Question 1

For $G(s)= 4/(s+2)$
This is a first order system with a single pole in left half-plane.

we know that a first-order system has the form:
$G(s)=1/(\tau s+1)$
By solving **$\tau=0.5$ sec**

Rise Time (10–90%) **$t_r = 2.2\tau = 1.1$ sec**

Settling Time (2%) **$t_s = 4\tau = 2$ sec**

FINAL VALUE THEOREM

$$y_{ss} = \lim_{s \to 0} s \cdot G(s) \cdot 1/s$$

$Y_{ss} = 4/2 = 2$.

Steady-State Error

$e_{ss} = r_{ss} - y_{ss} = 1-2 = -1$.
$e_{ss} = -1$.

increasing gain increases final value. Since the DC gain is greater than unity, the output settles above the reference value, resulting in a negative steady-state error.

Question 2

$G(s) = 10/(s(s+5))$
The system contains **one pole at the origin**, which corresponds to **one integrator**.
System Type = 1
Using Final Value Theorem:
$e_{ss} = \lim_{s\to 0} s(1/s - G(s).1/s) = 0$

e_{ss} = 0.(But this uses open circuit output G(s) but steady-state error is defined for the closed-loop system)
For a Unity feed back closed system the error signal is
$E(s)=R(s)/(1+G(s))$
For a unit step $R(s)=1/s$
$E(s)= 1/(s(1+G(s)))$
Now apply Final Value theorem $e_{ss} = \lim sE(s)$ as s tends to 0.
So, $e_{ss} = 0$. For closed loop.{as stents to 0, G(s) tends to infinity.}

Open-Loop Steady-State Error

Given:

$G(s) = 10 / [s(s + 5)]$
Unit step input:
$R(s) = 1 / s$

For an **open-loop system**, the output is simply: $Y(s) = G(s)R(s)$

Substitute:
$Y(s) = 10 / [s^2(s + 5)]$

Using the Final Value Theorem:

$y_{ss} = \lim (s \to 0) [sY(s)]$

As $s \to 0$, the denominator goes to zero, therefore:

$y_{ss} \to \infty$

Steady-state error is defined as:

$e_{ss} = r_{ss} - y_{ss}$

So:

$e_{ss}(\text{open-loop}) = 1 - \infty = -\infty$

In magnitude:

$e_{ss}(\text{open-loop}) = \infty$

Since the steady-state error is zero, the step response will reach exactly 1.

The system perfectly tracks a step input because the presence of an integrator eliminates steady-state error.

Question 3

Design Requirements

- $t_s<1.2$ seconds
- $e_{ss} = 0.1$

$t_s = 1/a$,

$a = 0.33$,

we know, $e_{ss} = 1/(1+K)$.   So, **$K = 9$.**

So, the modified new response will be

$G_n(s) = 9/(s+3.33)$

Predicted Response:

Pole moved further left meaning a faster response **and**

Higher gain so higher final value.

Question 4

Given, $G(s) = 3/(s+1)$

Requirement: $C(s) = K(s+z)$

Specifications : $t_s < 2$ sec, $M_p < 10\%$, $y_{ss} = 0.8$.

We can tell, A left half plane zero can

1. Reduce the rise time.
2. Speeds up response
3. May slightly increase overshoot.

To improve this we choose a zero as close to plant zero,

So, **z=1.**

Thus, $C(s) = K(s+1)$

$C(s)G(s) = 3K$.

$T(s) = C(s)G(s)/(1+C(s)G(s))$

So, $T(s) = 3K/(1+3K)$

But it only shows DC GAIN, and we know that the first order form is of order,$(A/(s+a))$

So, $T(s)=3K/(s+(1+3K))$

$\mathbf{s = -(1+3K)}$

Applying the controller cancels the plant pole and shifts the closed-loop pole to the left, resulting in a faster first-order system with improved steady-state performance.

First order system, closed loop pole at $(-(1+3K))$, Speed of response increases as K increases. Overshoot is zero. And setting time $t_s = 4/(1+3K)$.

Question 5

From Question 4, the controller and plant are:

$G(s) = 3 / (s + 1)$
$C(s) = (4/3)(s + 1)$

The input is a unit ramp:

$r(t) = t$

Open loop transfer function.

$C(s)G(s) = (4/3)(s + 1) \times 3 / (s + 1)$

Cancel $(s + 1)$:

$C(s)G(s) = 4$

The open-loop transfer function is a constant (4).

There is:

- No $1/s$ term
- No pole at $s = 0$

Therefore:

System Type = 0

From control system theory:

Type-0 system → finite step error, infinite ramp error

For a unit ramp:

$r(t) = t$
$R(s) = 1 / s^2$

Steady-state error is given by:

$e_{ss} = \lim_{s \to 0} [s \times E(s)]$

Where:

$E(s) = R(s) - T(s)R(s)$

So for a ramp input:

$e_{ss}(ramp) = \lim_{s \to 0} \{ s \times [(1 / s^2) - T(s)(1 / s^2)] \}$

Factor $1/s^2$:

$e_{ss}(ramp) = \lim_{s \to 0} [(1 - T(s)) / s]$

From Question 4, the closed-loop transfer function is:

$T(s) = 3K / [s + (1 + 3K)]$

With $K = 4/3$:

$T(s) = 4 / (s + 5)$

Substitute $s = 0$:

$T(0) = 4 / 5 = 0.8$

Substitute into the error expression:

$e_{ss}(ramp) = \lim_{s \to 0} [(1 - 0.8) / s]$
$e_{ss}(ramp) = \lim_{s \to 0} [0.2 / s]$

As $s \to 0$, the denominator goes to zero while the numerator is constant.

Therefore:

Steady-state ramp error = Infinite

The controller includes a zero $(s + 1)$.

From the control cheat-sheet:

- Zeros affect transient response (rise time, overshoot)
- System type depends only on integrators

Since the zero does not introduce a pole at $s = 0$:

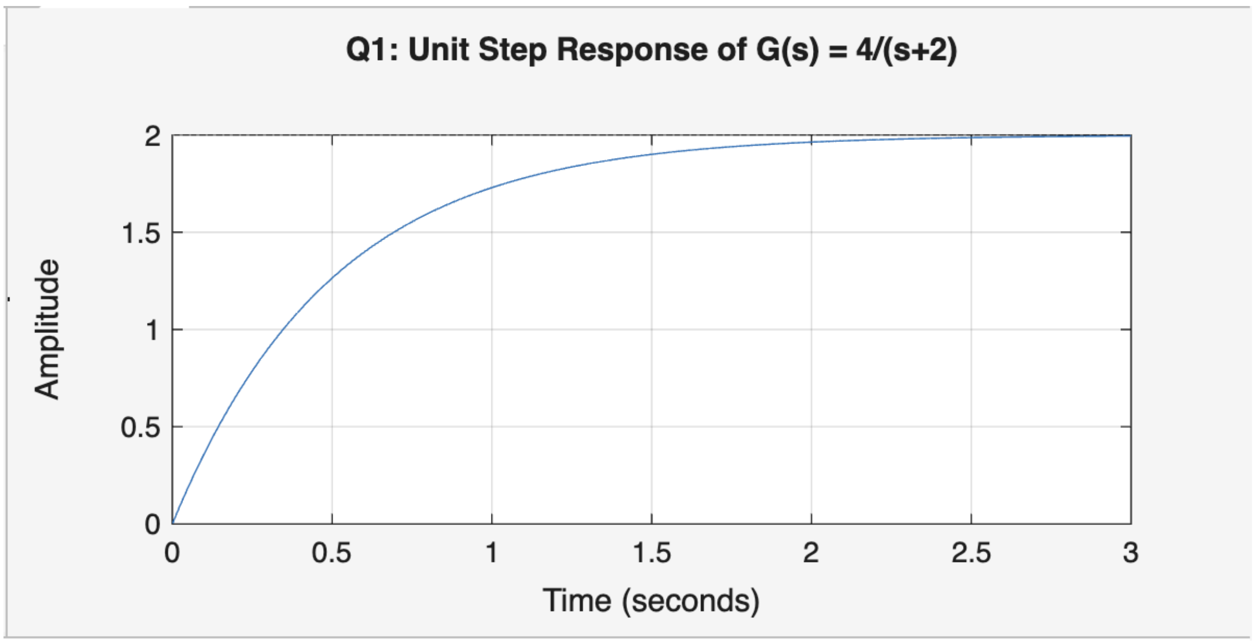
- System type does not change
- Ramp tracking does not improve

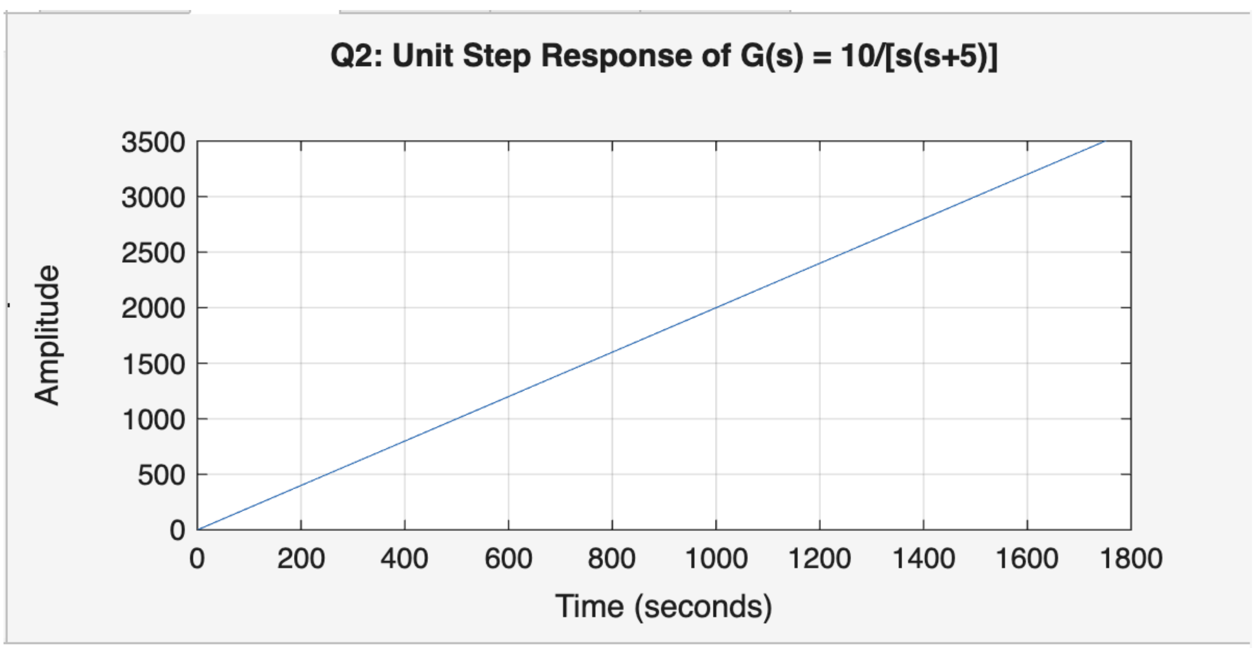
Final Conclusion

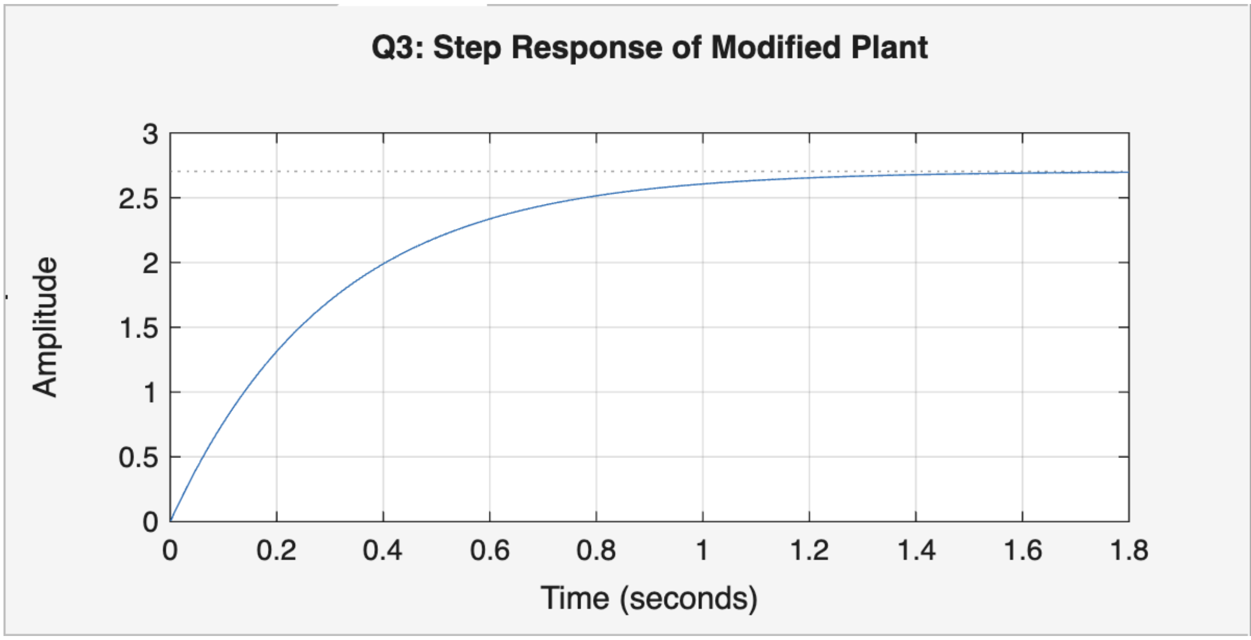
- The open-loop system has no integrators
- The system is Type-0

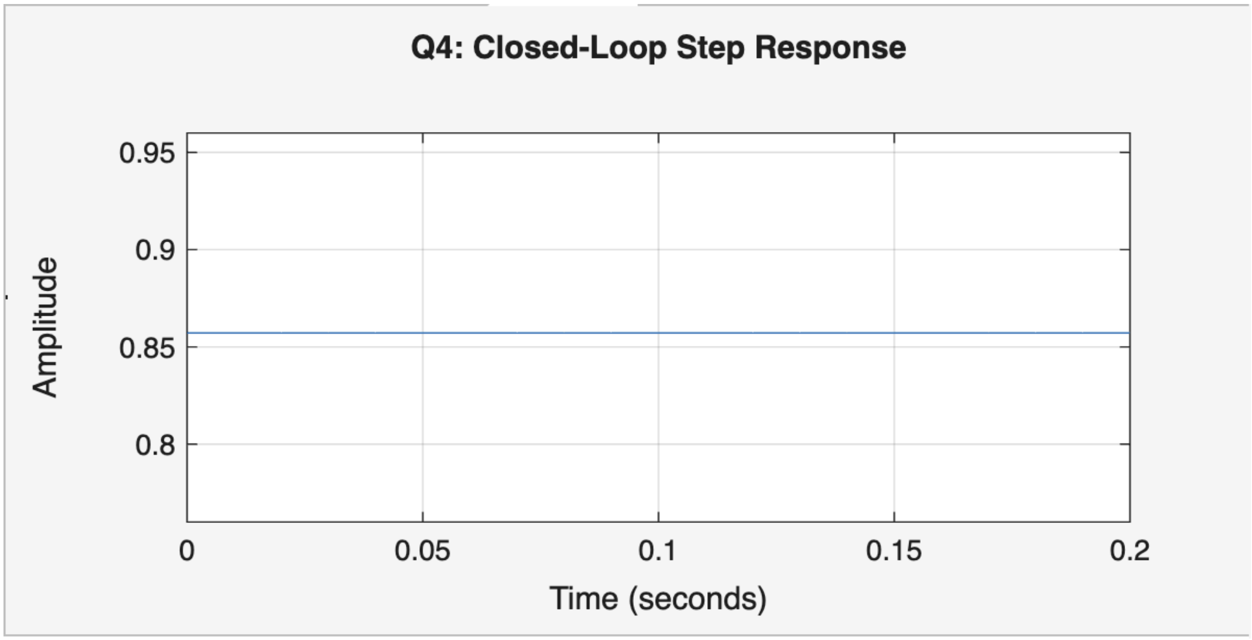
- Type-0 systems have infinite steady-state error for ramp inputs
- The Final Value Theorem confirms this result
- Adding a zero improves speed but does not help ramp tracking

Steady-state ramp error = Infinite.

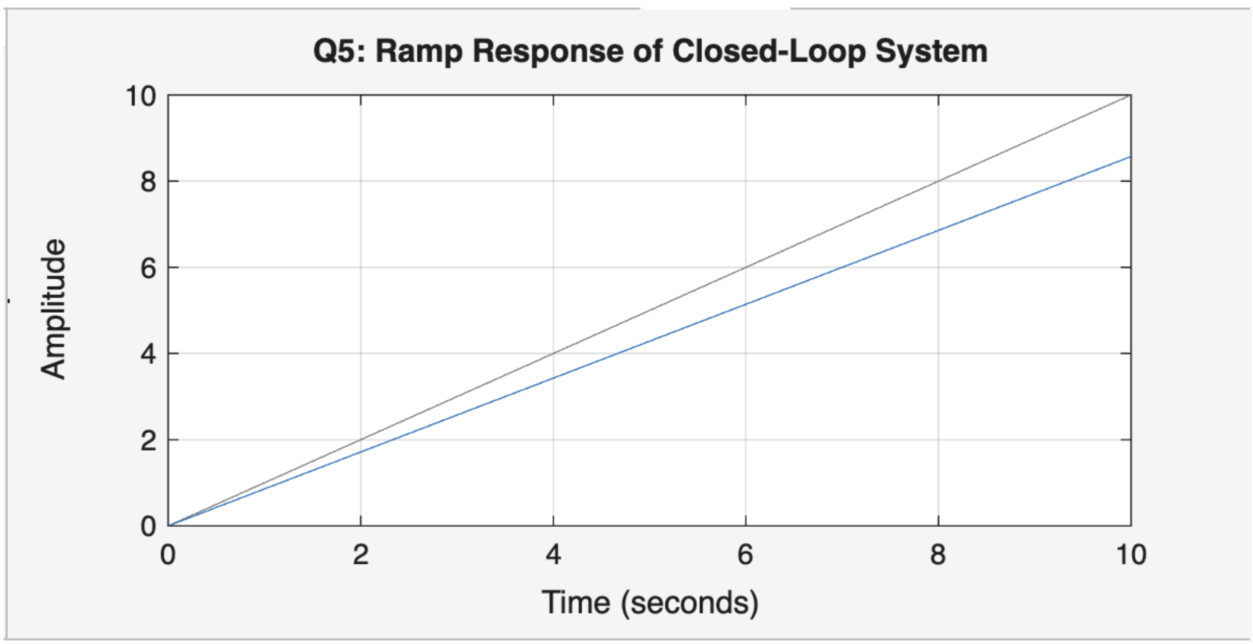





Q3: Step Response of Modified Plant
Amplitude
Time (seconds)
3
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8



Q4: Closed-Loop Step Response
Amplitude
0.95
0.9
0.85
0.8
0
0.05
0.1
0.15
0.2
Time (seconds)



Q5: Ramp Response of Closed-Loop System
Amplitude
10
8
6
4
2
0
0
2
4
6
8
10
Time (seconds)