测验, 8 个问题

## ✔ 恭喜!您通过了!

下一项



1/1分

1.

Which of the following (possibly more than one) must be true if random variable X is continuous with PDF f(x) ?

f(x) is a continuous function

未选择的是正确的

$$X>=0$$
 always

未选择的是正确的

$$oxed{ \lim_{x o\infty}f(x)=\infty}$$

未选择的是正确的

f(x) is an increasing function of x

未选择的是正确的

$$f(x) \geq 0$$
 always

正确



1/1分

2.

If  $X \sim \operatorname{Exp}(3)$  , what is the value of P(X>1/3) ? Round your answer to two decimal places.

0.37

#### 正确回答

This is 
$$P(X>1/3)=\int_{1/3}^{\infty}3e^{-3x}dx$$
  $=-e^{-3x}|_{1/3}^{\infty}$   $=0-(-e^{-3/3})=e^{-1}=0.368$ 



1/1分

3。

Suppose  $X \sim \mathrm{Uniform}(0,2)$  and  $Y \sim \mathrm{Uniform}(8,10)$  . What is the value of E(4X+Y) ?

13

## 正确回答

This is E(4X + Y) = 4E(X) + E(Y) = 4(1) + 9 .



## 8/8 分 (100%)

# 4。 Module 1 Hopuestions 4-7, consider the following: 测验, 8 个问题

Suppose  $X\sim N(1,5^2)$  and  $Y\sim N(-2,3^2)$  and that X and Y are independent. We have  $Z=X+Y\sim N(\mu,\sigma^2)$  because the sum of normal random variables also follows a normal distribution.

• What is the value of  $\mu$  ?

-1

### 正确回答

$$\mu = E(Z) = E(X + Y) = E(X) + E(Y) = 1 + (-2)$$



1/1分

5。

Adding normals:

• What is the value of  $\sigma^2$  ?

Hint: If two random variables are independent, the variance of their sum is the sum of their variances.

34

#### 正确同签

$$\sigma^2 = Var(Z) = Var(X+Y) = Var(X) + Var(Y) = 25 + 9$$

1/1分

6.

#### Adding normals:

## Module 1 Honors

测验, 8 个问题

If random variables X and Y are not independent, we still have E(X+Y)=E(X)+E(Y) , but now Var(X+Y)=Var(X)+Var(Y)+2Cov(X,Y) where Cov(X,Y)=E[(X-E[X])(Y-E[Y])] is called the covariance between X and Y.

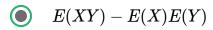
8/8 分 (100%)

 A convenient formula for calculating variance was given in the supplementary material:

 $Var(X)=E[(X-E[X])^2]=E[X^2]-(E[X])^2$  . Which of the following is an analogous expression for the covariance of X and Y?

Hint: Expand the terms inside the expectation in the definition of Cov(X,Y) and recall that E(X) and E(Y) are just constants.

$$\bigcirc \quad E[X^2] - (E[X])^2 + E[Y^2] - (E[Y])^2$$



正确

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$$
  
=  $E[XY - XE(Y) - E(X)Y + E(X)E(Y)]$   
=  $E[XY] - E[XE(Y)] - E[E(X)Y] + E[E(X)E(Y)]$   
=  $E[XY] - E(X)E(Y) - E(X)E(Y) + E(X)E(Y)$ 

$$(E[X^2] - (E[X])^2) \cdot (E[Y^2] - (E[Y])^2)$$

$$\bigcirc \quad E[Y^2] - (E[Y])^2$$



## 7。 Module 1 Hangrarmals: 测验, 8 个问题

• Consider again  $X\sim \mathrm{N}(1,5^2)$  and  $Y\sim \mathrm{N}(-2,3^2)$  , but this time X and Y are *not* independent. Then Z=X+Y is still normally distributed with the same mean found in Question 4. What is the variance of Z if E(XY)=-5?

Hint: Use the formulas introduced in Question 6.

28

#### 正确回答

$$egin{aligned} Var(Z) &= Var(X) + Var(Y) + 2Cov(X,Y) = 25 + 9 + 2Cov(X,Y) \ &= 34 + 2(E[XY] - E[X]E[Y]) \ &= 34 + 2(-5 - 1(-2)) = 34 - 2(3) \end{aligned}$$



1/1分

8.

Free point:

1) Use the definition of conditional probability to show that for events  $\boldsymbol{A}$  and  $\boldsymbol{B}$  , we have

$$P(A \cap B) = P(B|A)P(A) = P(A|B)P(B).$$

2) Show that the two expressions for independence P(A|B)=P(A) and  $P(A\cap B)=P(A)P(B)$  are equivalent.

Solution (1)

Write  $P(B|A) = rac{P(A\cap B)}{P(A)}$  and multiply both sides by P(A)

Solution (2)

Plug these expressions into those from (1).

测验, 8 个问题



