Lecture 3: Cost of Communications

September 27, 2022

How Long Does It Take to Communicate?

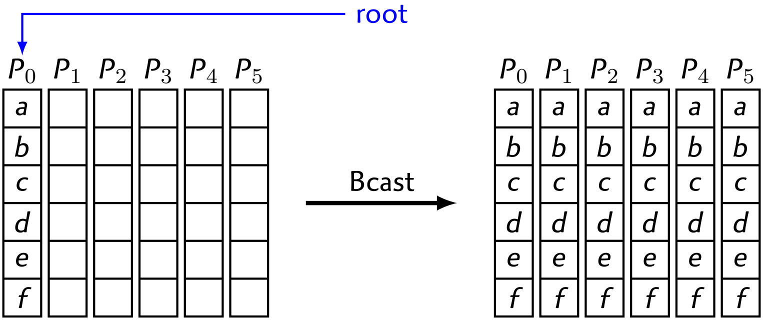
Sending a message along a network link

- Reasonable to assume $T = \alpha + n \times \beta$, where
  - n = message size (bits)
  - α = latency (s)
    - MPI layer, OS, NICs, Switches, ...
  - $\beta = 1/$ bandwidth (s / bit).
- Assumption: full-duplex
  - NICs can send & receive at the same time

Latency VS Bandwidth

Cluster	Year	Interconnect	$1/\beta$	α (μs)	$n\beta = \alpha$
PPTI	20??	1Gbit ethernet	64MB/s	50.0	3.2K
sagitaire	2006	1Gbit ethernet	116MB/s	65.0	7.5K
taurus	2012	10Gbit ethernet	1156MB/s	25.9	30K
gros	2019	25Gbit ethernet	2480MB/s	8.9	22K
grcinq	2013	56Gbit InfiniBand	2488MB/s	1.2	3K
grimoire	2016	56Gbit InfiniBand	4248MB/s	1.1	4.6K
drac	2015	100Gbit InfiniBand	7760MB/s	1.7	13K
grvingt	2018	100Gbit OmniPath	12100MB/s	0.9	11K

Broadcast

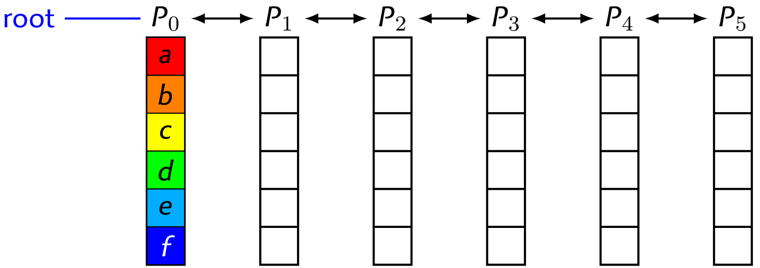


Lower bound

1. n elements exit from the root
   - $T \geq n\beta$ ("bandwidth term")
2. Reach everybody ("*disease propagation*")
   - $\lceil \log_2 p \rceil$ successive messages ("latency term")

$\Longrightarrow$ $T \geq \lceil \log_2 p \rceil \alpha + n\beta$

Naive Broadcast With Linear Network



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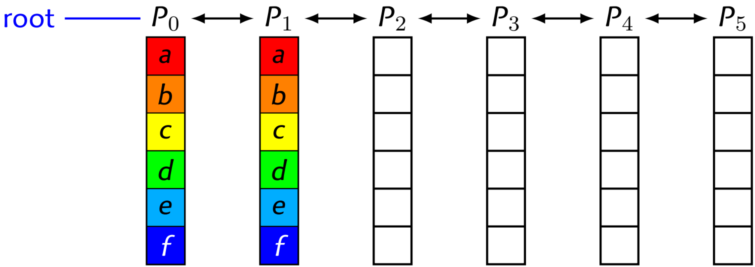

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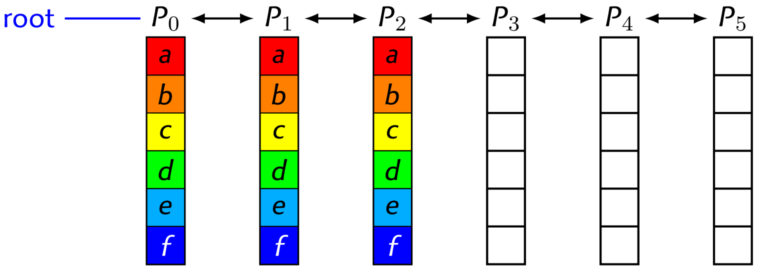

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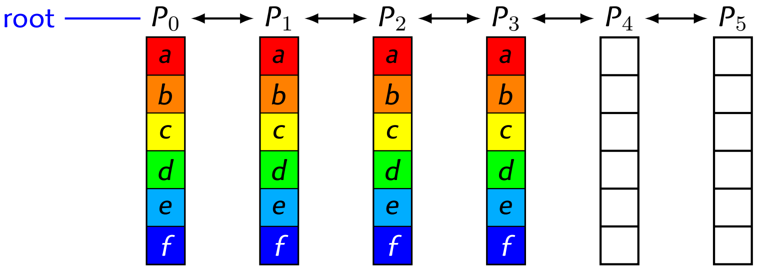

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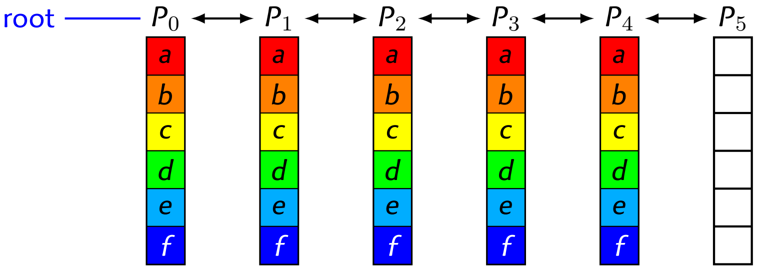

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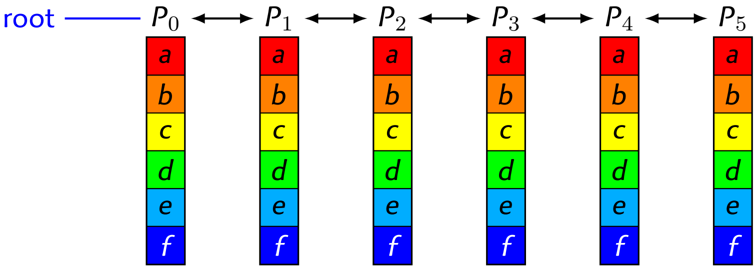

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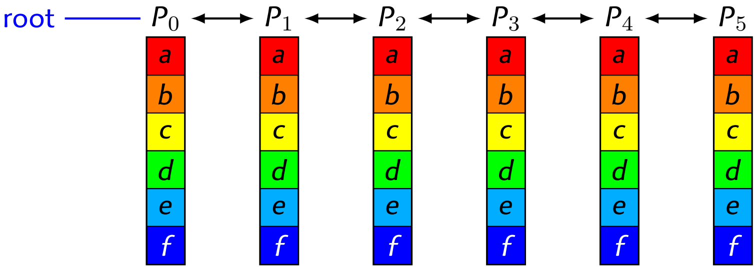
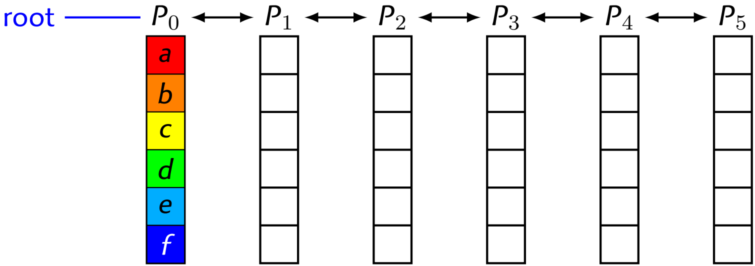


Summary

- $p-1$ successive messages of size n

$$T = (p-1)\alpha + (p-1)n\beta$$

Pipelined Broadcast With Linear Network



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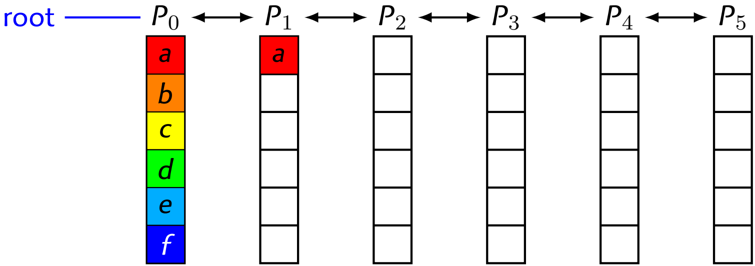
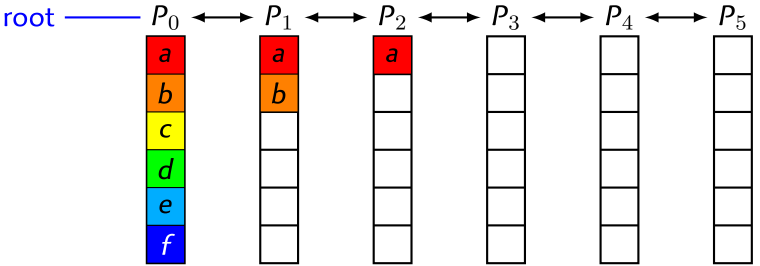


Pipelined Broadcast With Linear Network



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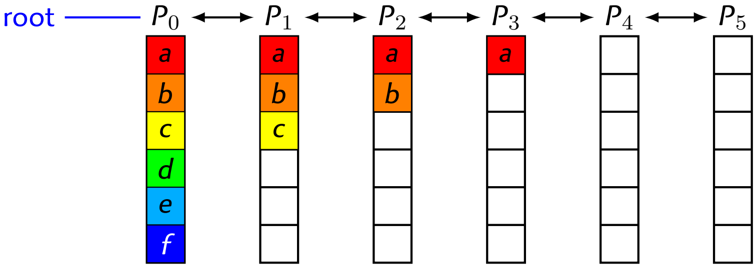

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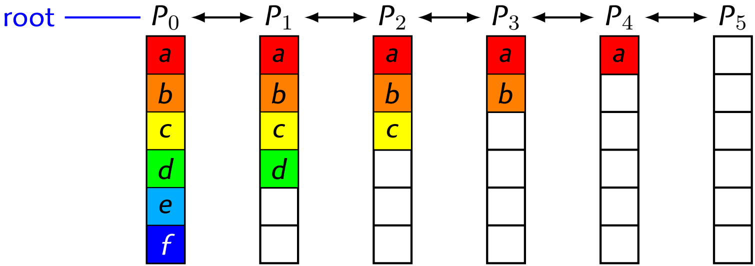

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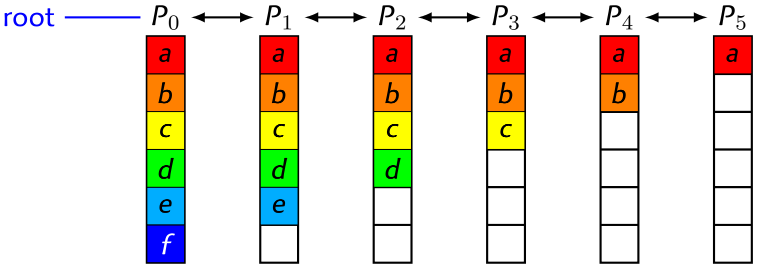

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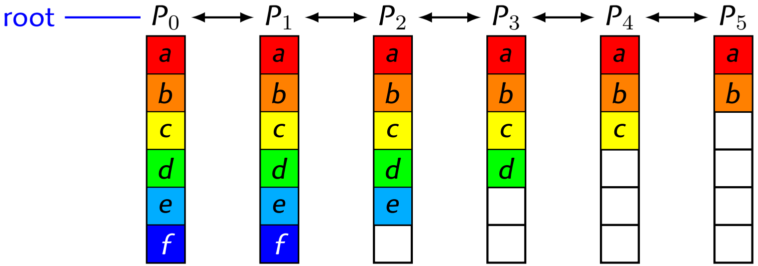

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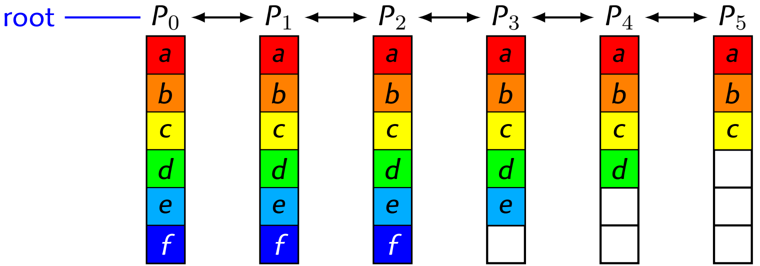

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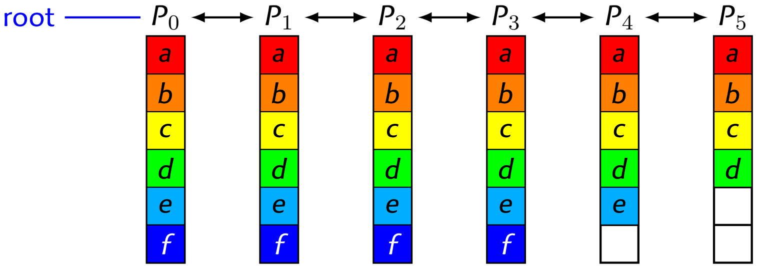

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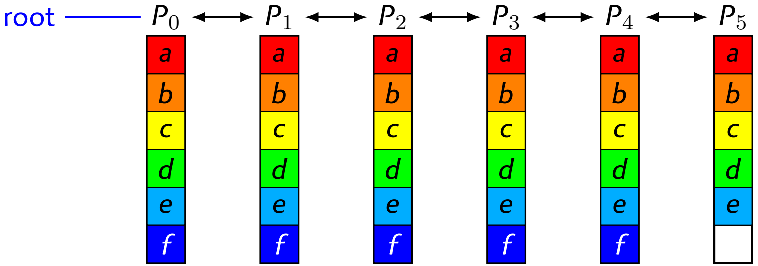

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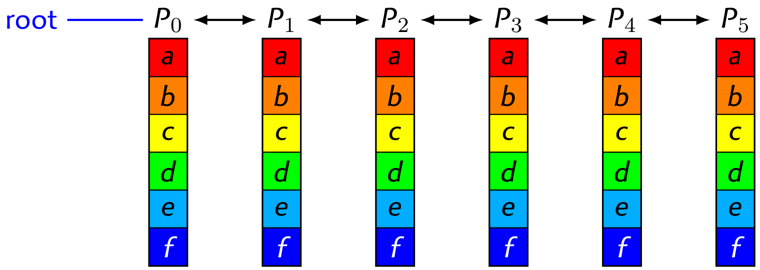
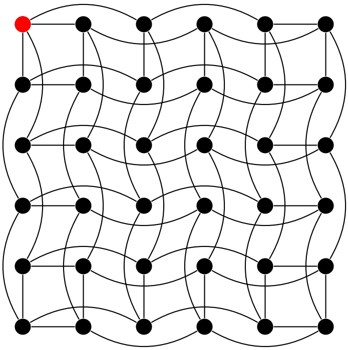


- Pipelining: k "phases"
  - Naive broadcasts of size n/k
- $k + p - 2$ successive messages of size n/k

$$T = (k + p - 2)\alpha + \left(1 + \frac{p-2}{k}\right) n\beta$$

$k = p - 2$ makes sense; best choice: $k = \sqrt{(p-2)\beta n/\alpha)}$

Naive Broadcast With 2D Torus Network



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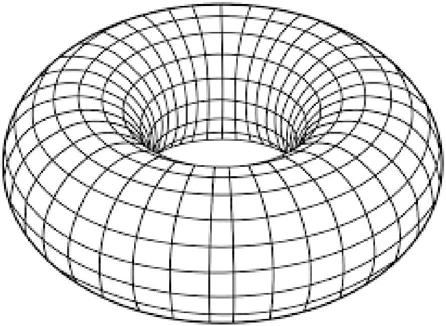

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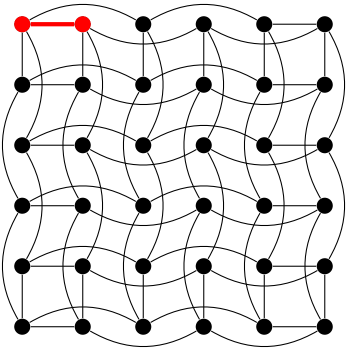

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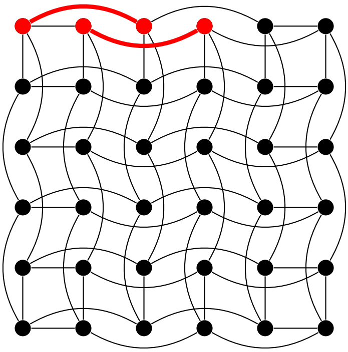

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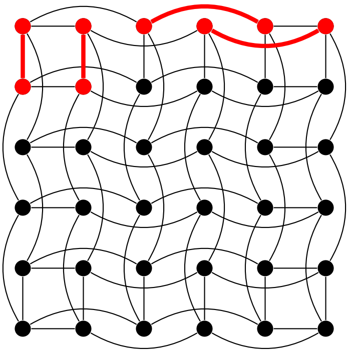

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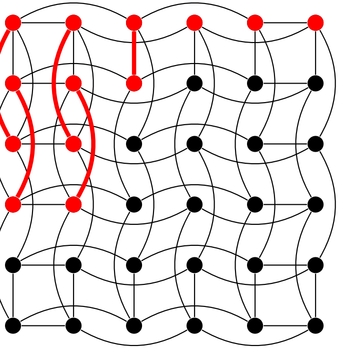

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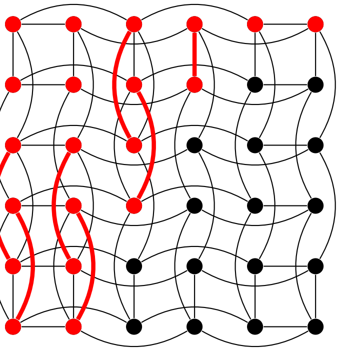

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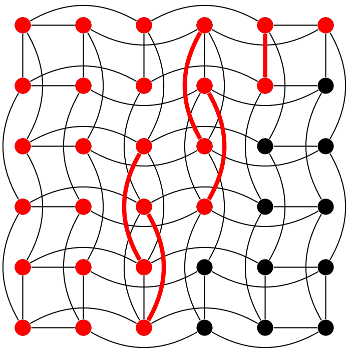

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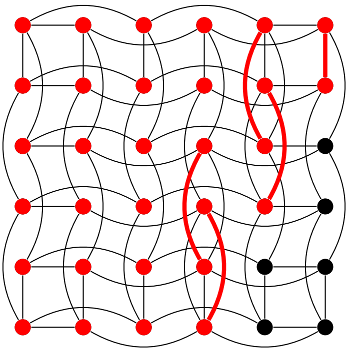

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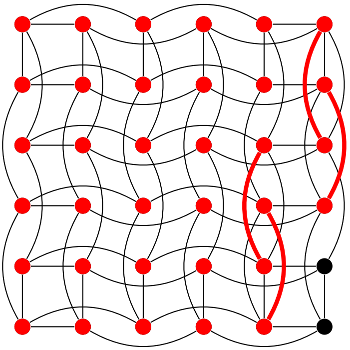

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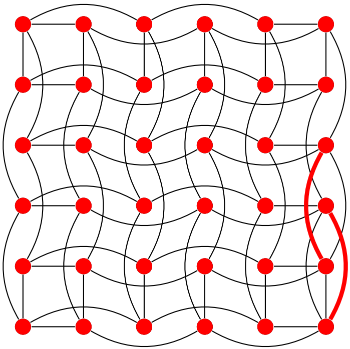
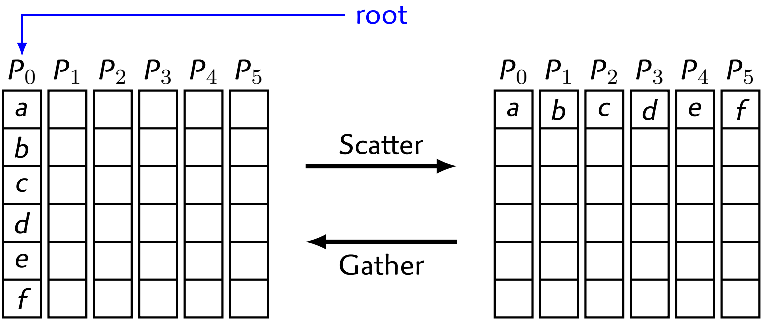


- $2\sqrt{p}$ successive messages of size n

$$T = 2\sqrt{p}\alpha + 2\sqrt{p}n\beta$$

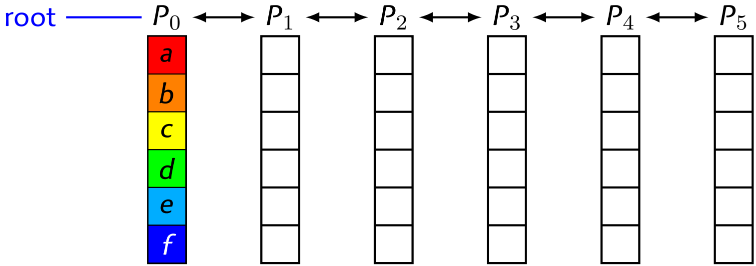
Scatter/Gather: Lower Bound



1. $(p-1)\frac{n}{p}$ elements exit from (resp. enter into) root
   - $T \geq (p-1)\frac{n}{p}\beta$ ("bandwidth term")
2. Reach everybody ("*disease propagation*")
   - $\lceil \log_2 p \rceil$ successive messages ("latency term")

$$\Longrightarrow T \geq \lceil \log_2 p \rceil \alpha + (p-1)\frac{n}{p}\beta$$

Scatter With Linear Network



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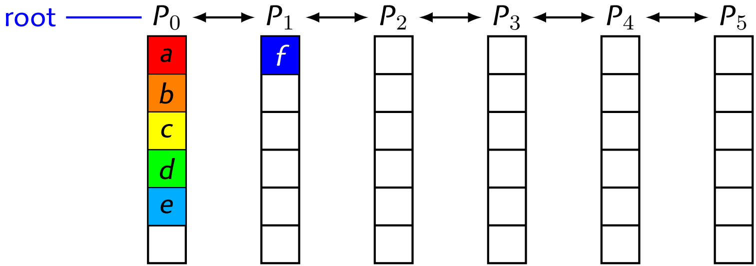

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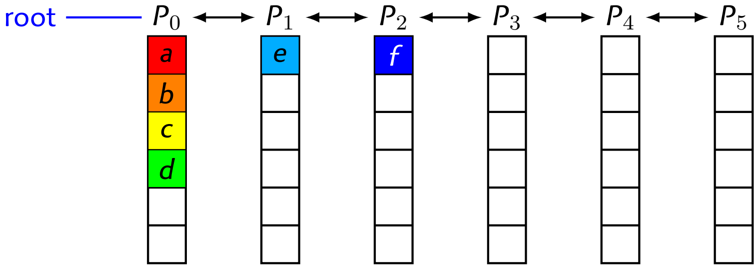

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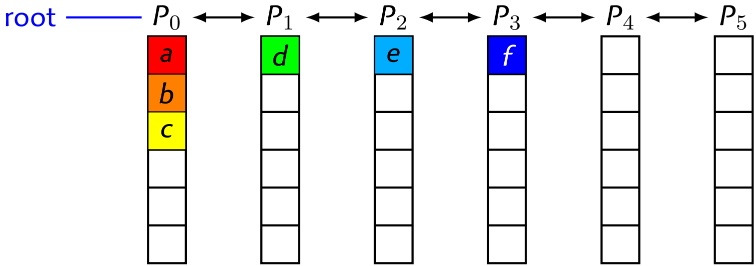

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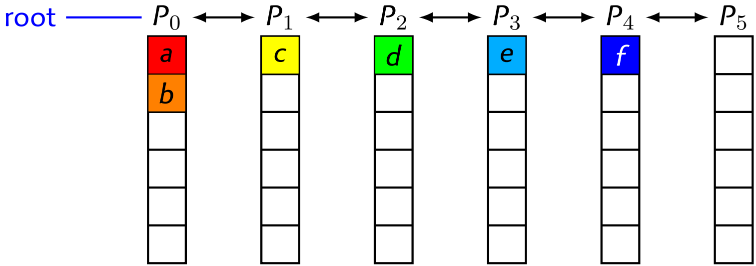

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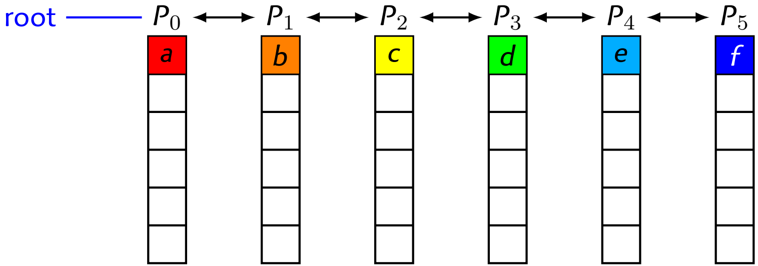
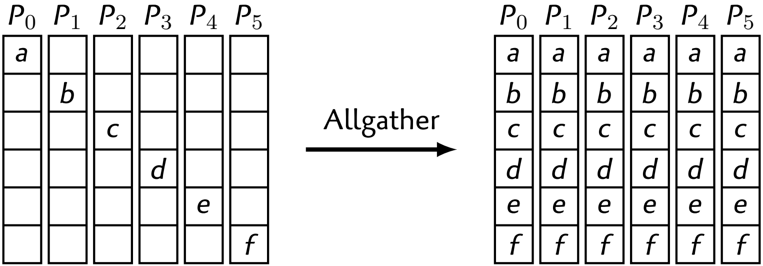


Summary

- $p-1$ successive messages of size n/p
- Parallel communications (“pipeline”)

$$T = (p-1)\alpha + (p-1)\frac{n}{p}\beta$$

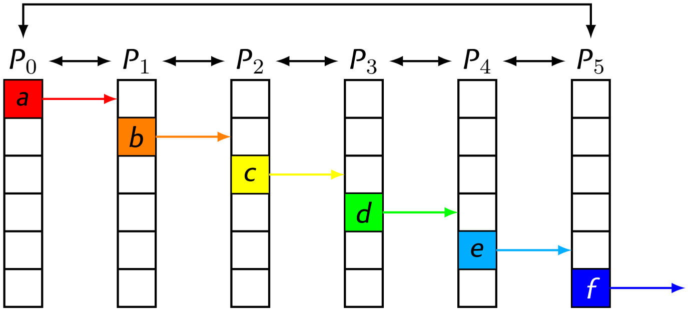
Gather-to-All



1. $(p-1)\frac{n}{p}$ elements exit from each node
   - $T \geq (p-1)\frac{n}{p}\beta$ ("bandwidth term")
2. Reach everybody ("*disease propagation*")
   - $\lceil \log_2 p \rceil$ successive messages ("latency term")

$$\Longrightarrow T \geq \lceil \log_2 p \rceil \alpha + (p-1)\frac{n}{p}\beta$$

Allgather on 1D Torus Network



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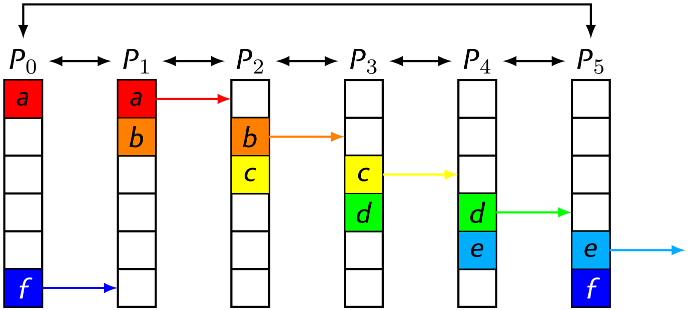

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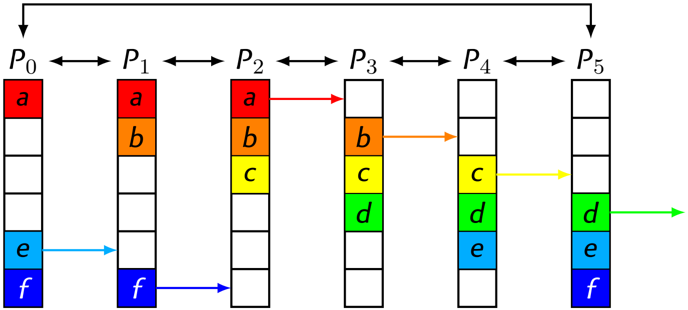

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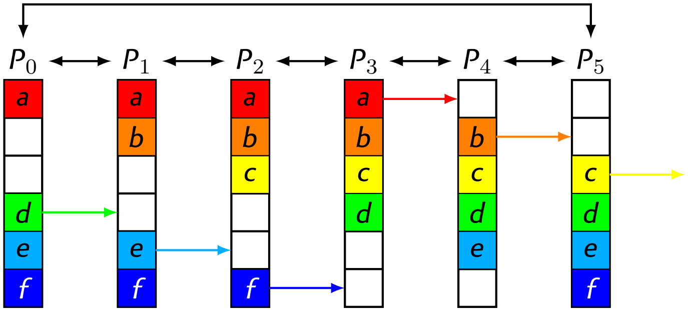

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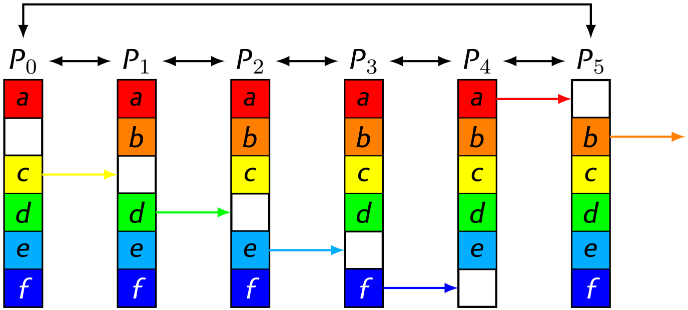

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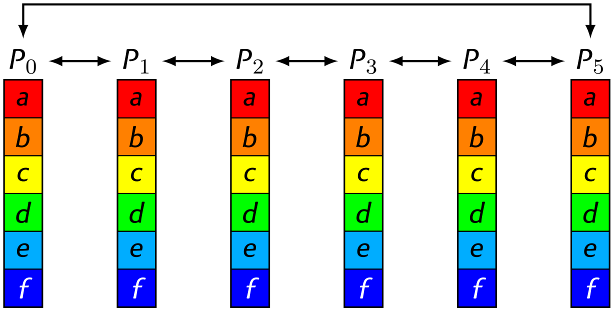


- Step 1: send you own data; step $i+1$: send what you just received
- $p-1$ successive messages of size n/p

$$T = (p-1)\alpha + \left(\frac{p-1}{p}\right) n\beta$$

Performance of collective operations…

… heavily affected by **network topology**

Topology

Bipartite graph with (NIC, Switch) nodes and link edges

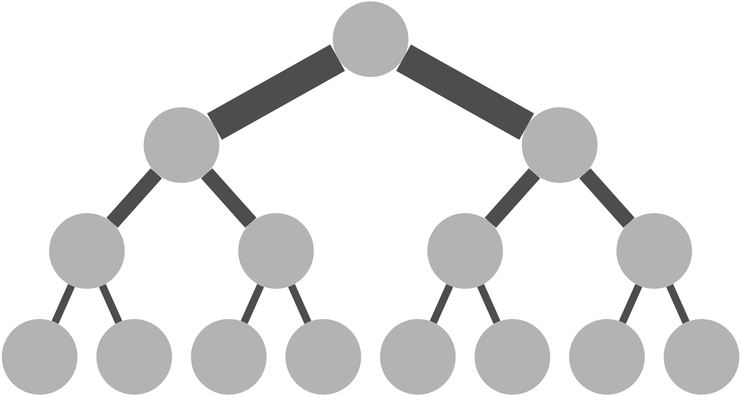
How Long Does It Take to Communicate (redux)?

P_i sends a message to **any other** process P_j

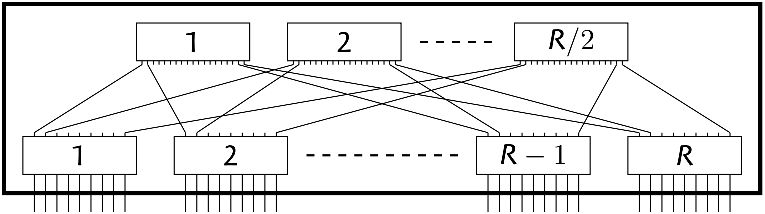
- Reasonable to assume $T = \alpha + n \times \beta$, where
  - n = message size (bits)
  - α = latency (s)
  - $\beta = 1/$ bandwidth (s / bit).
- Assumption: full-duplex
  - Processes can send & receive at the same time
- T independent of i, j (no weird topology)
- T independent from actions of other processes
  - No congestion

Realistic?

Fat Tree (Infiniband / Omnipath)



Fat tree (2 levels)



- **non-blocking** R-port "basic" switches
- $(R^2/2)$-port switch using $1.5R$ basic switches
- Infiniband : $R = 36 \rightsquigarrow$ 648 ports

Infiniband 648-port “Director” switch

image: (c) Mellanox

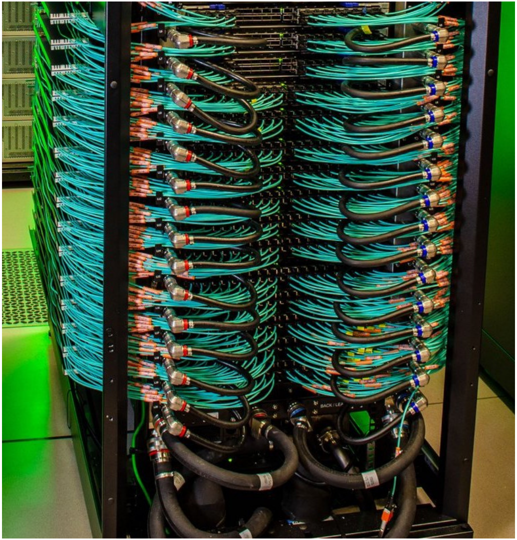
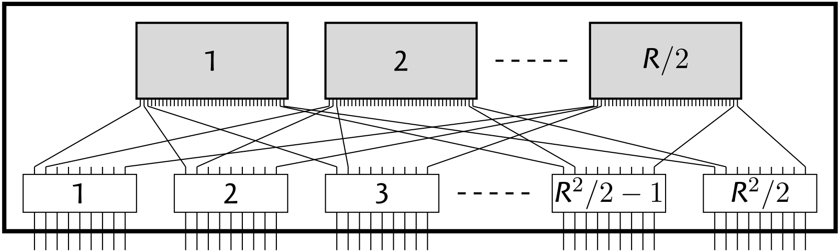


image: (c) UTexas

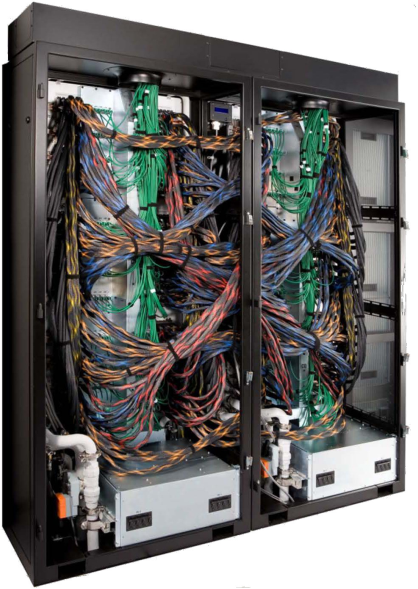
Fat tree (3 levels)



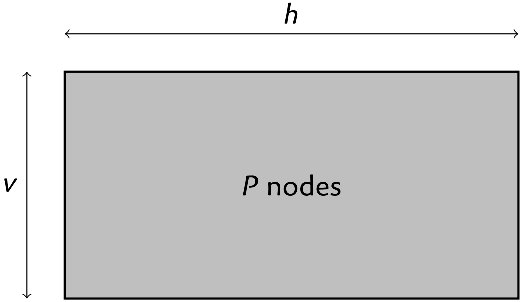
- Same technique, recursively
- $(R^3/4)$-ports with $1.25R^2$ basic R-port switches
- Infiniband : $R = 36 \rightsquigarrow 11\,644$ ports
- Used in `Summit` with $9\,216$ CPUs
  - 1 rack = 18 nodes = 36 CPUs = 36 NICs
  - Two 36-port switches per rack
  - 256 racks
  - 15 “director” (level-2) switches

Fat tree — High Cabling Costs

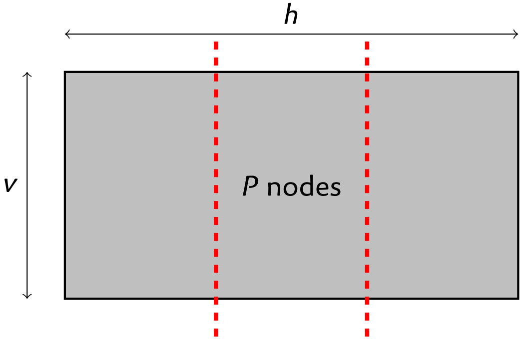
It ends up looking like this



Fat tree — High Cabling Costs

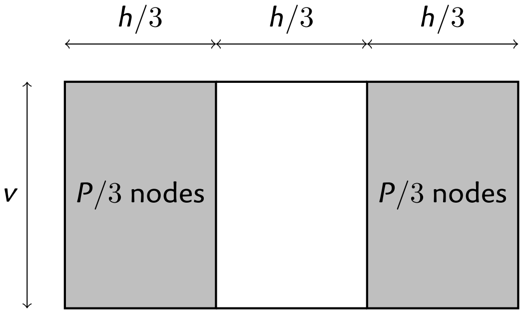


Fat tree — High Cabling Costs



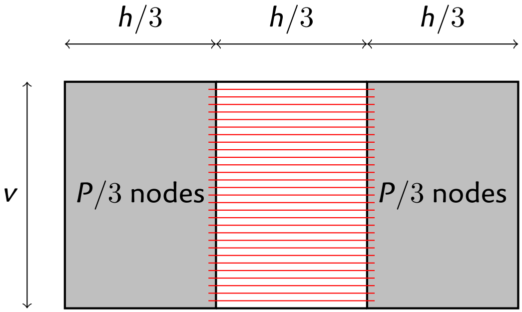
- Split machine in 3

Fat tree — High Cabling Costs



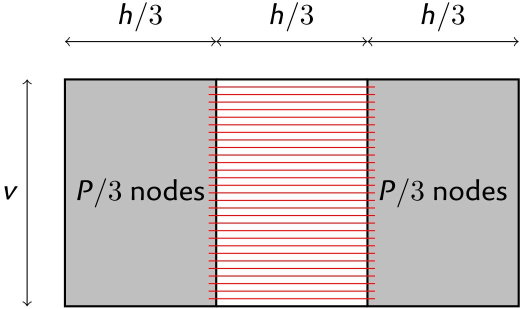
- Split machine in 3
- All nodes on the left send messages to nodes on the right

Fat tree — High Cabling Costs



- Split machine in 3
- All nodes on the left send messages to nodes on the right
- Assumption = no congestion
  - ⇝ Need $P/3$ parallel network links across the middle
  - ⇝ Length $\geq h/3$

Fat tree — High Cabling Costs



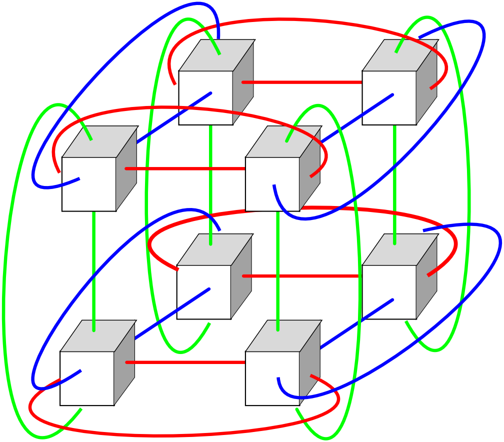
- Split machine in 3
- All nodes on the left send messages to nodes on the right
- Assumption = no congestion
  - ⇝ Need $P/3$ parallel network links across the middle
  - ⇝ Length $\geq h/3$

$\Longrightarrow$ cable length $\geq hP/9 = \Omega\left(P^{1.5}\right)$

- Fat tree a viable options with "small" #nodes ($\leq$ 10k)

Cost-Saving Alternatives

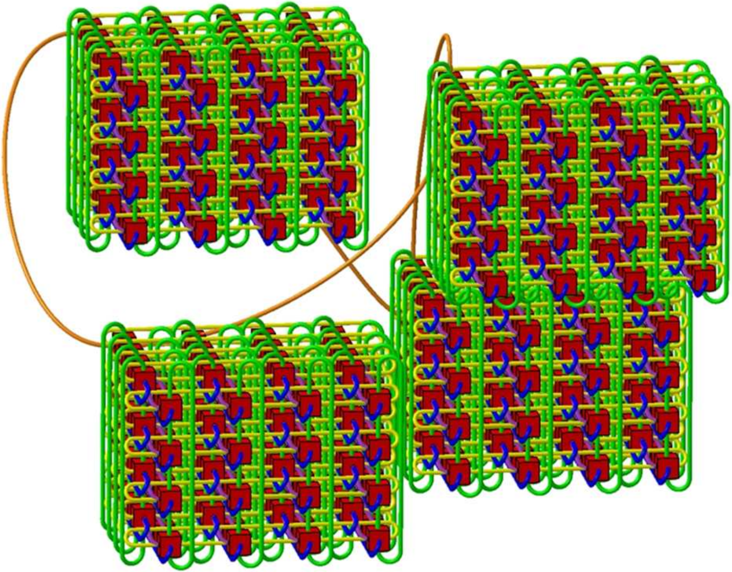
3D Torus (IBM BlueGene/P, Cray XT3, ...)



- Cable length proportional to #Nodes
- Used on very large machines
  - E.g. $48 \times 72 \times 24 \approx 100$k Nodes

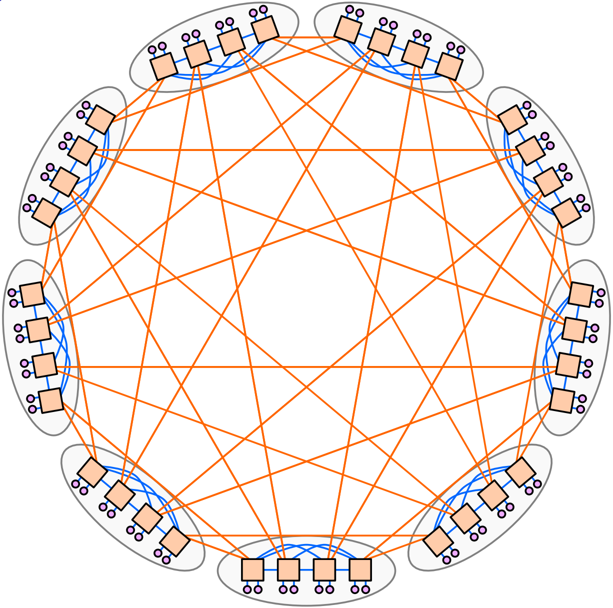
Cost-Saving Alternatives

5D Torus (IBM BlueGene/Q)



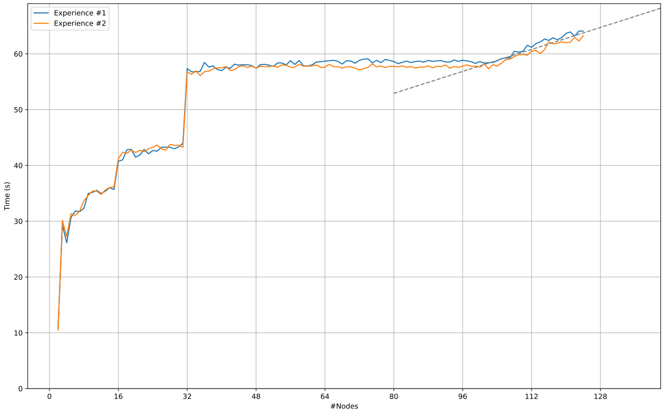
Compromise

Dragonfly (Cray "Aries" Interconnect)



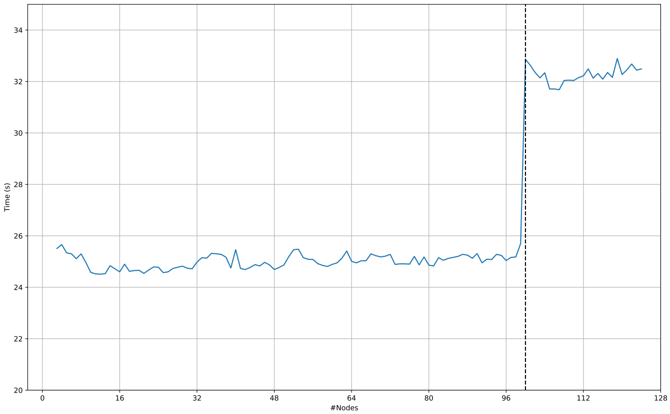
On Grid5000 ?

Except in special case, all nodes of a cluster connected to a switch
MPI All-to-All:

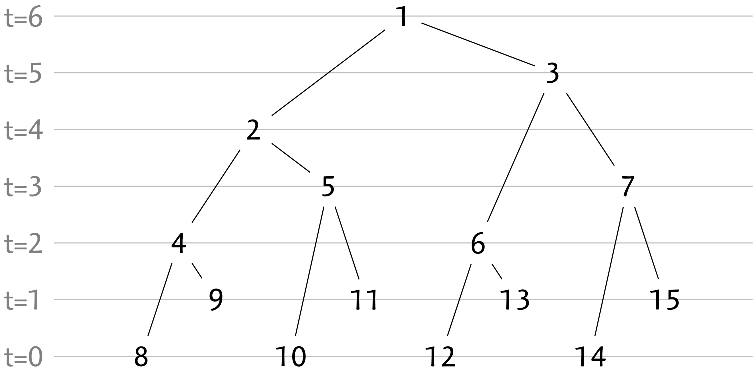


On Grid5000 ?

Ring algorithm:



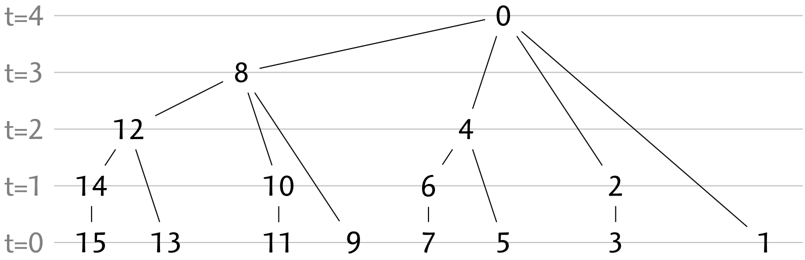
Scatter/Gather Again: Binary Tree Algorithm



- $2^{h+1} - 1$ nodes in total ; $2h$ successive messages
- 2^{h-i} nodes of height i. Each transfers:
  - Upwards $(2^{i+1} - 1)n/p$ items
  - Downwards $2 \times (2^{i} - 1)n/p$ items

$\Rightarrow$ $T \leq 2\lceil \log_2 P \rceil \alpha + 2(p-1)/pn\beta$

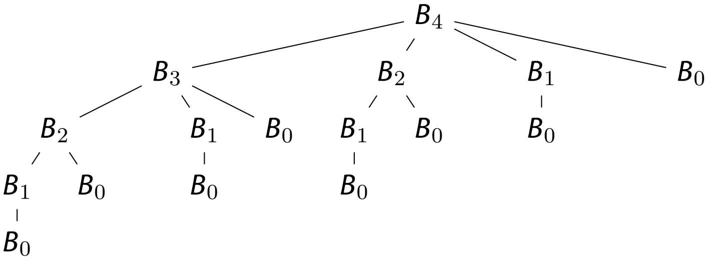
Scatter/Gather Again: **Binomial** Tree Algorithm



- 2^h nodes ; h successive messages
- Nodes of height i transfer $2^i n/p$ items (synchronously)

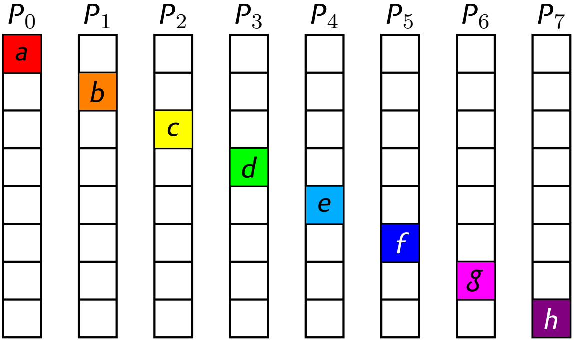
$$T = \text{message of size } 1 + \cdots + \text{message of size } n/2$$
$$= \alpha\lceil \log_2 p \rceil + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

Interlude: Binomial Tree



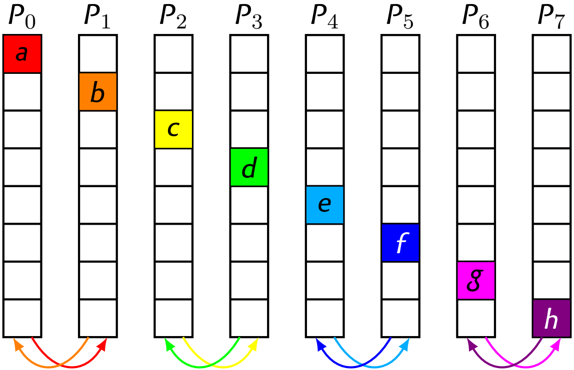
- B_i = root + $B_0 + B_1 + \ldots B_{i-1}$
- $B_i = B_{i-1} + B_{i-1}$
- $\binom{h}{i}$ nodes at height i

AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$
$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

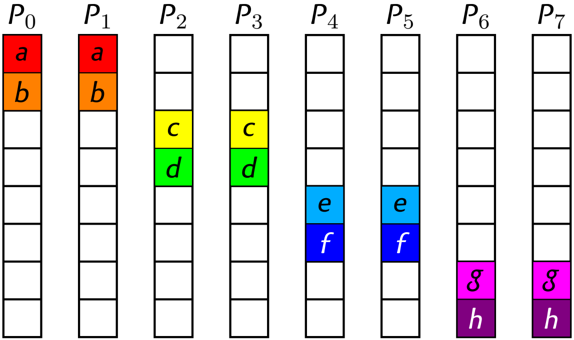
AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$

$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

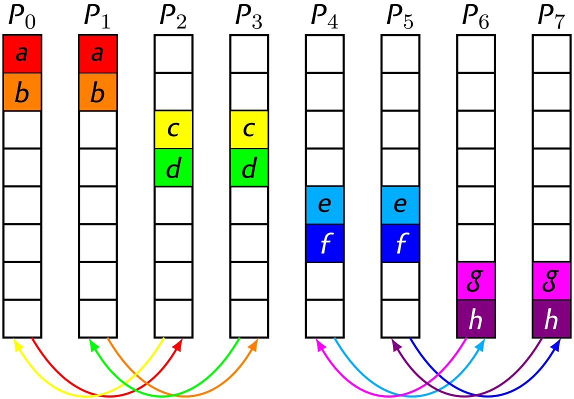
AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$

$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

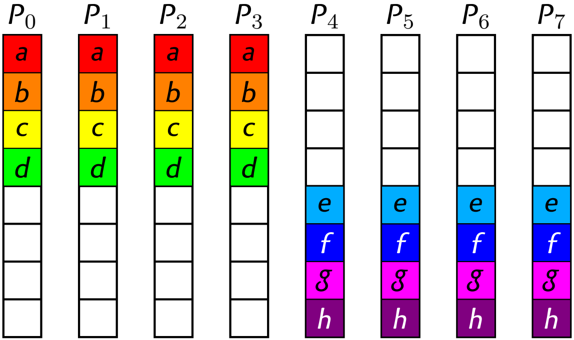
AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$

$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

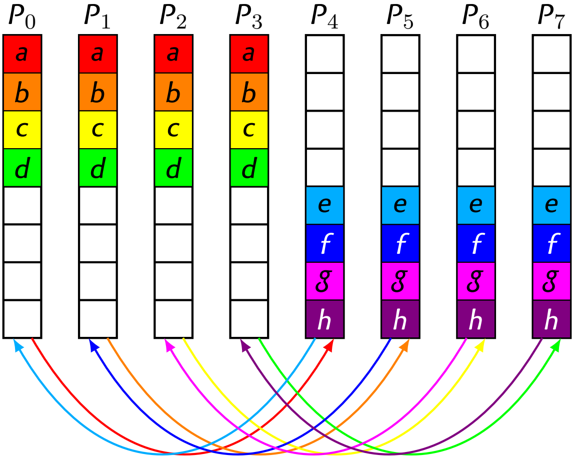
AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$

$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

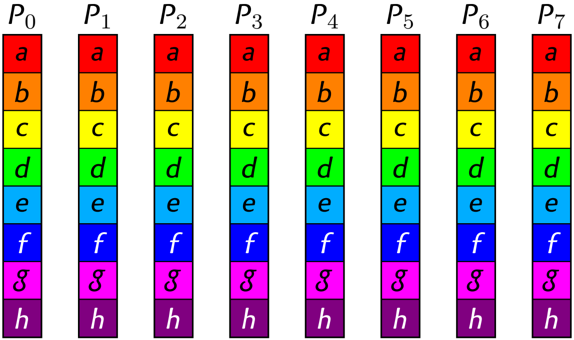
AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$

$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta$$ (optimal)

AllGather again: Recursive Doubling Algorithm



$$T = \text{msg of size } n/p + \cdots + \text{msg of size } n/4 + \text{msg of size } n/2$$

$$= \alpha \log_2 p + (p-1)\frac{n}{p}\beta \qquad \text{(optimal)}$$

Algorithms for Broadcast / Reduce

Binomial Tree

- $T = \lceil \log_2 p \rceil (\alpha + n\beta)$
- "Bandwidth" term sub-optimal
- (bad for large messages)

Van de Geijn

- Broadcast = AllGather ∘ Scatter
  - Recursive doubling / Binomial tree
- $T = 2(\log_2 p)\alpha + 2\left(1 - \frac{1}{p}\right) n\beta$

$\rightarrow$ $2\times$ lower bound