Module 3 Honors

测验, 4 个问题

4/4 分 (100%)

恭喜！您通过了！

下一项

1 / 1 分

1。

Identify which of the following conditions (possibly more than one) must be true for the sum of n Bernoulli random variables (with success probability p) to follow a binomial distribution.

- [x] each Bernoulli random variable is independent of all others

正确

- [x] p must be the same for each of the Bernoulli random variables

正确

- [ ] the sum must be greater than zero

未选择的是正确的

- [ ] p must be less than .5

未选择的是正确的

- [ ] the sum must exceed n

未选择的是正确的

1 / 1 分

2。

For Questions 2-4, consider the following:

In Lesson 6.3 we found the prior predictive distribution for a Bernoulli trial under a uniform prior on the success probability θ . We now derive the prior predictive distribution when the prior is any conjugate beta distribution.

There are two straightforward ways to to this. The first approach is the same as in the lesson. The marginal distribution of y is $f(y) = \int_0^1 f(y|\theta)f(\theta)d\theta$. Now $f(\theta)$ is a beta PDF, but the same principles apply: we can move constants out of the integral and find a new normalizing constant to make the integral evaluate to 1.

Another approach is to notice that we can write Bayes' theorem as $f(\theta|y) = \frac{f(y|\theta)f(\theta)}{f(y)}$. If we multiply both sides by $f(y)$ and divide both sides by $f(\theta|y)$, then we get $f(y) = \frac{f(y|\theta)f(\theta)}{f(\theta|y)}$ where $f(\theta)$ is the beta prior PDF and $f(\theta|y)$ is the updated beta posterior PDF.

- Both approaches yield the same answer. What is the prior predictive distribution $f(y)$ for this model when the prior for θ is $\mathrm{Beta}(a, b)$?

- ○ $f(y) = \frac{\Gamma(a+b)}{\Gamma(a+b+1)}\theta^y(1-\theta)^{1-y}$ for $y = 0, 1$
- ○ $f(y) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\theta^{a-1}(1-\theta)^{b-1}$ for $y = 0, 1$
- ○ $f(y) = \frac{\Gamma(a+b+1)}{\Gamma(a+y)\Gamma(b+1-y)}\theta^{a+y-1}(1-\theta)^{b+1-y}$ for $y = 0, 1$
- ○ $f(y) = \frac{\Gamma(a+y)}{\Gamma(a)} \cdot \frac{\Gamma(b+1-y)}{\Gamma(b)}$ for $y = 0, 1$
- ● $f(y) = \frac{\Gamma(a+b)}{\Gamma(a+b+1)} \cdot \frac{\Gamma(a+y)}{\Gamma(a)} \cdot \frac{\Gamma(b+1-y)}{\Gamma(b)}$ for $y = 0, 1$

正确

All the terms involving θ cancel out as they should.

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1 / 1 分

3。
Beta-Bernoulli predictive distribution:

Now suppose the prior for θ is $\mathrm{Beta}(2,2)$. What is the prior predictive probability that $y^* = 1$ for a new observation y^* ? Round your answer to one decimal place.

0.5

正确回答

$f(y) = \frac{\Gamma(2+2)}{\Gamma(2+2+1)} \cdot \frac{\Gamma(2+y)}{\Gamma(2)} \cdot \frac{\Gamma(2+1-y)}{\Gamma(2)}$ for $y = 0, 1$, so we have $f(1) = \frac{\Gamma(4)}{\Gamma(5)} \cdot \frac{\Gamma(3)}{\Gamma(2)} \cdot \frac{\Gamma(2)}{\Gamma(2)} = \frac{3!2!1!}{4!1!1!} = \frac{12}{24}$.

1 / 1 分

4。
Beta-Bernoulli predictive distribution:

After specifying our $\mathrm{Beta}(2,2)$ prior for θ , we observe 10 Bernoulli trials, 3 of which are successes.

- What is the posterior predictive probability that $y^* = 1$ for the next (11th) observation y^* ? Round your answer to two decimal places.

0.36

正确回答

We can use the same predictive distribution as before, replacing a and b with the parameters of the posterior $\mathrm{Beta}(5,9)$.

$f(y) = \frac{\Gamma(5+9)}{\Gamma(5+9+1)} \cdot \frac{\Gamma(5+y)}{\Gamma(5)} \cdot \frac{\Gamma(9+1-y)}{\Gamma(9)}$ for $y = 0, 1$, so we have $f(1) = \frac{\Gamma(14)}{\Gamma(15)} \cdot \frac{\Gamma(6)}{\Gamma(5)} \cdot \frac{\Gamma(9)}{\Gamma(9)} = \frac{13!5!8!}{14!4!8!} = \frac{5}{14}$.

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