Probabilities as Field-Energy Fractions

A deterministic scalar-field account of quantum statistics with a finite-environment test

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Abstract

A source-free scalar energy field $\phi(\mathbf{r}, t)$ reproduces the Schrödinger equation in the narrow-band limit. We show that the conserved quadratic form

$$\mathcal{E}[\Phi] = \int d^{3N}r \, |\Phi|^2$$

acts as the total field energy in configuration space. During a binary measurement the branch energies $\mathcal{E}_\uparrow$ and $\mathcal{E}_\downarrow$ remain constant and equal the Born probabilities. If the environment that produces decoherence contains only M modes, subsequent recoherence transfers energy between branches and violates standard quantum statistics by a factor $\delta P \approx e^{-\Lambda(M)t}$. A single-photon interferometer with a tunable cavity reservoir can vary M from 10^0 to 10^6 and detect deviations down to 10^{-3}. The energy-fraction interpretation is therefore falsifiable with present technology.

Scalar-field framework

The free wave equation

$$\partial_t^2 \phi = c^2 \nabla^2 \phi$$

with the ansatz

$$\phi = \Re\big[\psi(\mathbf{r}, t) \, e^{-i\omega_0 t}\big], \qquad \epsilon = \frac{|\partial_t \psi|}{\omega_0} \ll 1,$$

reduces to

$$i\partial_t \psi = -\frac{c^2}{2\omega_0} \nabla^2 \psi.$$

Setting $\hbar = E_{11}/\omega_{11}$ and $m = E_{11}/c^2$ gives the Schrödinger form

$$i\hbar\partial_t \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi.$$

For N particles we write $\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N, t)$. The conserved quantity

$$\mathcal{E}[\Phi] = \langle \Phi | \Phi \rangle = 1$$

is interpreted as total configuration-space field energy.

Measurement as branch-energy partition

A projective measurement produces $\Phi = \Phi_\uparrow + \Phi_\downarrow$ with $\langle \Phi_\uparrow | \Phi_\downarrow \rangle = 0$. Because $\mathcal{E}$ is quadratic, the fractions

$$P(\uparrow) = \langle \Phi_\uparrow | \Phi_\uparrow \rangle, \qquad P(\downarrow) = \langle \Phi_\downarrow | \Phi_\downarrow \rangle$$

satisfy $P(\uparrow) + P(\downarrow) = 1$ and reproduce the Born rule without extra postulates.

Finite-environment recoherence

Couple the pointer coordinate Q to M environmental oscillators (x_j, p_j) via

$$H_{\text{env}} = Q \sum_{j=1}^{M} g_j x_j.$$

Tracing out the environment yields an off-diagonal decay

$$c(t) = c(0)\, \exp\bigl[-\Lambda(M) t\bigr], \qquad \Lambda(M) = \frac{2 k_B T}{\hbar^2} (\Delta Q)^2 \sum_{j=1}^{M} m_j g_j^2.$$

For finite M, coherence revives at $t_{\text{rev}} \sim 1/\Lambda$. The branch energy then oscillates as

$$\mathcal{E}_\uparrow(t) = \mathcal{E}_\uparrow(0) + |c(0)|\, e^{-\Lambda t} \sin(\Omega t),$$

giving a probability deviation $\delta P \approx e^{-\Lambda t}$.

Dynamical derivation of probability drift

We model the combined qubit–pointer–bath Hamiltonian as

$$H = H_{\text{S}} + H_{\text{E}} + H_{\text{I}}, \qquad H_{\text{S}} = \frac{\omega_q}{2} \sigma_z + \frac{P^2}{2M} + V(Q),$$

$$H_{\text{E}} = \sum_{j=1}^{M} \Bigl(\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 x_j^2 \Bigr), \qquad H_{\text{I}} = Q \sum_{j=1}^{M} g_j x_j.$$

Initial state (after the ideal projective interaction but before bath coupling)

$$\rho(0) = \bigl[\ \alpha\, |\uparrow\rangle\langle\uparrow| \otimes \chi(Q - Q_0) + \beta\, |\downarrow\rangle\langle\downarrow| \otimes \chi(Q + Q_0) \bigr] \otimes \rho_{\text{th}},$$

where χ is a narrow Gaussian pointer packet and ρ_{th} is a thermal bath state.

Reduced evolution Tracing over the bath (second-order Born–Markov but **without** the continuum limit) gives, in the pointer basis,

$$\rho_{\uparrow\downarrow}(t) = \rho_{\uparrow\downarrow}(0)\,\exp\!\big[-\Gamma(M)t\big]\,\exp\!\big[i\varphi(t)\big],$$

$$\Gamma(M) = \frac{(\Delta Q)^2}{2\hbar^2}\sum_{j=1}^{M}\frac{g_j^2}{m_j\omega_j}\coth\!\Big(\frac{\hbar\omega_j}{2k_BT}\Big), \qquad \varphi(t) = \sum_{j=1}^{M}\frac{g_j^2}{m_j\omega_j^2}\sin(\omega_j t).$$

For a finite bath the phase $\varphi(t)$ periodically re-phases the two branches and converts part of the off-diagonal term back into diagonal population:

$$\delta P(t) = 2\,\mathrm{Re}\big[\rho_{\uparrow\downarrow}(0)\,e^{-\Gamma t}e^{i\varphi(t)}\big].$$

Taking equal coupling $g_j = g$, identical masses $m_j = m$, and $\omega_j = (j\pi/L)v$ (1-D cavity of length L) yields

$$\varphi(t) = \frac{2g^2}{m}\sum_{j=1}^{M}\frac{\sin(j\omega_1 t)}{j^2\omega_1^2}\ , \quad \omega_1 = \frac{\pi v}{L},$$

which approximates a **saw-tooth revival** with envelope

$$\delta P(t) \approx |\alpha\beta|\,e^{-\Gamma t}\,\frac{\sin\!\big(M\omega_1 t/2\big)}{M\sin(\omega_1 t/2)}.$$

At the first revival $t_{\mathrm{rev}} = 2\pi/\omega_1$ the sinc prefactor is $\simeq 1/M$, giving

$$\delta P_{\max} \approx |\alpha\beta|\frac{e^{-\Gamma t_{\mathrm{rev}}}}{M} \propto \frac{1}{M}.$$

Numerical estimate (cryogenic cavity, parameters from previous section): $M = 150,\ \Gamma t_{\mathrm{rev}} \approx 0.1 \Rightarrow \delta P_{\max} \sim 2\times10^{-3}$, in line with the 10^{-3} target.

Interpretation. Unitary dynamics **does** conserve total probability, but when the bath is finite the revival transfers weight between diagonal and off-diagonal sectors. Because the measurement record is read **before** full recoherence, the observed outcome frequencies drift by $\delta P(t)$ instead of remaining fixed at $|\alpha|^2$ and $|\beta|^2$. Taking the bath continuum limit ($M\to\infty$) restores orthodox statistics.

Experimental proposal

Element	Specification	Purpose
Mach–Zehnder interferometer	Single 1550 nm photons, SNSPD readout	Binary outcomes $\uparrow$ / $\downarrow$
Tunable cavity reservoir	Q factor 10^3–10^6 (mode count M)	Control $\Lambda(M)$
Cryostat	$T = 20$ K	Reduce thermal noise, lengthen t_{rev}
Optical delay line	Variable $t = 0$–$10\,\mu$s	Observe $\delta P(t)$

Predicted deviation: $\delta P \approx 10^{-3}$ for $M \sim 10^2$, $\Delta Q = 1\,\mu$m. Photon statistics of 10^7 counts reach 10^{-4} precision, sufficient to confirm or rule out the effect.

Implications

- Detecting δP validates deterministic field-energy probabilities and quantifies environmental decoherence.
- A null result beyond 10^{-4} rejects the model while leaving orthodox quantum mechanics intact.

Conclusion

Identifying Born weights with conserved field-energy fractions makes a clear, falsifiable prediction: finite environments induce measurable deviations from standard statistics. A tunable-reservoir single-photon experiment can perform the test now, deciding whether deterministic scalar-field physics underlies quantum probability.

References

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