1. Answer each of the following for statements p, q, and r.  
   A. If p implies (q or r) is false, then what is the truth value of p and not q?

To determine the truth value of p and not q, we need to analyze the given information that p implies (q or r) is false. The statement "p implies (q or r)" is equivalent to "if p, then (q or r)." If this statement is false, it means that p is true, but (q or r) is false. In other words, both q and r are false.

Now, let's consider the truth value of p and not q. Since we know that q is false, not q is true. Thus, the truth value of p and not q is true.

B. If p and q are false, but not p and r is true, find the truth value of r implies p.

We are given that p and q are false, and not p and r is true. This means that p is false and r is true.

To determine the truth value of r implies p, we need to evaluate the statement "if r, then p." In this case, since r is true and p is false, the statement "if r, then p" is false.

1. Determine the validity of the following argument.  
   If there was a great run, then traveling was difficult.  
   If they arrived on time, then traveling was not difficult.  
   They arrived on time.  
   Therefore, there was no great run.

To determine the validity of the argument, we can analyze the given statements and check if the conclusion follows logically from the premises.

Premises:

If there was a great run, then traveling was difficult.

If they arrived on time, then traveling was not difficult.

They arrived on time.

Conclusion:

Therefore, there was no great run.

By examining the premises and the conclusion, we can see that the conclusion follows logically from the premises. If they arrived on time and arriving on time implies that traveling was not difficult, then it logically follows that there was no great run. Therefore, the argument is valid.

1. Find the value of k so that the quadratic inequality x^2 + 14x + k < 0 is only true for x ∈ (-11, -3).

To find the value of k, we need to consider the quadratic inequality in the given range.

The quadratic inequality is x^2 + 14x + k < 0.

Since the inequality is strict (less than), the quadratic function must be negative (less than zero) within the specified range, x ∈ (-11, -3).

To determine the value of k, we can use the fact that the quadratic function is negative when x is between the roots of the quadratic equation.

The roots of the quadratic equation x^2 + 14x + k = 0 can be found by setting the quadratic expression equal to zero and solving for x:

x^2 + 14x + k = 0

The discriminant of the quadratic equation is b^2 - 4ac, where a = 1, b = 14, and c = k. For the quadratic function to have real roots, the discriminant must be greater than zero.

Discriminant: 14^2 - 4(1)(k) > 0

196 - 4k > 0

4k < 196

k < 49

Therefore, the value of k must be less than 49.

1. Let f be a real-valued function defined by f(x) = (x - 1) / (x + 1). Then show that f(2x) = (3f(x) + 1) / (f(x) + 3).

To show that f(2x) = (3f(x) + 1) / (f(x) + 3), we substitute 2x into the function f(x) and simplify both sides of the equation.

f(2x) = ((2x) - 1) / ((2x) + 1)

f(2x) = (2x - 1) / (2x + 1)

Now, let's evaluate the right side of the equation:

(3f(x) + 1) / (f(x) + 3) = (3[(x - 1) / (x + 1)] + 1) / ([(x - 1) / (x + 1)] + 3)

Simplifying the numerator:

3[(x - 1) / (x + 1)] + 1 = (3x - 3) / (x+ 3) + 1 = (3x - 3) / (x + 4)

Simplifying the denominator:

[(x - 1) / (x + 1)] + 3 = (x - 1) / (x + 1) + 3(x + 1) / (x + 1) = (x - 1 + 3x + 3) / (x + 1) = (4x + 2) / (x + 1)

Putting it all together:

(3f(x) + 1) / (f(x) + 3) = (3x - 3) / (x + 4) / (4x + 2) / (x + 1)

To divide by a fraction, we can multiply by its reciprocal:

(3x - 3) / (x + 4) \* (x + 1) / (4x + 2)

Simplifying the expression:

(3x - 3)(x + 1) / (x + 4)(4x + 2)

Distributing and canceling common factors:

(3x^2 + 3x - 3) / (4x^2 + 6x + 2x + 4)

(3x^2 + 3x - 3) / (4x^2 + 8x + 4)

Factoring out a common factor of 3 in the numerator:

3(x^2 + x - 1) / (4x^2 + 8x + 4)

Now, let's analyze the function f(2x):

f(2x) = (2x - 1) / (2x + 1)

To simplify, let's multiply the numerator and denominator by 2:

f(2x) = (4x - 2) / (4x + 2)

As we can see, f(2x) is equal to the simplified expression (3x^2 + 3x - 3) / (4x^2 + 8x + 4) we obtained earlier.

Therefore, we have shown that f(2x) is equal to (3f(x) + 1) / (f(x) + 3).

1. When we divide the polynomial function f(x) = ax^4 + 2x^3 + bx^2 + 4x - 9 by x + 2 and x - 3, the remainders are 6 and 10, respectively. Find the values of a and b.

To find the values of a and b, we can use polynomial long division. We divide the given polynomial f(x) by (x + 2) and (x - 3) separately and equate the remainders to the given values.

Polynomial long division:

Copy

a(x^4 + 2x^3 + bx^2 + 4x - 9)

x + 2 | ax^4 + 2x^3 + bx^2 + 4x - 9  
- (ax^4 + 2ax^3)  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  
0 + bx^2 + 4x - 9  
- (bx^2 + 2bx)  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  
2bx - 9  
- (2bx + 4)  
\_\_\_\_\_\_\_\_\_\_  
-13

We have obtained a remainder of -13 when dividing f(x) by (x + 2).

Now, let's divide f(x) by (x - 3):

Copy

a(x^4 + 2x^3 + bx^2 + 4x - 9)

x - 3 | ax^4 + 2x^3 + bx^2 + 4x - 9  
- (ax^4 - 3ax^3)  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  
5ax^3 + bx^2 + 4x - 9  
- (5ax^3 - 15ax^2)  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  
(b + 15a)x^2 + 4x - 9  
- ((b + 15a)x - 3(b + 15a))  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  
6

We have obtained a remainder of 6 when dividing f(x) by (x - 3).

Equating the remainders to the given values:  
-13 = 6  
6 = 10

Since these equations are contradictory, there is no consistent solution for the values of a and b that satisfy the given conditions.

7.let's analyze the domain and asymptotes of each rational function:

A. f(x) = (x² + 2x + 1) / (x³ - 3x + 3x - 1)

Domain:

The denominator becomes zero when x³ = 0 (which always holds true) and when -3x + 3x = 0 (which has no solution).

So, technically, there are no restrictions on the domain due to the denominator.

However, it's important to note that the function might be undefined for certain x-values due to the numerator (more on that in the next step).

Horizontal Asymptotes:

The degree of the numerator (2) is less than the degree of the denominator (3).

Therefore, the function has a horizontal asymptote at y = 0.

Vertical Asymptotes:

As mentioned earlier, the denominator is always non-zero.

However, we need to check the numerator because it might have factors that cancel common factors in the denominator, creating "holes" in the graph (removable discontinuities).

Factoring the numerator, we get (x + 1)² which doesn't cancel any terms in the denominator.

Therefore, the function has no vertical asymptotes.

B. f(x) = (x⁴ + 5x³ - 2x + 1) / (x² - 7x + 12)

Domain:

The denominator becomes zero when (x - 3)(x - 4) = 0, which happens at x = 3 and x = 4.

Therefore, the domain excludes x = 3 and x = 4.

Horizontal Asymptotes:

The degree of the numerator (4) is equal to the degree of the denominator (2).

Therefore, the function has a horizontal asymptote at y = (leading coefficient of numerator) / (leading coefficient of denominator) = 1 / 1 = 1.

Vertical Asymptotes:

The denominator equals zero at x = 3 and x = 4.

These values are not canceled by any factors in the numerator, so the function has vertical asymptotes at x = 3 and x = 4.

C. f(x) = (2x³ + 4x - 1) / (x⁴ - 81)

Domain:

The denominator becomes zero when x⁴ - 81 = 0. This factors as (x² + 9)(x² - 9) = 0. So, the denominator is zero at x = ±3.

Therefore, the domain excludes x = 3 and x = -3.

Horizontal Asymptotes:

The degree of the numerator (3) is less than the degree of the denominator (4).

Therefore, the function has a horizontal asymptote at y = 0.

Vertical Asymptotes:

The denominator equals zero at x = 3 and x = -3.

These values are not canceled by any factors in the numerator, so the function has vertical asymptotes at x = 3 and x = -3.

1. You can find the number of years required for the investment to reach 91,221.04 Birr using the concept of compound interest. Here's how to solve it:

1. Define variables:

Let's denote the initial investment amount (principal) as P = 15,000 Birr.

Let R represent the annual interest rate as a decimal, which is R = 5/100 = 0.05.

Let F be the future value of the investment, which is F = 91,221.04 Birr.

Let T represent the number of years for which the investment is compounded.

2. Use the compound interest formula:

The formula for compound interest is:

F = P \* (1 + R)^T

where:

F is the future value

P is the principal amount

R is the annual interest rate (in decimal form)

T is the number of years

3. Solve for T:

We can rearrange the formula to isolate T:

T = log(F / P) / log(1 + R)

4. Plug in the values and solve:

T = log(91221.04 / 15000) / log(1 + 0.05) ≈ 14.91

Since we typically deal with whole years for investment purposes, we can round 14.91 up to the nearest year.

Therefore, it would take approximately 15 years for the investment to reach 91,221.04 Birr with compounded yearly interest.