1. The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one trading day?

   The standard deviation of the percentage price change in time Δt is $\sigma\sqrt{\Delta t}$ where σ is the volatility. Assuming 252 trading days in one year,

   $$\Delta t = 1/252 = 0.004$$

   and

   $$\sigma\sqrt{\Delta t} = 0.3\sqrt{0.004} = 0.019$$

   So the standard deviation of the percentage price change in one trading day is 1.9%.

2. If the volatility of a stock is 18% per annum, estimate the standard deviation of the percentage price change in (a) one day, (b) one week, and (c) one month.

   Similar to the previous question, 1 year = 12 months = 52 weeks = 252 trading days.

   (a) $\sigma\sqrt{\Delta t_1} = 0.18\sqrt{1/252} = 1.13\%$

   (b) $\sigma\sqrt{\Delta t_2} = 0.18\sqrt{1/52} = 2.50\%$

   (c) $\sigma\sqrt{\Delta t_3} = 0.18\sqrt{1/12} = 5.20\%$

3. Suppose that observations on a stock price (in dollars) at the end of each of 15 consecutive weeks are as follows:

   $$30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2$$

   Estimate the stock price volatility. What is the standard error of your estimate?

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prices = c(30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6,
           33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2)
logret = diff(log(prices))      # log returns
sigma_w = sd(logret)            # volatility per week
sigma_w
# 0.02883609

sigma = sigma_w * sqrt(52)      # volatility per annum
sigma
# 0.20794

se = sigma / sqrt(2 * 14)       # Hull P325 Example 15.4
se
# 0.03929697
```

   The stock price volatility is 0.21, and the standard error of the estimate is 0.04.

4. An investor enters into a short forward contract to sell 100,000 British pounds for U.S. dollars at an exchange rate of 1.5000 U.S. dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.4900 and (b) 1.5200?

   Since we enter a short position here,

   (a)
   $$-(1.4900 - 1.5000) \times 100,000 = \$1,000$$
   So the investor will gain $1,000.

   (b)
   $$-(1.5200 - 1.5000) \times 100,000 = -\$2,000$$
   So the investor will lose $2,000.

5. A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound; (b) 51.30 cents per pound?

   Since we enter a short position here,

   (a)
   $$-(0.4820 - 0.5000) \times 50,000 = \$900$$
   So the trader will gain $900.

   (b)
   $$-(0.5130 - 0.5000) \times 50,000 = -\$650$$
   So the trader will lose $650.

6. Suppose the current USD/euro exchange rate is 1.2000 dollar per euro. The six-month forward exchange rate is 1.1950. The six-month USD interest rate is 1% per annum continuously compounded. Estimate the six-month euro interest rate.

   Consider the following two ways to obtain a euro in six month:

   - Invest $Ke^{-r_{US}T}$ dollar in a us bank to have funds available in six month to buy 1 euro, and sign a forward to buy 1 euro for K dollars.
   - Buy now $e^{-r_{EU}T}$ euro and invest it in a European bank to cash 1 euro in six month.

   Based on arbitrage pricing theory (APT), knowing $K = 1.1950$ and $T = 6/12 = 0.5$ we have
   $$1.1950\, e^{-0.01\times 0.5} = 1.2000\, e^{-0.5 r_{EU}}$$

   Then
   $$r_{EU} = 2\left[\ln\left(\frac{1.2000}{1.1950}\right) + 0.01 \times 0.5\right] = 1.835\%$$

   So the six-month euro interest rate is 1.835%.

7. A stock is expected to pay a dividend of \$1 per share in 2 months and in 5 months. The stock price is \$50, and the risk-free rate of interest is 8% per annum with continuous compounding for all maturities. An investor has just taken a short position in a 6-month forward contract on the stock.

   (a) What are the forward price and the initial value of the forward contract?

   (b) Three months later, the price of the stock is \$48 and the risk-free rate of interest is still 8% per annum. What are the forward price and the value of the short position in the forward contract?

   (a) Consider the following two ways to get the stock in six month:

   - Borrow $e^{-rt_1} + e^{-rt_2}$ from a bank and buy the stock now with \$50. Pay back \$1 at t_1 with the first dividend and another \$1 at t_2 with the second dividend.
   - Invest Ke^{-rT} in a bank to cash K at time T, and sign a forward contract to buy the stock at time T with K dollars.

   Based on APT, the cash flows for the above two methods should be exactly the same. Then

$$Ke^{-rT} = S_0 - (e^{-rt_1} + e^{-rt_2})$$

   So we get

$$K = (50 - e^{-0.08\times 2/12} - e^{-0.08\times 5/12})e^{0.08\times 6/12} = \$50.01$$

   By design, the forward price $F_0 = K = \$50.01$ so that the initial value $V_0 = 0$.

   (b) Similar to part (a), we can establish the following equation based on APT

$$F_{t_3}e^{-r(T-t_3)} = S_{t_3} - e^{-r(t_2-t_3)}$$

   Then we have

$$F_{t_3} = (48 - e^{-0.08\times(5/12-3/12)})e^{0.08\times(6/12-3/12)} = \$47.96$$

   Since the value of the forward contract is

$$V_{t_3} = (F_{t_3} - K)e^{-r(T-t_3)} = (47.96 - 50.01)e^{-0.08\times(6/12-3/12)} = -\$2.01$$

   the value of the short position is \$ 2.01.