Module 1 Honors

测验, 8 个问题

8/8 分 (100%)

恭喜！您通过了！

下一项

1 / 1 分

1。

Which of the following (possibly more than one) must be true if random variable X is continuous with PDF $f(x)$?

- [ ] $f(x)$ is a continuous function

未选择的是正确的

- [ ] $X >= 0$ always

未选择的是正确的

- [ ] $\lim_{x\to\infty} f(x) = \infty$

未选择的是正确的

- [ ] $f(x)$ is an increasing function of x

未选择的是正确的

- [x] $f(x) \geq 0$ always

正确

- [x] $\int_{-\infty}^{\infty} f(x)dx = 1$

正确

1 / 1 分

2。

If $X \sim \text{Exp}(3)$, what is the value of $P(X > 1/3)$? Round your answer to two decimal places.

0.37

正确回答

This is $P(X > 1/3) = \int_{1/3}^{\infty} 3e^{-3x}dx$

$$= -e^{-3x}|_{1/3}^{\infty}$$

$$= 0 - (-e^{-3/3}) = e^{-1} = 0.368$$

1 / 1 分

3。

Suppose $X \sim \text{Uniform}(0, 2)$ and $Y \sim \text{Uniform}(8, 10)$. What is the value of $E(4X + Y)$?

13

正确回答

This is $E(4X + Y) = 4E(X) + E(Y) = 4(1) + 9$.

1 / 1 分

4。
For Questions 4-7, consider the following:

Suppose $X \sim \mathrm{N}(1, 5^2)$ and $Y \sim \mathrm{N}(-2, 3^2)$ and that X and Y are independent. We have $Z = X + Y \sim \mathrm{N}(\mu, \sigma^2)$ because the sum of normal random variables also follows a normal distribution.

- What is the value of μ ?

-1

正确回答
$\mu = E(Z) = E(X+Y) = E(X) + E(Y) = 1 + (-2)$

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1 / 1 分

5。
Adding normals:

- What is the value of σ^2 ?

Hint: If two random variables are independent, the variance of their sum is the sum of their variances.

34

正确回答
$\sigma^2 = Var(Z) = Var(X+Y) = Var(X) + Var(Y) = 25 + 9$
.

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6。

Adding normals:

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If random variables X and Y are not independent, we still have $E(X+Y)=E(X)+E(Y)$, but now $Var(X+Y)=Var(X)+Var(Y)+2Cov(X,Y)$ where $Cov(X,Y)=E[(X-E[X])(Y-E[Y])]$ is called the covariance between X and Y.

- A convenient formula for calculating variance was given in the supplementary material: $Var(X)=E[(X-E[X])^2]=E[X^2]-(E[X])^2$. Which of the following is an analogous expression for the covariance of X and Y?

Hint: Expand the terms inside the expectation in the definition of $Cov(X,Y)$ and recall that $E(X)$ and $E(Y)$ are just constants.

- ○ $E[X^2]-(E[X])^2+E[Y^2]-(E[Y])^2$
- ◉ $E(XY)-E(X)E(Y)$

正确

$$Cov(X,Y)=E[(X-E[X])(Y-E[Y])]$$
$$=E[XY-XE(Y)-E(X)Y+E(X)E(Y)]$$
$$=E[XY]-E[XE(Y)]-E[E(X)Y]+E[E(X)E(Y)]$$
$$=E[XY]-E(X)E(Y)-E(X)E(Y)+E(X)E(Y)$$

- ○ $(E[X^2]-(E[X])^2)\cdot(E[Y^2]-(E[Y])^2)$
- ○ $E[Y^2]-(E[Y])^2$

✔ 1 / 1 分

7。
Adding normals:

- Consider again $X \sim \mathrm{N}(1, 5^2)$ and $Y \sim \mathrm{N}(-2, 3^2)$, but this time X and Y are *not* independent. Then $Z = X + Y$ is still normally distributed with the same mean found in Question 4. What is the variance of Z if $E(XY) = -5$?

Hint: Use the formulas introduced in Question 6.

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正确回答

$$Var(Z) = Var(X) + Var(Y) + 2Cov(X,Y) = 25 + 9 + 2Cov(X,Y)$$
$$= 34 + 2(E[XY] - E[X]E[Y])$$
$$= 34 + 2(-5 - 1(-2)) = 34 - 2(3)$$

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1 / 1 分

8。
Free point:

1) Use the definition of conditional probability to show that for events A and B , we have $P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$.

2) Show that the two expressions for independence $P(A|B) = P(A)$ and $P(A \cap B) = P(A)P(B)$ are equivalent.

- [x] Solution (1)

Write $P(B|A) = \frac{P(A \cap B)}{P(A)}$ and multiply both sides by $P(A)$

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- [x] Solution (2)

Plug these expressions into those from (1).

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