概率论与数理统计 作业(六)　　邹思瑞　42404455

§2.5

T6. 解：设圆盘直径为X，则 $X \sim U(a,b)$

$$\therefore E(X^2) = \int_a^b \frac{x^2}{b-a}dx = \frac{1}{3}(a^2+ab+b^2)$$

$$\therefore S = \frac{\pi E(X^2)}{4} = \frac{\pi(a^2+ab+b^2)}{12}$$

T11 解：$X \sim Exp(\frac{1}{5})$

则 $F_X(x) = 1-e^{-\frac{1}{5}x}$

则 $P(X>10) = 1-P(X\leq 10) = 1-(1-e^{-2}) = e^{-2}$

$P(Y\geq 1) = 1-P(Y=0) = 1-(1-e^{-2})^5 = 0.5167$

T12 解：$X \sim Exp(\frac{1}{600})$

则 $P(X>200) = 1-P(X\leq 200) = 1-(1-e^{-\frac{1}{3}}) = e^{-\frac{1}{3}}$

$\therefore P(\text{至少有一个元件损坏}) = 1-(e^{-\frac{1}{3}})^3 = 1-e^{-1} = 0.6321$

T17 解(1)：$\because P(X\leq 70) = 0.5$　　$\therefore \mu = 70$

$\because \frac{X-70}{\sigma} \sim N(0,1)$　　$\therefore P(\frac{X-70}{\sigma} \leq \frac{-10}{\sigma}) = 0.25$

$\therefore 0.25 = 1-\Phi(\frac{10}{\sigma})$

$\frac{10}{\sigma} = 0.675$

$\sigma = 14.81$

解(2)：$P(X>65) = \Phi(\frac{70-65}{14.81}) = \Phi(0.3376) = 0.6324$

$\therefore P(\text{至少2人} X>65) = 1-0.3676^5 - 5\times 0.3676^4 \cdot 0.6324$

$= 0.94$

T26 解: $p_1 = P\{X \le u-4\} = P(\frac{X-u}{4} \le \frac{u-4-u}{4}) = \Phi(-1)$

$p_2 = P\{Y \ge u+5\} = P(\frac{Y-u}{5} \ge \frac{u+5-u}{5}) = 1-\Phi(1) = \Phi(-1)$

$\therefore p_1 = p_2$ ，两者一样大.

§2.6

T1 解: $Y = X^2$ 时

Y	4	1	0	1	9
P	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{11}{30}$

整理得

Y	0	1	4	9
p	$\frac{1}{5}$	$\frac{7}{30}$	$\frac{1}{5}$	$\frac{11}{30}$

$Z = |X|$ 时

Z	2	1	0	1	3
P	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{11}{30}$

整理得

Z	0	1	2	3
P	$\frac{1}{5}$	$\frac{7}{30}$	$\frac{1}{5}$	$\frac{11}{30}$

T15. 解(1): $Y = 2X+1$ 为严格单增 $y = g(x) = 2x+1$ $x = h(y) = \frac{y-1}{2}$

$\therefore p_Y(y) = p_X(h(y))|h'(y)|$

$= p(X = \frac{y-1}{2}) \cdot \frac{1}{2} = \frac{1}{2}e^{-\frac{y-1}{2}} \quad (y>1)$

$$\therefore p_Y(y) = \begin{cases} \frac{1}{2}e^{-\frac{y-1}{2}}, & y>1 \\ 0, & \text{其它} \end{cases}$$

解(2): $Y = e^X$ 可能取值为 $(1, +\infty)$ 且 $y = g(x) = e^x$ 严格单增

$x = h(y) = \ln y \qquad h'(y) = \frac{1}{y}$

$$\therefore p_Y(y) = \begin{cases} p_X(\ln y) \cdot |\frac{1}{y}|, & y>1 \\ 0, & \text{其它} \end{cases}$$

$$\therefore p_Y(y) = \begin{cases} \frac{1}{y^2}, & y>1 \\ 0, & \text{其它} \end{cases}$$

解(3)：$Y=X^2$ 取值范围为 $(0,+\infty)$ 且 $y=g(x)=x^2$ 在 $(0,+\infty)$ 严格单增

$\therefore \quad h(y)=\sqrt{y} \qquad h'(y)=\frac{1}{2}y^{-\frac{1}{2}}$

$\therefore \quad y>0$ 时 $\quad p_Y(y)=p_X(\sqrt{y})\cdot\left|\frac{1}{2}y^{-\frac{1}{2}}\right|=\frac{1}{2}e^{-\sqrt{y}}\,y^{-\frac{1}{2}}$

综上 $\quad p_Y(y)=\begin{cases}\frac{1}{2}e^{-\sqrt{y}}\,y^{-\frac{1}{2}}, & y>0\\ 0, & \text{其它}\end{cases}$

T17 解：$\because X\sim LN(\mu,\sigma^2)$

$\therefore \quad p_X(x)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\,x\sigma}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, & x>0\\ 0, & x\leq 0\end{cases}$

$\because$ $Y=\ln X$ 取值范围为 $(-\infty,\infty)$ 且 $y=g(x)=\ln x$ 严格单增

$\therefore \quad h(y)=e^y \qquad h'(y)=e^y$

$\therefore \quad p_Y(y)=p_X(e^y)\,|e^y|$

$$=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(y-\mu)^2}{2\sigma^2}} \qquad y\in(-\infty,+\infty)$$

$\therefore \quad Y=\ln X\sim N(\mu,\sigma^2)$

§2.7

T3 解(1): $$0.5 = F(x_{0.5}) = \int_a^{x_{0.5}} \frac{1}{b-a} dx$$

$$x_{0.5} = \frac{a+b}{2}$$

解(2): $X \sim N(\mu, \sigma^2)$

$\because$ $P(X \le \mu) = \Phi\left(\frac{\mu-\mu}{\sigma}\right) = 0.5$

$\therefore$ $x_{0.5} = \mu$

解(3): $X \sim LN(\mu, \sigma^2)$

令 $Y = \ln X$ ，则 $Y \sim N(\mu, \sigma^2)$

$\therefore$ $y_{0.5} = \mu$

则 $x_{0.5} = e^{y_{0.5}} = e^{\mu}$