The Static Optimization Toolbox in OpenSim

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Note: This document is based on data culled from multiple sources cited in the OpenSim literature/the OpenSim users forum and my own experience. However, *none of the explanations presented here have been verified by any of the* OpenSim *maestros* so take everything with a grain of salt (and let me know if you find any errors so I won't keep disseminating false information). :)

1 The Objective Function for Static Optimization

The Static Optimization toolbox is used in OpenSim to resolve the net joint forces and moments into individual muscle forces subject to either of the following muscle activation-to-force conditions

$$\sum_{m=1}^{n} a_m F_m^o \rho_{m,j} = \tau_j, \tag{1}$$

or

$$\sum_{m=1}^{n} \left[a_m f(F_m^o, l_m, v_m)\right] \rho_{m,j} = \tau_j, \tag{2}$$

while minimizing the objective function

$$J = \sum_{m=1}^{n} (a_m)^p \tag{3}$$

where a_m, F_m^o, l_m, v_m is the activation level, maximum isometric force, muscle length and muscle velocity of muscle m, $\rho_{m,j}$ is the moment arm of muscle m about joint axis j, τ_j is the joint moment acting about the jth joint axis, and p is some user defined constant. The first of these assumes that the muscle is an "ideal force generator" condition and is non-physiological while the send provides a more physiological representation of the muscle with the force generated constrained by the force-length-velocity properties of the muscles.[1]

Most researchers have focused on objective functions that minimize muscle stress σ_m raised to some power, p. This is most likely because minimizing the function σ_m^p has been shown to be physiologically analogous to minimizing muscle fatigue when $1.4 \leq p \leq 5.1$ [Crowninshield and Brand, 1981]. However, the StaticOptimization toolbox in OpenSim utilizes an objective function that minimizes the muscle activations instead since minimizing $\sum_{m=1}^{n}(a_m)^p$ resulted in predicted muscle activations that were in

[1] Anderson and Pandy [2001] in their gait analysis study, found that the force trajectories were similar for the physiological and non-physiological static optimization muscle activation-to-force conditions. The small impact of the force-length-velocity muscle properties on the static solutions was probably because the force required from each muscle was well within the limits described by the force-length-velocity surface. However, we are uncertain if these findings can be extrapolated to the muscles spanning the lumbar spine.

much better agreement with EMG data for muscles approaching the extremes of their force length curve [Anderson and Pandy, 2001]. Nevertheless, depending on the muscle activation-to-force condition used, the muscle activations can be related directly to the muscle stress. For the non-physiological case (1), the muscle activation is some constant multiplied by the muscle stress

$$a_m = \frac{F_m}{F_m^o} = k\left(\frac{F_m}{PCSA}\right) = k\sigma_m \tag{4}$$

while, for the physiological case,

$$a_m = \frac{F_m}{f(F_m^o, l_m, v_m)} \approx k\left(\frac{F_m}{PCSA}\right) \quad \text{when} \quad l_m \approx l_m^o, v_m \approx 0. \tag{5}$$

Hence, minimizing $\sum_{m=1}^{n}(a_m)^p$ is approximately equivalent to minimizing $\sum_{m=1}^{n}(\sigma_m)^p$.

Different muscles and activities have different p values for the objective function J given in Eqn. 3. A very good literature review of the different objective functions used is given in Table 1 of Erdemir et al. [2007].

2 Deciphering the Output

When running the StaticOptimization toolbox, the messages window outputs values for "Performance" and "Constraint Violation." The constraint violation is the acceleration constraint matching error. That is,

$$\sum_{i=1}^{n}\left(F_i^{tot} - m_i a_i\right) = \text{Constraint violation}, \tag{6}$$

and

$$\sum_{m=1}^{n}(a_m^p) = \text{"Performance"}, \tag{7}$$

where F_i^{tot} is the total force acting on body i and a_i is the acceleration of body i. Large Contraint violation values signify that the model requires additional forces to balance out the large accelerations produced by the motion while small values (on the order of 10^{-10} or less) indicate that all is well with the world and, *this year*, you'll actually have the time to go home for Christmas/Thanksgiving/Hanukkah/President's Day/National Blueberry Day/etc. In general, a smaller "Performance" value is better since this is the function that is to be minimized.

References

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