UNIT-III

Formal Grammar:

* Introduction
* classification of formal Grammar
    1. chomsky Hirearchy.
    2. Types.

* Introduction :—

Mathematically A formal Grammar is a tuple like

$G = (V, T, P, S)$ where,

V = finite and non empty set of non-terminal symbols (or) variables.

variables are represented by upper case letters

T = finite and non empty set of Terminal symbols represented by lower case letters and some special symbols are there.

P = It is a set of production rules are of the form

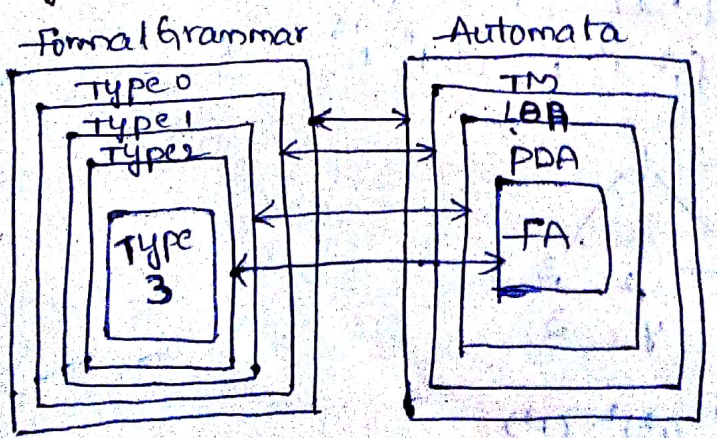
$$P \rightarrow \alpha \rightarrow \beta$$
$$\alpha \in V$$
$$\beta \in (V \cup T)^*$$

S → It is the starting symbol of the Grammar is always, a variable which is $S \in V$.

NOTE:— Grammar's are used to describe a language

* classification of Grammar :—

- using chomsky hierarchy.



Type 3 Grammar :—

* It is also called as Regular grammar.
* Type 3 Grammar is defined as $G = (V, T, P, S)$ where,

V → set of variables.
T → set of Terminals
P → set of production rules are of the form

$A \to Ba$
$A \to a$

— According to left linear grammar

(or)

$A \to aB$
$A \to a$

— According to right linear grammar.

Ex :— $A \to aB$
$A \to Ba$
$A \to a$
$A \to \epsilon$

where.

$(A, B) \in V$
$a \in T^*$

* Type 3 Grammar is used to generating Regular language
* Regular languages are recognised (or) accepted by finite automata. i.e; NFA (or) DFA.

Type 2 Grammar :—

* It is also called as Context-free grammar.
* Type 2 context-free grammar is defined as $G = (V, T, P, S)$

where V → finite set of variables
T → finite set of Terminals
P → finite set of production rules are of the form

$\alpha \to \beta$

where $\alpha \in V$
$\beta \in (V \cup T)^*$

Ex :— $S \to aSa$
$S \to bSb$
$S \to ab$
$S \to \epsilon$

* Context-free grammars are used to generate context-free language.

* context-free language recognised (or) Accepted by pushdown Automata.

Type 1 Grammar :—

* It is also called as context - Sensitive Grammar.
* A CSG is defined as $G=(V,T,P,S)$ where

V = finite set of variables

T = finite set of Terminals.

P = set of production rules are of the form $\alpha \rightarrow \beta$

where, $\alpha \in (VUT)^+$

$\beta \in (VUT)^*$

length of $|\alpha| \leq$ length of $|\beta|$

Ex:- $S \rightarrow aBb$

$bB \rightarrow aa$

$\beta \rightarrow b$

* CSG is used to generating Context - Sensitive language
* CSL recognised (or) Accepted by Linear Bounded Automata

Type 0 Grammar :—

* It is also called also Recursive - Grammar (or) Recursive Enumerable grammar. (or) phrase structured Grammar.
* mathematically Recursive grammar is defined as $G=(V,T,P,S)$ where

$V \rightarrow$ finite set of variables

$T \rightarrow$ finite set of Terminals

$P \rightarrow$ set of production rules are of the form.

$\alpha \rightarrow \beta$

$\alpha \in (VUT)^{*}{}^{+}$

$\beta \in (VUT)^*$

$|\alpha| \geq |\beta|$

Ex:- $S \rightarrow aAbB$

$aAbB \rightarrow aB$

$abB \rightarrow aA$

$A \rightarrow \epsilon$

* Recursive Grammars are used to generating recursive language (or) Recursive-enumerable language (or) phrase structured language.

* Recursive languages are recognised are accepted by Turing machine.

Relationship b|w formal grammar and automata :—

1. Type 3 ⊆ Type 2 ⊆ Type 1 ⊆ Type 0
2. FA ⊆ PDA ⊆ LBA ⊆ TM

Context - Free Grammar :

* Introduction
* Design of CFL
* closure properties of CFL

*Introduction) :—

Context-free Grammar is a Grammar which is defined by four tuples like $G=(V, T, P, S)$ where,

V – It is finite and non-empty set of non-terminal symbols (or) variables.

T – finite and non-Empty set of Terminal symbols.

P – finite and non-Empty set of production rules are of the form $\alpha \rightarrow \beta$

$\alpha \in V$

$\beta \in (V \cup T)^*$

Ex:- $S \rightarrow asa$

$S \rightarrow bSb$

$S \rightarrow aa|bb$

$S \rightarrow \epsilon$

S → It is starting symbol.

Context free -language :—

Let $G=(V, T, P, S)$ be a Context-free grammar. The CFG generating a language 'L' is called Context-free langua

*It is denoted by L(G).

*Context-free languages are organized by PDA.

Design of CFL :—

1) Construct a CFL for the following set $\{\epsilon, a, aa, aaa, \cdots a^n\}$

Sol:- Given set $\{\epsilon, a, aa, aaa, aaaa, \cdots\cdots a^n\}$

minimum string = ϵ

Next minimum string = a

Maximum string = a^n

$S \rightarrow a^n$

$\Downarrow$

$a \cdot a^{n-1} \Rightarrow S \rightarrow aS$

$\Downarrow$

$a \cdot a \cdot a^{n-2}$ $\quad S \rightarrow \epsilon$

$\Downarrow$ $\quad S \rightarrow a$

$a \cdot a \cdot a \cdot a^{n-3}$

CFG:- $\quad S$

$S \rightarrow aS$

$S \rightarrow \epsilon$

$S \rightarrow a$

$L = \{a^n \mid n \geq 0\}$ //.

2) Construct a CFL for the following set $\{\epsilon, ab, aabb, \cdots\}$

Sol:- Minimum String = ϵ

Next minimum string = ab

Maximum string = $a^n b^n$

$S \rightarrow a^n b^n$

$S \rightarrow a a^{n-1} \cdot b^{n-1} b$

$S \rightarrow aa a^{n-2} b^{n-2} bb$

$\therefore S \rightarrow aSb$

$S \rightarrow \epsilon$

$S \rightarrow ab$

$\therefore$ CFG :- $S \rightarrow aSb$

$S \rightarrow \epsilon$

$S \rightarrow ab$

$\therefore L = \{a^n b^n \mid n \geq 0\}$

3) Construct a CFL for the following set {a, b, ab, aabb, aaabbb, ...}

Sol:- Minimum string = ab

Maximum string = $a^n b^n$

$S \rightarrow a^n b^n$

$\rightarrow a a^{n-1} b^{n-1} b$ $\Rightarrow$ $S \rightarrow aSb$

$\rightarrow aaa^{n-2} b^{n-2} bb$ $S \rightarrow ab$

$S \rightarrow b$.

$\therefore$ CFG $S \rightarrow asb$

$S \rightarrow a$

$S \rightarrow b$

$\therefore L = \{ a^n b^n \mid n \geq 1 \}$

4) Construct a CFG to generate the language $L = \{a^n b^{2n} / n \geq 1\}$

Sol:- Minimum string = abb

Maximum string = $a^n b^{2n}$

$S \rightarrow a^n b^{2n}$

$S \rightarrow a a^{n-1} b^{2n-2} bb$ $\Rightarrow$ $S \rightarrow aSbb$

$S \rightarrow abb$.

$\therefore$ CFG = $S \rightarrow aSbb$

$S \rightarrow abb$.

5) Construct CFG for the following CFL

$L = \{ 0^i 1^{i+1} \mid i \geq 0 \}$

Sol: $L = \{ 0^i 1^{i+1} \mid i \geq 0 \}$

$= 0^i 1^i . 1$

$S \rightarrow A1$

$A \rightarrow 0^i 1^i$

$\rightarrow 0 0^{i-1} 1^{i-1} 1$

$\rightarrow 0A1$

CFG: $S \rightarrow A1$

$A \rightarrow 0A1$

$A \rightarrow \epsilon$

$A \rightarrow 01$

6) Construct a CFL from the following Language.

$L = \{ a^m b^m c^n \mid m, n \geq 0 \}$

Sol:

$\underbrace{a^m b^m}_{A} \underbrace{c^n}_{B}$

$A \to a^m b^m$

$\to a\, a^{m-1} b^{m-1} b$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to c^n$

$B \to c c^{n-1}$

$B \to cB$

$B \to \epsilon$

$B \to c$

CFG:- $S \to AB$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to cB$

$B \to \epsilon$

$B \to c$

Closure properties of CFL:-

- context free languages are closed under union
- " " " " " " concatenation.
- " " " " " " Kleene closure
- " " " " " " Reversal
- context free languages are not closed under Complement
- " " " " " " Intersection
- " " " " " " difference.

Derivation:-

* Introduction * Types of Derivation * Derivation tree

Derivation is a process of generating a string from a given grammar.

Derivation process can be represented graphically is called Derivation tree (or)

* left most derivation * Rightmost derivation.

Left most derivation:- With example

In this, we can replace a left most Variable to obtain the given input string.

Right most derivation:-

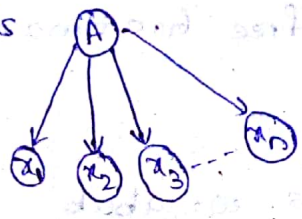
In this, we can replace a Right most variable to obtain the given input string.

Derivation Tree :—

Let $G = (V, T, P, S)$ be a CFG. Then there is a derivation tree for G. If and only if.

* the root node of the Tree is labelled with start symbol of G
* All leaf nodes of Tree are labelled by Terminals (or) special symbols of G.
* the interior nodes are labelled by variables of G.
* If any production rule in G. is are the form.

$A \rightarrow x_1 x_2 x_3 \cdots x_n$ then the derivation tree is



find the i) left most derivation
ii) Right most derivation
iii) parse tree for the i/p string id+id*id
from the following grammar. $E \rightarrow E+E$
$E \rightarrow E*E$
$E \rightarrow id$.

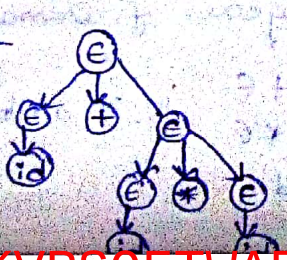
Sol:- the given grammar is $E \rightarrow E+E$
$E \rightarrow E*E$
$E \rightarrow id$.

Input string id+id*id.

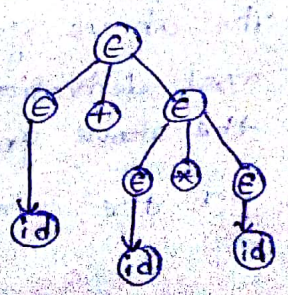
LMD:- $E \rightarrow E+E$
$\Rightarrow E+E*E$
$\rightarrow id+E$
$\rightarrow id+E*E$
$\rightarrow id+id*E$
$\rightarrow id+id*id$.

RMD:- $E \rightarrow E+E$
$\rightarrow E+E*E$
$\rightarrow E+E*id$
$\rightarrow E+id*id$
$\rightarrow id+id*id$

Parse tree:-



Parse tree:-



Ambiguious Grammar:-

* A CFG $G=(V,T,P,S)$ which generates two (or) more parse trees for given i/p string is called Ambiguious grammar.

* that means an Ambegeous grammar has two or more left most derivations (or) right most derivation (or) parse tree.

Ex:- Prove that $S \to aSbS$, $S \to bSaS$, $S \to \epsilon$ is ambegeous for the i/p string abab

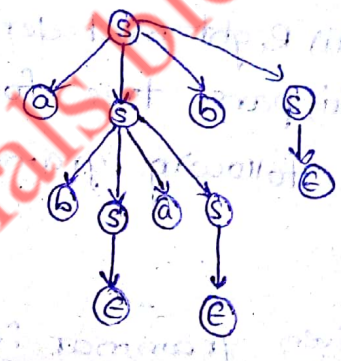
sol:- The given content free Grammar is $S \to aSbS$, $S \to bSaS$, $S \to \epsilon$

the input string is $w = abab$

① LMD:

$S \to aSbS$
$\to abSaSbS$
$\to ab\epsilon aSbS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

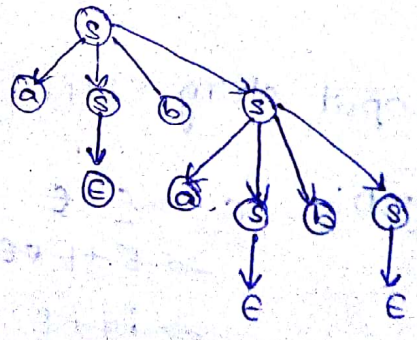
parsetree



② LMD:

$S \to aSbS$
$\to a\epsilon bS$
$\to abS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

parsetree



∴ the above grammar generates two parsetrees (or) two left most derivation for the same i/p string $w=abab$. Hence the above grammar is ambegeous grammar.

2) P.T the grammar $E \to E+E$, $E \to E*E$, $E \to id$ is ambegeous for i/p string id+id*id.

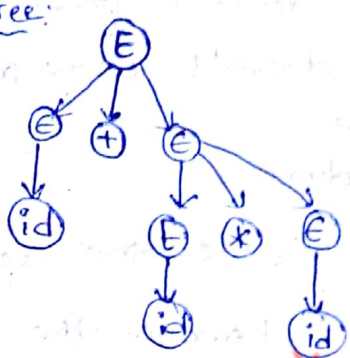
Sol:- The given context free grammar is $E \to E+E$
$E \to E*E$
$E \to id$

The input string is $w = id+id*id$.

① LMD:

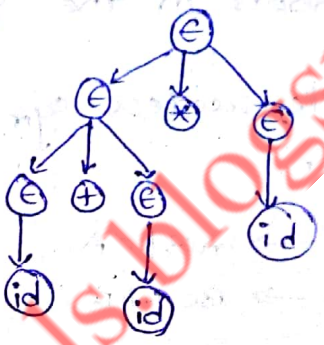
$E \to E+E$
$\to id+E$
$\to id+E*E$
$\to id+id*E$
$\to id+id*id.$

Parse tree:



② LMD:-

$E \to E*E$
$\to E+E*E$
$\to id+E*E$
$\to id+id*E$
$\to id+id*id.$



** (Im) Simplification of CFG:

* Introduction
* Methods
  1. Elimination of useless symbols.
  2. Elimination of ϵ-productions
  3. Elimination of unit productions.

Introduction:-

It's means minimizing the no. of productions in the given CFG. that is reducing size of CFG. size of CFG is equal to no. of productions.

Methods:- $S \to AB$
$A \to a$
$A \to aA$
$B \to SB$

Elimination of useless symbols:-

Useful symbol:- A variable is said to be useful if and only if

* It generates a terminal string.
* It is used in derivation of a string at least one time

useless symbol :-

* A variable is said to be useless if and only if.
  * It doesn't generates a terminal string.
* It doesn't used in derivation of a string at least one time.

Procedure :-

step 1 :- Determine useless symbols in the grammar.

step 2 :- Remove the productions which contains useless symbols in the grammar.

Ex :- eliminate useless symbols from the following grammar.

$S \rightarrow AB \mid CA$
$B \rightarrow BC \mid AB$
$A \rightarrow a$
$C \rightarrow aB \mid b$

sol :- The given CFG is

$S \rightarrow AB$
$S \rightarrow CA$
$B \rightarrow BC$
$B \rightarrow AB$
$A \rightarrow a$
$C \rightarrow aB$
$C \rightarrow b$

In the given grammar 'B' doesn't generating a terminal string.

$S \rightarrow AB$
$S \rightarrow aB$
$\rightarrow aBC$
$\rightarrow aABC$
$\rightarrow aaBC$
$\rightarrow aaBb$
$\rightarrow aaABb$
$\rightarrow aaaBb$
:

∴ so, 'B' is useless symbol.

so, we can eliminate the productions which contains 'B'.

∴ The reduced CFG is

$S \rightarrow CA \mid C \rightarrow b.$
$A \rightarrow a$

2) Elimination of ∈-production :-

∈-production :- A production is of the form $A \to \epsilon$ is called ∈-production (or) NULL production.

procedure :-

step 1 :- If the grammar contains $A \to \epsilon$ then replace 'A' with ∈ in the remaining productions.

step 2 :- Remove $A \to \epsilon$ from the grammar.

ex :- Remove ∈-productions from the following grammar

$A \to 0B1 \mid 1B1$

$B \to 0B \mid 1B \mid \epsilon$

sol :- the given CFG is $A \to 0B1$

$A \to 1B1$

$B \to 0B$

$B \to 1B$

$B \to \epsilon$

$A \to 0B1$
$\to 0\epsilon1$
$\to 01$

$\therefore A \to 0B1$
$A \to 01$

$B \to 0B$
$\to 0\epsilon$
$\to 0$

$\therefore B \to 0B$
$B \to 0$

$B \to 1B1$
$\to 1\epsilon1$
$\to 11$

$\therefore A \to 1B1$
$A \to 11$

$B \to 1B$
$\to 1\epsilon$
$\to 1$

$\therefore B \to 1B$
$B \to 1$

After eliminating $B \to \epsilon$ the resultant CFG is

$A \to 0B1$
$A \to 01$
$A \to 1B1$
$A \to 11$
$B \to 0B$
$B \to 0$
$B \to 1B$
$B \to 1$

14m
** Normal forms :-

*Introduction

*Types of Normal forms

1. chomsky Normal form (CNF)
2. Greibach Normal form (GNF)

Introduction :-

In CFG each production of the form $\alpha \rightarrow \beta$ where $\alpha \in V$ that means β contains any no. of non-terminal symbols ($\beta \in (V \cup \Sigma)^*$) and any no. of terminal symbols. But, we need to have a grammar in specific form i.e; we can decide the no. of non-terminals and terminals on RHS of the grammar. This can be implemented by using "normalization of CFG".

Normalization :-

The process of Arranging the grammar with fixed no. of

non-terminals and terminals on R.H.S of CFG is called normalization.

normal forms are classified into two types

i) chomsky normal form.
ii) Greibach normal form

chomsky normal form :—

It is defined as $\alpha \rightarrow \beta$

> non-terminal → Non-terminal. Non-terminal.
>
> (or)
>
> Non-terminal → Terminal.

conversion of CFG to CNF :—

procedure :—

step 1 :- simplify the CFG

step 2 :- convert the simplified CFG to CNF.

Ex:- convert the following CFG into chomsky normal form.

$S \rightarrow aaaaS$

$S \rightarrow aaaa$

Sol:- The given grammar is $S \rightarrow aaaaS$

$S \rightarrow aaaa$

Consider a non-terminal $A \Rightarrow$ that derives terminal a.

∴ The production rule is $A \rightarrow a$. is in CNF

$S \rightarrow aaaaS$

$S \rightarrow AAAAS$ can be replaced by P_1

$S \rightarrow AP_1$ is in CNF.

$P_1 \rightarrow AAAS$ can be replaced by P_2

$P_1 \rightarrow AP_2$ is in CNF

$P_2 \rightarrow AAS$ can be replaced by P_3

$P_2 \rightarrow AP_3$ is in CNF

$P_3 \rightarrow AS$ is in CNF

$S \rightarrow aaaaa$

$S \rightarrow AAAA$ can be replaced by P_4

$S \to AP_4$ is in CNF

$P_4 \to A[AA]$ can be replaced by P_5.

$P_4 \to AP_5$ is in CNF

$P_5 \to AA$ is in CNF.

The resultant Grammar CNF is

$S \to AP_1$ $\quad P_5 \to AA$

$S \to AP_4$ $\quad A \to a$

$P_1 \to AP_2$

$P_2 \to AP_3$

$P_3 \to AS$

$P_4 \to AP_5$

2) Convert the given CFG to CNF. $S \to aSa$

$S \to bSb$

$S \to a$

$S \to b$.

Sol:- The given grammar is $S \to aSa$

$S \to bSb$.

$S \to a$

$S \to b$

It is already in simplified form.

Consider a non-terminal A that derives a terminal a and the non-Terminal B that derives the terminal b.

∴ The production rules is $A \to a$ are in CNF.

$B \to b$.

(i) $S \to aSa$

$S \to A[SA]$ can be replaced by P_1

$S \to AP_1$ is in CNF.

$P_1 \to SA$ is in CNF.

(ii) $S \to bSb$

$S \to B[SB]$ can be Replaced by P_2

$S \to BP_2$ is in CNF

$P_2 \to SB$ is in CNF

(iii) $S \to a$ is in CNF

$S \to b$ is in CNF.

∴ The resultant grammar in CNF is

$S \to AP_1$

$S \to BP_2$

$S \to a$

$S \to b$

$P_1 \to SA$

$P_2 \to SB$

$A \to a$

$B \to b$.

Greibach Normal form (GNF) :-

GNF is defined as

Non-terminal $\longrightarrow$ Terminal . any no. of nonterminals

Non-terminal $\longrightarrow$ Terminal.

Lemma 1 :

Let CFG be $G = (V, T, P, S)$ and there is a production rule $A \to \alpha B$ and $B \to \beta_1 | \beta_2 | \beta_3 | \dots | \beta_n$ then add the new production rule to GNF, $A \to \alpha B_1 | \alpha\beta_2 | \alpha\beta_3 | \dots | \alpha\beta_n$ to GNF.

$\therefore$ B is replaced by $B \to \beta_1 | \beta_2 | \dots | \beta_n$

Lemma 2 :

Let CFG be $G = (V, T, P, S)$ and there is production rule

$A \to A\alpha_1 | A\alpha_2 | \dots | A\alpha_n | \beta_1 | \beta_2 | \dots | \beta_n$ then the production rules are added to GNF.

$A \to \beta_1 | \beta_2 | \beta_3 | \dots | \beta_n$

$A \to \beta_1 Z | \beta_2 Z | \beta_3 Z | \dots | \beta_n Z$

$Z \to \alpha_1 | \alpha_2 | \alpha_3 | \dots | \alpha_n$

$Z \to \alpha_1 Z | \alpha_2 Z | \alpha_3 Z | \dots | \alpha_n Z$

Converting CFG into GNF :-

Procedure :-

step 1 :- Simplify the CFG.

step 2 :- Converting simplified CFG into GNF.

Ex :- Convert the given CFG to GNF $S \to ABA$

$A \to aA | \epsilon$

$B \to bB | \epsilon$

Sol :- The Given CFG is $S \to ABA$

$A \to aA$

$A \to \epsilon$

$B \to bB$

$B \to \epsilon$

Simplified of given CFG :-

(a) elimination of ϵ-productions :-

$A \to \epsilon$ $\quad$ $B \to \epsilon$

(1) $S \to ABA$
$S \to \epsilon BA$
$S \to BA$

(2) $S \to ABA$
$S \to AB\epsilon$
$S \to AB$

(3) $S \to ABA$
$S \to A\epsilon A$
$S \to AA$

(4) $S \to ABA$
$S \to \epsilon\epsilon A$
$S \to A$

(5) $S \to ABA$
$S \to \epsilon B\epsilon$
$S \to B$

$A \to aA$ $\quad$ $B \to bB$
$A \to a\epsilon$ $\quad$ $B \to b\epsilon$
$A \to a$ $\quad$ $B \to b$

∴ After eliminating $A \to \epsilon$, $B \to \epsilon$ from the grammar the resultant grammar is

$S \to ABA$ $\quad$ $A \to aA$
$S \to BA$ $\quad$ $A \to a$
$S \to AB$ $\quad$ $B \to bB$
$S \to AA$ $\quad$ $B \to b$
$S \to A$ ✓
$S \to B$ ✓

Elimination of unit productions :-

The above grammar has two unit productions like

$S \to A$ ×
$S \to aA$
$S \to a$

$S \to B$ ×
$S \to bB$
$S \to b$

[∴ since $A \to aA$, $A \to a$, $B \to bB$, $B \to b$]

∴ After elimination unit productions $S \to A$, $S \to B$ from the grammar. The resultant grammar is

$S \to ABA$
$S \to BA$
$S \to AB$
$S \to AA$
$S \to aA$
$S \to a$
$S \to bB$
$S \to b$

$A \to aA$
$A \to a$
$B \to bB$
$B \to b$

there is no useless production.

∴ The simplified CFG is

S → ABA
S → BA
S → AB
S → AA
S → aA
S → a
S → bB
S → b
A → aA
A → a
B → bB
B → b

Converting simplified CFG to GNF :

i) S → ABA
S → aABA ✓
S → aBA ✓
A → aA ✓
A → a ✓
B → bB ✓
B → b ✓

ii) S → BA
S → bBA ✓
S → bA ✓

iii) S → AB
S → aAB
S → aB ✓

iv) S → AA
S → aAA
S → aA

v) S → aA ✓
S → a ✓

vi) S → bB ✓
S → b ✓

∴ The resultant grammar is in GNF. is

S → aABA | aBA | bBA | bA | aAB | aB | aAA | aA | bB | a | b
A → aA | a
B → bB | b.

(2) Convert the following CFG into GNF S → AA | 0
A → SS | 1

Sol: Given Grammar S → AA
S → 0
A → SS
A → 1

The simplified CFG is S → AA
S → 0
A → SS
A → 1

① $S \to AA|0$
$S \to SA|0$

$S \to 0$
$S \to 0Z$
$Z \to SA$
$Z \to SAZ$
$Z \to SA$
$Z \to 0A$
$Z \to 0ZA$
$Z \to 1AA$
$Z \to 0A$

② $S \to AA|0$
$S \to 1A|0$

$S \to 1A$
$S \to 0$

$Z \to SAZ$
$Z \to 0AZ$
$Z \to 0ZAZ$
$Z \to 1AAZ$
$Z \to 0AZ$

③ $A \to Ss$
$A \to 0S$
$A \to 0ZS$
$A \to 1AS$
$A \to 0S$

∴ The resultant grammar is

$S \to 0|0Z|1A$

$Z \to 0A|0ZA|1AA|0A|0AZ|0ZAZ|1AAZ$

$A \to 0S|0ZS|1AS|1$

③ convert the given CFG to GNF $S \to CA$
$A \to a$
$C \to aB|b$

⇒ Given CFG is not a simplified Grammar

After eliminating the useless symbols the resultant CFG is. $S \to CA$
$A \to a$
$C \to b$

By Applying Lemma 1 $S \to CA$
$S \to bA$

∴ The resultant GNF is $S \to bA$
$A \to a$
$C \to b$

④ Convert the given CFG to GNF $S \to SS$
$S \to 0S1|01$

The given CFG is a simplified CFG

The resultant grammar is $S \to SS$
$S \to 0S1$
$S \to 01$

Replaced 0 by A, 1 by B

Then productions are $A \to 0$
$B \to 1$

S → SS

S → ASB

S → AB

Applying Lemma ①

① S → SS
S → ASBS
S → OSBS

② S → SS
S → ABS
S → OBS

② S → ASB
S → OSB

S → AB
S → OB

The resultant grammar GNF is

S → OSBS | OBS | OSB | OB

A → O

B → 1

Pumping Lemma for CFL :—

pumping Lemma is used for proving the given language is CFL (or) not.

Lemma :- let 'L' be any CFL, then there is a constant 'n' which depends only a part 'L' such that there exists a string.

$w = uvxyz$ such that 1. $|vy| \geq 1$
2. $|vxy| \leq n$
3. for $\forall i \geq 0$ $uv^i xy^i z$ is in 'L'

Then 'L' is said to be CFL. otherwise it is not a CFL.

① Prove that $L = \{a^n b^n c^n | n \geq 0\}$ is not a CFL.

The given language $L = \{a^n b^n c^n | n \geq 0\}$.

$L = \{\epsilon, abc, aabbcc, ----\}$

consider a constant n and the string $w = a^n b^n c^n$

consider a string $w \in L$

$w = abc$ for $n = 1$

$|w| = 3n$

for $i = 1$ $w = abc$

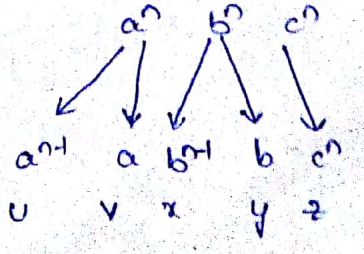
for $i = 2$

$w = uv^i xy^i z$

$w = uv^2 xy^2 z$

$w = a^{n-1} a^2 b^{n-1} b^2 c^n$

$w = a^{n+1} b^{n+1} c^n \notin L$



∴ The given language is not a CFL.

② show that the language $L = \{ ss^T / s \in \{a,b\}^* \}$

Given language $L = \{ ss^T / s \in \{a,b\}^* \}$

$L = \{ \epsilon,$