, we show that *dimension* and *space* can similarly be derived when these causal distances can be embedded into manifold-like structures. We provide a more formal treatment in logical and mathematical terms, defining an *effective dimensionality* based on approximate metric embeddings. This formalism clarifies how conventional spatial dimensions may arise from deeper, purely relational dynamics.

1 Introduction

Standard physics posits a background of D-dimensional spacetime as the arena for matter and fields. Recent discrete or relational approaches instead propose that spacetime is *emergent*, constructed from more fundamental elements like causal sets [2, 3, 4], spin networks, or dynamical graphs [5].

In [1], a single "stable Node" N was introduced with constant total energy (or analogous conserved property). Within N, subnodes interact via cause-effect links. A partial order $\succ$ encodes causality:

$$n_i \succ n_j \quad \Longrightarrow \quad \text{“}n_i \text{ can trigger a change in } n_j\text{”}.$$

Time emerges from this partial order, and *distance* emerges by counting minimal causal-step chains between subnodes. No absolute space is assumed.

The present paper formalizes how *dimension* can be seen as a large-scale, emergent feature of these causal relations. We introduce a definition of *effective dimensionality* via approximate embeddings into $\mathbb{R}^D$. If arbitrarily large subsets of subnodes can be embedded with small metric distortion, we call the system effectively D-dimensional.

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2 Recap of the Node Framework

2.1 Structure of the Node

- **Stable Node:** A single entity N with globally fixed total property (e.g. total energy), unaffected by internal rearrangements.
- **Subnodes:** A (possibly uncountable) set of "subnodes" $\{n_i\}_{i \in I}$ that can locally interact without changing the global total.
- **Causality:** A partial order $\succ \subseteq I \times I$, where $n_i \succ n_j$ means a causal influence from n_i to n_j.

2.2 Distance from Minimal Chains

For $n_i, n_j \in \{n_i\}_{i \in I}$, define a causal *chain* C of length k from n_i to n_j as:

$$n_i \succ n_{a_1} \succ \ldots \succ n_{a_{k-1}} \succ n_j.$$

The *distance* $d(n_i, n_j)$ is taken to be the minimal k such that such a chain exists. If no chain exists, $d(n_i, n_j)$ can be infinite or undefined. This d acts like a discrete metric derived from causal steps.

2.3 Time as Partial Order

From $\succ$, we interpret $n_i \prec n_j$ as "n_i occurs before n_j." Iterated cause-effect loops also allow local *clock* definitions, where each cycle's step count becomes a measure of duration. Detailed aspects appear in [1].

3 Formalizing Emergent Dimension

3.1 Metric-Like Properties

The function $d(\cdot, \cdot)$ from subnodes to $\mathbb{N} \cup \{\infty\}$ behaves like a graph distance, where edges correspond to immediate causal links. Although not necessarily symmetric or guaranteed to satisfy the triangle inequality in the usual sense, there is an effectively *metric-like* structure whenever $d(n_i, n_j)$ is finite.

Proposition 1 (Pseudo-Metric)**.** *If the causal relation is acyclic and well-defined, then d induces a pseudo-metric on those pairs (n_i, n_j) for which a causal path exists.*

3.2 Approximate Embeddings and Effective Dimension

At large scales, if these distances can be faithfully embedded in $\mathbb{R}^D$, one sees a D-dimensional geometry. We formalize this with an *effective dimensionality* concept, adapting manifold-embedding ideas to discrete or partially discrete sets.

Definition 1 (Effective Dimensionality)**.** *Let* $\mathbf{N}$ *be our set of subnodes, with distance function* $d(\cdot,\cdot)$*. A finite subset* $S = \{n_{i_1}, \dots, n_{i_m}\} \subset \mathbf{N}$ embeds *in* $\mathbb{R}^D$ *with fidelity* $\epsilon \geq 0$ *if there exists a mapping*

$$\Phi : S \ \to \ \mathbb{R}^D$$

such that for any $n_{i_p}, n_{i_q} \in S$*,*

$$\big| \|\Phi(n_{i_p}) - \Phi(n_{i_q})\| - d(n_{i_p}, n_{i_q}) \big| \ \leq \ \epsilon.$$

If arbitrarily large subsets of $\mathbf{N}$ *embed in* $\mathbb{R}^D$ *with arbitrarily small* ϵ*, we say* $\mathbf{N}$ *is* effectively D-dimensional*.*

Intuitively, if you can map large pieces of the causal distance graph into $\mathbb{R}^D$ so that metric distortions remain negligible, then geometry *looks* D-dimensional at those scales.

3.3 Connections to Manifold-Like Behavior

In many emergent-spacetime scenarios, the large-scale limit of a causal network exhibits manifold-like features (local neighborhoods approximate Euclidean or Minkowski geometry). Our definition captures this by requiring consistent low-distortion embeddings of bigger and bigger chunks. If such embeddings exist for $D = 3$ (or 4 for spacetime), we interpret that as the system being effectively 3D or 4D, respectively.

4 Implications and Discussion

4.1 Space vs. Distance

Here, space is not fundamental. Instead, we measure:

- **Distance:** The discrete length of minimal causal chains.
- **Embedding:** A representation of those distances in some $\mathbb{R}^D$.

Space, in effect, is the *continuous manifold* we superimpose on top of these discrete causal distances once they become large-scale and dense enough to warrant a dimension-based description.

4.2 Origin of Dimensionality

Many quantum gravity models [6] suggest dimension might change with scale (e.g. dimensional reduction at high energies). In the Node framework, different scale regimes might embed well into different $\mathbb{R}^D$ spaces or fail to embed at all. Thus, dimension can:

- Be *scale-dependent*,
- Possibly transition from fractal-like at small scales to integer at large scales,
- Appear as a *phenomenological emergent* property rather than an absolute parameter.

4.3 Beyond Geometry: Matter and Fields

While this paper focuses on geometry, the approach in [1] alludes to embedding additional physical properties (like mass or charge) into the causal substructure. Future research may define mappings that preserve not only distances but also other field-like attributes, allowing emergent gauge fields or matter excitations from purely relational data.

5 Conclusions

In this formalized extension of the Node framework, we show how dimension is derived rather than assumed. By defining *effective dimensionality* through approximate embeddings of causal distances into $\mathbb{R}^D$, we capture the notion that a large-scale manifold can *emerge* from a purely cause-effect, distance-based system.

This reaffirms the broader claim: time, distance, space, and dimension are not separate fundamentals but are all conceptual layers built atop a deeper stable Node with internal causal dynamics. Observers interpret that internal web in geometric terms whenever those cause-effect patterns admit low-distortion embeddings into a dimension-based continuum.

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