

## UNIT - 2

### Chapter - 1

### PROBLEM SOLVING

- | Problem Solving - State space Search & Control strategies - introduction - General problem solving, - characteristics of problems - Exhaustive searches - heuristic search techniques - iterative deepening A\* - constraint satisfaction.

#### \* Problem Solving:      Introduction

- It can be done by building an AI system to solve that particular problem.
- To do this one need to define the problem statement first & generating the solution by keeping the condition in mind.
- Some of the most popularly used problem solving with the help of AI are: chess, Travelling salesman problem, Towers of Hanoi problem, water jug problem, N-Queen problem....,

#### \* Problem Searching:

- ↳ in general searching refers to as finding information one needs.
- ↳ searching is the most commonly used technique of problem solving in AI.
- ↳ The process of solving a problem in general consists of
  - ↳ • Define the problem
  - ↳ • Analyzing the problem
  - ↳ • identification of solution
- ↳ • choosing the solution
- ↳ • implementation

#### \* Searching is most commonly used technique in problem solving using AI.

#### \* Properties of Search algorithms:

- ↳ The essential properties of search algorithm to complete the efficiency of these algorithms.
- Completeness - A search algorithm is said to be complete if it guarantees to return a solution if at least any solution exist for random input
- Optimality - If a solution found for an algorithm is guaranteed to the best solution (lowest path cost) among all other solutions, then

such solution for it is said to be optimal solution.

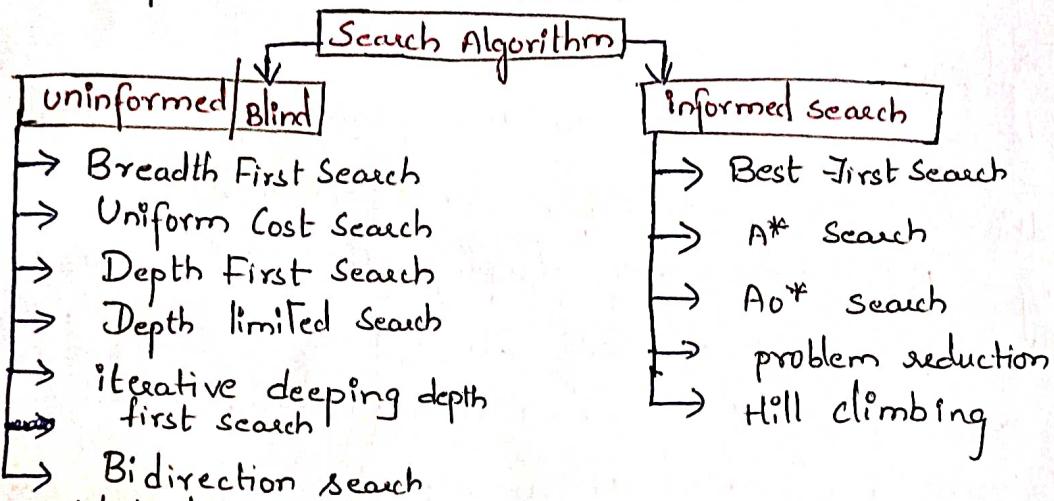
- Time Complexity - It is a measure of time for an algorithm complete its task.
- Space Complexity - It is the maximum storage required at point during the search, as the complexity of the problem.

#### \* Types of Search algorithms:

Based on the search problems we can classify search,

into Uninformed search (Blind search)

Informed search (Heuristic search)



- \* Uninformed / Blind Search: Does not contain any domain knowledge.
  - it operates in brute force way, as it only includes information how to traverse the tree & how to identify leaf & goal nodes.
  - it applies a way in which search tree is searched without for the goal, so it is called blind search. It examines each node until it achieves the goal node.

- \* Informed Search: Uses domain knowledge & problem info is available.
  - Informed search strategies can find a solution more efficiently than uninformed. It is also called heuristic search.
  - A heuristic is a way which might not always be guaranteed the best solution but guaranteed to find a good solution in less time.
  - It can solve much complex problems which could not be solved another way.

## State Space Search:

$S : \{S, A, \text{Action}(s), \text{Result}(s,a), \text{Cost}(s,a)\}$

↓ Set of all possible actions

Total no of states  $\rightarrow S = \text{start, intermediate, Goal}$

- Method of problem representation that facilitates easy search
- With this one can also find a path from start state to goal state while solving a problem.
- State space basically consists of four components
  - Set  $S$  starting state of problem
  - A set  $G_1$  containing goal states of problem
  - Set of nodes (states) in graph/tree. Each node represents the state in problem solving process.
  - Set of arcs connecting nodes. Each arc corresponds to operator that is a step in a problem solving process.

A solution path is a path through the graph from a node in  $S$  to a node in  $G_1$ .

The main objective of search algorithms is to determine a solution path in the graph.

There may be more than one solution paths, as there may be more than one ways of solving the problem.

Commonly used approach is to apply appropriate operator to transfer one state of problem to another.

Example: Eight puzzle problem:

Problem statement: The eight-puzzle problem has a  $3 \times 3$  grid with 8 randomly numbered (1 to 8) tiles arranged on it with one empty cell. At any point, the adjacent tile can move to the empty cell creating a new empty cell. Solving this problem involves arranging tiles such that we get the goal state from the start state.

Start state		
3	7	8
5	1	2
4	□	8

Goal state		
5	3	6
7	□	2
4	1	8

Eight puzzle problem.

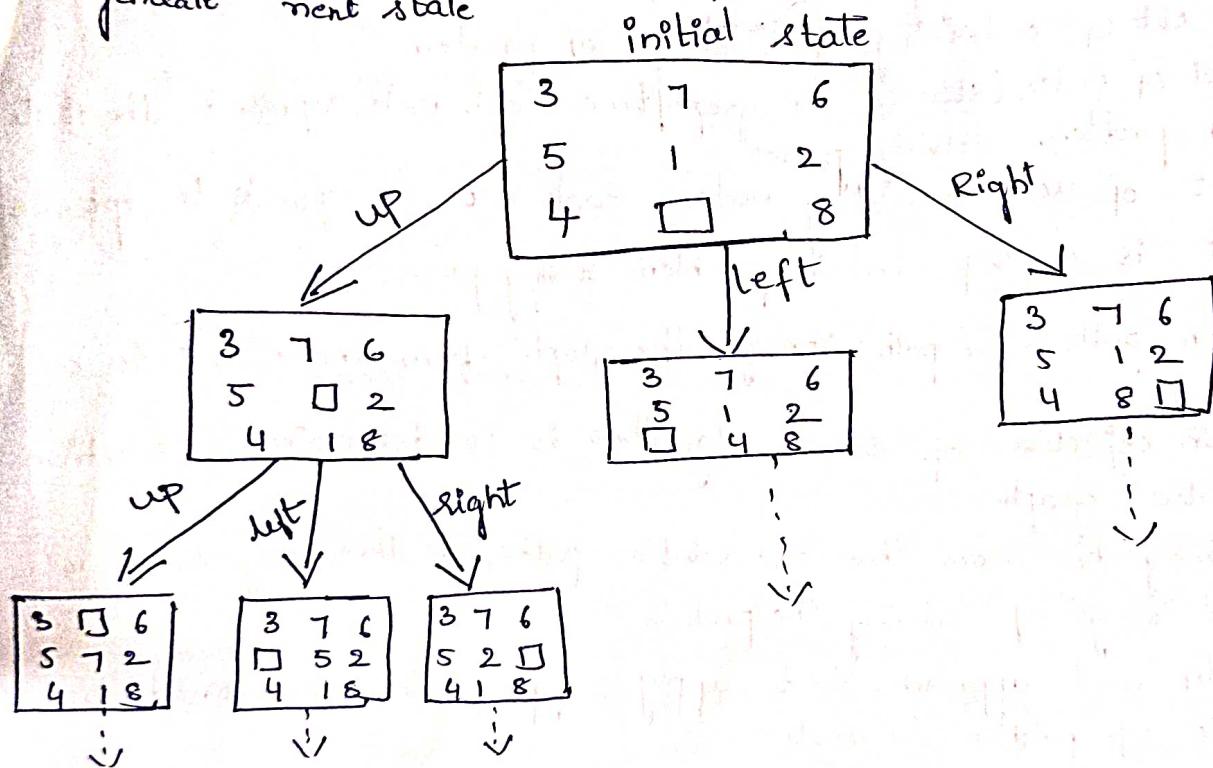
A state for this problem should keep track of the position of a tiles on the game board, with 0 representing the blank (empty) position on the board. The start & goal states may be represented as follows with each list representing corresponding row:

(1). Start state :  $[3, 7, 6]; [5, 1, 2]; [4, 0, 8]$

(2). The goal state could be represented as :  $[5, 3, 6], [7, 0, 2], [4, 1, 0]$

(3). The operators can be thought of moving [Up, Down, Left, Right], the direction in which blank space efficiently moves.

To simplify, a search tree up to level 2, we use operators to generate next state



— Continue the searching like this till we reach the goal!

The exhaustive search can proceed using depth-first search/breadth first search strategies.... Some intelligent searches can be made to find solution faster.

## Control Strategies:

- Control strategy is one of the most important components of problem solving that describes the order of application of the rules to the current state.
- These should cause motion towards solution.
- Better choose best control strategy rather than simple control strategy in the rule list.  
It should explore the solution space in systematic manner.
- To solve real world problems, effective control strategy must be used.
  - \* The problem can be solved by searching for a solution, the main thing is to find correct search strategy for a given problem\*/  
There are two directions in which search should proceed.
    - o Data Driven Search [forward chaining] from start state
    - o Goal driven Search [backward chaining] from goal state

### \* Forward chaining:

- ↳ The process of forward chaining begins with known facts & works towards a conclusion
- ↳ In this case we begin building a tree of move sequences with the root of tree as start state
- ↳ The states of next level of the tree are generated by finding all rules whose left sides match with root & use their right side to create new states  
Eg: Eight puzzle problem
- ↳ This process is continued until a configuration that matches the goal state is generated.
- ↳ Language OPS5 uses forward reasoning rules. Rules are expressed in the form of if-then rules.

### \* Backward chaining:

- ↳ It is a goal directed strategy that begins with goal state & continues working backward, generating more subgoals that must also be satisfied to satisfy main goals until we reach to start state
- ↳ Prolog language uses this strategy.

- He can use both data driven & goal strategies for problem solving, depending on the nature of problem
- If there are large number of explicit goal states & one start state, then it would not be efficient to solve using back chaining strategy because we do not know which goal state is closest to the start state.
- Therefore, the general observations are that move from a set of states to the larger set of states & proceed in the direction with lower branching factors (the average number of nodes that can be reached directly from single nodes).
  - \* So depending upon the given problem the control strategy forward/backward chaining is to be considered \*.

- o Characteristics of Problem:

Before starting modelling the search and trying to find solution for the problem one must analyze it along several key characters initially. Some of them are:

- \* Type of problem - There are two types of problems in real life
  - Ignorable, Recoverable, Irrecoverable.
- o Ignorable: These are problems where one can ignore the solutions. These can be solved using simple control strategy.
- o Recoverable: These are problems where solution steps can be undone. These problems are generally puzzles played by a single player. Such problems can be solved using backtracking using push-downs.
- o Irrecoverable: The problems where solution steps cannot be undone. Two players playing games. Problems solved using planning process. The major characteristics are

→ Decomposability of a problem: Divide the problem into a set of independent small sub problems, solve them & ~~contribute~~ combine the solutions to get final solutions. The process of dividing sub problems continues till we get the set of smallest sub problems for which a small collection of specific rules are used.

Divide & Conquer technique is commonly used method for solving such problems. It is an important character to solve & handle different problems. These problems are generally solved using parallel processing.

→ Role of knowledge: Knowledge plays an important role in solving any problem. Knowledge could be in the form of rules & facts which help in generating search space for finding the solution.

→ Consistency of knowledge Base used in solving problem: Make sure that knowledge base used to solve problem is consistent. Inconsistent knowledge base will lead to wrong solution.

→ Requirement of solution: He should analyze the problem whether solution required is absolute (exact) or relative (reasonable). The path problems are best path & any-path problems.  
(travelling sales man problem) (watching)  
absolute relative  
Best path problems are relatively harder than any path problems.

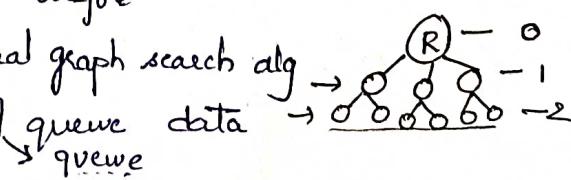
### 2. Exhaustive Searches:

Some of systematic uninformed exhaustive searches are

- \* BFS
- \* DFS
- \* Depth first iterative deepening / deepening
- \* Bidirectional Search

#### \* Breadth First Search:

- It is most common search strategy for traversing a tree or graph.
- This algorithm searches breadthwise in a tree / graph, so it is called BFS
- BFS algorithm starts searching from root node of the tree & expands all successor nodes at the current level before
- BFS alg is an example of general graph search alg
- BFS implemented using FIFO queue data structure.



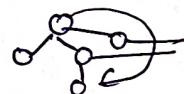
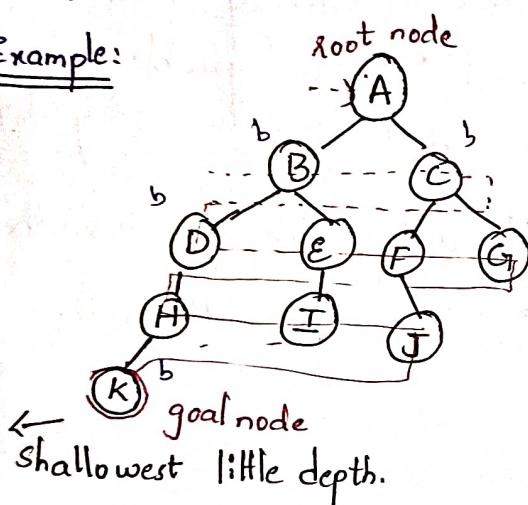
## Advantages:

- BFS will provide a solution if any solution exists
- If there are more than one solution for a given problem, the BFS will provide minimal solution which requires minimum number of steps.

## Disadvantages:

- It requires lots of memory since each level of the tree is saved into memory to expand the next level.
- BFS needs lots of time if the solution is far away from root node.

## Example:



level 0

Time complexity

level 1

$d = \text{depth of shallowest node}$

level 2

$b$  is a node at every level

level 3

$$T(b) = b^0 + b^1 + b^2 + b^3$$

level 4

$$= 1 + 2 + 4 + 8$$

$$T(b) = b^0 + b^1 + b^2 + \dots + b^d$$

$$\boxed{T(b) = O(b^d)}$$

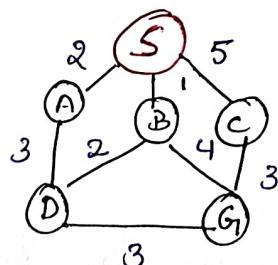
order of

Space Complexity

$$S(b) = O(b^d)$$

• Completeness - BFS is complete (Reached goal node)

• Optimality - optimal (if path cost is non decreasing)



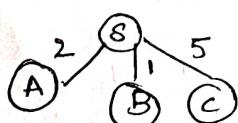
Find the route from S to G using BFS

Step 1:

Visit root node

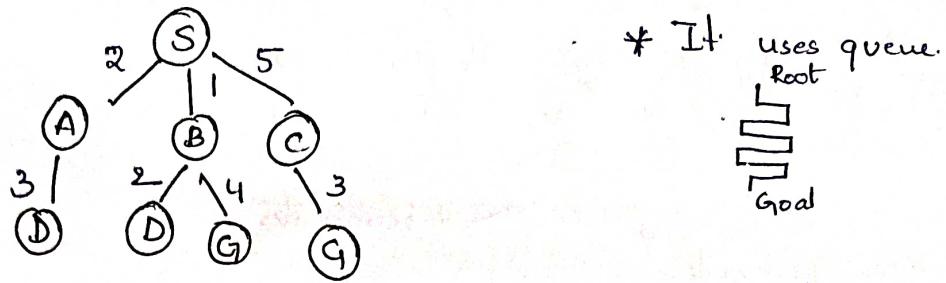
(S)

Step 2: Visit all successor nodes of root node

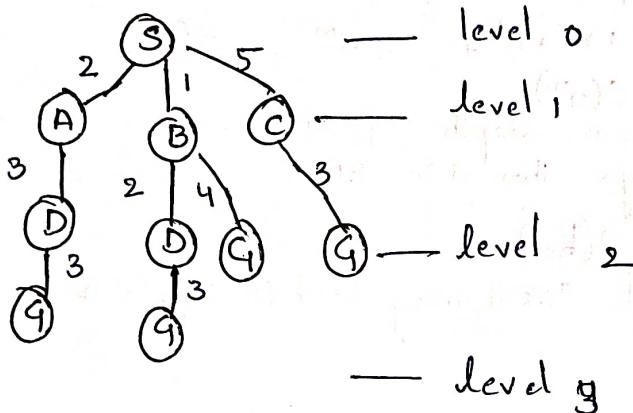


Aim: To find G, start from S

Step 3: Expand the tree with successor nodes



Step 4: Continue the expansion.



Path:

SBG<sub>1</sub>

(or)

SG<sub>1</sub>G<sub>1</sub>

1 + 4  
(5)

5 + 3  
(8)

S to G<sub>1</sub> is reached

shorted path is SBG<sub>1</sub>

\* Depth First Search:

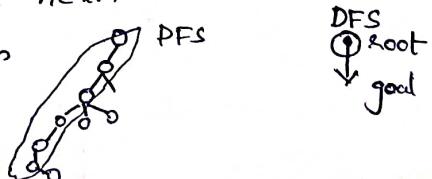
- It is a recursive algorithm for traversing a tree (or) graph DS.
- It is called DFS because it starts from the root & follows. Each path to its greatest depth node before moving to the next.
- DFS uses a stack DS for implementation.
- The process is similar to BFS algorithm.

o Advantages:

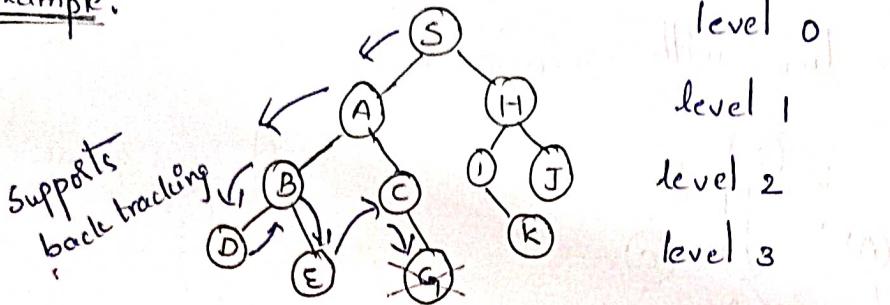
- It requires very less memory as it only needs to store a stack of nodes on the path from root node to the current node.
- It takes less time to reach to the goal node than BFS algorithm.

o Disadvantages:

- There is the possibility that many states keep re-occurring, & there is no guarantee of finding solution.
- DFS algorithm goes for deep down searching & sometimes it may go to the infinite loop.



• Example:



S is source node  
G is goal node

→ it terminates

Completeness - It is complete within finite state space

Time complexity -  $T(n) = 1 + n^r + n^3 + \dots + n^m = O(n^m)$

$$T(n) = O(n^m)$$

$m$  - maximum depth of any node  
much larger than  $d$  in BFS.

Space Complexity -  $S(c) = O(bm)$

Optimal - non optimal (as it may lead to infinite loop)

• Algorithm:

push

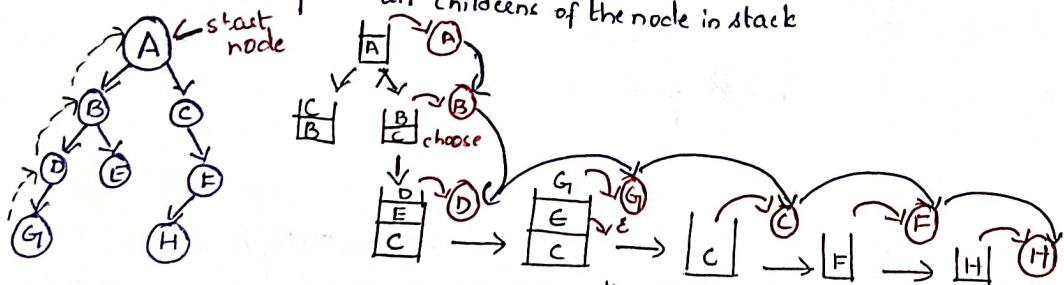
- Enter Root node on stack

- Do until stack is not empty

    └ (a) Remove node

        └ (i) if node is Goal node stop

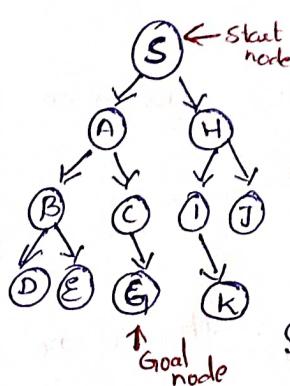
        └ (ii) push all children of the node in stack



$A \rightarrow B \rightarrow D$

Back track

DFS



Space complexity =  $O(bd)$

Time complexity =  $O(b^d)$

Branching factor

$S \rightarrow A \rightarrow B \rightarrow D \downarrow E \downarrow C \downarrow G$

## Depth limited Search: (DLS)

It is similar to DFS with a predetermined limit. It can solve the drawback of infinite path in DFS. In this algorithm, the node at the depth limit will treat as it has no successor nodes together.

Depth limited Search can be terminated with two conditions of failure:

- ↳ standard failure value (indicates that problem does not have any solution).

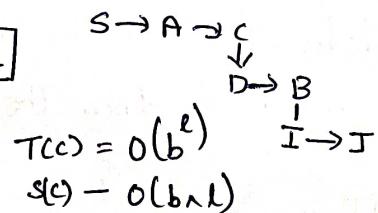
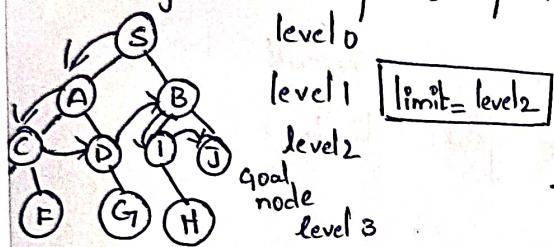
- ↳ cutoff failure value (defines no soln for the problem within a given depth limit)

### Advantages:

- memory efficient

### Disadvantages:

- incompleteness
- it may not be optimal if the problem has more than one solution.



## Depth First Iterative Deepening: [DFID]

Combination of both DFS & BFS

Best depth limit is found out by gradually increasing limit initially  $d = 0$

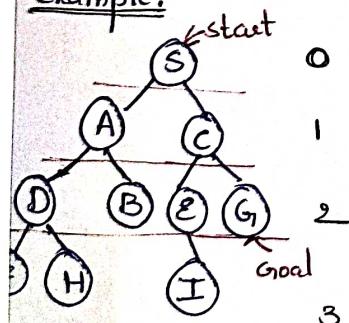
Advantages: for every iteration increase by 1

incorporates benefits of both BFS + DFS

Disadvantages: fast search less memory

Repeat the work / process.

### Example:



1st iteration  $d=0$  [s]

2nd iteration  $d=0+1$  [s → A → C]

3rd iteration  $d=1+1$  [s → A → D → B → C → E → G]

if not  
4th iteration.



Goal node

Space complexity —  $O(d)$

Time complexity —  $O(bd)$  Always less deeper than DFS

## \* Bidirectional Search:

- It runs two simultaneous searches, one from initial state called as search & other from goal node called backward search to find path.
  - It replaces one single search graph with two small subgraphs which one starts the search from initial vertex & other starts from vertex.
  - The search stops when these two graphs intersect each other.
  - Bidirectional search can use search techniques such as BFS, DFS, A\* Search.

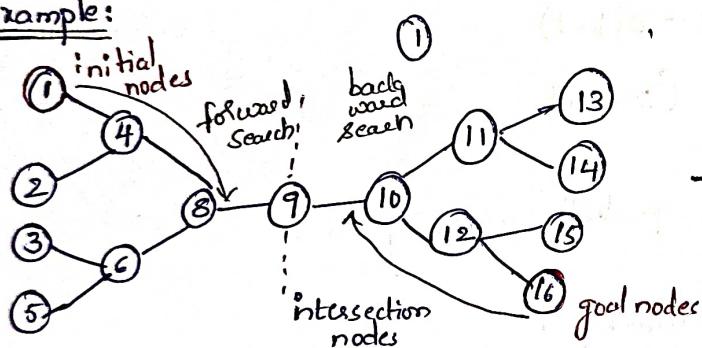
- Advantages:

- Bidirectional search is fast
  - Requires less memory.

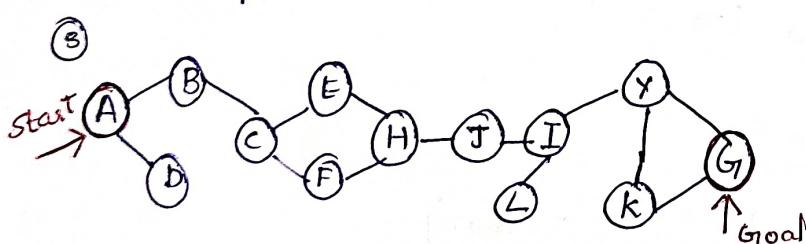
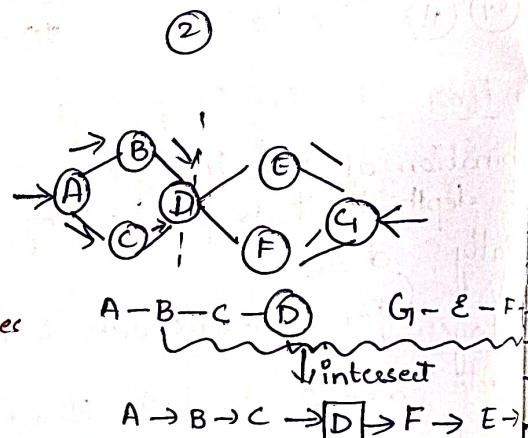
### Disadvantages:

- Disadvantages: implementation of the bidirectional search tree is difficult.
  - in bidirectional search, one should know the goal state in advance.

• Example:



Search [STOP] conditions when [Graph] intersects



~~1 (1)~~ forward

A

ABD

ABDSC

ABDCEF

A B O C E F H

## Backward

G

Gky.

GKYI

GKYILJ

GKYILJH

A → B → D → C → E → F → [H] → J → L → I → Y → K → G

*forward*                                   *backward.*

## Heuristic Search Techniques:

Heuristic technique is a criterion for determining which among several alternatives will be most effective to achieve some goal.

This technique improves the efficiency of a search process possibly by sacrificing claims of systematic & completeness.

It no longer guarantees to find the best solution but almost always finds a very good solution.

Using good heuristics, we can hope to get good solution to hard problems in less than exponential time.

Two types of heuristics

General purpose

Special purpose

General purpose heuristics:

A general purpose heuristics for combinatorial problem is nearest neighbor algorithm that works by selecting the locally superior alternative.

For such algorithm, it is often possible to prove an upper bound on the error.

It provides reassurance that we are not paying too high a price in accuracy for speed.

In many AI problems, it is often difficult to measure precisely the goodness of a particular solution.

For real world problem, it is often useful to introduce heuristic on the basis of relatively unstructured knowledge.

- In AI approaches, behavior of algorithms is analyzed by running them on computer as contrast to analysing algorithm mathematically.

- There are at least many reasons for such adhoc approaches in AI

\* It is so fun to see a program do something intelligent than to prove it.

\* Since AI problem domains are usually complex. It is generally not possible to produce analytical proof that a procedure will work.

\* It is not even possible to describe the range of problems well enough to make statistical analysis of program behaviour meaningful.

- It is important to keep performance in mind while designing algorithm.

- One of the most important analysis of the search process is to

find no. of nodes in a complete search tree of depth 'd'.  
ching factor 'f': that is  $f \times d$ .

- This simple analysis motivates to look for improvement in the exhaustive search procedure & to find an upper bound on search time which can be compared with exhaustive search.
- The searches will use some domain knowledge are called Informed Search Strategies.

#### \* Branch & Bound Search (Uniform Cost Search):

[It is a way to combine space saving of depth first search with information.

- ↳ Optimal solution is chosen among many solution paths.
- ↳ Idea in Branch & Bound search is to maintain lowest path to goal found so far, and its cost.
- ↳ Typically used with Depth first Search
- ↳ Two parts
  - Branch : several choices are found
  - Bound : setting bound on solution quality
- ↳ Pruning: Trimming off branches where solution is poor.]
- Used for Weighted tree/graph Traversal
- Goal is to path finding to goal node with lowest cumulative cost.
- Node expansion is based on path costs.
- Priority queue is used for implementation.
- Used to find optimal path
- Priority queue — high priority to minimum cost
- Supports back tracking.

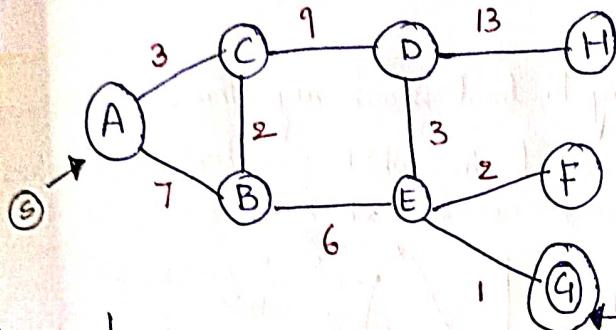
- o Advantages:

Optimal solution

- o Disadvantages:

↳ Struck in infinite loop

Example:



— cost function denoted by  
 $g(x)$

Start

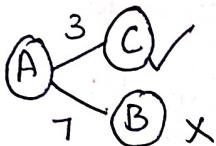
- Generate a possible solution
  - Test if it is a goal
  - If not go to start else quit
- end

Goal node

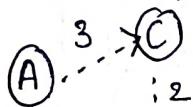
He should choose path with lowest cumulative cost i.e path  
Optimal path

& Based on priority queue, high priority should be given to  
path with minimum cost

if



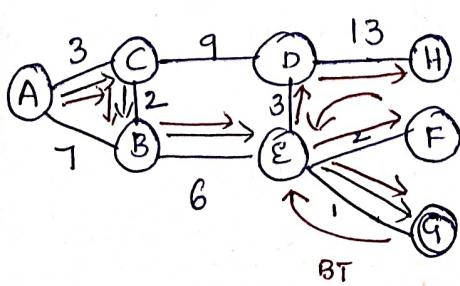
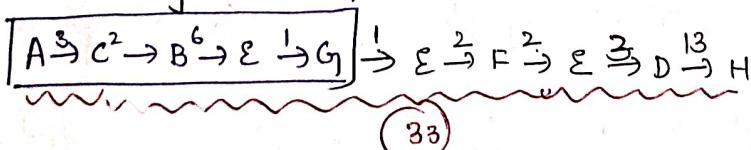
Given  $G_1$  is goal node ; the path is



path to goal node  
 $A \xrightarrow{3} C \xrightarrow{2} B \xrightarrow{6} E \xrightarrow{1} G$

But you should also visit  $H$  &  $F$

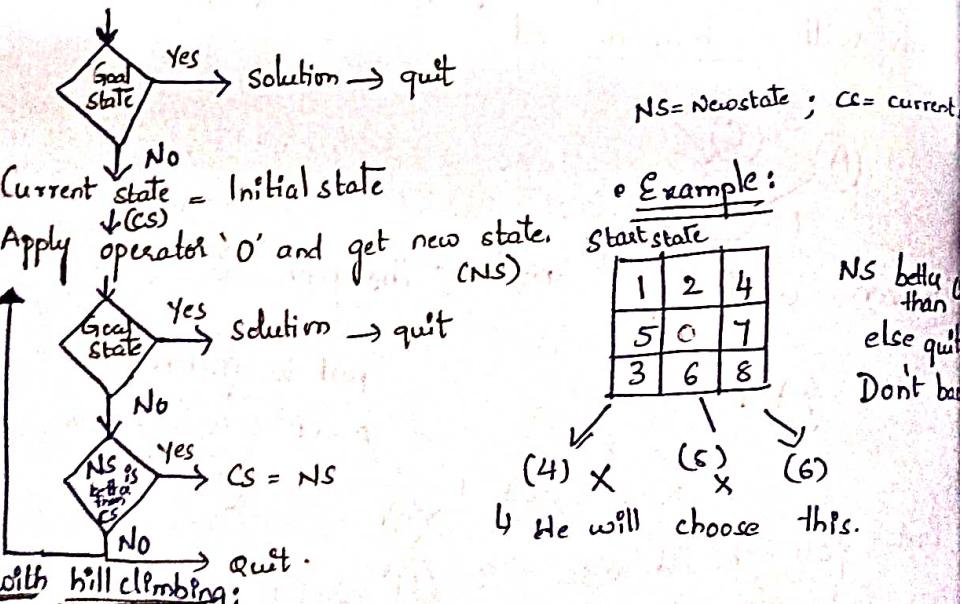
You should not go to first as we already started &  
the traversal is at node  $G_1$  you should move from  $G$  to  $H$  by  
covering  $F$  in between as the search is uniform cost search. He also uses  
back tracking. So path is



## \* Hill Climbing:

- Quality measurement turns DFS into Hill climbing (variant of greedy strategy).
- Optimization technique belonging to local search algorithm.
- Variant of generate & test method in which feedback from test space is used to help generator decide which direction to move.
- Follows local search algorithm
- Greedy approach
- No backtracking

### o Flowchart :

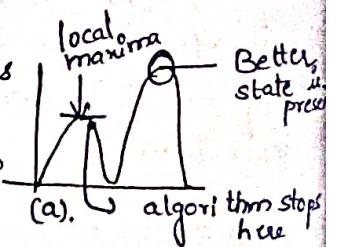


### o Limitations with hill climbing:

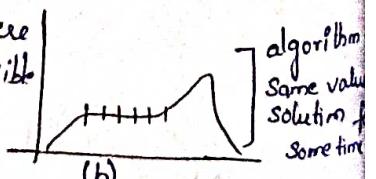
The search process may reach to a position that a solution but from there no move improves the situation. This happens if we have reached local maximum, a plateau, or a ridge.

(a) local maxima: It is a state better than all its neighbours but not better than some other which are far away.

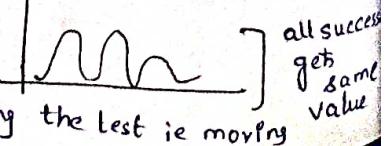
From this state all moves looks to be worse. In such situation, backtrack to some earlier state & try going in different direction to find a solution.



(b). Plateau: It is a flat area of search space where all neighbours states has the same value. It is not possible to determine the best direction. In such situation make a big jump to some directions & try to get to new section of search space.



(c). Ridge: It is an area of search space that is higher than surrounding areas but that cannot be traversed by single moves in any one direction. It is a special kind of local maxima. Thus apply two more rules before doing the last ie moving in several directions at once.



## Beam Search:

Optimized Version of Best First Search

Heuristic search algorithm

Explores a graph by expanding the most promising node in a limited set.

Reduces memory requirement

Greedy algorithm.

Only predetermined no of best partial solutions are kept as candidates

Beam width ( $B$ )

Given Beam Value ( $B$ ) =

Predetermined  
no of best partial soln  
are kept as candidates

straight line distance

$$A \rightarrow G_1 = 40$$

$$B \rightarrow G_1 = 32$$

$$C \rightarrow G_1 = 25$$

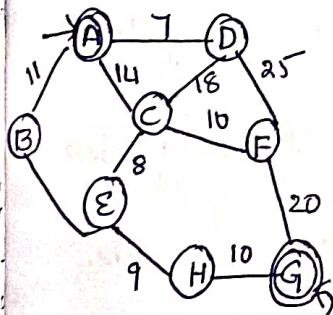
$$D \rightarrow G_1 = 35$$

$$E \rightarrow G_1 = 19$$

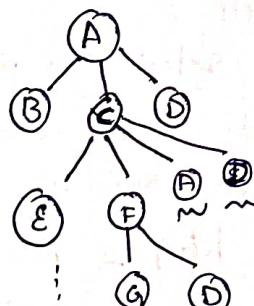
$$F \rightarrow G_1 = 17$$

$$H \rightarrow G_1 = 10$$

$$G_1 \rightarrow G_1 = 0$$



Best first search



uses sorting priority  
~~A & BD~~

$$C \text{ to } G_1 = 25$$

choose C

C children  
~~E, F, A, D~~

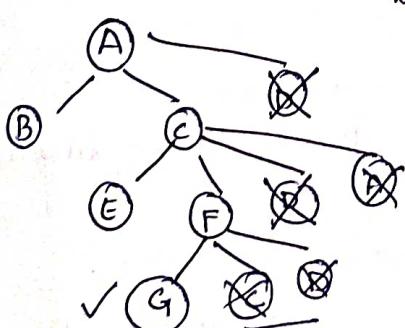
already visited  
~~E~~  $E \rightarrow G_1 = 17$   
choose F

optional.

## Beam Search

Predetermined value of candidate are considered.

Consider  $(B=2)$



$$B \rightarrow G_1 = 32 ; D \rightarrow G_1 = 35, C \rightarrow G_1 = 25$$

in B & C, E is better.

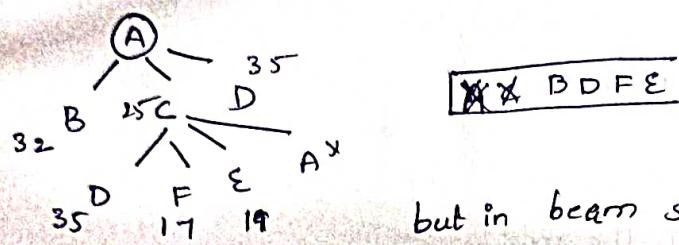
D, A already visited  $\Rightarrow$  optional

$$E \rightarrow G_1 = 19$$

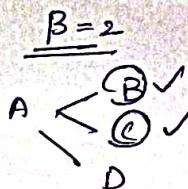
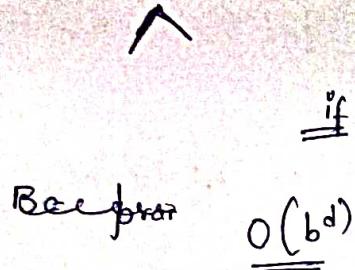
$$F \rightarrow G_1 = 17 \checkmark$$

C, D already visited

It is similar to Best first search but we search only using beam value



but in beam search



as  $B=2$   
you can consider best 2 states.  
With this you can minimize

Only concentrates on space complexity.

### \* Best First Search:

- It is expanding the best partial path from current node to goal node.
- Greedy Search
- It always selects the path which appears best at that moment.
- Combination of both BFS & DFS.
- It uses the heuristic function  $h(n) \leq h^*(n)$  & searches
  - $h(n)$  = heuristic cost
  - $h^*(n)$  = estimated cost
- The greedy best first algorithm is implemented by priority queue.

### ◦ Algorithm steps: (in general)

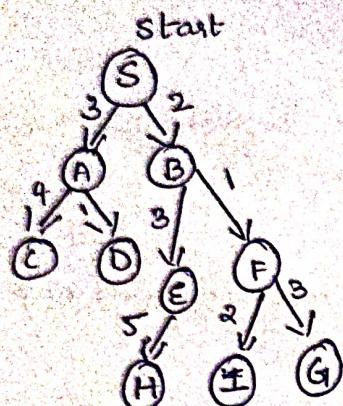
1. Place starting node into open list.
2. if the open list is empty, stop & return failure.
3. Remove the node  $n$ , from open list which has lowest value of  $h(n)$ , & put it in the closed list.
4. Expand node  $n$ , & generate the successors of node  $n$ .
5. check each successor of node  $n$  & find whether the node is goal node or not. If any successor node is goal node, then return success & terminate the search, else return to further step.
6. for each successor node, algorithm checks for evaluation function & then check if the node has not been in both list, then add it to list
7. Return to step 2.



Space Complexity  $O(b^d)$

b is branching factor  
d is depth.

• Example 2



node(n)	H(n)	open	closed
A	12	[B, A]	[S]
B	4		
C	7	[F, E, A]	[S, B]
D	3		
E	8		
F	2		
H	4		
I	9		
S	13		
G	0	[G, I, E, A]	[S, B, F]

P

path =  $S \rightarrow B \rightarrow F \rightarrow G$

$\frac{2+1+3}{6}$

\* A\* Algorithm: (A score algorithm)

- Uses heuristic function  $h(n)$  & cost to reach the node 'n' from state  $g(n)$
- Finds shortest path through search space.
- Fast & optimal result.

$$f(n) = g(n) + h(n)$$

node

f scores are generated from graph

→ Brand & Best Bound Search

- has dynamic programming principles
- uses queue

- Algorithm:
  - (i) Enter starting node in OPEN list.
  - (ii) If OPEN list is empty return FAIL
  - (iii) Select node from OPEN list which has smallest value of  $f(n)$ .
  - ↳ if node = Goal, return success
  - (iv) Expand node 'n' & generate all successors.
  - ↳ compute  $g+n+h$  for each successor node
  - (v) if node 'n' is already OPEN/CLOSED, attach to backpointer
  - (vi) Goto (iii).

• Advantages:

- ↳ Best searching algorithm
- ↳ Optimal & complete
- ↳ Solves complex problems

• Disadvantages:

- ↳ Doesn't always produce shortest path
- ↳ Some complexity issues
- ↳ Requires more memory.

## Optimal Solution by A\* algorithm:

A\* algorithm finds optimal solution if heuristic function is carefully designed & is underestimated.

### Underestimation:

If you can guarantee that  $h$  never overestimates actual value from current to goal, then A\* algorithm ensures to find an optimal path to a goal if one exists.

By underestimating heuristic value, we have ~~to waste~~ some effort but eventually we can discover goal node.

### Overestimation:

If we are overestimating heuristic value of each node in a graph/tree, then it is not guaranteed to find the shortest path.

### Admissibility of A\*:

A search algorithm is admissible, if for any graph, it always terminates in an optimal path from start state to goal state if path exists. A\* algorithm always terminates with the optimal path in case  $h$  is an admissible heuristic function.

### Monotonic Function:

A heuristic function  $h$  is monotonic if (neither increasing nor decreasing)

- i) It states  $x_i$  &  $x_j$  such that  $x_j$  is successor of  $x_i$ .
- ii)  $h(x_i) - h(x_j) \leq \text{cost}(x_i, x_j)$  i.e. actual cost is going from  $x_i$  to  $x_j$ .
- iii)  $h(\text{goal}) = 0$

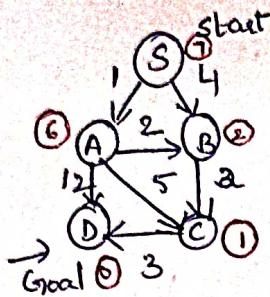
Hence heuristic is locally admissible, i.e. consistently finds the minimal path to each state they encounter in search.

Monotonic property is search space which is everywhere locally present consistent with heuristic function employed i.e. reaching each state with shortest path from its ancestors.

A cost function  $f$  is monotone if  $f(N) \leq f(\text{succ}(N))$

for any admissible cost function  $f$ , we can construct a monotonic admissible function

• Example :



$$f(n) = g(n) + \frac{h(n)}{\text{child}}$$

$$S \rightarrow A = 1 + 6 = 7$$

$$S \rightarrow B = \frac{4 + 2}{\text{child}} = 6$$

$$S \rightarrow B \rightarrow C = \frac{4 + 2 + 1}{\text{child}} = 7$$

$$S \rightarrow B = 4 \quad \text{child}$$

$$B \rightarrow C = 2 \quad h = 1$$

$$\begin{aligned} S \rightarrow B \rightarrow C \rightarrow D &= 4 + 2 + 3 + 0 = 9 \\ &\quad (S \rightarrow B) \quad (B \rightarrow C) \quad (C \rightarrow D) \quad (D \text{ child}) \\ &\quad \hline (g) \quad h=0 \end{aligned}$$

$$S \rightarrow B \rightarrow C \rightarrow D = 9$$

Route 1

To check optimality let us check another path ie 'A'

$$S \rightarrow A \rightarrow B = \frac{1+2}{g} + \frac{2}{h} = 5$$

$$S \rightarrow A \rightarrow C = \frac{1+5}{g} + \frac{1}{h} = 7$$

$$S \rightarrow A \rightarrow D = \frac{1+12}{g+h} = 13 \quad \therefore S \rightarrow A \rightarrow D = 13$$

Route 2

$$S \rightarrow A \rightarrow B \rightarrow C = \frac{1+2+2}{g} + \frac{1}{h} = 6$$

$$S \rightarrow A \rightarrow B \rightarrow C \rightarrow D = \frac{1+2+2+3}{g} + \frac{0}{h} = 8$$

$$\therefore S \rightarrow A \rightarrow B \rightarrow C \rightarrow D = 8$$

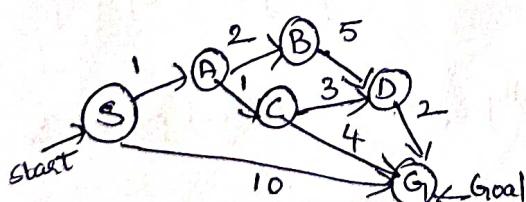
Route 3

$$S \rightarrow A \rightarrow C \rightarrow D = \frac{1+5+3}{g} + \frac{0}{h} = 9 \quad \therefore S \rightarrow A \rightarrow C \rightarrow D = 9$$

Route 4

So the shortest route is

$$R_3 = S \rightarrow A \rightarrow B \rightarrow C \rightarrow D = 8$$



State	$h(n)$
S	5
A	3
B	4
C	2
D	6
G	0

is optimal cost.

$$R_2 = S \rightarrow A \rightarrow C \rightarrow G$$

$$= 6$$

$$S \rightarrow A = 1 + 3 = 4$$

$$S \rightarrow G = 10 + 0 = 10$$

$$S \rightarrow G = 10 \quad R_1$$

$$S \rightarrow A \rightarrow B = 1 + 2 + 4 = 7$$

$$S \rightarrow A \rightarrow C = 1 + 1 + 2 = 4$$

$$S \rightarrow A \rightarrow C \rightarrow G = 1 + 1 + 3 + 6 = 11$$

$$S \rightarrow A \rightarrow C \rightarrow G = 1 + 1 + 4 + 0 = 6 \quad R_2$$

$$\begin{aligned} S \rightarrow A \rightarrow B \rightarrow D &= 1 + 2 + 5 + 6 = 14 \\ S \rightarrow A \rightarrow B \rightarrow D \rightarrow G &= 1 + 1 + 3 + 2 + 0 = 7 \\ S \rightarrow A \rightarrow B \rightarrow D \rightarrow G &= 7 \quad R_3 \end{aligned}$$

$$S \rightarrow A \rightarrow B \rightarrow D \rightarrow G = 10 \quad R_4$$

## Iterative Deepening A\*:

It is a combination of depth first iterative deepening & A\* algorithm (IDA\*).

Successive iterations are corresponding to increasing values of the total cost of a path rather than increasing depth of a search.

### Algorithm:

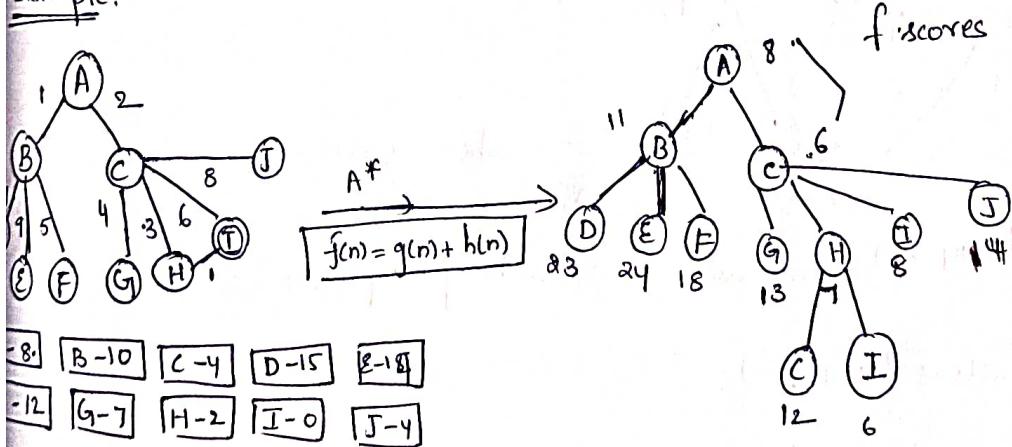
For each iteration, perform a DFS pruning off a branch when its total cost ( $g+h$ ) exceeds a given threshold.

The initial threshold starts at the estimate cost of start state & increases for each iteration of algorithm.

The threshold used for the next iteration is the minimum cost of all values exceeded the current threshold.

These steps are repeated until we find a goal node.

### Example:



Step-1: Consider least value as threshold

Threshold = 6

$$\rightarrow \textcircled{A}^8 \quad (8 > 6)$$

f-value (8)

Step-2:

Threshold = 7

$$\rightarrow \textcircled{A}^8 \quad (8 > 7)$$

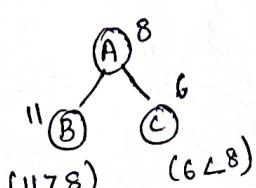
f-value (8)

Step-3:

Threshold = 8

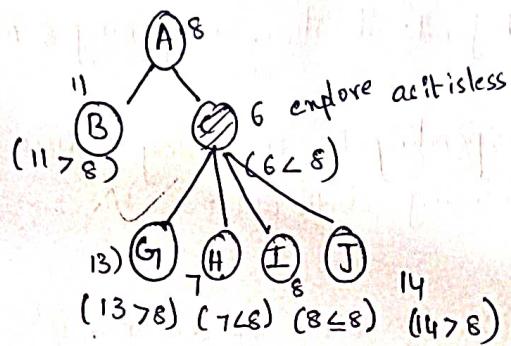
$$\textcircled{A}^8 \quad (8 = 8)$$

then expand

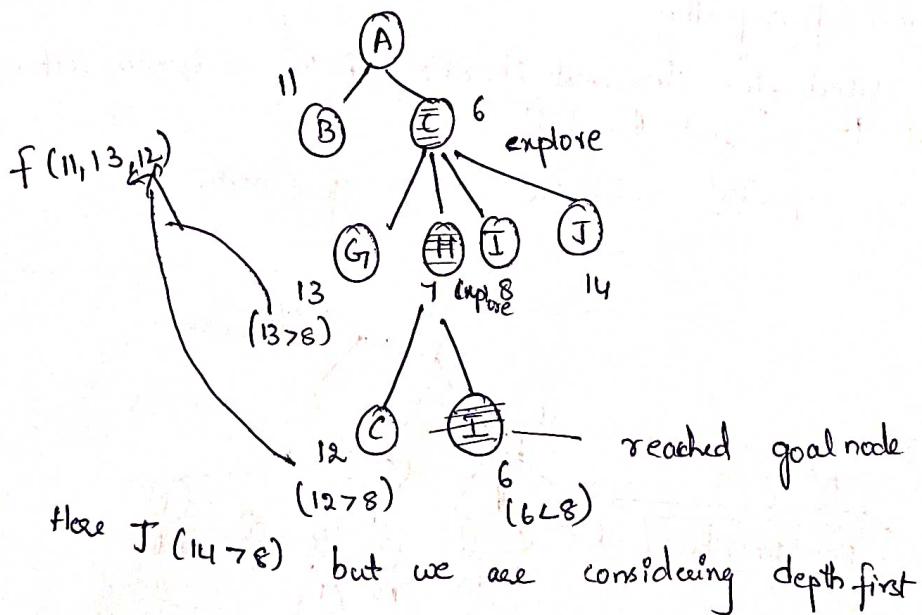


f-value (11)

Value of B | nodes of B



f values  $(11, 13, \dots)$ , Now, explore depth wise.



So f values  $(11, 13, 12)$

Here we need not to store all values as in A\* to reach goal node but values of threshold is calculated ftree  $(11, 13, 12)$  is required to reach path  $A \rightarrow C \rightarrow H \rightarrow I$

#### Advantages:

- Optimal path.
- Both advantages of DFID & A\*

#### Disadvantages:

- Suffers from too little memory.

## Constraint Satisfaction:

Many AI problems can be viewed as problems of constraint satisfaction, which the goal is to solve<sup>some</sup> problem state that satisfies a given set of constraints instead of finding optimal path to solution. Such problems are called constraint satisfaction (CS) problems.

Search can be made easier in those cases in which the solution is required to satisfy local consistency conditions. Some examples are

A cryptography problem: A number puzzle problem in which a group of arithmetical operations has some/all of its digits replaced by letters & the original digits must be found. In such a puzzle each letter represents unique digit.

A N-Queen problem: The condition is that on the same row/column/diagonal, no two 'b' queens attack each other.

A map colouring problem: Given a map, color regions of map using three colors, blue, red, & black such that no two neighbouring countries have the same colour.

In general we can define constraint satisfaction problem as follows:

- o set of variables  $\{x_1, x_2, x_3 \dots, x_n\}$  with each  $x_i \in D_i$  with possible values and
- o set of constraints, i.e. relations, that are assumed to hold between the values of variables.

The problem is to find, for each  $i, 1 \leq i \leq n$ , a value of  $x_i \in D_i$ , so that all constraints are satisfied. A CS problem is usually represented as an undirected graph, called constraint Graph in which the nodes are the variables & the edges are the binary constraints. We can easily see that a CSP can be given an incremental formulation as standard search problem.

Start state: the empty assignment, i.e. all variables are unassigned.

Goal state: all the variables are assigned values while satisfy constraints.

Operator: assigns values to any unassigned variable, provided that it does not conflict with previously assigned variable.

Every solution must be a complete assignments & therefore appears as

depth  $n$  if there are variables. Further search tree extends only to depth  $n$  & hence depth-first search algorithms are most popular for CSPs.

- Algorithm:

- Start

- until a complete solution is found (or) all paths have lead to dead ends

- {
- select an unexpanded node of the search graph;
- apply the constraint interface rules to the selected node to generate all possible new constraints.
- if the set of constraints contain a contradiction, then report that this path is dead end.
- if the set of constraints describes a complete solution, then report success.
- if neither a contradiction nor a complete solution has been found, then apply the problem space rules to generate new partial solutions that are consistent with the current set of constraints. Insert these partial solutions into the search graphs.
- }

- Stop

Example:

Crypt Arithmetic puzzle:

