Numerical Algorithms (MU4IN910)

Final exam - 15 March 2023

Documents allowed. Calculators allowed - Duration: 1h30

The points are given as an indication

The subject is divided into 4 independent exercises. The quality of the writing, the clarity and the precision of the reasoning will play an important part in the evaluation.

Exercise 1 (Floating-point arithmetic (4 points))**.**

1. You have a *sorted* array of *nonnegative* floating-point numbers. You are asked to add them together. In what order do you perform the operation and why?
2. The hyperbolic sine function $\sinh(x)$ is defined by

$$\sinh(x) = \frac{e^x - e^{-x}}{2}.$$

Why is it a bad idea to implement the function $\sinh(x)$ with the following code in MATLAB? Give an argument x for which a problem can be observed.

```
function res = my_sinh (x)
  res = 0.5 * (exp(x) - exp(-x));
```

Exercise 2 (Extrema (6 points))**.** Do the following functions, defined on $\mathbb{R}^2$, admit local extrema? If so, specify their coordinates and their nature:

1. $f_1(x, y) = x^2 - 4xy + y^2$;
2. $f_2(x, y) = (x - y)^4 + x^2 - y^2 - 2x + 2y + 1$.

Exercise 3 (Optimization (5 points))**.** We seek to maximize the area of a rectangle of given perimeter equal to 2.

1. Formalize this problem as a constrained minimization problem.
2. Show that this problem has at least one solution.
3. Compute a solution.

Exercise 4 (Newton's method (5 points))**.** In this exercise, we want to efficiently compute $\sqrt{a}$.

1. Propose an algorithm to calculate an approximation of $1/\sqrt{a}$ with $a \in \mathbb{R}^+$.
2. What is the remarkable difference with the formula to calculate $\sqrt{a}$? Deduce an algorithm to compute $\sqrt{a}$ using only addition and multiplication.