Name Key Period__________ Score__________

AP Calculus AB **Chapter 4 More Derivatives Review**

1. Given $y = \frac{1}{2}(2x+5)^3$. Determine $\frac{dy}{dx}$.

(A) $\frac{3}{2}(2x+5)^3$ $\boxed{\text{(B) } 3(2x+5)^2}$ (C) $3(2x+5)$

(D) $\frac{3}{2}(2x+5)$ (E) $6(2x+5)$

$\frac{dy}{dx} = 3 \cdot \cancel{\tfrac{1}{2}}(2x+5)^2 \cdot \cancel{2}$

$\frac{dy}{dx} = 3(2x+5)^2$

$v(t) = y$

2. If $v(t) = \ln(t^2+t+1)$, then $v'(1) =$

(A) $\frac{1}{3}$ (B) $-\frac{2}{3}$ $\boxed{\text{(C) } 1}$ (D) $\frac{4}{3}$ (E) 3

$v'(t) = \frac{1}{t^2+t+1} \cdot 2t+1 = \frac{2t+1}{t^2+t+1}$

$v'(1) = \frac{2(1)+1}{(1)^2+(1)+1} = \frac{2+1}{1+1+1} = \frac{3}{3} = 1$

3. Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	$1/2$	-2	$3/2$	-1
1	$1/3$	1	$5/3$	$2/3$
2	1	$-1/2$	$1/4$	-4
3	-1	2	0	-3
4	3	$-1/3$	$-4/5$	$-1/3$

$2(3)-3$
$= 6-3$
$= 3$

If $p(x) = g(x) \cdot f(x) - g(2x-3)$, then $p'(3) =$

$p'(x) = g(x) \cdot f'(x) + f(x) \cdot g'(x) - g'(2x-3) \cdot 2$

$p'(3) = g(3) \cdot f'(3) + f(3) \cdot g'(3) - g'(2(3)-3) \cdot 2$

$p'(3) = \boxed{2 \cdot 0 + (-1) \cdot (-3) - (-3) \cdot 2} = 3+6 = \boxed{9}$

4. Find the **second** derivative of $f(x)$ if $f(x) = (2x^2 + 5)^3$

$f'(x) = 3(2x^2+5)^2 \cdot 4x = 12x(2x^2+5)^2$

$f''(x) = \boxed{12x \cdot 2(2x^2+5)\cdot 4x + (2x^2+5)^2 \cdot 12}$

$f''(x) = \boxed{96x^2(2x^2+5) + 12(2x^2+5)^2}$

Use the given information about differentiable functions $f(x)$ and $g(x)$ at $x = 1$ and $x = 2$ for problem 5.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	12	-8
2	$\sqrt{7}$	π	-9	10

5. Find $\frac{d}{dx}\{f(g(x))\}$ at $x = 1$

$= f'(g(x))\cdot g'(x)$

$= f'(g(1))\cdot g'(1)$

$= f'(2)\cdot(-8)$

$= \boxed{(-9)\cdot(-8)}$

$= \boxed{72}$

For problems 6 – 11, find the derivative. Do not leave negative or rational exponents in your answer.

6. $s(x) = \sqrt{\frac{4+x}{5-x}} = \left(\frac{4+x}{5-x}\right)^{1/2}$

★ can also be: $\boxed{\frac{9}{2\sqrt{4+x}\,(\sqrt{5-x})}}$

$s'(x) = \frac{1}{2}\left(\frac{4+x}{5-x}\right)^{-1/2}\cdot \frac{(5-x)(1)-(4+x)(-1)}{(5-x)^2}$

$\boxed{s'(x) = \frac{1}{2\sqrt{\frac{4+x}{5-x}}}\cdot\frac{(5-x)(1)-(4+x)(-1)}{(5-x)^2}} = \frac{1}{2\sqrt{\frac{4+x}{5-x}}}\cdot\frac{5-\cancel{x}+4+\cancel{x}}{(5-x)^2} = \boxed{\frac{9}{2(5-x)^2\sqrt{\frac{4+x}{5-x}}}}$

7. $y = \cos\left(\frac{1}{2}x\right) - \tan^2(2x) = \cos\left(\frac{1}{2}x\right) - (\tan(2x))^2$

$\boxed{y' = -\sin\left(\tfrac{1}{2}x\right)\cdot\left(\tfrac{1}{2}\right) - 2\tan(2x)(\sec^2(2x))\cdot 2}$

$y' = -\frac{\sin(\frac{1}{2}x)}{2} - 4\,\frac{\sin(2x)}{\cos(2x)}\cdot\frac{1}{\cos^2(2x)} = -\frac{\sin(\frac{1}{2}x)}{2} - \frac{4\sin(2x)}{\cos^3(2x)}$

$\boxed{y' = -\frac{\sin(\frac{1}{2}x)\cos^3(2x) + 8\sin(2x)}{2\cos^3(2x)}}$

★ This can go on forever, so I would stop after the first box.

8. $y = e^{5x-3} \cdot x^4$

$\boxed{y' = e^{5x-3} \cdot 4x^3 + x^4 \cdot e^{5x-3} \cdot 5} = 4x^3 e^{5x-3} + 5x^4 e^{5x-3}$

$\boxed{y' = x^3 e^{5x-3}(5x+4)}$

9. $y = \sin^{-1}(2-x)$

$\boxed{y' = \frac{1}{\sqrt{1-(2-x)^2}} \cdot (-1)} = -\frac{1}{\sqrt{1-(2-x)^2}}$

$y' = -\frac{1}{\sqrt{1-4+4x-x^2}} = \boxed{-\frac{1}{\sqrt{-x^2+4x-3}}}$

10. $y = 13^{-2x}$

$y' = 13^{-2x}(\ln 13)(-2) = (13^{-2})^x(\ln 13)(-2) = \boxed{\left(\frac{1}{13^2}\right)^x(\ln 13)(-2)}$

$y' = -2\ln 13 \cdot \left(\frac{1}{13^2}\right)^x = \boxed{-2\ln 13 \cdot \left(\frac{1}{169}\right)^x}$

11. $y = \log_4(3x+2)$

$\boxed{y' = \frac{1}{(3x+2)\ln 4} \cdot 3} = \frac{3}{(\ln 2^2)(3x+2)} = \boxed{\frac{3}{(2\ln 2)(3x+2)}}$

12. Let g be the function defined by $g(x) = x^5 + x$. If $f(x) = g^{-1}(x)$ and $g(1) = 2$, what is the value of $f'(2)$?

$g'(x) = 5x^4 + 1$

$g'(1) = 5(1)^4 + 1$

$g'(x) = 6$

$g(x)$	$g^{-1}(x)$ $f(x)$
$(1,2), m=6$	$(2,1), m=\frac{1}{6}$

$\boxed{f'(2) = \frac{1}{6}}$

*again, this could go on forever

13. If $\tan(x^2y) = 2x$, then $\frac{dy}{dx} =$

$$\frac{d}{dx}\left(\tan(x^2y)\right) = \frac{d}{dx}(2x)$$

$$\sec^2(x^2y)\cdot\left(x^2\frac{dy}{dx} + 2xy\right) = 2$$

$$x^2\sec^2(x^2y)\frac{dy}{dx} + 2xy\sec^2(x^2y) = 2$$

$$x^2\sec^2(x^2y)\frac{dy}{dx} = 2 - 2xy\sec^2(x^2y)$$

$$\boxed{\frac{dy}{dx} = \frac{2-2xy\sec^2(x^2y)}{x^2\sec^2(x^2y)}}$$

$$\frac{dy}{dx} = \frac{2}{x^2\sec^2(x^2y)} - \frac{2xy\,\cancel{\sec^2(x^2y)}}{x^2\,\cancel{\sec^2(x^2y)}}$$

$$\frac{dy}{dx} = \frac{2\cos^2(x^2y)}{x^2} - \frac{2xy}{x^2}$$

$$\frac{dy}{dx} = \frac{2\cos^2(x^2y) - 2xy}{x^2}$$

$$\frac{dy}{dx} = \frac{2\left(\cos^2(x^2y) - xy\right)}{x^2}$$

$$\boxed{\frac{dy}{dx} = 2\left(\frac{\cos^2(x^2y)}{x^2} - \frac{y}{x}\right)}$$

14. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$$\frac{d}{dx}(x^2+xy) = \frac{d}{dx}(10)$$

$$2x + x\frac{dy}{dx} + y = 0$$

$$x\frac{dy}{dx} = -y-2x$$

$$\frac{dy}{dx} = \frac{-y-2x}{x}$$

$$2^2 + 2y = 10$$
$$4 + 2y = 10$$
$$2y = 6$$
$$y = 3$$

$$\left.\frac{dy}{dx}\right|_{(2,3)} = \boxed{\frac{-3-2(2)}{2}}$$

$$= \frac{-3-4}{2} = \boxed{-\frac{7}{2}}$$

15. What is the slope of the line tangent to the curve defined by $x^4 - 3x^2y^2 + 4y^2 = 5$ at the point (1,2)?

$$\frac{d}{dx}(x^4-3x^2y^2+4y^2) = \frac{d}{dx}(5)$$

$$4x^3 - 3x^2(2y)\frac{dy}{dx} - 6xy^2 + 8y\frac{dy}{dx} = 0$$

$$-6x^2y\frac{dy}{dx} + 8y\frac{dy}{dx} = 6xy^2 - 4x^3$$

$$\frac{dy}{dx}(-6x^2y + 8y) = 6xy^2 - 4x^3$$

$$\frac{dy}{dx} = \frac{6xy^2 - 4x^3}{-6x^2y + 8y}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{6(1)(2)^2 - 4(1)^3}{-6(1)^2(2) + 8(2)}}$$

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{24-4}{-12+16} = \frac{20}{4} = \boxed{5}$$

16. Given the circle: $x^2 + y^2 = 100$

a) Find where the graph has horizontal tangents.

slope of 0

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(100)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$\frac{dy}{dx} = -\frac{x}{y} = 0$$

$$-x = 0 \rightarrow x = 0$$

$$0^2 + y^2 = 100$$

$$\sqrt{y^2} = \sqrt{100}$$

$$y = \pm 10$$

$$\boxed{\begin{matrix}(0,10)\\(0,-10)\end{matrix}}$$

b) Find $\frac{d^2y}{dx^2}$ evaluated at $(-6,8)$.

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{x}{y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x\left(-\frac{x}{y}\right)}{y^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-6,8)} = \boxed{\frac{-8-6\left(\frac{6}{8}\right)}{8^2}} = \frac{-8-\frac{36}{8}}{64} = \frac{-\frac{64}{8}-\frac{36}{8}}{64} = \frac{\frac{-100}{8}}{64}$$

$$= \frac{-100}{8} \cdot \frac{1}{64} = \boxed{\frac{-100}{512}}$$