

11. Prove and demonstrate that binary search takes $O(\log(n))$.

Binary Search calls itself/iterates through the while loop a number of times based on what power of 2 is it less than or equal to.

If its $2^1 = 2$ elements it's found in one loop

If its $2^2 = 4$ elements it'll take 2 loops to find the individual elements when breaking up by 2

If its $2^3 = 8$ elements it'll take 3 loops splitting it into two to find each individual

And so on

We're looking for what power of 2, 2^x can be used to divide N to be less than or equal to 1

Aka

$$1 = n / (2^x)$$

Multiply both sides

$$2^x = n$$

The definition of logarithms would let you do

$$\text{Log}(\text{Base}2)(n) = x$$

So the amount of times the function loops is based on $\text{Log}(\text{Base}2)(n)$

Which falls under the $O(\log(n))$ value