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HW-3

(F)

Problem 17.10.

Given $S_0 = 250$

$q = 0.04$

$r = 0.06$

$k = 245$

European call $= 10$

$T = 3/12 = 1/4 = 0.25$

From formula of In general put-call parity given an index (17.3) given as

$$S_0 e^{-qT} = C - P + Ke^{-rT}$$

$$250 \times e^{(-0.04)(0.25)} = 10 - P + (245)e^{-(0.06)(0.25)}$$

$$247.5125 = 10 - P + 241.3524$$

$$P = 10 + 241.3524 - 247.5125$$

$P = 3.84$ — 3 month put option

Problem 17.13.

Given forward price equals strike price.

$$f_0 = K \quad \text{—①}$$

put-call parity on futures

$$F_0 e^{-rT} = C - P + Ke^{-rT}$$

From ① $\Rightarrow$ $\cancel{Ke^{-rT}} = C - P + \cancel{Ke^{-rT}}$

$$\boxed{C = P}$$

Hence shown that European call option on currency has same price as corresponding European put option.

Problem 15.22:

The probability that the European call option will be exercised when $S_T > K$

According to black-scholes formula return in risk neutral world

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \sigma^2/2\right)T, \sigma^2 T\right]$$

The probability of $S_T > k$ is similar to $\ln S_T > \ln k$

$$\Rightarrow 1 - N\left[\frac{\ln k - \ln S_0 - (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

$$\Rightarrow N\left[\frac{\ln[S_0/k] + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

$\Rightarrow N(d_2)$ from black-scholes formula for call on a stock.

The value of derivative pays \$100 when stock at time T greater than k is $100\,N(d_2)$.

From risk-neutral evaluation value $\Rightarrow 100\,e^{-rT}N(d_2)$

Problem 14.10:

Stock price follows brownian motion with expected return 'μ' and volatility σ.

$$dS = \mu S\,dt + \sigma S\,dz$$

Given to find if S^n follows a Geometric brownian motion

$\Rightarrow G$ in $(S, t) = S^n$

$$\frac{\partial G}{\partial S} = n \cdot S^{n-1} \qquad \frac{\partial^2 G}{\partial S^2} = n(n-1)S^{n-1} \qquad \frac{\partial G}{\partial t} = 0$$

From Ito lemma

$$dF = \left[\frac{\partial F}{\partial t} + \frac{\partial F}{\partial S}(\mu r S) + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(\sigma S)^2\right]dt + \frac{dF}{dS}(\sigma S)dz$$

$$dG \Rightarrow \left[0 + \cancel{\mu G}^{\,\mu S} + \frac{1}{2}(n)(n-1)\sigma^2 S^2\right]dt + \sigma S\,dz$$

$$\Rightarrow dG \Rightarrow \left[0 + \mu n G + \frac{1}{2}n(n-1)\sigma^2 G\right]dt + \sigma G\,dz$$

expected value is given by $S_0^n(e)^{\left[\mu n + \frac{1}{2}(n)(n-1)\sigma^2\right] \times T}$

Problem 4·14

drift rate = 0·1 per month
Variance rate = 0·16 per month
initial cash position = 2·0

(a) probability distribution after one month $\phi(2.0+0.1, 0.16\times1)$

$\Rightarrow \phi(2.1, 0.16)$

after 6 months $= \phi(2.0+0.6, 0.16\times6)$

$= \phi(2.6, 0.96)$

after 1 year ⇒ 12 months $= \phi(2.0+1.2, 0.16\times12)$

$= \phi(3.2, 1.92)$

(b) After 6 months $\Rightarrow \phi(2.6, 0.96)$ since negative

$N\left[\frac{-2.6}{\sqrt{0.96}}\right] = N[-2.70]$

$\Rightarrow 0.003466974$

$\Rightarrow 0.34\%$ Pnorm(−2.70) from R.

After one year $\Rightarrow \phi(3.2, 1.92)$ is $N\left[\frac{-3.2}{\sqrt{1.96}}\right]$

$\Rightarrow N[-2.30]$

$\Rightarrow 0.01072411 = 1.07\%$ [pnorm(−2.3)]

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(c) At what time probability of -ve cash position will be greatest

$\Rightarrow \phi(2.0+(0.1)x, 0.16 \times x)$

probability of -ve cash option $\Rightarrow N\left[-\frac{2.0+0.1x}{\sqrt{0.16 \times x}}\right]$

To find the maximized value, value should be minimum.

$$\Rightarrow \frac{2.0}{\sqrt{0.16x}} + \frac{0.1x}{\sqrt{0.16x}}$$

$$\Rightarrow \frac{2}{0.4\sqrt{x}} + \frac{0.1x}{0.4\sqrt{x}}$$

$$\Rightarrow 5/\sqrt{x} + 0.25\sqrt{x}$$

To find minimal value, $dy/dx = 0$.

$$\Rightarrow dy/dx = \frac{d}{dx}\left[5 \cdot x^{-1/2} + 0.25\, x^{1/2}\right]$$

$$= -\frac{1}{2} \times 5x^{-3/2} + \frac{1}{2}(0.25)\, x^{-1/2}$$

$$\Rightarrow -2.5x^{-3/2} + 0.125x^{-1/2} = 0.$$

gives the value for x after which the negative cash flow is maximized for $t > x$.