ECE-533 Homework 1

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1. Introduction

This assignment estimates the user's position using the least-squares estimation technique. At a minimum, 4 satellites are required to solve for state vector (3 for spatial distance, 1 for temporal bias), using the least-squares technique all the available data can be considered in the estimation process, thereby converging to an over-determined solution.

2. Background

The estimation technique is based on the trilateration process, which uses the user's relative position with respect to satellites to determine its global position. It uses Linear Least-squares to update the estimated position based on the difference in estimated pseudorange and actual pseudorange.

3. Methodology

State Vector,

$$X = \begin{bmatrix} x_u & y_u & z_u & ct_u \end{bmatrix}'$$

Pseudoranges, m $\geq$ 4

$$\begin{aligned}
\rho_u &= \begin{bmatrix} \rho_{1u} & \rho_{2u} & \dots & \rho_{mu} \end{bmatrix}'; \rho_{ij}(\text{pseudorange of } j_{th} \text{ receiver from } i_{th} \text{ satellite}) \\
&= F(X_u) \\
&= \begin{bmatrix} \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + ct_u \\ \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + ct_u \\ \vdots \\ \sqrt{(x_m - x_u)^2 + (y_m - y_u)^2 + (z_m - z_u)^2} + ct_u \end{bmatrix} = \begin{bmatrix} r_1 + ct_u \\ r_2 + ct_u \\ \vdots \\ r_m + ct_u \end{bmatrix} \\
\nabla F_x &= \begin{bmatrix} \frac{x_u - x_1}{r_1} & \frac{y_u - y_1}{r_1} & \frac{z_u - z_1}{r_1} & 1 \\ \frac{x_u - x_2}{r_2} & \frac{y_u - y_2}{r_2} & \frac{z_u - z_2}{r_2} & 1 \\ \vdots & \ddots & \vdots & \\ \frac{x_u - x_m}{r_m} & \frac{y_u - y_m}{r_m} & \frac{z_u - z_m}{r_m} & 1 \end{bmatrix} \\
X_{true} &= X_{estimate} + \triangle X \\
\rho_{estimate} &= \rho_{true} + \triangle \rho \\
\triangle \rho &= \rho_{estimate} - \rho_{true} \\
&= \rho_{estimate} - F(X_{true}) \\
&= \rho_{estimate} - F(X_{estimate} + \triangle X) \\
&\approx \rho_{estimate} - F(X_{estimate}) - \nabla F \triangle X \\
&\approx \rho_{estimate} - \rho_{estimate} - \nabla F \triangle X \\
&\approx -\nabla F \triangle X \\
H &= -\nabla F \\
\Rightarrow \triangle \rho &\approx H \triangle X
\end{aligned}$$

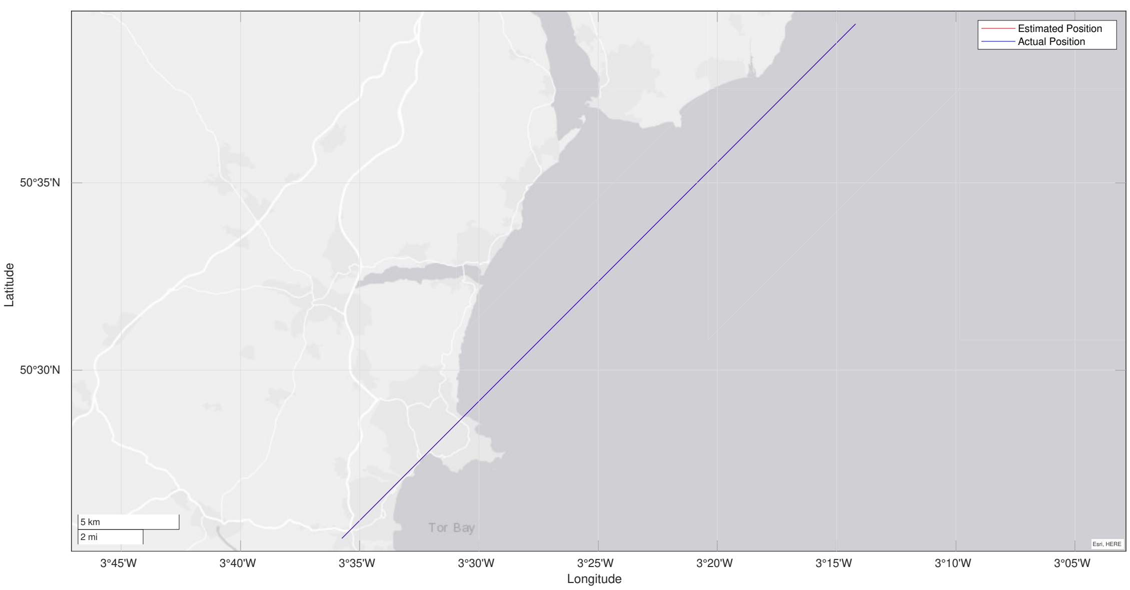
Solving for $\triangle X$ using Moore-Penrose inverse.

$$\triangle X = (H'H)^{-1}H\triangle\rho$$

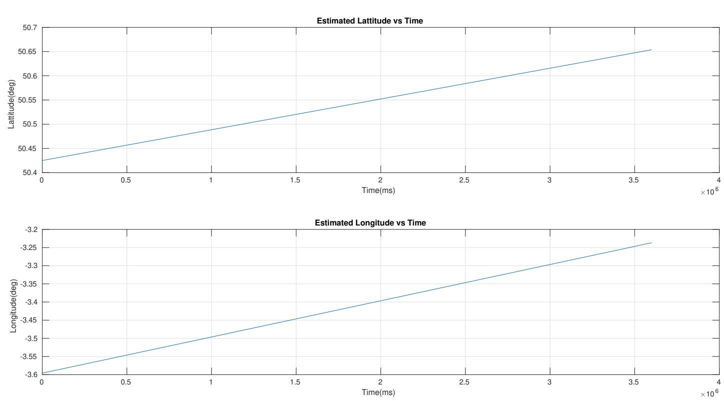
for the cold start $X_{estimate} = 0_{4x1}$, then using the pseudorange and satellite position information $\triangle X$ is computed, which updates the $X_{estimate}$, this is repeated until $\|\triangle X\|_2 \geq 10^{-8}$. Same is done for all the timestamps with one noticeable change, for consecutive time stamps, intial $X_{estimate}(t+1)$ is taken to be $X_{estimate}(t)$.

4. Result

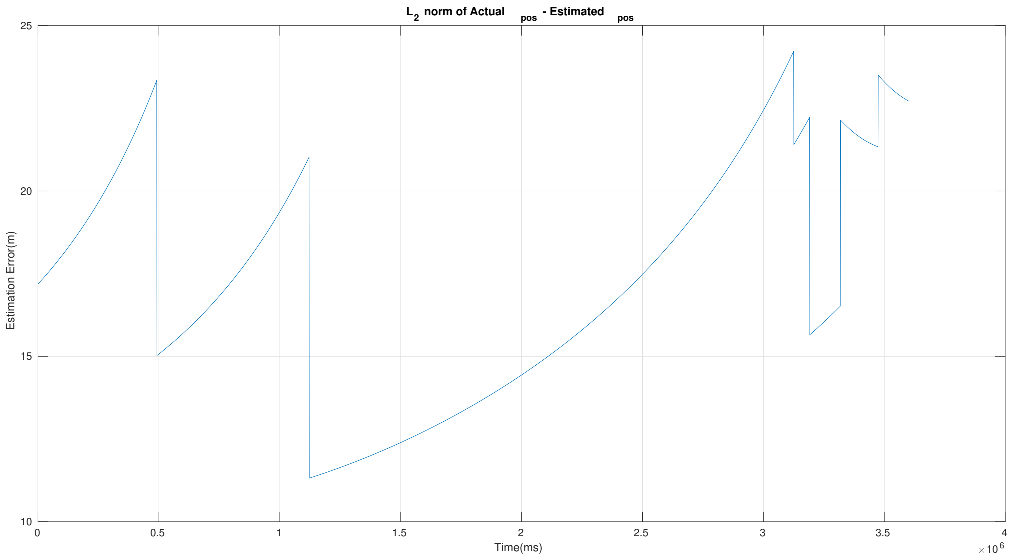
Estimated Position and Actual Position on Map



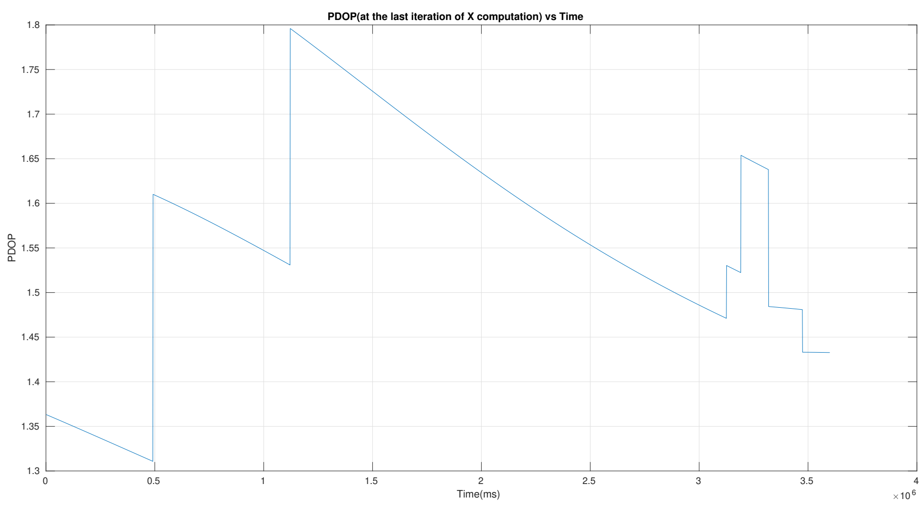
(Lattitude, Longitude) vs Time



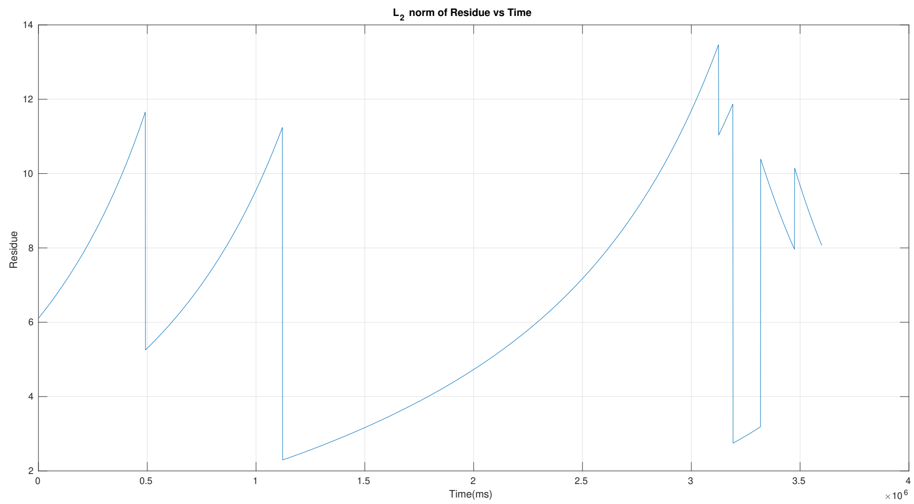
Estimation Error



PDOP vs Time



L_2 norm of Residue vs Time



Remark

- On increasing number of satellite(nSat) in least square estimation, $\|\text{residual error}\|_2$ increases and so does the $\|\text{estimation error}\|_2$. Currently, all the satellites are considered in estimation(except the ones which have zero rows), however, reducing the nSat can reduce the estimation error.

- $\triangle X$ is computed until its norm become smaller 10^{-8}
- Matlab Version: R2020a