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Parameter Estimation

Q1 Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Sample of size n $\rightarrow$ $x_1, x_2, x_3 \ldots x_n$.

$$L(x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} f(x_i)$$

taking log on both sides

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2}$$

$\rightarrow$ ~~taking~~ Differentiate w.r.t. μ.

$$\frac{\partial \ln L}{\partial \mu} = 0 + \sum_{i=1}^{n} +\frac{2(x_i-\mu)}{2\sigma^2} = 0$$

$$\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

mean $= \theta_1 = \bar{x}$ is sample mean.

$\rightarrow$ Differentiating w.r.t. σ^2

$$\frac{\partial \ln L}{\partial \mu} = -\frac{n \times \cancel{2}\cancel{\sigma}}{2\times\cancel{2}\pi\sigma^{\cancel{2}}} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2(-2)}{\cancel{2}\sigma^3} = 0$$

$$= \frac{-n}{2\pi\sigma} + \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^3} = 0$$

$$= \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^{\cancel{3}2}} = \frac{n}{2\pi\cancel{\sigma}}$$

$$= \sum (x_i-\mu)^2 = \frac{n\sigma^2}{2\pi}$$

$$\sigma^2 = \frac{2\pi}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\theta_2 = \frac{2\pi}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

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Q2 Binomial distribution → ${}^{n}C_{x_i}\theta^{x_i}(1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\theta^{x_i}(1-\theta)^{n-x_i}$$

Taking log

$$\log(L) = \sum_{i=1}^{n}\left(\log {}^{n}C_{x_i} + \log\theta^{x_i} + \log(1-\theta)^{n-x_i}\right)$$

$$= \sum_{i=1}^{n}\log {}^{n}C_{x_i} + \log\theta\sum_{i=1}^{n}x_i + \log(1-\theta)\sum_{i=1}^{n}(n-x_i)$$

∴ Differentiate wrt. θ.

$$\frac{\partial \log L}{\partial\theta} = 0 + \frac{1}{\theta}\Sigma x_i + \frac{(-1)}{1-\theta}\cdot\Sigma(n-x_i) = 0$$

$$\therefore \frac{\Sigma x_i}{\theta} - \frac{1}{(1-\theta)}\Sigma(n-x_i) = 0$$

$$\frac{\Sigma x_i}{\theta} - \frac{n^2}{1-\theta} + \frac{\Sigma x_i}{1-\theta} = 0.$$

$$\frac{\Sigma x_i}{\theta(1-\theta)} = \frac{n^2}{(1-\theta)}$$

$$\boxed{\theta = \frac{\Sigma x_i}{n^2}}$$