ME 455 – Active Learning

Homework 3, due date: May 8, 2023.

Joris Chomarat, jorischomarat2024@u.northwestern.edu

Problem 1

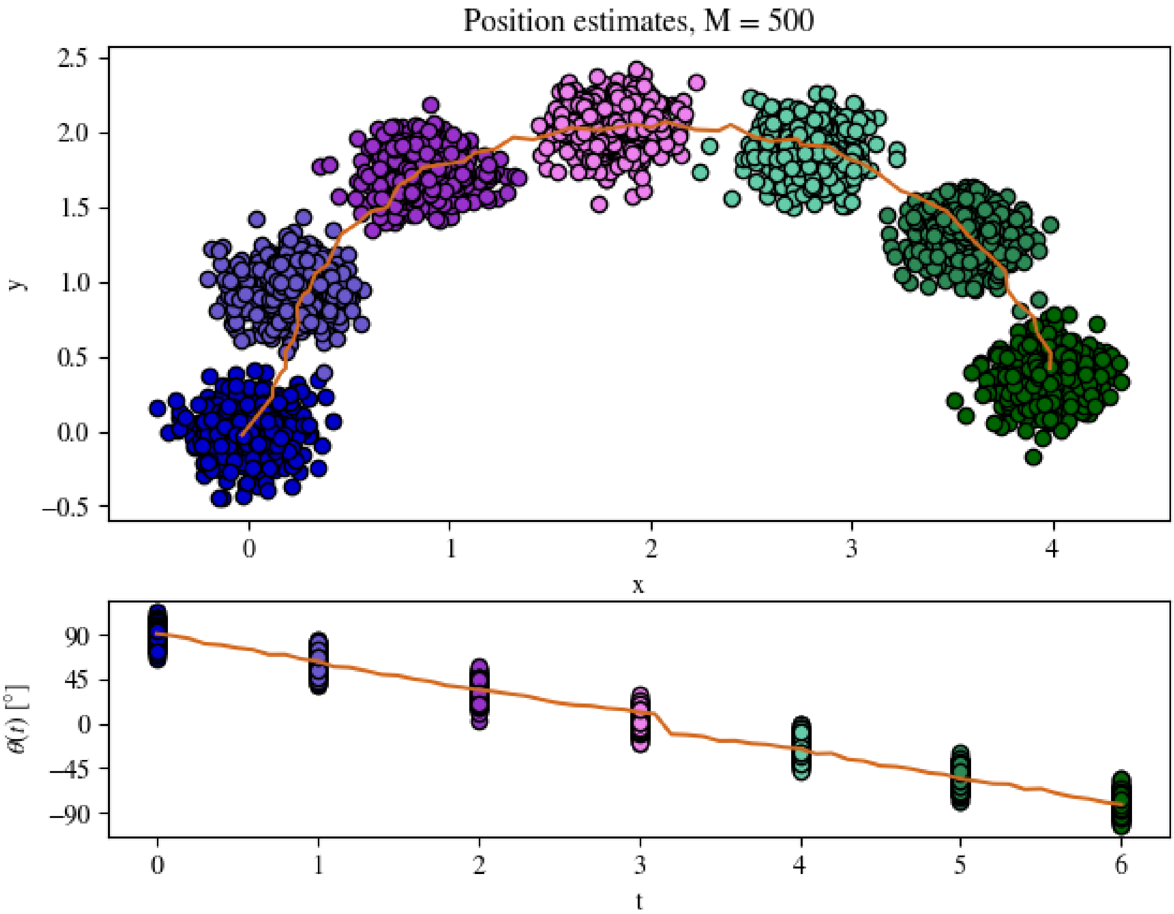

Figure 1: Particle filter, implemented for the semi-circle trajectory with process and measurement noise with a variance of 0.02. 500 measurements are collected at each time step, the particles being displayed every 1 second.

Problem 2

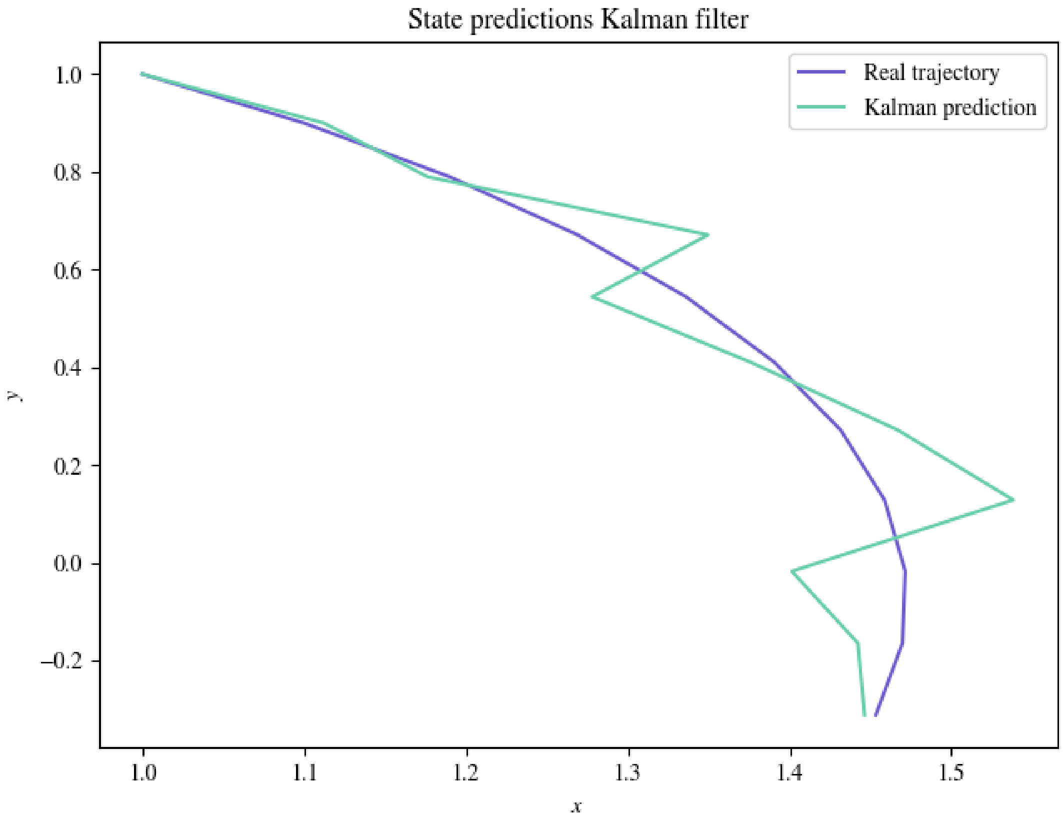

Figure 2: Evolution of the state prediction using a Kalman filter, compared to the real trajectory. Measurement and process noise with a variance of 0.1, time steps are of $dt = 0.1$ up to a total time of $T = 1$.

The following matrices represent the evolution of the covariance of the state prediction:

$$
\begin{aligned}
\Sigma(t = 0) &= 0.100 \times \mathbb{1} \\
\Sigma(t = 0.1) &= 0.067 \times \mathbb{1} \\
\Sigma(t = 0.2) &= 0.075 \times \mathbb{1} \\
\Sigma(t = 0.3) &= 0.080 \times \mathbb{1} \\
\Sigma(t = 0.4) &= 0.083 \times \mathbb{1} \\
\Sigma(t = 0.5) &= 0.086 \times \mathbb{1} \\
\Sigma(t = 0.6) &= 0.088 \times \mathbb{1} \\
\Sigma(t = 0.7) &= 0.089 \times \mathbb{1} \\
\Sigma(t = 0.8) &= 0.090 \times \mathbb{1} \\
\Sigma(t = 0.9) &= 0.091 \times \mathbb{1} \\
\Sigma(t = 1) &= 0.092 \times \mathbb{1}
\end{aligned}
$$

Problem 3

The 10 "nearby" filters were chosen by multiplying the Kalman gain with a constant $\neq 1$. Figure 3 shows the chosen gains along the x axis, and the respective average error of the filter for 100 sample paths of 1 second duration ($dt = 0.1$) on the y axis.

The Kalman filter indeed seems to be the optimal filter, gains smaller and larger than K all leading to increasingly greater average errors. The standard deviation increases as well, as the deviation from K becomes greater.

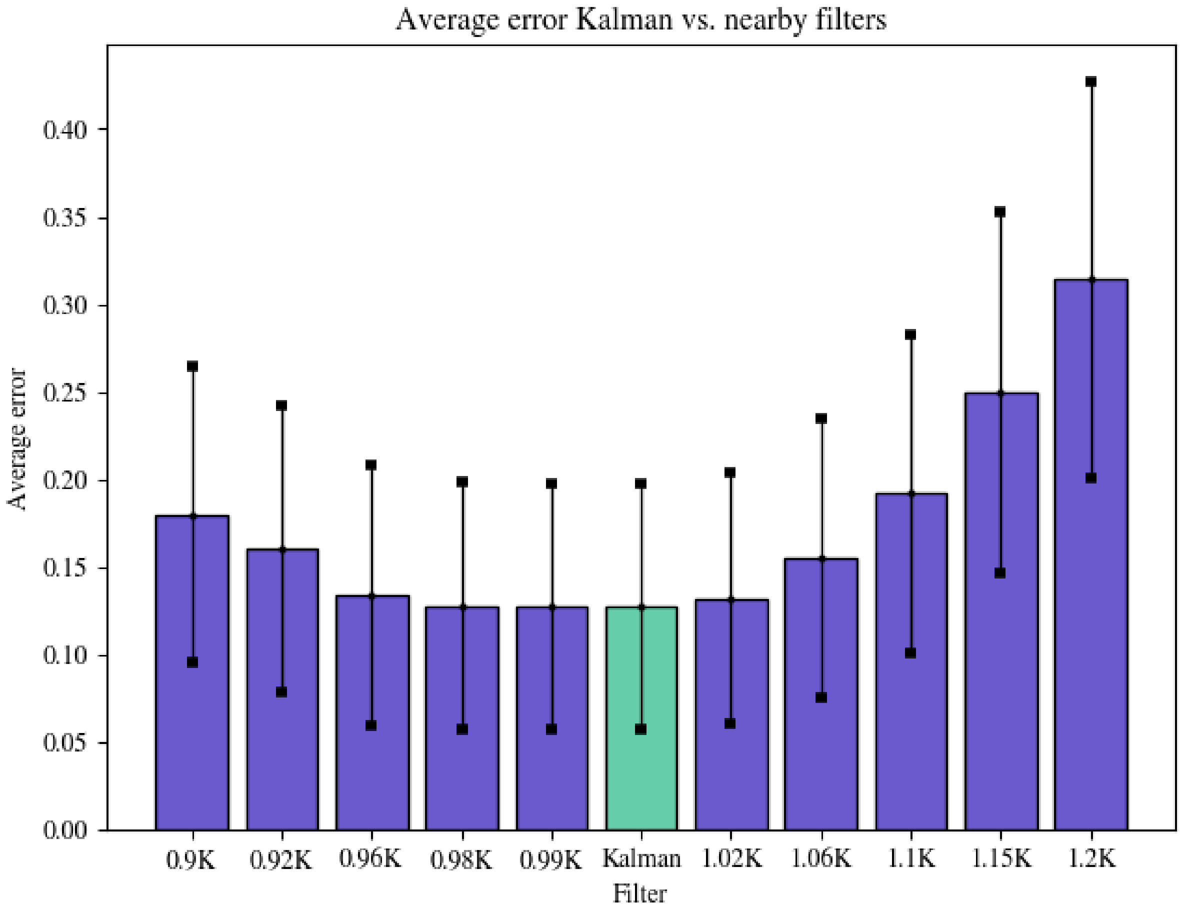

Figure 3: Average error and standard deviation of filter vs. real trajectory for the Kalman filter and a choice of 10 nearby filters.