Homework 1: Control

1. (35 pts) Apply finite-dimensional numerical optimization to the differential drive vehicle for a length of time $T = 2\pi sec$ using $(x_d, y_d, \theta_d) = (\frac{4}{2\pi}t, 0, \pi/2)$ subject to the dynamics,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta)u_1 \\ \sin(\theta)u_1 \\ u_2 \end{bmatrix}, \quad (x(0), y(0), \theta(0)) = (0, 0, \pi/2).$$

   Use a semi-circle as an initial trajectory. You can get this by simulating the system forward using $[u_1(t), u_2(t)[= [1, -1/2]$. Hint: Use a tool like MATLAB's fmincon(), SNOPT, etc. **Turn in:** Plots of the initial trajectory, the final optimized trajectory, and the optimzied control signal.

2. (35 pts) Compute the control $u(t)$ that minimizes

$$J = \frac{1}{2}\int_0^{10} x^T \begin{bmatrix} 2 & 0 \\ 0 & 0.01 \end{bmatrix} x + u^T[0.1]u dt + \frac{1}{2}x(10)^T \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix} x(10)$$

   subject to the constraint that

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1.6 & -0.4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad x(0) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

   by solving the Two Point Boundary Value Problem. Evaluate the directional derivative of J at $(x_{sol}(t), u_{sol}(t)$ in 10 directions $\zeta(t) = (z(t), v(t))$ of your choosing. If you have found the optimizer, these values should be ≈ 0 regardless of the directions you choose. Make a table of the directions and values of the derivative. Hint: A direction you might take is $A\ sin(Bt + C) + D$. **Turn in:** Plots of the resulting $x(t)$ and $u(t)$ and your table of the directional derivatives.

3. (30 pts) Compute the control $u(t)$ that minimizes

$$J = \frac{1}{2}\int_0^{10} x^T \begin{bmatrix} 2 & 0 \\ 0 & 0.01 \end{bmatrix} x + u^T[0.1]u dt + \frac{1}{2}x(10)^T \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix} x(10)$$

   subject to the constraint that

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1.6 & -0.4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad x(0) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

   by solving the Riccati Equation. **Turn in:** Plot of the *difference* between $x(t)$ computed this way and $x(t)$ computed using the TPBVP as well as the *difference* between $u(t)$ computed this way and $u(t)$ computed using the TPBVP. Make sure to scale your vertical axis appropriately!