Lesson 6

测验, 6 个问题

6/6 分 (100%)

✔ **恭喜！您通过了！**

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✔ 1 / 1 分

1。

For Questions 1-2, consider the following experiment:

Suppose you are trying to calibrate a thermometer by testing the temperature it reads when water begins to boil. Because of natural variation, you take several measurements (experiments) to estimate θ , the mean temperature reading for this thermometer at the boiling point.

You know that at sea level, water should boil at 100 degrees Celsius, so you use a precise prior with $P(\theta = 100) = 1$. You then observe the following five measurements: 94.6 95.4 96.2 94.9 95.9.

- What will the posterior for θ look like?

○ Most posterior probability will be concentrated near the sample mean of 95.4 degrees Celsius.

○ Most posterior probability will be spread between the sample mean of 95.4 degrees Celsius and the prior mean of 100 degrees Celsius.

◉ The posterior will be $\theta = 100$ with probability 1, regardless of the data.

正确

Because all prior probability is on a single point (100 degrees Celsius), the prior completely dominates any data. If we are 100% certain of the outcome before the experiment, we learn nothing by performing it.

Clearly this was a poor choice of prior, especially in light of the data we collected.

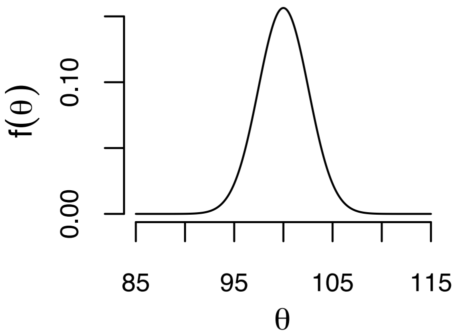
None of the above.

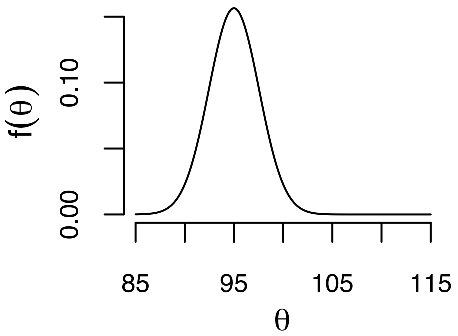
1 / 1 分

2。
Thermometer:

Suppose you believe before the experiments that the thermometer is biased high, so that on average it would read 105 degrees Celsius, and you are 95% confident that the average would be between 100 and 110.

- Which of the following prior PDFs most accurately reflects this prior belief?





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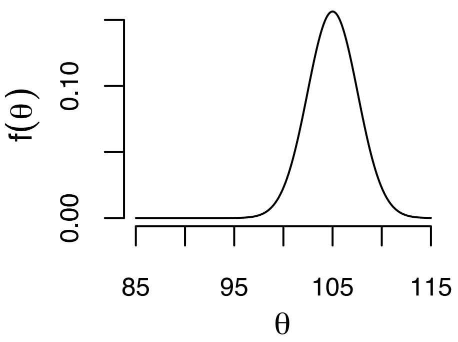
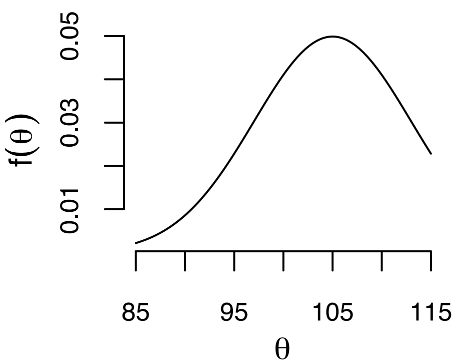


正确

The prior mean is 105 degrees Celsius and approximately 95% of the prior probability is assigned to the interval (100, 110).



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3。

Recall that for positive integer n , the gamma function has the following property: $\Gamma(n) = (n-1)!$.

What is the value of $\Gamma(6)$?

120

正确回答
This is $\Gamma(6) = 5! = 120$.

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1 / 1 分

4。
Find the value of the normalizing constant, c , which will cause the following integral to evaluate to 1.

$\int_0^1 c \cdot z^3 (1-z)^1 dz$.

Hint: Notice that this is proportional to a beta density. We only need to find the values of the parameters α and β and plug those into the usual normalizing constant for a beta density.

- (selected) $\frac{\Gamma(4+2)}{\Gamma(4)\Gamma(2)} = \frac{5!}{3!1!} = 20$

正确
$\alpha = 3 + 1$ and $\beta = 1 + 1$.

- $\frac{\Gamma(1)}{\Gamma(z)\Gamma(1-z)} = \frac{0!}{(z-1)!1!}$
- $\frac{\Gamma(3+1)}{\Gamma(3)\Gamma(1)} = \frac{3!}{2!0!} = 3$

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1 / 1 分

5。

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Consider the coin-flipping example from Lesson 5. The likelihood for each coin flip was Bernoulli with probability of heads θ , or $f(y \mid \theta) = \theta^y (1-\theta)^{1-y}$ for $y = 0$ or $y = 1$, and we used a uniform prior on θ .

Recall that if we had observed $Y_1 = 0$ instead of $Y_1 = 1$, the posterior distribution for θ would have been $f(\theta \mid Y_1 = 0) = 2(1-\theta) I_{\{0 \le \theta \le 1\}}$. Which of the following is the correct expression for the posterior predictive distribution for the next flip $Y_2 \mid Y_1 = 0$?

- [ ] $f(y_2 \mid Y_1 = 0) = \int_0^1 2(1-\theta) d\theta$ for $y_2 = 0$ or $y_2 = 1$.

- [x] $f(y_2 \mid Y_1 = 0) = \int_0^1 \theta^{y_2} (1-\theta)^{1-y_2} 2(1-\theta) d\theta$ for $y_2 = 0$ or $y_2 = 1$.

正确

This is just the integral over likelihood $\times$ posterior. This expression simplifies to

$\int_0^1 2\theta^{y_2}(1-\theta)^{2-y_2} d\theta I_{\{y_2 \in \{0,1\}\}} = \frac{2}{\Gamma(4)} \Gamma(y_2+1)\Gamma(3-y_2) I_{\{y_2 \in \{0,1\}\}}$

$= \frac{2}{3} I_{\{y_2=0\}} + \frac{1}{3} I_{\{y_2=1\}}$

- [ ] $f(y_2 \mid Y_1 = 0) = \int_0^1 2\theta^{y_2} (1-\theta)^{1-y_2} d\theta$ for $y_2 = 0$ or $y_2 = 1$.

- [ ] $f(y_2 \mid Y_1 = 0) = \int_0^1 \theta^{y_2} (1-\theta)^{1-y_2} d\theta$ for $y_2 = 0$ or $y_2 = 1$.

✔ 1 / 1 分

6。

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The prior predictive distribution for X when θ is continuous is given by $\int f(x \mid \theta) \cdot f(\theta) d\theta$. The analogous expression when θ is discrete is $\sum_\theta f(x \mid \theta) \cdot f(\theta)$, adding over all possible values of θ .

Let's return to the example of your brother's loaded coin from Lesson 5. Recall that he has a fair coin where heads comes up on average 50% of the time ($p = 0.5$) and a loaded coin ($p = 0.7$). If we flip the coin five times, the likelihood is binomial: $f(x \mid p) = \binom{5}{x} p^x (1-p)^{5-x}$ where X counts the number of heads.

Suppose you are confident, but not sure that he has brought you the loaded coin, so that your prior is $f(p) = 0.9 I_{\{p=0.7\}} + 0.1 I_{\{p=0.5\}}$. Which of the following expressions gives the prior predictive distribution of X ?

- (selected) $f(x) = \binom{5}{x} .7^x (.3)^{5-x} (.9) + \binom{5}{x} .5^x (.5)^{5-x} (.1)$

  正确

  This is a weighted average of binomials, with weights being your prior probabilities for each scenario (loaded or fair).

- $f(x) = \binom{5}{x} .7^x (.3)^{5-x} (.1) + \binom{5}{x} .5^x (.5)^{5-x} (.9)$
- $f(x) = \binom{5}{x} .7^x (.3)^{5-x} + \binom{5}{x} .5^x (.5)^{5-x}$
- $f(x) = \binom{5}{x} .7^x (.3)^{5-x} (.5) + \binom{5}{x} .5^x (.5)^{5-x} (.5)$