Constraint Violation Probability Minimization for a Robot Manipulator

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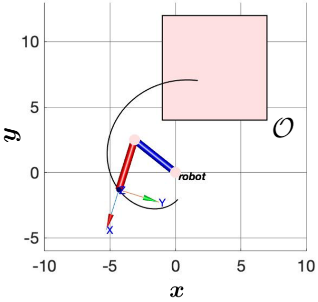
Motivation

- Safety critical systems: constraints
- Model Predictive Control (MPC): efficient algorithm for reference trajectory task
- Presence of uncertainties
- Constraint Violation Probability Minimization with underlying MPC method (CVPM-MPC)

Problem Settings

Goal

- Follow reference trajectory
- Minimize probability of collision with the obstacle set $\mathcal{O}$

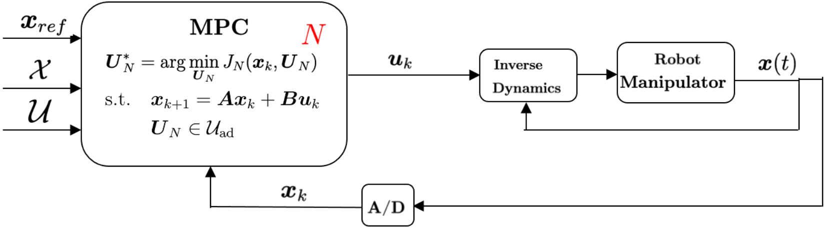


Related Work

- CVPM Approach [Brüdigam+ 2020]
    - Avoid an obstacle with an uncertain position
- Robots Model and Dynamics [Siciliano+ 2010]

Multi-loop Control Scheme: MPC

- Inverse Dynamics: Linearise dynamics of robot
- Standard linear MPC: follow $\boldsymbol{x}_{\text{ref}}$



Inverse Dynamics Approach

- Dynamics of the robot:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{V}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) \tag{1}$$

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- Control law for feedback linearization:

$$\boldsymbol{\tau} = \hat{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{u} + \hat{\boldsymbol{V}}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \hat{\boldsymbol{G}}(\boldsymbol{q}) \tag{2}$$

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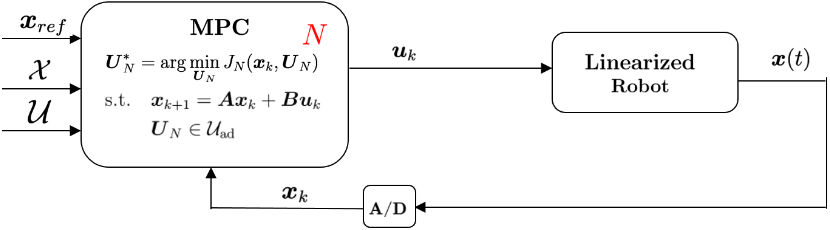
- Good Estimation Parameters:

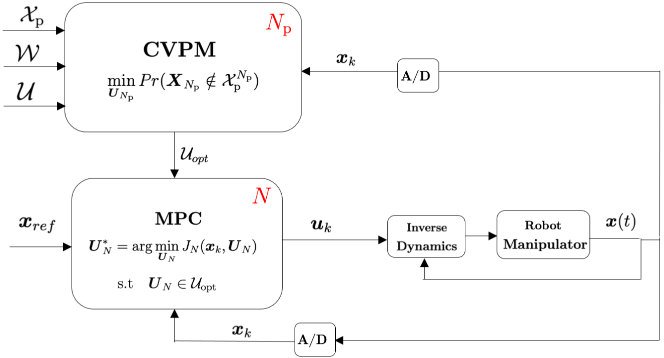
$$\ddot{\boldsymbol{q}} = \boldsymbol{u} \tag{3}$$

Multi-loop Control Scheme: MPC

- Inverse Dynamics: Linearise dynamics of robot
- Standard linear MPC: follow x_{ref}



Multi-loop Control Scheme: CVPM-MPC



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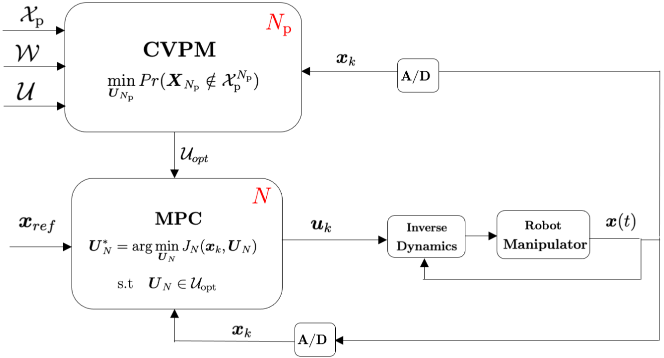

- Probabilistic Set $\mathcal{X}_p$: collision-free set

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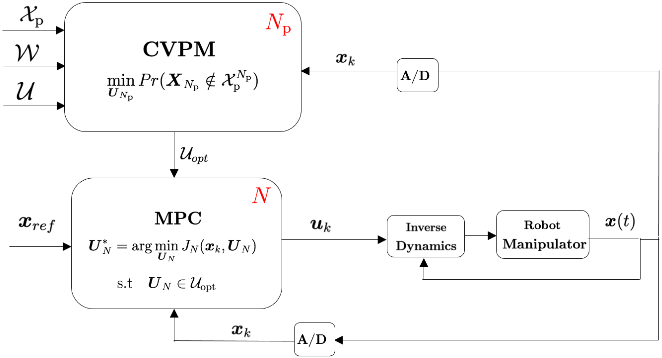

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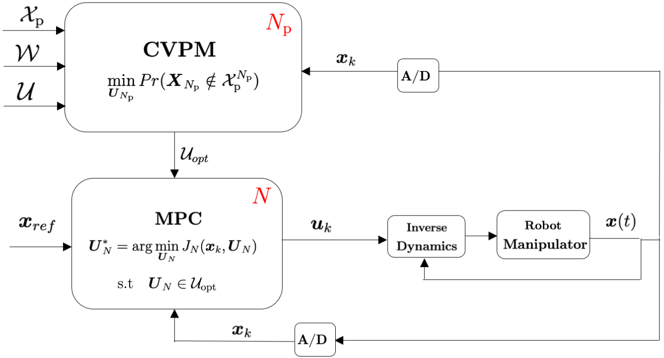
Multi-loop Control Scheme: CVPM-MPC



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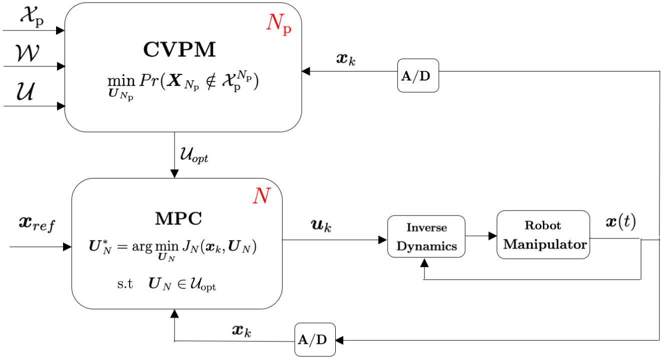
  $P = \mathrm{Pr}\left(\boldsymbol{X}_{N_\mathrm{p}} \notin \mathcal{X}_\mathrm{p}^{N_\mathrm{p}}\right)$

Multi-loop Control Scheme: CVPM-MPC



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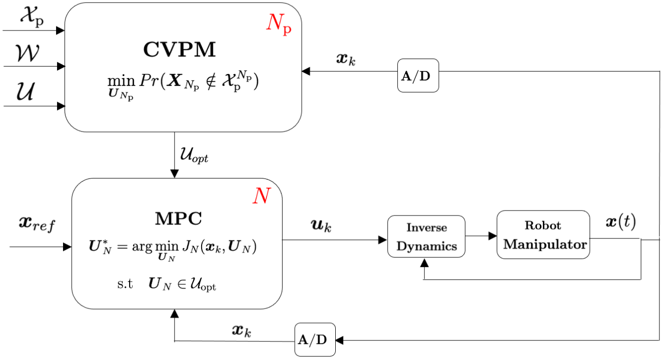
1. CVPM:

Multi-loop Control Scheme: CVPM-MPC



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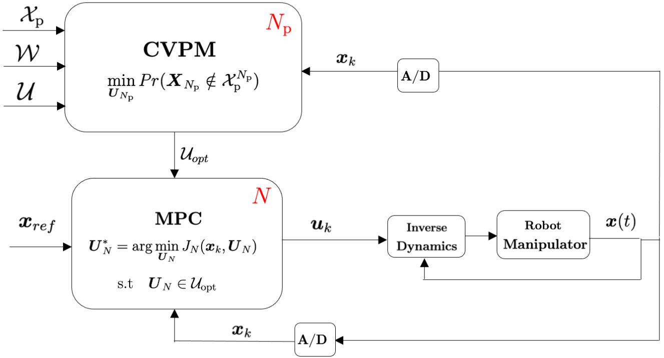
1. CVPM:
   - Case 1: $\exists \boldsymbol{U}_{N_\mathrm{p}}$ s.t. $P = 0$

Multi-loop Control Scheme: CVPM-MPC



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1. CVPM:
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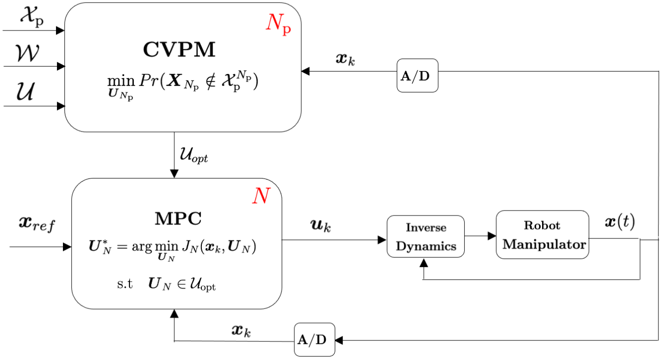
Multi-loop Control Scheme: CVPM-MPC



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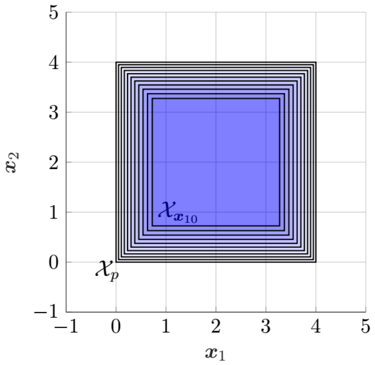
   - Case 1: $\exists \boldsymbol{U}_{N_{\mathrm{p}}}$ s.t. $P = 0$
   - Case 2: Otherwise $\min_{\boldsymbol{U}_{N_{\mathrm{p}}}} P$
2. MPC

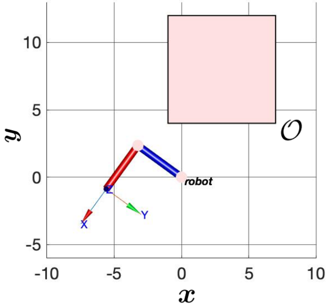
Target Set $\mathcal{X}_\mathsf{T}$

- Case 2 optimization: Minimize distance between the states and the target set.
- $\mathcal{X}_\mathsf{T}$: tightens probabilistic set $\mathcal{X}_\mathsf{p}$ w.r.t disturbances.
- $N_\mathsf{p} = 10$
- Probabilistic set $\mathcal{X}_\mathsf{p}$: collision-free set

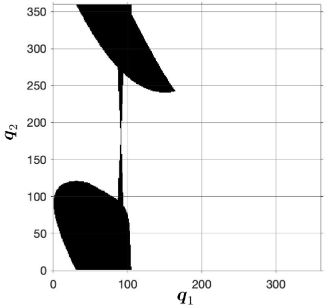


Configuration Space

- Configuration Space (C-Space): Point representation of the robot



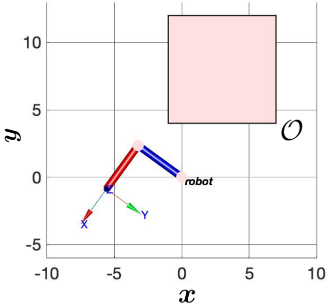
(a) Workspace



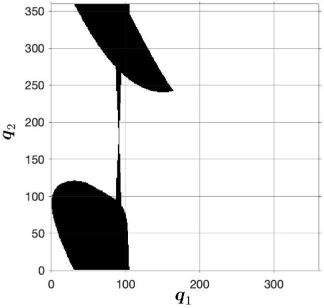
(b) C-Space

Configuration Space

- C-Space: Grid of sample 0.1°
- Forward Dynamics of each configuration pair of grid
- Check collison in workspace



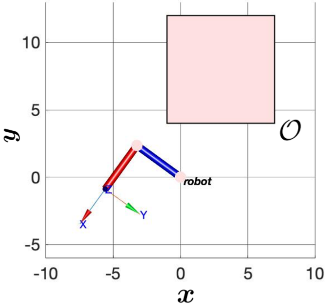
(c) Workspace



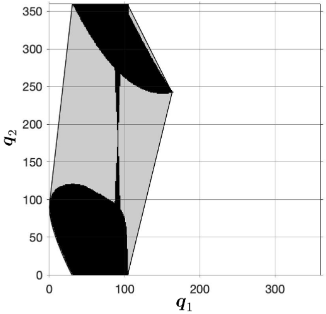
(d) C-Space

Configuration Space

- enclose the C-obstacle in a convex polyhedron



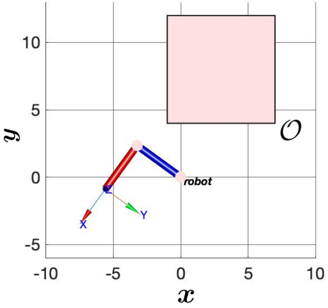
(e) Workspace



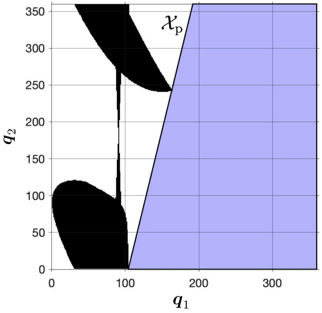
(f) C-Space

Configuration Space

- Set $\mathcal{X}_{\mathrm{p}}$ as the largest convex polyhedron in free space



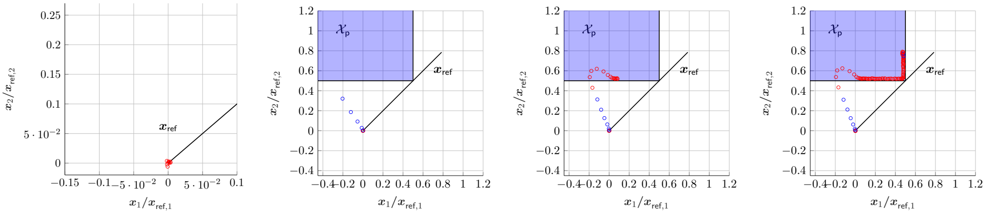
(g) Workspace



(h) C-Space

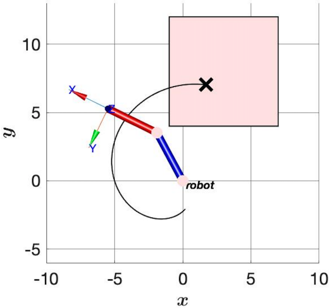
CVPM-MPC

- Case 1, Case 2
- $t_p = 1$ change of probabilistic set
- $N = N_{\mathsf{p}} = 10$

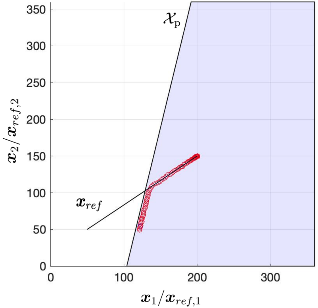


(i) $t = 0.9$ s　(j) $t = 2$ s　(k) $t = 4$ s　(l) $t = 10$ s

CVPM-MPC in Configuration & Work-Space



(m) Workspace, $t = 10$ s



(n) C-Space, $t = 10$ s

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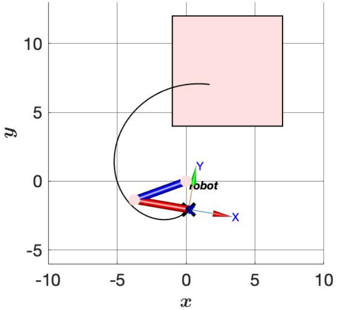

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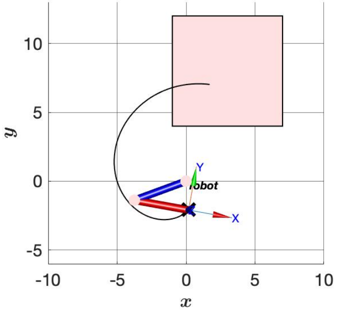

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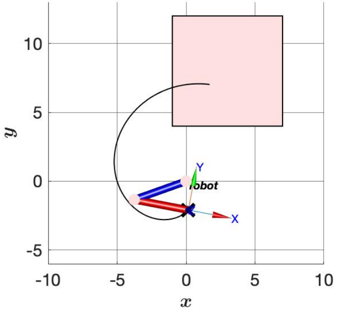

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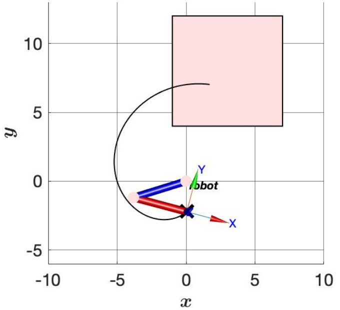

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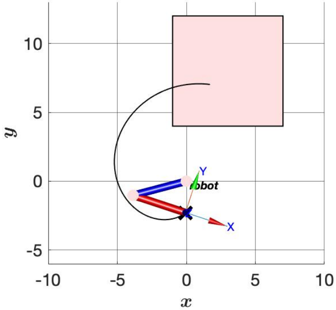

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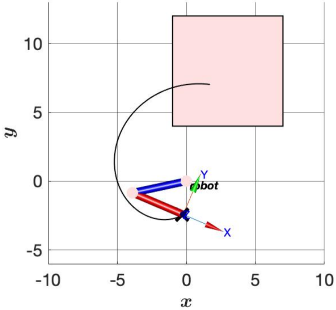

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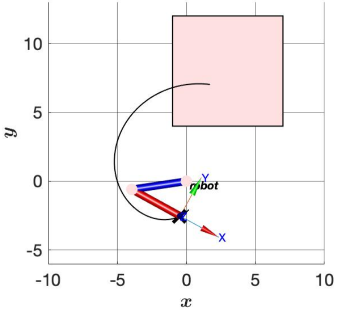

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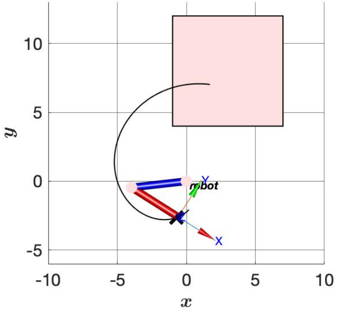

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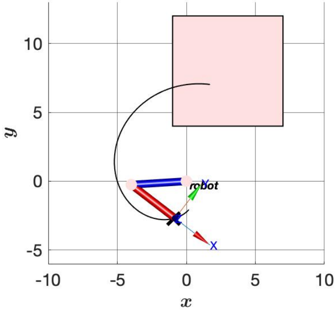

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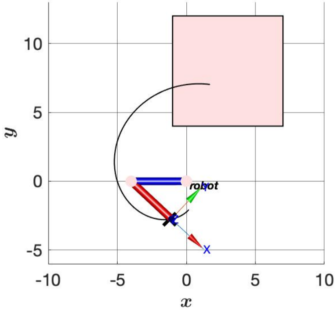

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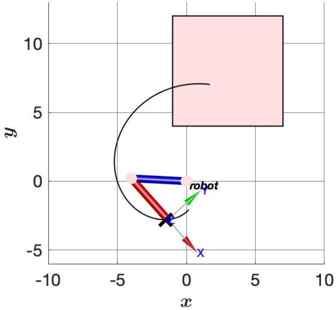

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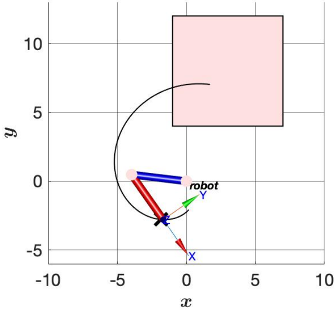

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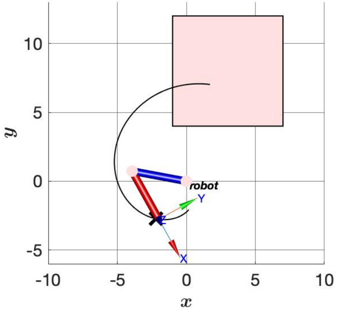

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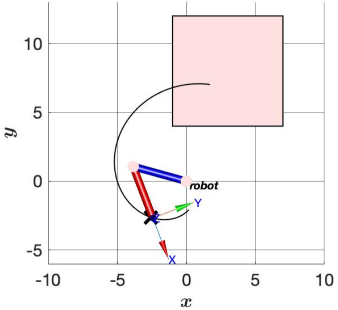

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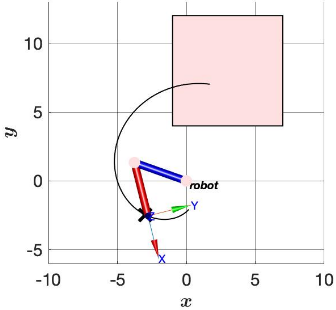

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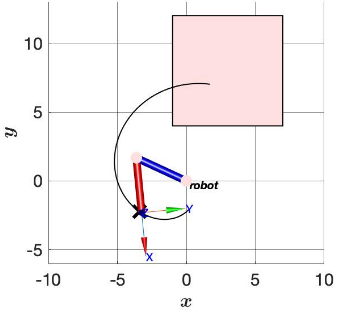

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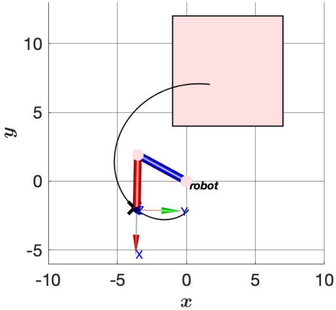

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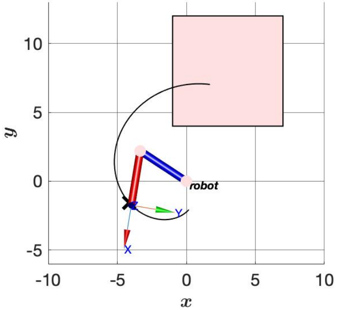

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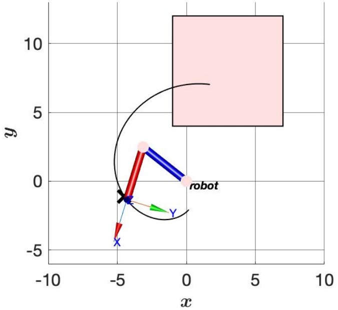

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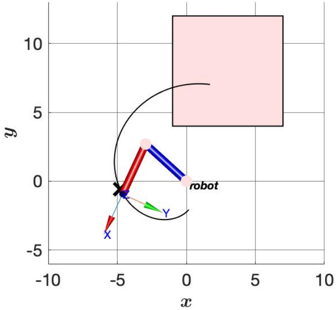

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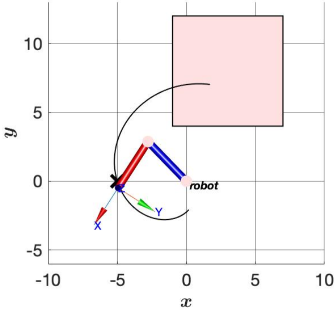

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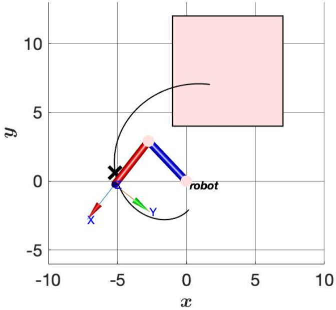

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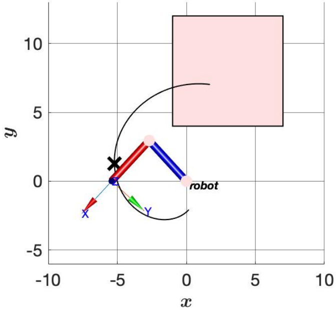

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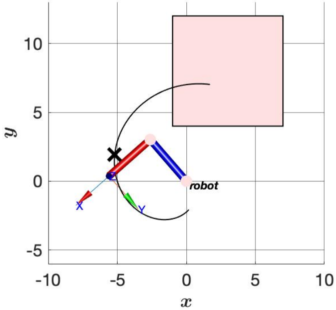

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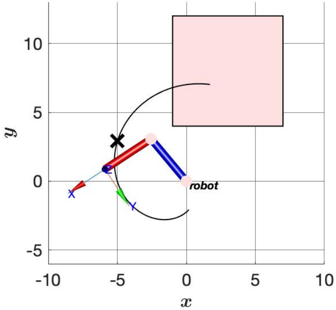

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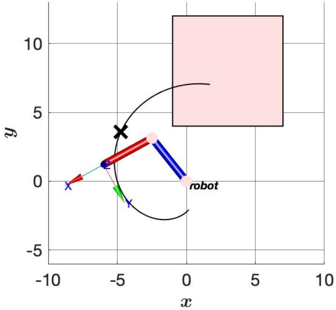

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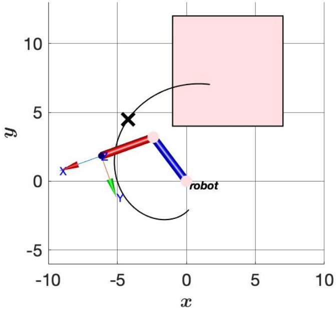

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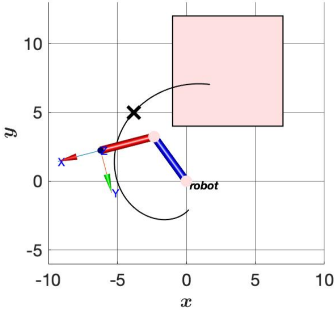

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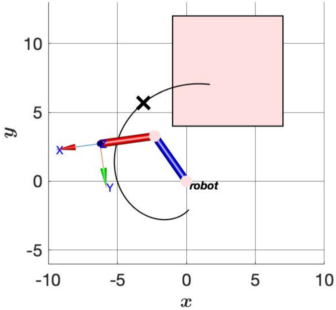

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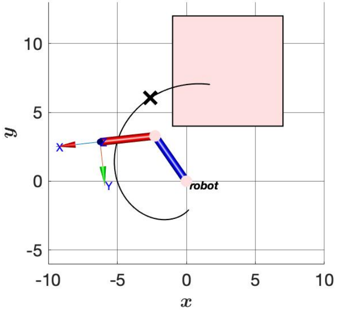

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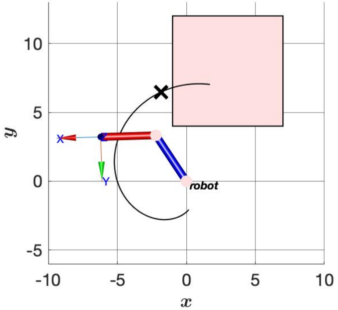

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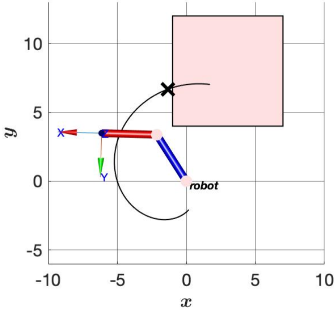

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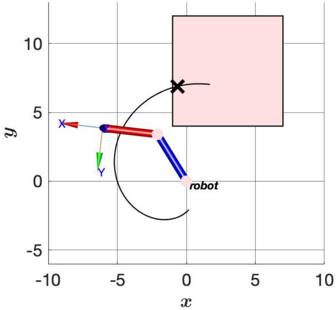
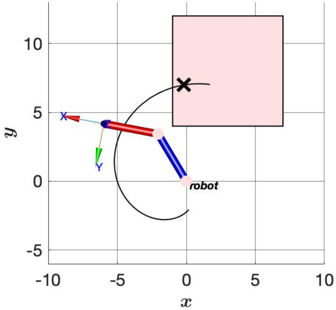


CVPM-MPC in Configuration & Work-Space



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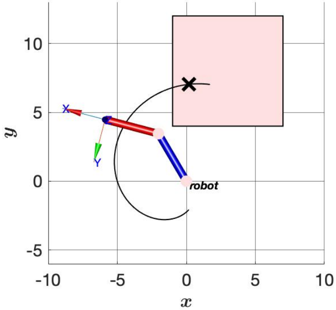

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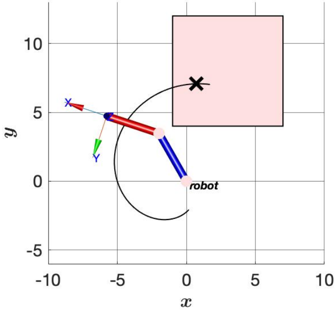

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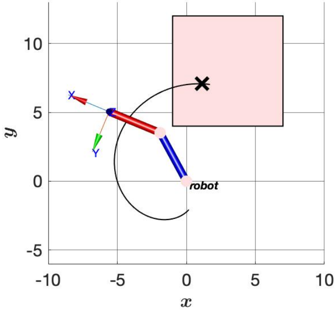

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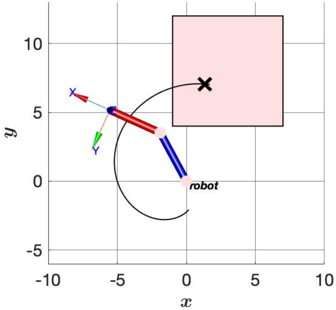

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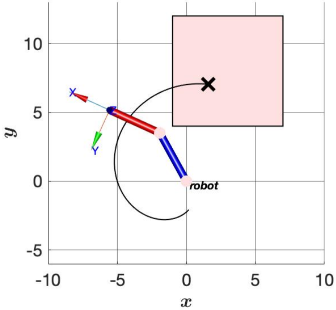

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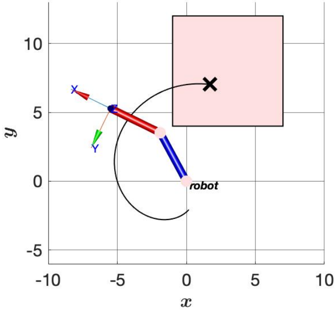

Summary

- CVPM-MPC
  - Handle uncertainties and sudden change of probabilistic set
  - Minimize the probability of collision
- Future Work:
  - Manipulator with higher degrees of freedom
  - Extend approach for a moving obstacle

References

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