Introduction to robotics

hw -2

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עבודת בית 2 רובו

סעיף 1:

an open kinematic chain robot with 3 degrees of freedom.

סעיף 2:

The end effector has three degrees of freedom.

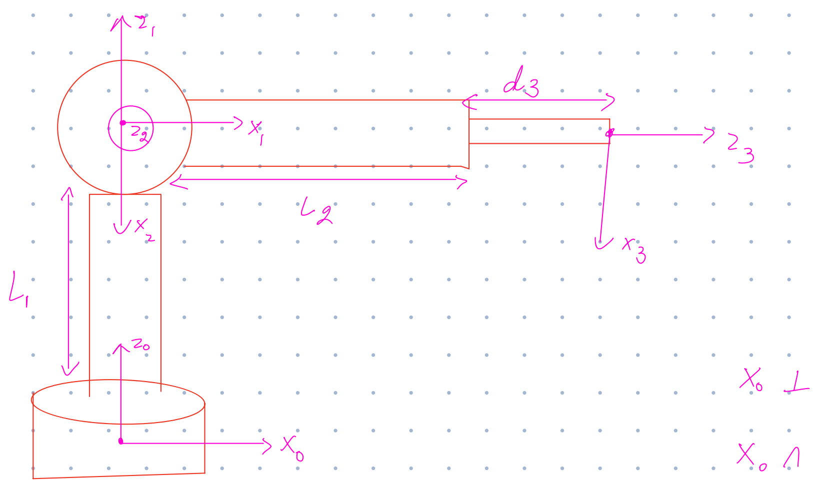
סעיף 3:

The end effector can only translate.

סעיף 4:

There may be one or more DH frames

סעיף 5:



$X_0 \perp Z_1$

$X_0 \wedge Z_1$

$X_1 = Z_1 \times Z_2$

$X_2 \perp Z_3$ $X_2 \wedge Z_3$

joint i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	90	0	0	$-90+\theta_2$
3	90	0	L_2+d_3	0

לפי תרגול נציב בנוסחה. הי $^{i-1}_{i}T$

$$ {}^{0}_{1}T = \begin{pmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad {}^{1}_{2}T = \begin{pmatrix} C_{\theta_2-90}, & -S_{\theta_2-90} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_{\theta_2-90} & C_{\theta_2-90} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}_{3}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_2+d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T $$

$$ \sin(\theta_2 - 90) = -\cos\theta_2 $$

$$ \cos(\theta_2 - 90) = \sin\theta_2 $$

$$ {}^{0}_{3}T = \begin{pmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -C\theta_2 & S\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_2+d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

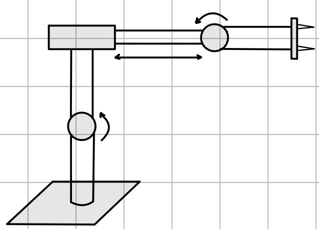
$$ = \begin{pmatrix} C\theta_1 S\theta_2 & -S\theta_1 & C\theta_1 C\theta_2 & C\theta_1 C\theta_2 (L_2+d_3) \\ S\theta_1 S\theta_2 & C\theta_1 & S\theta_1 C\theta_2 & S\theta_1 C\theta_2 (L_2+d_3) \\ -C\theta_2 & 0 & S\theta_2 & S\theta_2 (L_2+d_3) + L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

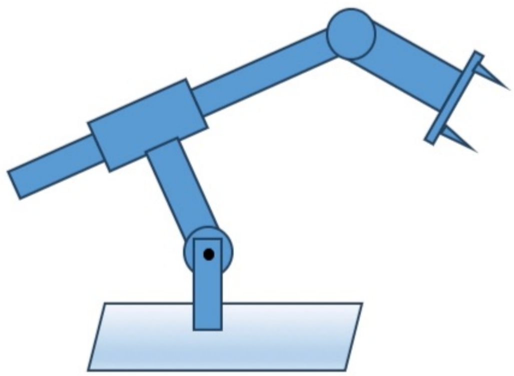
$\alpha = \theta_1$ $\quad$ $\gamma = \theta_3$

$\beta = \theta_2$

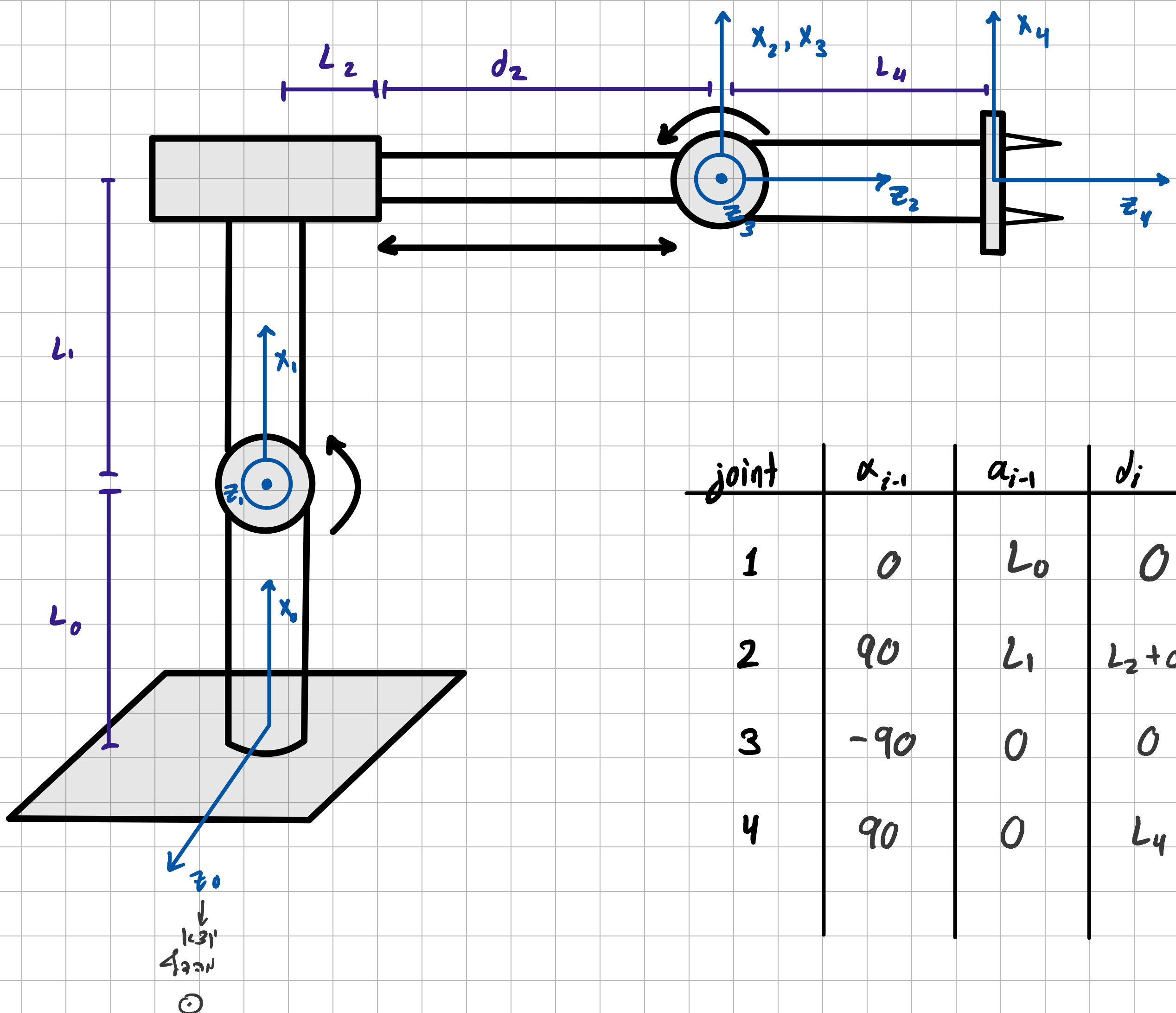
$\omega = \theta_4$ $\quad$ $-\sin\alpha_2 < 1$

5. Compute the DH and the Transformation Matrix for the following three manipulators:





since z_2 and z_3 are parallel, x_2 can be selected



joint	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	L_0	0	θ_1
2	90	L_1	L_2+d_2	0
3	-90	0	0	θ_3
4	90	0	L_4	θ_4

$$
{}^{0}_{1}T = \begin{pmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & L_0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{1}_{2}T = \begin{pmatrix} 1 & 0 & 0 & L_1 \\ 0 & 0 & -1 & -(L_2+d_2) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{2}_{3}T = \begin{pmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_{\theta_3} & -C_{\theta_3} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{3}_{4}T = \begin{pmatrix} C_{\theta_4} & -S_{\theta_4} & 0 & 0 \\ 0 & 0 & -1 & -L_4 \\ S_{\theta_4} & C_{\theta_4} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{0}_{2}T = {}^{1}_{0}T \; {}^{2}_{1}T = \begin{pmatrix} c_{\theta_1} & 0 & s_{\theta_1} & -s_{\theta_1}(-d_2-L_2)+L_0+L_1 c_{\theta_1} \\ s_{\theta_1} & 0 & -c_{\theta_1} & -c_{\theta_1}(d_2+L_2)+L_1 s\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{0}_{3}T = {}^{0}_{2}T \; {}^{3}_{2}T = \begin{pmatrix} c_{\theta_3} c_{\theta_1} - s_{\theta_3} s_{\theta_1} & -s_{\theta_3} c_{\theta_1} - c_{\theta_3} s_{\theta_1} & 0 & s_{\theta_1}(d_2+L_2)+L_0+L_1 c_{\theta_1} \\ s_{\theta_3} c_{\theta_1} + c_{\theta_3} s_{\theta_1} & c_{\theta_3} c_{\theta_1} - s_{\theta_3} s_{\theta_1} & 0 & -c_{\theta_1}(d_1+d_2)+L_1 s_{\theta_1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{0}_{4}T = {}^{0}_{3}T \; {}^{3}_{4}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \; {}^{3}_{4}T =
$$

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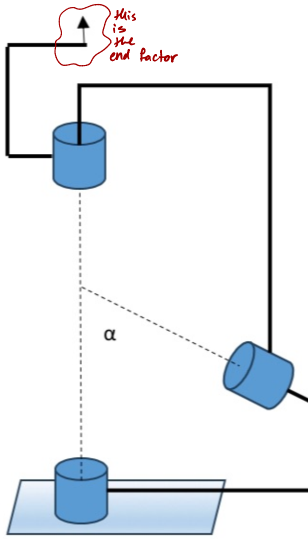
את ט אלה

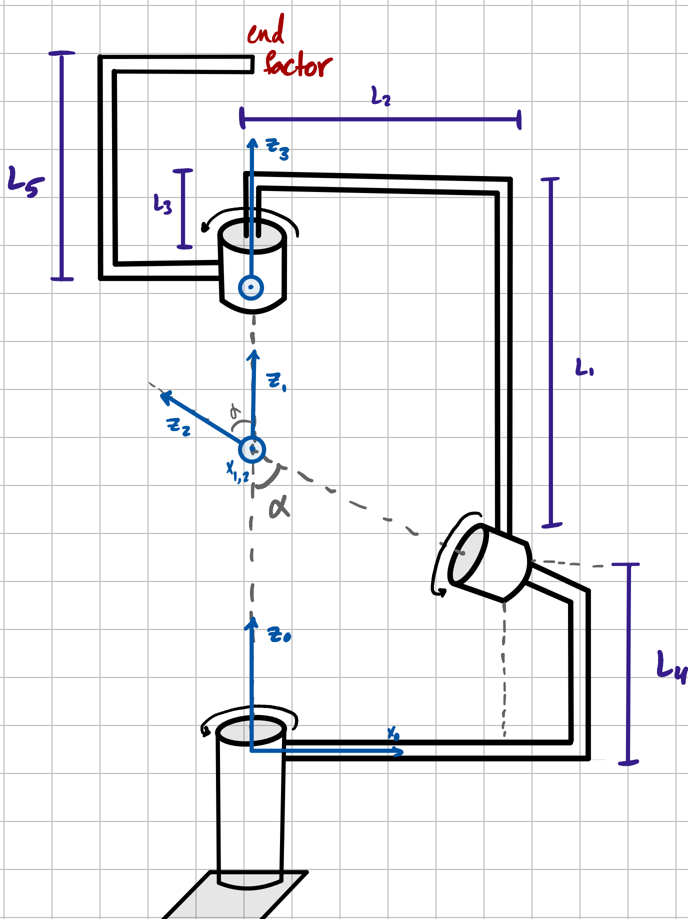
$$
\left({}^{4}_{0}T\right) = \left({}^{0}_{4}T\right)^{-1}
$$

5. Compute the DH and the Transformation Matrix for the following three manipulators:

the TA said the arrow is where the end factor is, so we assumed the arrow itself is irrelevant and the end factor is the arrow's origin.



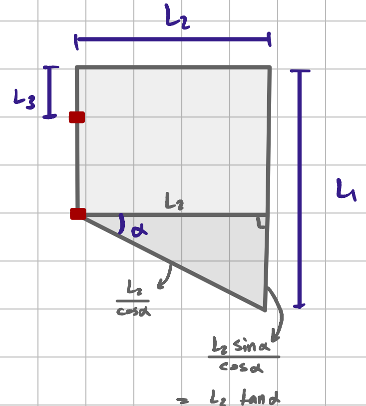
also, we assume that the joints are small enough that their additions to d_i, a_{i-1} are negligible.



note that we can shift the origin of a frame along its rotation access. (O_1 , O_2)

joint	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	$L_4+L_2\tan\alpha$	θ_1
2	α	0	0	θ_2
3	$-\alpha$	$L_1-L_3-L_2\tan\alpha$	$L_1-L_3-L_2\tan\alpha$	θ_3

side calcs:



the distance between the two red points is:

$$L_1 - L_3 - L_2\tan\alpha$$

Translation matrix between frame 3 and the end factor: $${}^{3}_{\text{end factor}}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{0}_{1}T = \begin{pmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & 0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & L_4 + L_2 \tan\alpha \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

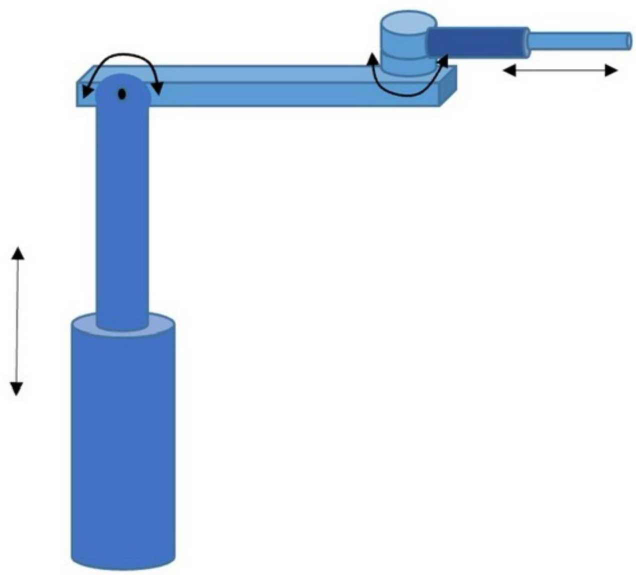
$${}^{1}_{2}T = \begin{pmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & 0 \\ S_{\theta_2} C_\alpha & C_{\theta_2} C_\alpha & -S_\alpha & 0 \\ S_{\theta_2} S_\alpha & C_{\theta_2} S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

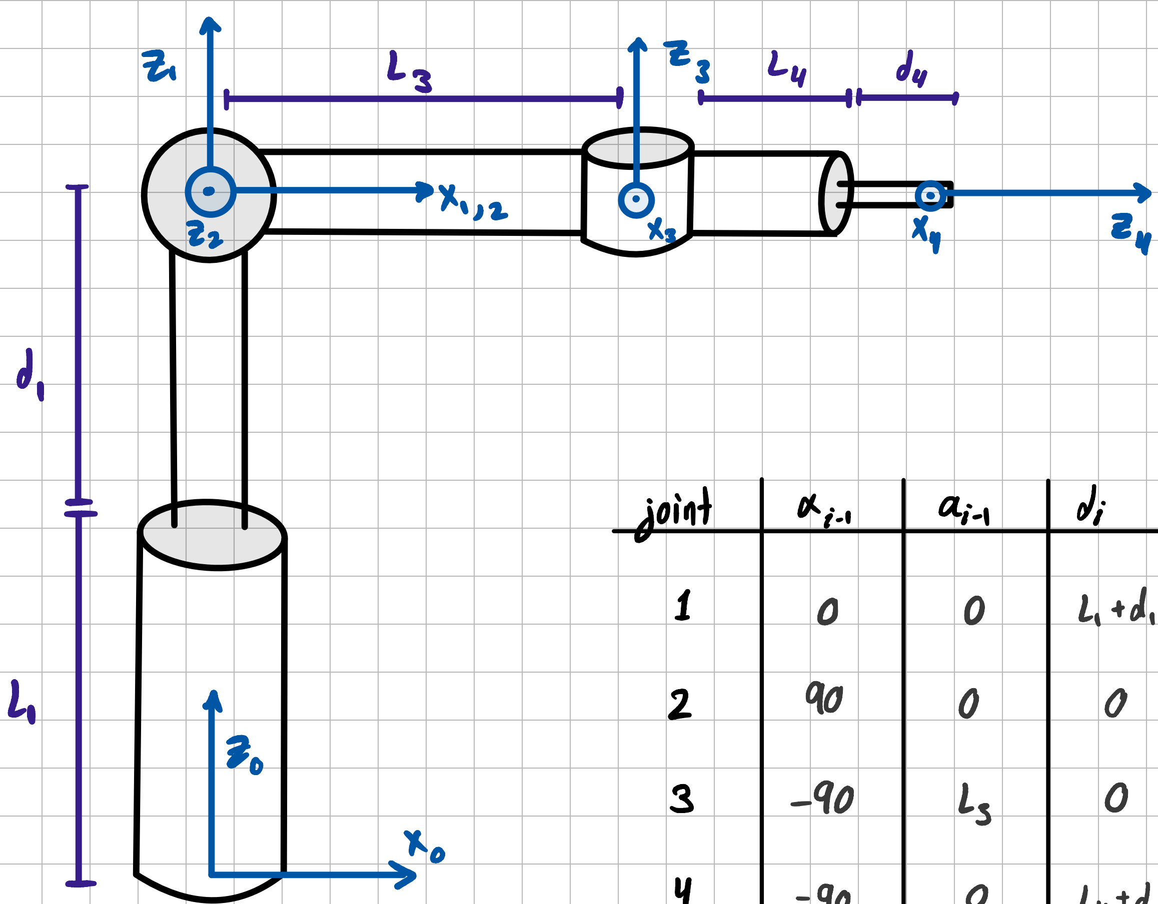
$${}^{2}_{3}T = \begin{pmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & L_1 - L_3 - L_2 \tan\alpha \\ S_{\theta_3} C_{-\alpha} & C_{\theta_3} C_{-\alpha} & -S_{-\alpha} & -S_{-\alpha}(L_1 - L_3 - L_2 \tan\alpha) \\ S_{\theta_3} S_{-\alpha} & C_{\theta_3} S_{-\alpha} & C_{-\alpha} & C_{-\alpha}(L_1 - L_3 - L_2 \tan\alpha) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{0}_{\text{end factor}}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \; {}^{3}_{\text{end factor}}T$$

$$\left({}^{\text{end factor}}_{0}T\right) = \left({}^{0}_{\text{end factor}}T\right)^{-1}$$

6. For the 4R robot in the figure, compute the DH and the Transformation matrix.





joint	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	$L_1 + d_1$	0
2	90	0	0	θ_2
3	-90	L_3	0	$-90 + \theta_3$
4	-90	0	$L_4 + d_4$	0

$$ {}^{0}_{1}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & & 0 \\ 0 & 0 & 1 & L_1+d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{1}_{2}T = \begin{pmatrix} c_{\theta_2} & -s_{\theta_2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_{\theta_2} & c_{\theta_2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}_{3}T = \begin{pmatrix} c_{\theta_3-90} & -s_{\theta_3-90} & 0 & L_3 \\ 0 & 0 & +1 & 0 \\ -s_{\theta_3-90} & -c_{\theta_3-90} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{3}_{4}T = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -(L_4+d_4) \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ \rightarrow \quad {}^{0}_{4}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \; {}^{3}_{4}T $$

$$ \left({}^{4}_{0}T\right) = \left({}^{0}_{4}T\right)^{-1} $$

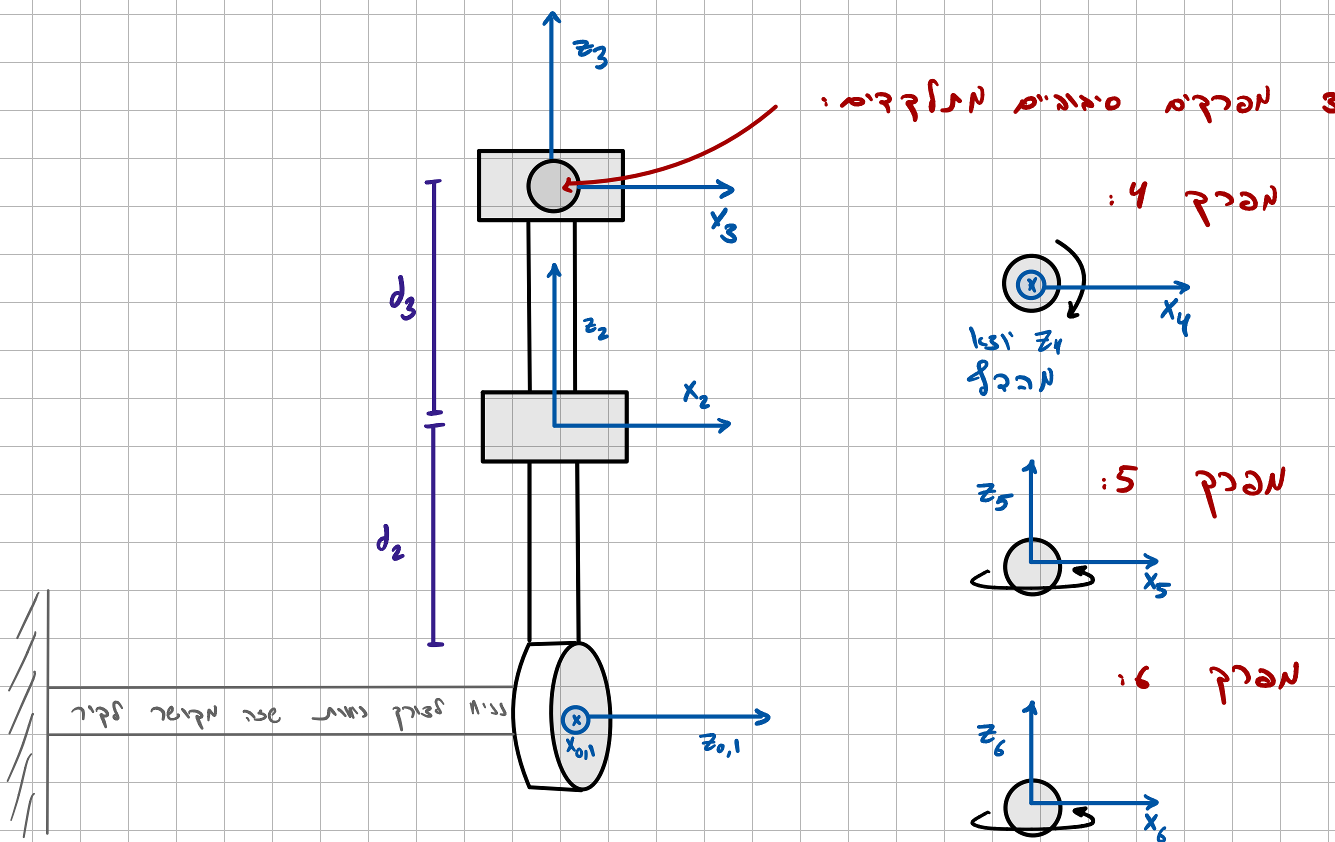
prismatic

rotation

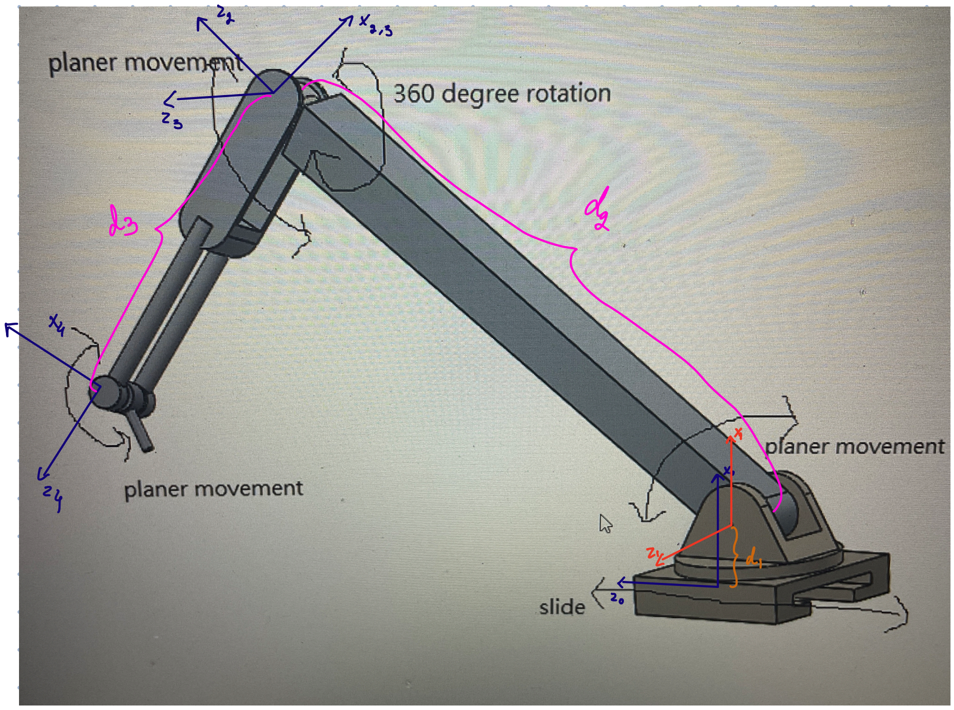
7. A reverse engineering problem. Consider the following DH table:

Joint i	θ_i	a_{i-1}	α_{i-1}	d_i
1	θ_1	0	0	0
2	0	0	-90°	d_2
3	0	0	0	d_3
4	θ_4	0	-90°	0
5	θ_5	0	90°	0
6	θ_6	0	0	0

Sketch the corresponding robot.



שאלה 8



Joint i	α_{i-1}	a_{i-1}	d_i	θ_i
1	90	d_1	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	0	θ_3
4	0	d_3	0	$\theta_4 + 90$

$$
{}^{0}_{1}T = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & d_1 \\ \sin\theta_1 \cos 90 & \cos\theta_1 \cos 90 & -\sin 90 & 0 \\ \sin\theta_1 \sin 90 & \cos\theta_1 \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{1}_{2}T = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 \cos(-90) & \cos\theta_2 \cos(-90) & -\sin(90) & -\sin(-90) d_2 \\ \sin\theta_2 \sin(-90) & \cos\theta_2 \sin(-90) & \cos(-90) & \cos(-90) d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

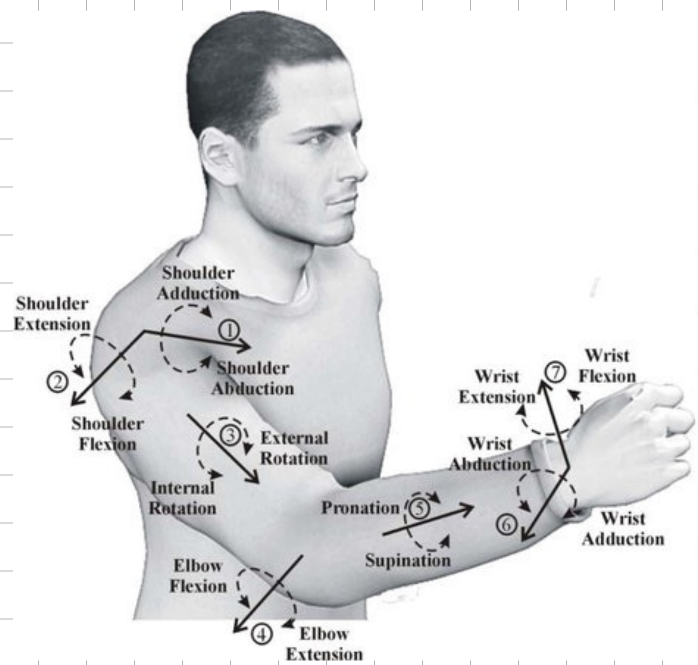
$$
{}^{2}_{3}T = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 \cos 90 & \cos\theta_3 \cos 90 & -\sin 90 & 0 \\ \sin\theta_3 \sin 90 & \cos\theta_3 \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{3}_{4}T = \begin{pmatrix} \cos(90+\theta_4) & -\sin(90+\theta_4) & 0 & d_3 \\ \sin(90+\theta_4) \cos 0 & \cos(90+\theta_4) \cos(0) & 0 & 0 \\ \sin(90+\theta_4) \cos 0 & \cos(90+\theta_4) \sin 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{0}_{4}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T \cdot {}^{3}_{4}T
$$

$$
{}^{4}_{0}T = \left({}^{0}_{4}T\right)^{-1}
$$

9. Model the human arm from the shoulder to the wrist and compute the corresponding DH matrix.



The shoulder has 3 DoFs:

1. Rotation
2. Abduction / Adduction
3. Flexion / Extension

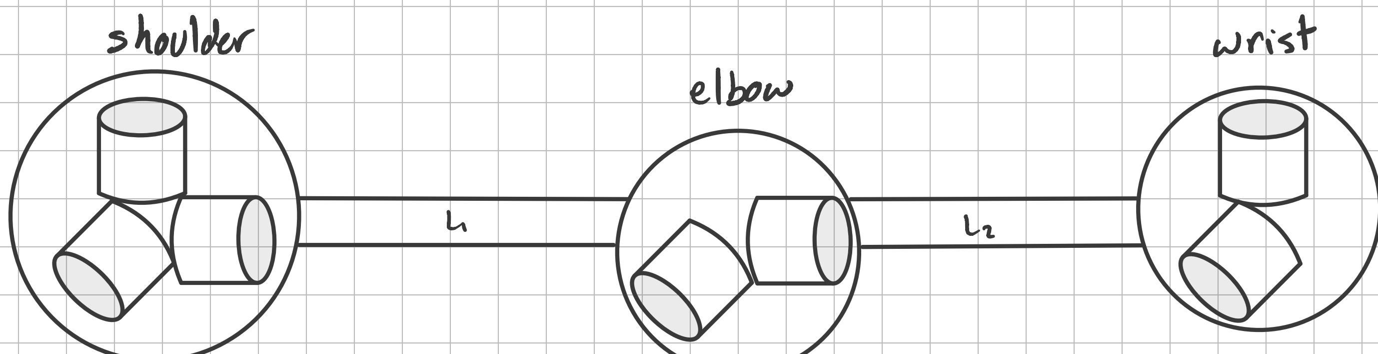
The elbow has 2 DoFs

1. Flexion / Extension
2. Pronation / Supination

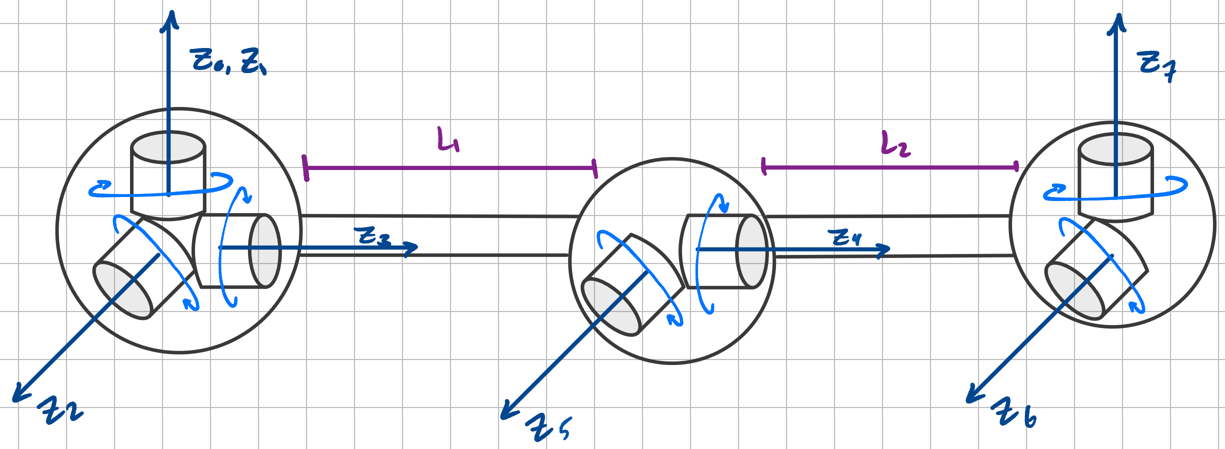
The wrist has 2 DoFs:

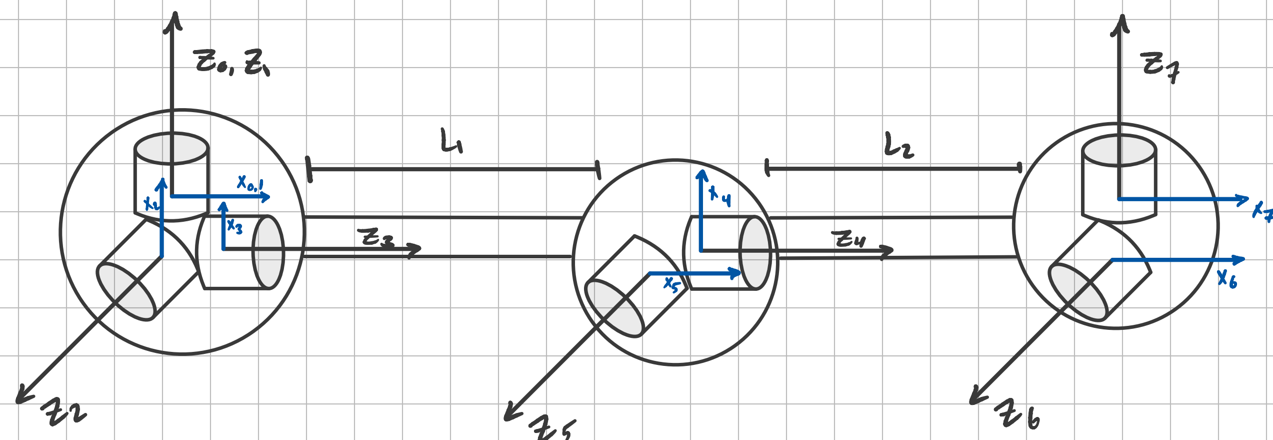
1. Abduction / Adduction
2. Flexion / Extension

we drew each motion as a seperate joint, but all the joints in the shoulder share an origin point, the same goes for the elbow and the wrist.



the rotations are as follows:





we get the following DH parameters:

joint	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	$90+\theta_1$
2	90	0	0	θ_2
3	90	0	0	θ_3
4	0	0	L_1	θ_4
5	-90	0	0	θ_5
6	0	0	L_2	θ_6
7	-90	0	0	θ_7

$$ {}^{0}_{1}T = \begin{pmatrix} C_{90+\theta_1} & -S_{90+\theta_1} & 0 & 0 \\ S_{90+\theta_1} & C_{90+\theta_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{1}_{2}T = \begin{pmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_{\theta_2} & C_{\theta_2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}_{3}T = \begin{pmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_{\theta_3} & C_{\theta_3} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{3}_{4}T = \begin{pmatrix} C_{\theta_4} & -S_{\theta_4} & 0 & L_1 \\ S_{\theta_4} & C_{\theta_4} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{4}_{5}T = \begin{pmatrix} C_{\theta_5} & -S_{\theta_5} & 0 & 0 \\ 0 & 0 & +1 & 0 \\ -S_{\theta_5} & -C_{\theta_5} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{5}_{6}T = \begin{pmatrix} C_{\theta_6} & -S_{\theta_6} & 0 & L_2 \\ S_{\theta_6} & C_{\theta_6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{6}_{7}T = \begin{pmatrix} C_{\theta_7} & -S_{\theta_7} & 0 & 0 \\ 0 & 0 & +1 & 0 \\ -S_{\theta_7} & -C_{\theta_7} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ \rightarrow {}^{0}_{7}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T\,{}^{6}_{7}T \quad , \quad {}^{7}_{0}T = \left({}^{0}_{7}T\right)^{-1} $$