FFT Decimation in Frequency (8-point sequence)

For N-point sequence: $X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}, \quad k = 0, 1, 2, \ldots, N-1$$

∴ Here we are using 8-point sequence

$$X[k] = \sum_{n=0}^{7} x[n]\, e^{-j\left(\frac{2\pi}{8}\right)kn} = \{X(0), X(1), \ldots, X(7)\}.$$

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$N = 8$ $\quad 2^3 \Rightarrow 3$, ∴ we have going to have 3 stages.

$$W_8^0 = e^{-j\left(\frac{2\pi}{8}\right)(0)} = 1$$

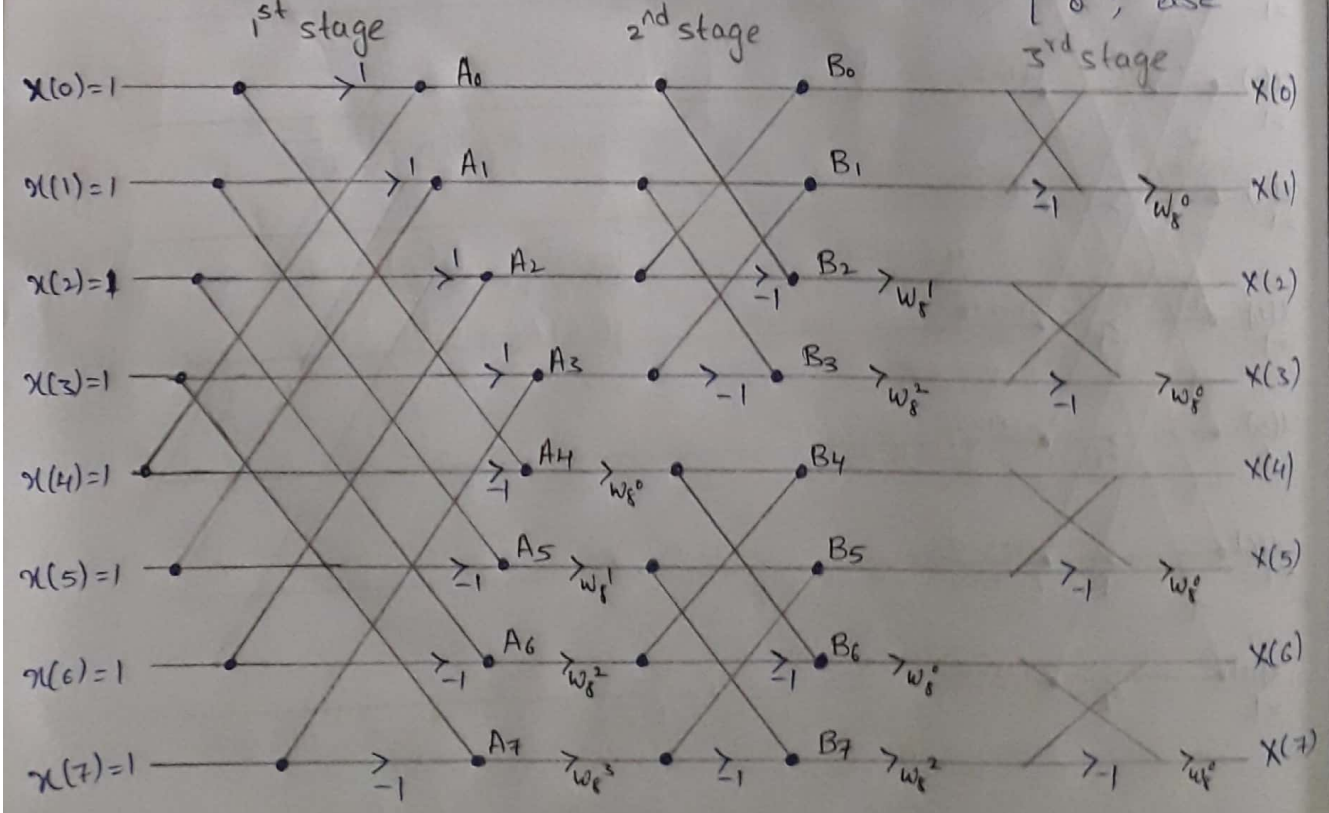
$$W_8^1 = e^{-j\left(\frac{2\pi}{8}\right)(1)} = e^{-j\pi/4} = 0.707 - j0.707$$

$$W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)(2)} = e^{-j\pi/2} = -j.$$

$$W_8^3 = e^{-j\left(\frac{2\pi}{8}\right)(3)} = e^{-j\frac{3\pi}{4}} = 0.707 - j0.707.$$

Butterfly Diagram

$$x(n) = \begin{cases} 1 & ; 0 \le n \le 7 \\ 0 & ; \text{else} \end{cases}$$



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o/p of 1st stage:-

$A_0 = x(0) + x(4) = 2$

$A_1 = x(1) + x(5) = 2$

$A_2 = x(2) + x(6) = 2$

$A_3 = x(3) + x(7) = 2$

$A_4 = [x(0) - x(4)] W_8^0 = 0$

$A_5 = [x(1) - x(5)] W_8^1 = 0$

$A_6 = [x(2) - x(6)] W_8^2 = 0$

$A_7 = [x(3) - x(7)] W_8^3 = 0$

2nd stage

$B_0 = A_0 + A_2 = 4$

$B_1 = A_1 + A_3 = 4$

$B_2 = [A_0 - A_2] W_8^0 = 0$

$B_3 = [A_1 - A_3] W_8^2 = 0$

$B_4 = [A_4 + A_6] = 0$

$B_5 = A_5 + A_7 = 0$

$B_6 = [A_4 - A_6] W_8^0 = 0$

$B_7 = [A_5 - A_7] W_8^2 = 0$

3rd stage :-

$x(0) = B_0 + B_1 = 8$

$x(4) = [B_0 - B_1] W_8^0 = 0$

$x(2) = [B_2 + B_3 = 0$

$x(6) = B_2 - B_3 = 0$

$x(1) = B_4 + B_5 = 0$

$x(5) = [B_4 - B_5] W_8^0 = 0$

$x(6) = B_6 + B_7 = 0$

$x(7) = [B_6 - B_7] W_8^0 = 0$

$\therefore x[k] = \{8, 0, 0, 0, 0, 0, 0, 0\}$

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