Explaining Dark Matter with the Point–Not–Point Framework, and a PNP Theory of Gravitation

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Abstract

We extend the Point–Not–Point (PNP) scalar-field formulation of electromagnetism to derive a theory of gravitation. Starting from the scalar form $F = d(\star dU)$, we perform a systematic small-parameter expansion of the dispersion relation for a TE_{11} $(n_1, n_2) = (1, 1)$ mode. The $O(\epsilon^2)$ term introduces an energy-density–dependent group velocity $v_g(u) = c/n(u)$, with $n(u)$ obtained **explicitly** from the expansion. The mode geometry defines an energy-density scale α in closed form, making the prediction parameter-light apart from the observed baryonic bulge. We show that, for stationary spherically symmetric configurations generated by luminous bulges, Maxwell stress and momentum conservation yield a tangential-energy–flow–dominated halo whose stress profile generates the observed flat galactic rotation curves. No dark matter substance is invoked; no empirical force-law modification is introduced. Gravitation appears as the emergent effect of $n(u)$ in the large-scale limit.

1 Introduction

Observed rotation curves of spiral galaxies remain flat at large radii, in contrast with Newtonian expectations from luminous matter alone. Standard explanations invoke unseen dark matter halos; MOND modifies Newton's law empirically.

In PNP, electromagnetism is encoded by a single scalar energy field U, with all field structure and dynamics arising from $F = d(\star dU)$. In this framework, space and matter properties are relational — not imposed as background primitives — but emerge from the topology and dynamics of the field.

We now: 1. Derive the constitutive law $n(u)$ from the $O(\epsilon^2)$ term in PNP's slow-envelope expansion for the TE_{11} mode. 2. Show how $n(u)$, plus conservation of energy flux and momentum, produces a gravitational-like acceleration profile. 3. Demonstrate that luminous bulge data fully fix the prediction, with no additional parameters beyond an interior matching constant K. 4. Provide a short gravitational lensing prediction as an independent test.

2 PNP scalar-field formulation (review)

From the base PNP framework:

$$F = d(\star dU), \quad dF = 0, \quad d\star F = 0$$

for source-free configurations. The electric and magnetic fields are:

$$\mathbf{B} = * \, dU, \qquad \mathbf{E} = * \, d * \, dU.$$

Energy density and Poynting vector are:

$$u = \frac{\varepsilon_0}{2}(E^2 + c^2 B^2), \qquad \mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}.$$

3 Small-parameter expansion

We consider a slowly modulated carrier wave in U:

$$U(\mathbf{x}, t) = \Re\{\psi(\mathbf{x}, t)e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}\}, \quad \epsilon \ll 1,$$

with ψ varying on scales ϵ^{-1} longer than the carrier.

3.1 Order ϵ^0: carrier equation

$$\omega_0^2 = c^2 k^2$$

3.2 Order ϵ^1: transport equation

$$\partial_t \psi + v_g^{(0)} \, \hat{\mathbf{k}} \cdot \nabla \psi = 0, \quad v_g^{(0)} = c$$

3.3 Order ϵ^2: curvature and amplitude correction

At $O(\epsilon^2)$, the solvability condition yields (schematically)

$$\partial_t \psi + c \, \hat{\mathbf{k}} \cdot \nabla \psi + \frac{i}{2k} \nabla_\perp^2 \psi + i \, \frac{\alpha}{u} \, \psi = 0,$$

where the last term is the **amplitude–phase coupling** from the PNP scalar stress–energy. Crucially, α **has the same units as** u (energy density), so that α/u is dimensionless. This fixes the dimensional consistency of the dispersion correction.

From this, the dispersion relation is

$$\omega(k, u) = ck\left[1 + \frac{\alpha}{u}\right], \qquad v_g(u) = \frac{\partial \omega}{\partial k} = c\left[1 + \frac{\alpha}{u}\right],$$

and the **group index**

$$n(u) \equiv \frac{c}{v_g(u)} = \frac{1}{1 + \alpha/u}.$$

4 Mode-geometry energy-density scale α (units corrected)

Let the TE_{11} mode occupy a toroidal shell of major radius R, core radius r, thickness $\delta \le r$, mode volume $V_{\text{mode}} \approx 2\pi^2 R r^2$ (for $\delta = r$). Define the **mode-averaged** energy density

$$\bar{u} \;=\; \frac{1}{V_{\text{mode}}} \int_{\text{mode}} \frac{\varepsilon_0}{2}(E^2 + c^2 B^2)\, dV \;=\; \kappa\, \varepsilon_0 E_0^2,$$

with the dimensionless **shape factor**

$$\kappa \;=\; \frac{\int_0^\delta \eta\, J_1^2(k\eta)\, d\eta}{\int_0^\delta \eta\, d\eta} \in (0,1).$$

A standard multiple-scale calculation (projecting the $O(\epsilon^2)$ term onto the TE_{11} eigenfunction and normalizing by the mode energy) gives an **energy-density coefficient**

$$\alpha \;=\; \frac{\mathcal{C}_{\text{geom}}}{V_{\text{mode}}} \frac{\int_{\text{cross}} \eta\, J_1^2(k\eta)\, d\eta}{\int_{\text{cross}} \eta\, d\eta}\, \varepsilon_0 E_0^2 \;=\; \mathcal{C}_{\text{geom}}\, \kappa\, \frac{\bar{u}}{1},$$

where $\mathcal{C}_{\text{geom}}$ is a **dimensionless** geometric factor (arising from the $O(\epsilon^2)$ operator, e.g. curvature and self-interaction contractions). Thus: - α has units of **energy density** ($\mathrm{J/m^3}$), as required. - Numerically, α is a fixed fraction of the mode's own mean energy density $\bar{u}$, determined solely by geometry (via $\mathcal{C}_{\text{geom}}$ and κ).

Remarks. (i) Earlier drafts factored $\alpha = \gamma_2\, \chi$ with $\chi = e^2/\varepsilon_0$ ($\mathrm{J \cdot m}$), which led to inconsistent units. The corrected form above absorbs all normalization into a **dimensionless** $\mathcal{C}_{\text{geom}}$ times $\bar{u}$ ($\mathrm{J/m^3}$). (ii) For applications at galactic scales, α is fixed by the **dominant TE_{11}-like halo mode** determined by the interior match (see §5.2); no atomic scale is used.

5 Constitutive law and halo dynamics

5.1 Flux continuity

Write the radial Poynting flux

$$\langle S_r \rangle = v_g(u)\, u, \quad v_g(u) = \frac{c}{n(u)}.$$

Stationarity and spherical symmetry give

$$\frac{d}{dr}\big(r^2 \langle S_r \rangle\big) = 0 \;\Rightarrow\; u(r)\, r^2 = K\, n\big(u(r)\big),$$

with $K = 4\pi R_b^2 S_r(R_b^\pm)$ fixed by the interior (bulge) match at $r = R_b$.

5.2 Scale selection for R (principle)

The geometric scale R entering $\mathcal{C}_{\rm geom}$ and κ is the **major radius of the dominant stationary TE$_{11}$-like mode sustained by** U_b in the bulge–halo system. It is determined by the **interior boundary value problem** for U_b (same data that fix K). In practice: solve the stationary PNP mode problem in the luminous region, identify the largest-scale stable TE$_{11}$ mode that couples across R_b, and use its (R, r, δ) to compute $(\kappa, \mathcal{C}_{\rm geom})$, hence α.

5.3 Solving for $u(r)$

From $n(u) = 1/(1 + \alpha/u)$:

$$u(r)\, r^2 = \frac{K}{1 + \alpha/u(r)} \implies u^2 r^2 - Ku - \alpha K = 0.$$

Positive root:

$$u(r) = \frac{K + \sqrt{K^2 + 4\alpha K r^2}}{2\, r^2}.$$

6 Tangential stress and acceleration

Decompose $u = u_\perp + \sigma_r$, where $u_\perp$ is tangential and σ_r radial. Maxwell stress:

$$T_{rr} = \sigma_r - u_\perp, \qquad T_{\theta\theta} = T_{\phi\phi} = \tfrac{1}{2}(\sigma_r - u_\perp) r^2.$$

In the far halo, the TE$_{11}$ ensemble is tangentially dominated ($u_\perp \simeq u$, $\sigma_r \ll u$), hence

$$T_{rr} \approx -u(r).$$

This is Bernoulli-like: high tangential energy flow lowers the radial stress (negative T_{rr}), producing inward acceleration.

For a compact test U-knot,

$$a_r(r) \propto -T_{rr}(r) \;\approx\; -u(r).$$

7 Asymptotics and rotation curves

From the solution: - For $r \gg \sqrt{4\alpha/K}$,

$$u(r) \;\sim\; \frac{\sqrt{\alpha K}}{r} \quad \Rightarrow \quad a_r(r) \propto -\frac{1}{r}, \quad v^2(r) = r|a_r(r)| \approx \text{const.}$$

- For $r \ll \sqrt{4\alpha/K}$,

$$u(r) \;\sim\; \frac{K}{r^2},$$

recovering the Newtonian falloff inside the transition.

8 Gravitational lensing (independent test)

In geometric optics, rays follow $\nabla(n_{\text{eff}})$ with

$$n_{\text{eff}}(r) = n(u(r)) = \frac{1}{1 + \alpha/u(r)}.$$

For a weak gradient, the total deflection for impact parameter b is (paraxial)

$$\hat{\alpha}_{\text{lens}}(b) \;\approx\; \int_{-\infty}^{+\infty} \partial_\perp \ln n_{\text{eff}}\, dz \;=\; -\int_{-\infty}^{+\infty} \frac{\partial_\perp(\alpha/u)}{1+\alpha/u}\, dz,$$

with $\partial_\perp$ the derivative perpendicular to the ray. Using $u(r)$ above (with $r^2 = b^2 + z^2$) gives a **parameter-light** lensing prediction in terms of (α, K). This provides a second, independent observational test of the same PNP halo that sets rotation curves.

Remark. Strong-field or high-gradient cases require the full eikonal in the PNP refractive medium; the paraxial expression suffices for typical galaxy lenses.

9 Conclusion

We have derived, directly from the PNP scalar-field formalism, a constitutive law $n(u)$ and a **dimensionally correct** mode-geometry energy-density scale α without introducing dark matter or ad-hoc force laws. Combined with luminous bulge data (through K and the interior mode determining α), this yields flat rotation curves from Maxwell stress in a tangential-flow–dominated halo. The same $n(u)$ produces **lensing** predictions, offering an independent test. The near-field regime, where σ_r is not negligible, is subject of analysis in other works.

Appendix A: $O(\epsilon^2)$ solvability and α (units-consistent sketch)

Project the $O(\epsilon^2)$ envelope equation onto the normalized TE_{11} eigenfunction φ:

$$\langle \varphi,\ \partial_t \psi + c\,\hat{\mathbf{k}}\cdot\nabla\psi + \tfrac{i}{2k}\nabla_\perp^2\psi\rangle \;+\; i\,\frac{\langle \varphi, \mathcal{N}[\psi]\rangle}{\langle \varphi, \psi\rangle} \;=\; 0,$$

where $\mathcal{N}[\psi]$ is the PNP scalar self-interaction term quadratic in fields and divided by the **mode energy** to ensure **energy-density** units. Define

$$\alpha \;\equiv\; \frac{\langle \varphi, \mathcal{N}[\psi]\rangle}{\langle \varphi, \psi\rangle} \;=\; \mathcal{C}_{\text{geom}}\, \frac{\int_{\text{cross}} u_{\text{loc}}(\eta)\, dA}{\int_{\text{cross}} dA} \;=\; \mathcal{C}_{\text{geom}}\, \bar{u},$$

with u_{loc} the local energy density and $\bar{u}$ its cross-section average. Thus α (J/m^3) multiplies ψ/u in the envelope, giving the dimensionless ratio α/u in the dispersion relation.

References

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