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The Physics of Quantum Information

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Rapid ongoing progress in quantum information science makes this an apt time for a Solvay Conference focused on The Physics of Quantum Information. Here I review four intertwined themes encompassed by this topic: Quantum computer science, quantum hardware, quantum matter, and quantum gravity. Though the time scale for broad practical impact of quantum computation is still uncertain, in the near future we can expect noteworthy progress toward scalable fault-tolerant quantum computing, and discoveries enabled by programmable quantum simulators. In the longer term, controlling highly complex quantum matter will open the door to profound scientific advances and powerful new technologies.

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1. Introduction

This Solvay Conference on Physics provides a welcome opportunity to assess recent scientific progress and to reflect on the challenges and opportunities before us. Solvay Conferences had a stirring influence on advances in quantum physics during the 20th century, going back to the very first in 1911. Those advances transformed our understanding of nature, and also led the way to remarkable technologies such as lasers, atomic clocks, magnetic resonance imaging, and billions of transistors on a single microchip.

Such technologies, though undeniably impressive and impactful, barely scratch the surface of how quantum theory reshapes our view of what's possible in the universe. Now, for the first time in human history, we are developing and perfecting the tools to create and precisely control very complex states of many interacting particles, states so complex that we cannot efficiently simulate them with our most powerful existing computers or anticipate their behavior using currently known theoretical ideas. As our ability to control the quantum world matures, profound scientific discoveries and powerful technologies will surely ensue.

The rapidly unfolding developments in quantum information science make now a particularly apt time for a Solvay Conference on The Physics of Quantum Information. This topic encompasses four intertwined themes to be targeted in subsequent sessions: Quantum computer science,[1,2] quantum hardware,[3,4] quantum matter,[5,6] and quantum gravity.[7,8] For each theme I will provide some historical background, then comment on the current status and future prospects.

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2. Background

Modeling computation

The fundamental theory of computation builds on foundations erected by Turing in the 1930s.[9] Turing defined a computation in terms of an idealized physical process involving manipulation of symbols on a movable tape, and his model became widely accepted as a correct characterization of functions that are in principle computable in the physical world, an assertion known as the Church-Turing thesis.

A more refined notion, efficient computation, drew attention in the 1970s, igniting the theory of computational complexity.[10–12] It became accepted that a problem can be solved efficiently if the number of steps on a Turing machine scales like a polynomial in the size of the input to the problem, the so-called extended Church-Turing thesis. By broad consensus, these are the problems that are feasible to solve in practice. They are said to belong to a complexity class called P, for polynomial time.

Problems in the complexity class NP are those such that a solution, once found, can be efficiently verified by a Turing machine, and it's generally believed that NP contains hard problems that are outside P. It was noticed that a large family of problems, like combinatorial optimization problems, are in a class called NP-complete – these may be regarded as the hardest problems in the class NP.[10–12]

We believe that NP also contains problems outside P which are not NP-complete. Finding the prime factors of a large composite integer is a famous problem thought to be of this type.

As a practical application of complexity theory, public key cryptosystems were proposed in the 1970s, based on problems like factoring that are outside P but not NP-complete.[13,14] These schemes are heavily used today to protect the privacy of electronic communication, and are based on the presumption that a computation that could break the protocol is too hard to carry out in practice.

Quantifying information

Information theory builds on foundations erected by Shannon in the 1940s.[15] Shannon quantified the information conveyed by a message according to how much the message could be compressed to fewer bits without any loss of content. He also quantified how much information can be transmitted from sender to receiver through a noisy communication channel such that the information can be decoded with negligible probability of error by the receiver.

This theory led to the notion of an error-correcting code which can protect redundantly encoded information against the damaging effects of noise;[16] this in turn led to results establishing that computations can be performed reliably even when the computing hardware is imperfect.[17] Error-correcting codes are also vitally important in modern communication systems, like mobile phone cellular networks.

Quantum information

The origins of quantum information theory can be traced back to observations by Einstein and collaborators in the 1930s,[18] who noticed that correlations among parts of a quantum system can have counterintuitive properties, a phenomenon called "quantum entanglement" by Schrödinger.[19] John Bell formalized this notion in the 1960s[20] by establishing that players who share quantum entanglement can win a cooperative game with a higher success probability if they share entangled qubits as opposed to classically correlated bits.[21,22] In this sense quantum entanglement is a valuable resource that can be consumed to perform useful tasks.

In the 70s and 80s it was recognized that quantum communication, for example by sending photons through optical fiber or free space, can be advantageous in cryptography, as security can be based on principles of quantum physics rather than limitations on the computational resources available to potential adversaries.[23–25] The crucial principle is that unknown quantum states, in contrast to classical bits, cannot be copied accurately,[26,27] and in fact acquiring information about the content of quantum signals produces an unavoidable disturbance which is in principle detectable.

Also in the 70s, the general theory of measuring and processing quantum states was developed, including fundamental limits on how much classical information can be acquired when a quantum system is measured.[28,29]

Quantum computation

That properties of a complex highly correlated quantum system of many particles are hard to compute is an old observation already known to the pioneers of quantum mechanics. In the early 1980s, Feynman[30] and Manin[31] articulated the idea that properties that are hard to calculate with a conventional computer might be easy if we compute with a quantum device instead. This gave rise to a revision of the extended Church-Turing thesis, which in its revised form can be informally stated as, "A quantum computer can efficiently simulate any process that occurs in nature."[32] It is now widely believed, though not proven from first principles, that quantum computers have an exponential advantage over conventional computers for some problems, potentially including problems of interest in chemistry and materials science. That is, computations that can be performed in a time that scales polynomially with system size using a quantum computer require a time that scales exponentially using a conventional computer.

It was also found theoretically that quantum algorithms have a superpolynomial advantage over the best known classical algorithms for problems of interest in modern cryptography, such as finding the prime factors of a large composite integer.[33–35] In addition, it is known that quantum computers can speed up exhaustive search for a solution to a combinatorial optimization problem, but in that case the speedup is quadratic, meaning that the quantum time to solution scales like the square root of the classical time.[36,37]

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What is a quantum computer?

A mathematical model of an ideal quantum computer was formulated, the quantum circuit model, which has these five essential ingredients.[38] (1) A physical system harboring many qubits, such that the qubit number can be scaled upward as needed to solve problems of increasing size. (2) The ability to prepare simple standard initial states of the qubits, in effect to cool the system to a state with low entropy. (3) A universal set of entangling quantum operations, called quantum gates, each acting on two or more qubits, universal meaning that by composing many such gates in succession we can approximate arbitrary unitary transformations acting on many qubits. (4) A classical computer that efficiently translates a problem into a suitable circuit of quantum gates. And (5) the ability to measure qubits in a standard basis to read out classical bits providing the result of the computation. The efficiently solvable problems are those that can be solved with high success probability using a number of quantum gates that scales polynomially with the problem's input size.

Other physically reasonable models of quantum computation were also studied, such as the topological[39,40] and adiabatic[41,42] models, and shown to be equivalent to the quantum circuit model, thus lending further support to the quantum version of the extended Church-Turing thesis.

One should appreciate that all the features of the quantum circuit model can be simulated by an conventional classical computer, if equipped by a random number generator to capture the nondeterministic nature of the final quantum measurement. All the classical computer needs to do is keep track of a vector in a Hilbert space as we act on the vector with a sequence of matrices. For the final readout, it projects the vector onto a standard set of axes, and assign probabilities to the different measurement outcomes accordingly. Since a (randomized) classical computer can do whatever a quantum computer does, there is no difference in *computability* — whatever is computable by a quantum computer is computable by a classical computer as well.

The important distinction between the quantum and classical models is all about *efficiency*. In general, for the classical computer to simulate the quantum computer, it has to deal with vectors in a space whose dimension is exponential in the number of qubits. For the hardest problem instances, all known classical methods for doing this simulation require resources that scale exponentially with the number of qubits.

Quantum hardware

After the discovery of Shor's factoring algorithm in 1994, interest in quantum computing exploded, igniting pursuit of possible approaches to constructing hardware that could meet the five criteria enumerated above, at least to a reasonable approximation. And it was noticed that some technologies that were already being developed for other reasons could be adapted for the purpose of coherent quantum information processing.

For example, motivated by the quest for more precise clocks, tools had been

developed for cooling and manipulating individual electrically charged atoms using laser fields, which led to ion-trap quantum processors. [43,44] Josephson junctions, nonlinear elements in superconducting circuits which were being used in high precision magnetometers, led to superconducting quantum processors. [45–47] Experience with nanoscale electrical circuits resulted in the ability to isolate and manipulate spins of single electrons. [48,49] High efficiency sources and detectors for single photons opened the possibility of processors based on photonics. [50] Methods for trapping and cooling neutral atoms led to tunable simulations of strongly interacting quantum matter. [51,52] Later, optical tweezers provided an opportunity to build programmable simulators based on arrays of highly excited neutral atoms. [53–56] These and other approaches to quantum hardware are still being developed and are steadily advancing.

Currently the two most advanced quantum computing technologies are ion traps [3,57] and superconducting circuits. [4,58] In an ion trap, each qubit is a single electrically charged atom, which can be in either its ground state or a long-lived excited state. Tens of qubits can be stored in a linear array, and the state preparation, readout, and single-qubit quantum gates can all be achieved by addressing an ion with a stable laser. To perform entangling two-qubit gates, one manipulates the normal modes of vibration of ions in the trap using laser fields — a two-qubit gate can be executed on any pair of ions in tens of microseconds.

To scale up to larger systems, one envisions modular trapping regions, connected together by optical interconnects or by shuttling ions from one trapping region to another.

In a superconducting quantum computer, of order 100 qubits can be arranged in a two-dimensional array, with nearest-neighbor couplings among the qubits. These qubits, called transmons, are in effect artificial atoms that must be carefully fabricated and frequently calibrated. One reads out a transmon by coupling it to a microwave resonator, and single-qubit quantum gates are executed by addressing the qubit with microwave pulses. Two-qubit gates can be performed by various means, for example by tuning the frequencies of a pair of qubits into and out of resonance, or by driving one qubit at the frequency of another. A two-qubit gate takes tens of nanoseconds.

For scaling up to larger systems, one must address the challenge of dealing with many microwave control lines, and to improve gate fidelities, better materials, fabrication quality, and possibly alternative qubit designs would all be helpful.

By now, quantum processors have advanced to the stage where they can perform tasks that are challenging to simulate using classical computers. In particular, one can sample from the output probability distribution of randomly chosen circuits with 60 qubits and over 20 cycles of entangling two-qubit gates. [59–61] Though this specific task is not of intrinsic practical interest, such experiments have been useful, by providing new benchmarks for circuit fidelity, solidifying our understanding of the global features of circuit noise, and provoking improvements in classical simulation

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methods.[62]

Quantum error correction

When Shor's algorithm was discovered, and interest in quantum computing surged, there was widespread and understandable skepticism regarding whether large-scale quantum computing would ever be practical.[63–65] Quantum systems have the inconvenient property that observing a quantum state inevitably disturbs the state in an uncontrolled way, so interactions with the environment cause quantum information to decay rapidly, a phenomenon called decoherence. To execute a quantum computation reliably we must keep the information we process nearly perfectly isolated from the outside world to prevent decoherence, which is quite difficult because our hardware can never be perfect.

It was quickly discovered that, at least in principle, hardware imperfections can be overcome with suitable software based on what we call quantum error-correcting codes.[66–69] The crucial idea is that we can protect quantum information by storing it nonlocally, encoding it in a very highly entangled form such that when the environment interacts with the parts of the system locally, it acquires negligible information about the encoded quantum state and so need not damage the state. Furthermore, we learned how to process efficiently quantum information that is encoded in this highly entangled way.[70] It follows that, if errors in a quantum computer are sufficiently rare and not too strongly correlated, we can simulate an ideal quantum computation efficiently using a noisy quantum computer.[71–77]

The most promising protocol for error-corrected quantum computing in the relatively near term is based on Kitaev's surface code,[78] which has two advantages: it can tolerate a relatively high physical error rate,[79–81] and it requires only geometrically local processing in a two-dimensional layout. Even so, the overhead cost of error correction, in both the number of physical qubits needed and the number of physical gates, is quite daunting. One can plausibly anticipate running algorithms for a few hundred protected logical qubits that would surpass the best conventional computers for some problems of practical interest, but to achieve sufficiently reliability the number of physical qubits might be in the millions. That's a big leap from the devices we expect to have in the next few years, with hundreds of physical qubits.

Quantum matter

Deep connections between quantum information and quantum matter emerged with the discovery of topological order[82] (initially in fractional quantum Hall systems[83,84]), which we now recognize as a manifestation of long-range entanglement in a quantum phase of matter.[85] By long-range entanglement, we mean that the time needed to prepare the quantum phase, using spatially local operations in a quantum computer and starting with an unentangled state, scales with the total

size of the system. Furthermore, topologically ordered phases of matter may be fruitfully viewed as quantum error-correcting codes which conceal nonlocally encoded quantum information.[78] Symmetry-protected topological phases were also discovered,[86–88] for which the time to prepare the quantum state scales with system size if all local operations in the quantum circuit are required to satisfy specified symmetries.

It was discovered that ground states of quantum systems often obey an entanglement "area law," meaning that the amount of entanglement between the particles inside and outside of a specified ball-shaped region scales not like the total number of particles in the region, but rather like the number of particles near the boundary of the region.[89–91] This led to new methods for simulating quantum many-body systems on classical computers based on tensor networks, which exploit this entanglement structure to improve substantially on previous methods.[92–95] And it was noticed that entanglement, when quantified by the entropy of the marginal quantum state for the particles in a region, has universal properties that can be used to identify distinct quantum phases of matter.[96–98]

The computational hardness of preparing quantum ground states was studied, and it was argued convincingly that in some cases this is a hard problem for quantum computers;[99] this hardness can persist even for translationally invariant one-dimensional systems,[100] though admittedly such computationally intractable quantum many-body systems may have exotic interactions that are not necessarily of practical physical interest. At any rate, according to the quantum extended Church-Turing thesis, these ground states that are intractable for quantum computers could not arise in nature by any feasible physical process.

Quantum information has also provided a fresh perspective on the behavior of strongly chaotic quantum systems, which we now view through the lens of entanglement dynamics.[101] Information that is imprinted locally on a quantum system quickly spreads, becoming encoded in the form of quantum entanglement shared by many particles, and hence invisible to local observers who have access to just a few particles at a time. This entanglement spreading can be efficiently simulated by quantum computers,[102] but is beyond the reach of known classical computational methods, which cannot succinctly encode or efficiently simulate highly entangled many-particle quantum states.

Quantum gravity

The connection between quantum gravity and quantum information can be traced back to Hawking's 1974 discovery that due to quantum effects black holes emit thermal radiation, arising because of quantum entanglement between the inside and outside of the black hole's event horizon.[103] This led to a quantitative relationship between the area of the event horizon and the black hole's entropy,[104] a measure of how much quantum information the black hole can store. These results anticipated the area law for entanglement entropy in condensed matter physics that

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would be discovered years later. Furthermore, the entropy of a black hole is astonishingly large — for example, the entropy of a solar mass black hole, which is just a few kilometers across, is 20 orders of magnitude larger than the entropy of the sun. Indeed black holes, though remarkably simple objects as described by classical gravitation theory,[105,106] are quantum-mechanically the most complex objects nature allows, as quantified by the black hole's information storage capacity.

Holographic duality, discovered in the 1990s, established that, at least in negatively curved anti-de Sitter space, quantum gravity in bulk quantum spacetime is equivalent to a nongravitational quantum field theory in one lower dimension that resides on the boundary of the spacetime.[107] And it turned out that the bulk geometry is encoded on the boundary by the structure of the quantum entanglement in the boundary theory.[108] Furthermore, the holographic dictionary which maps local bulk observables to the corresponding highly nonlocal observables on the boundary was recognized as the encoding map of a kind of quantum error-correcting code.[109,110] So we can regard the geometry of spacetime itself as an emergent feature arising from underlying quantum entanglement, which is intrinsically robust with respect to some deformations of the boundary theory.

The entanglement dynamics of black holes was studied, and it was conjectured that black holes are the most efficient scramblers of quantum information that nature will allow.[111,112] Here, too, studies of information scrambling that originated in studies of black hole physics stirred growing interest in how information becomes scrambled in other quantum many-body systems that are more accessible in the laboratory.[113–115]

The entropy of the Hawking radiation emitted by an evaporating black hole, which tracks the evolving quantum entanglement of the radiation with the hole, was studied quantitatively,[116,117] and calculations confirmed that the entropy evolves as expected if the evaporation process is correctly described by unitary quantum theory.[118] Rather unexpectedly, this unitary behavior can be captured by semiclassical computations without reference to the microscopic details of quantum gravity. Such results indicate that black hole physics is profoundly nonlocal; one can in principle access the black hole interior by manipulating radiation that is far away, but only by performing quantum operations which are of such high computational complexity as to be infeasible in practice.[119]

Connections

The discussion so far has already illustrated the many cross-connections among the scientific themes that are represented at this conference. For example, information scrambling is now studied in quantum computing circuits, in chaotic many-particle systems, and in black holes. Quantum error correction, introduced for the purpose of extending quantum computing to large systems, is also relevant to topological phases of matter, and to the holographic correspondence in quantum gravity. Computational complexity, the study of the hardness of computational problems, turns

out to be relevant to preparation of topological quantum phases of matter, and also to the geometry of the black hole interior. These are a few examples among many such connections.

3. Status and prospects

Where are we now?

Coming back to quantum computing technology, what is the status today? There are two central questions about quantum computing, which could already have been articulated 40 years ago. How will we scale up to quantum computing systems that can solve hard problems? And how can we best make use of that computational power in science and in industry? In my view, both questions are wide open. How we direct our effort should be guided by that realization.

One may ask, what should we do with the noisy intermediate-scale quantum computers that we have now?[120] Two obvious answers are: We should use near-term quantum computers to learn how to build more powerful quantum computers that can have a practical impact. And we should seek a clearer understanding of how those practical quantum computers can eventually be used.

Even if broadly useful quantum computers are still a ways off, much can be accomplished over the next five years or so. In that time frame we can expect to see encouraging progress toward scalable fault-tolerant quantum computing. And we can anticipate scientific discoveries enabled by programmable quantum simulators and circuit-based quantum computers.

Progress toward quantum error correction

What would constitute notable progress toward fault-tolerant quantum computing? We need to be able to do repeated rounds of accurate error syndrome measurement for quantum error correction. And we would like to see concrete evidence that quantum memory times continue to improve sharply as we include more and more physical qubits to encode each protected logical qubit.

The ion trappers could justifiably protest that they don't care very much about quantum memory times, because their atomic qubits already have extraordinarily long lifetimes. That's true. But for all currently foreseen platforms it is crucial to achieve much higher fidelity for entangling two-qubit logical gates — only then will we be able to run powerful quantum algorithms. What we may hope to see in the near term are logical two-qubit gates, protected by quantum error correction, with much higher fidelity than our best physical two-qubit gates, as well as solid evidence that logical gate fidelities continue to improve sharply as the code block increases in size. That has not yet been accomplished.

What is the status now? There has been exciting recent progress toward quantum error correction; I'll highlight two contributions, one from Google and one from

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Honeywell (now called Quantinuum).[a]

Google investigated the quantum repetition code, using up to 21 qubits in the Sycamore processor, 11 in the code block and 10 ancilla qubits for error syndrome readout.[121] Importantly, this is not a full-fledged quantum error-correcting code — it protects against dephasing errors but not against bit flips. Nonetheless, it was an impressive demonstration. They did up to 50 consecutive rounds of syndrome measurement, each taking about 1 microsecond, with most of that time devoted to resetting the ancilla qubits to prepare for the next round of syndrome measurement. They observed that the rate of logical errors due to dephasing decreased by about a factor of 10 each time the code distance increased by 4, for example as the code length increased from 3 to 7 and from 7 to 11. That was in accord with expectations given the noise in their device.

Quantinuum demonstrated error correction for a 7-qubit code that can correct an arbitrary error acting on any one qubit out of the 7.[122] They did up to 6 consecutive rounds of error correction, each taking 200 milliseconds. Note the much different cycle times for the superconducting and ion trap devices. As quantum computing advances, and the time on the wall clock for running an algorithm becomes an increasingly important consideration, that difference may loom large. Quantinuum uses an architecture in which ions are transported to processing zones where rather high fidelity operations can be performed in parallel, and after movement they use another ion species to sympathetically cool the motional state. That cooling enables them to do the repeated rounds of syndrome measurement, but also accounts for much of the time budget of their circuit.

Unfortunately, gate error rates in the Google and Honeywell machines, and other current devices,[123,124] are still too high for quantum error correction to improve two-qubit logical gate fidelities.

Fault-tolerance with the surface code

The best currently known prospect for scaling up quantum computing in the relatively near term is based on the surface code, introduced by Alexei Kitaev over 25 years ago.[78] As already noted, the two great virtues of the surface code are that error syndromes can be extracted using only geometrically local processing in a two-dimensional layout, and that each syndrome bit can be read out using a simple quantum circuit involving only four data qubits. As a result, the surface code can tolerate higher error rates than other feasible quantum codes.[79–81]

Despite being more effective than other codes, error correction with the surface code still carries a rather hefty overhead cost in the number of qubits and gates needed. Let's suppose we can do physical controlled-NOT gates with an error rate of .1%. That's better than we have now in multiqubit devices, but might

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[a]This discussion reflects progress that had already been reported at the time of the Solvay Conference in May 2022.

plausibly be reached in the near future. Perhaps we will start to see quantum advantage using circuits with hundreds of protected qubits and millions of high-fidelity quantum (Toffoli) gates. To execute those circuits, we're likely to need at least tens of thousands of physical qubits. For breaking public key cryptosystems using Shor's algorithm, it is estimated that 20 million physical qubits are needed.[125] If we can somehow do controlled-NOT gates with four 9s of fidelity, an improvement by another order of magnitude, that will reduce the overhead cost, but we're still likely to want at least hundreds of physical qubits per logical qubit to see significant quantum advantage running algorithms we currently know about. These numbers are surely daunting from the perspective of currently available technology.

In an exciting recent development, quantum codes have been discovered that are far more efficient than the surface code.[2,126,127] Someday we might use these codes to reduce significantly the overhead cost of fault-tolerant quantum computing. As best we currently understand, though, to perform well these codes require much lower physical error rates than the surface code, and so are not likely to be useful until much better quantum hardware is available.

Much better gate error rates?

It would pay off handsomely to have much improved physical gate error rates in quantum hardware, but that's very hard to achieve. A particularly visionary proposal is topological quantum computing, where qubits are encoded in an exotic material that provides physical protection against noise.[128] High fidelity topologically protected quantum gates, if realized, would be a genuine milestone for quantum many-body physics, aside from any implications for future information technologies. Though the theoretical idea is compelling, up to now experimental progress has been slow.[129]

There are other potential ways to incorporate better protection against noise into the hardware itself. Some promising ideas exploit precise manipulation of bosonic modes, such as microwave resonators in superconducting circuits, harmonic motion of trapped ions, or optical modes in photonic devices. For example, GKP-encoded states of bosonic modes have a periodic grid structure in phase space, allowing slight shifts in phase space to be corrected.[130–132] Bosonic cat codes use superpositions of coherent states to provide strong protection against bit flips, resulting in highly biased physical noise that can be corrected by quantum codes at a reduced overhead cost.[133–135] Fluxonium qubits[136,137] and zero-pi qubits[138,139] use strong nonlinearity resulting from large inductance in a superconducting circuit to suppress noise. In the arena of superconducting qubits, all of these schemes are more complex than the relatively simple transmon; they are still at a comparatively early stage, and we can't say yet how they'll pan out. But it is important to continue pursuing these and other challenging approaches offering the potential for a leap forward in performance, because significantly lower physical gate error rates will bring us closer to useful applications of quantum computation.

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Creating quantum states of matter

The quantum technology we already have is exciting, as it provides new tools for exploring the physics of many entangled particles. On this front, too, we've seen significant recent progress, including unprecedented studies of new quantum phases of matter. I'll highlight two examples.

The Harvard/MIT group, using a Rydberg atom platform with 219 qubits, recently created and detected a novel highly entangled phase of quantum matter, a quantum spin liquid.[140] Theorists have speculated about quantum spin liquids for nearly 50 years,[141] but convincing experimental evidence for this type of quantum state had never been seen before, for two main reasons. First, one needs a material with suitable properties for qubits to seek a ground state with long-range quantum entanglement. In nature, such materials seem to be rare. Second, the features of a long-range-entangled state are very elusive to observe because one needs to make collective observations on many qubits at once. The Rydberg platform is highly programmable and sufficiently versatile to simulate the right kind of material. And one can measure nonlocal observables with sufficient fidelity to identify signatures of long-range entanglement.

Guided by university condensed matter physicists from Stanford, Princeton, the Max Planck Institute and elsewhere, 20 superconducting qubits in the Google Sycamore processor were used to create and observe a discrete time crystal.[142] This is a novel phase of matter in a periodically driven system, which oscillates indefinitely at a frequency different from that of the periodic drive. The idea of a time crystal had been suggested for the first time 10 years ago,[143] and there had been previous experiments which were partially successful in validating the phenomenon,[144–146] but the high fidelity gates and accurate single-qubit readout and control in Sycamore made a more convincing demonstration possible.

Take note of two things. First, five years ago Rydberg atoms were not so much on the radar screen of quantum platforms, yet now they are advancing rapidly. That reminds us that we are still in the early days of quantum technology, and big surprises may continue to arise. Second, the Google experiment was done on a gate-based quantum computer, while the Harvard/MIT experiment was done in a programmable analog mode. That reminds us that these two approaches to studying quantum matter are complementary, and both are valuable to pursue.

These are encouraging signs that we're acquiring tools to create and investigate a variety of other new quantum phases of matter in the near future, both in equilibrium like a quantum spin liquid, or driven far from equilibrium like a discrete time crystal. There are good reasons to be impressed by these developments. First because, of the applications of quantum computing that we currently foresee, those to materials and chemistry are the ones that seem to have the greatest potential to benefit humanity broadly speaking, and it's exciting that we already have tools in the current era that can advance our understanding of quantum matter. And second because studies of topological phases could ignite new approaches to quantum error

correction and fault tolerance that will pay off in the longer run. Looking ahead, we can glimpse opportunities to create states of matter beyond what is known to occur in nature, which can have both scientific and technological value.

Opportunities in quantum simulation

What will be the long term impact of quantum computing on society? No one knows that. Nor should we be expected to envision clearly how quantum computing might change the world. As I've said, of the applications we currently most clearly foresee, what we imagine is most likely to broadly benefit humanity are applications to chemistry and materials, which could improve human health, energy production, agriculture, and the sustainability of our planet.

Can we be more concrete about that expected impact? That's quite difficult for several reasons. We seek applications of quantum computing that meet three criteria. The problems of interest should be too hard to solve with conventional computing, efficiently solvable by quantum computers, and the solutions should be of scientific and/or practical value.

There are methods for simulating complex molecules and highly correlated materials using conventional computers which are actually rather good, and getting better fast, not just because conventional computers are becoming more powerful, but even more importantly because the classical algorithms are getting better.[5] For computing properties of ground states and other low-energy states, the classical methods are heuristic, without rigorous performance guarantees. But numerical evidence suggests that the resources needed to obtain accurate results using classical methods such as those based on tensor networks and neural networks scale reasonably with system size for typical molecules or materials that are of scientific interest, because these systems are not so profoundly entangled. If that's true, the advantage enjoyed by quantum computers may scale polynomially rather that exponentially for such problems.[147] And the competing quantum methods are also heuristic, because to obtain accurate results efficiently we must be able to prepare states in the quantum computer that have a substantial overlap with the targeted quantum states, which is not rigorously guaranteed. The general purpose method for performing the state preparation task is the adiabatic method, which can be very expensive in systems where there are multiple competing phases separated by first-order phase transitions, as is often true in cases of interest.

Exponential quantum advantage can be expected in quantum simulation of dynamics, if we consider easily prepared initial excited states which become highly entangled as they evolve, for example in highly inelastic collisions of fundamental particles in a quantum field theory.[148] What scientific opportunities that may entail is an issue worthy of further investigation.

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Challenges in quantum gravity

Circling back to quantum gravity, what are some of the challenges where we can realistically expect to make substantial progress in the reasonably near future?

In the case of quantum gravity in anti-de Sitter space, we still lack a good handle on why local quantum physics provides an excellent approximation on distance scales that are small compared to the scale of the spatial curvature. In addition, the spacetime we live in is not anti-de Sitter, and we need better tools for describing quantum gravity in spacetimes that are asymptotically flat or positively curved. Anti-de Sitter space has the convenient feature that spacetime has a boundary, and we can define the observables of the theory by making reference to that boundary. But de Sitter space, which is relevant to early-universe inflationary cosmology, does not have that convenient feature, which makes quantum gravity in de Sitter space intrinsically harder to think about.

Despite remarkable recent progress,[7,8] we don't have an adequate way, in quantum gravity, to describe the experience of an observer who falls into a black hole, and we especially don't know what happens to observers who encounter the singularity in the black hole interior.

Holographic duality is very empowering, but we have analytic control over how it works only in a limited number of special cases. Can we understand more systematically under what conditions a nongravitational boundary theory admits a holographic dual which is useful for describing quantum gravitational phenomena?

And can we learn more concretely what resources we'll need to simulate quantum gravity with quantum computers, and compute observable properties of scientific interest?

Quantum gravity: can experiments help?

Eventually, we might hope to make progress on some of these questions by using quantum computers and quantum simulators; in particular, by simulating strongly-coupled quantum many-body systems and leveraging holographic duality, we might probe the dual quantum geometry by measuring features of quantum entanglement on the boundary. We might, for example, learn about locality in the bulk spacetime though linear response measurements that yield information about the commutators of boundary observables. Studying the entanglement dynamics of strongly chaotic systems can reveal how quantum information gets scrambled, which might unveil signatures of string theory in the bulk. Or we might in other contexts be able to measure quantum corrections to semiclassical gravity that would be hard to compute analytically or by using classical computers. Simulations of very-high-energy scattering in the bulk could be especially instructive.

Perhaps guidance from simulations can help us to grasp holographic dual descriptions of spacetimes beyond anti-de Sitter space. And we may find that some otherwise opaque features of strongly-coupled dynamics can be more easily interpreted using the lens of bulk quantum gravity. One already much studied example

is a mysterious type of coherent quantum teleportation in the boundary theory which has a quite natural alternative interpretation in terms of quantum information transmitted through a traversable spatial wormhole in the bulk theory.[149–152]

4. Some things I haven't mentioned

There are some important things I have not yet had time to mention in this talk, of which I list four here.

The discovery of Shor's algorithm will have a disruptive effect on electronic commerce, as the public key cryptosystems we now rely on to protect our privacy will no longer be secure when powerful quantum computers are readily available. The world is responding by developing new classical cryptosystems that are widely believed to be resistant to attack by quantum computers.[153] It will be a necessary task, but a prolonged and expensive one, to deploy these new systems.

An alternative approach to protecting our privacy is to distribute secure private keys through quantum communication, presumably by sending photons through optical fiber or free space.[24,25] Here the security rests on principles of quantum physics, rather than assumptions about the computational power of our adversaries, and in fact there are protocols which are provably secure even if we don't trust the equipment we use to distribute the key.[154] It's not clear to what extent the world will demand quantum cryptography for secure communication; in any event, quantum key distribution on a global scale will require new technologies which are now nascent, like quantum repeaters to extend the range of quantum communication, which in turn are likely to rely on transduction of single-photon signals from optical to microwave frequencies and back.[155] As is the case for quantum computing, we still lack a clear understanding of what will be the most impactful future applications of quantum networking.

Advancing quantum technology will also enhance the sensitivity and resolution of sensors, which can be expected to have widespread applications including inertial sensors for navigation, gravity gradiometers for surveying, magnetometers for noninvasive nanoscale imaging of living mater, and many others.[156] There are also applications of fundamental interest, including looking for symmetry violations in searches for new physics, dark matter detection, detection of gravitational waves with enhanced sensitivity, and long-baseline optical interferometry enabled by quantum teleportation within a network of telescopes. These improvements will be based on advanced quantum strategies, which exploit squeezing, entanglement, and quantum error correction.

Another important issue is: how can we be sure that a quantum computation gives the right answer? In some cases, like when factoring a large number, the answer once found can be easily checked with a classical computer, but that's not the case if, for example, we are simulating the properties of a complex quantum many-body system. Yet clever protocols have been developed for verifying that a quantum computer really performs an assigned task; these leverage the power of

quantum-resistant cryptography.[157] One important challenge is to reduce the cost of these verification protocols so we can make use of them in the relatively near term, if for example we send a job to a quantum server in the cloud, and want to be confident that the answer we receive can be trusted.

5. Conclusions

To conclude, we might have a long road ahead to practical commercial applications of quantum computing, and quantum error correction is most likely the key to getting there eventually. But the next five years should be exciting, marked by progress toward fault-tolerant quantum computing and unprecedented opportunities to explore exotic properties of quantum matter.

As this conference has illustrated, The Physics of Quantum Information provides unifying concepts and powerful technologies for controlling and exploring complex many-particle quantum systems of both practical and fundamental interest. Communication among the practitioners of quantum computer science, quantum hardware, quantum matter, and quantum gravity sparks new ideas and insights, making life sweeter for all of us who investigate the elusive properties of highly entangled quantum systems.

For the longer term quantum science and technology face enormous challenges, and many advances in both basic research and systems engineering will be needed to fulfill our aspirations. We've only just begun.

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