Homework 4: Information

1. (50pts) Compute the ergodic metric of a trajectory of $\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -b \end{bmatrix} x$ with respect to the normally distributed distribution $\phi(x) = \det(2\pi\Sigma)^{\frac{-1}{2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right) = \mathcal{N}(x;\mu,\Sigma)$ for $\mu = 0$, $\Sigma = Diag(2,2)$, $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, a time horizon of $T = 100$ s, and $b = 0$. **Turn In:** A plot of the ergodic metric as a function of b. Is there a most ergodic choice of b? What is the most ergodic choice if you can choose both b and the time horizon T?

2. (50pts) Implement infotaxis for the localization problem of a door. (We will say it is a Tardis or whatever type of magical/science fiction door you like that does not have walls around it, but still does something really cool.) In the figure below, D is the location of the door and the values in the rectangles around the door denote the probability of measuring a 1 in those locations. For all other locations, the probability of measuring a 1 is 1/100.

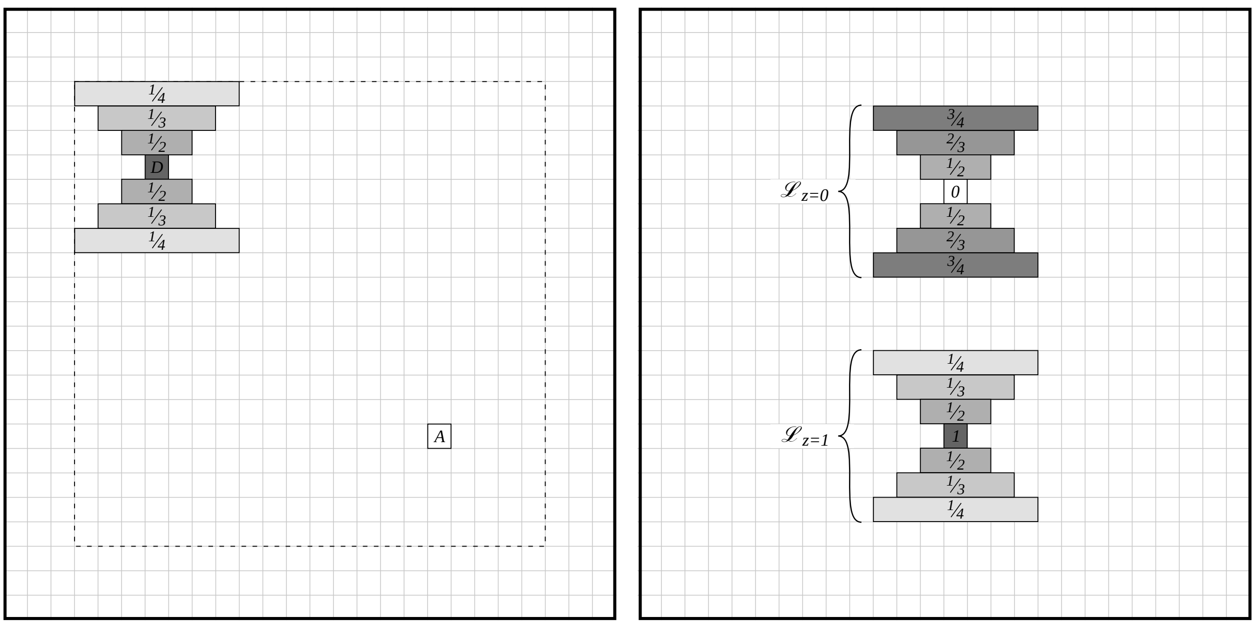

Figure 32: Left: Infotaxis problem setup. Right: **(Hint)** This model results in a likelihood function that looks like the upper example if $z = 0$ is measured and the lower example if $z = 1$ is measured. For all other squares in the grid, the likelihood of measuring $z = 1$ is 1/100 and the likelihood of measuring $z = 0$ is 99/100.

Turn In: Four plots of trajectories generated using Infotaxis when you start from 4 randomly chosen locations of door and agent within the 25×25 grid.