THE TRANSITION FROM THE ARS ANTIQUA TO THE ARS NOVA: EVOLUTION OR REVOLUTION?

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The long-standing conviction that the fourteenth century marks a turning point in the history of music has been seriously questioned in recent studies. In particular, the traditional recognition of Jehan de Meur and Philippe de Vitry as the initiators of a new music has given way to an interpretation that denies the *ars nova* was a revolution, and instead stresses a gradual evolution from *ars antiqua*.[1]

The purpose of this paper is to rehabilitate the traditional conviction by pointing to the radically new elements in the music theory of Jehan de Meur's *Notitia artis musicae*. These new elements, first apparent in the musical terminology, imply a deeper shift in the understanding of music, a shift from cosmological processes to earthly, physical ones. In order to see these implications of Jehan's rhythmic theories, we need to read his *Notitia* in the broader context of the shift from thirteenth-century Scholasticism and Thomism to the new fourteenth-century schools of mathematics and philosophy. More specifically, I shall demonstrate parallels between Jehan's tacit

[1] Ulrich Michels claims that Jehan's aim in his *Notitia Artis Musicae* of 1320 was not to legitimize imperfect rhythmical values. Instead, he contends, Jehan "demands something which has long before been there, namely, the Petronian division of the *brevis*, and has not touched the perfect mensuration of the *ars antiqua*.... On the other hand, [he] did introduce some innovations, in particular the enlargement of the old perfect system and its arrangement into four divisions." (Ulrich Michels, *Die Musiktraktate des Johannes de Muris*, Beihefte zum Archiv für Musikwissenschaft 8 [Wiesbaden, 1970], pp. 74-75.) L. Gushee's various studies on Jehan de Meur's theories led him to an inconclusive view regarding the revolutionary aspects of Jehan's *Notitia artis musicae*: "The reader will understandably look for those things which might be termed revolutionary, in accordance with the at least partly mythical musical revolution called *ars nova*. He must look carefully, however. The tact, one might almost say deviousness, with which Jehan brought in the idea of fundamental binary relations between the four levels of note values is most striking" (Lawrence Gushee, "Jehan de Murs", *The New Grove Dictionary of Music and Musicians*, Vol. 9, p. 598). An attempt to demystify the achievements of early fourteenth-century theorists is also made by Sarah Fuller, in her recent study of the group of treatises associated with the *ars nova* of Philippe de Vitry. Fuller supports Max Haas' view that *ars antiqua* and *ars nova* should not be interpreted as opposites but "mesh in an unbroken continuum in which the Nova is a complementary extension of the *ars vetus*." (Sarah Fuller, "A Phantom Treatise of the Fourteenth Century? The Ars Nova," *Journal of Musicology* 4 [1985/86], p. 47).

philosophical presuppositions, his terminology and his operative principles, and the revolutionary mathematical physics which was developed at Merton College in Oxford during the first half of the fourteenth century.[2]

The link between Aristotelian philosophy and mensural musical theories of the thirteenth century has been discussed by Lawrence Gushee[3] and Max Haas.[4] Gushee had pointed to the effect of Aristotelian ideas on the overall organization of Franco's *Ars cantus mensurabilis* in terms of its order of presentation.[5] Haas has restored the place of music within medieval universities,[6] accentuating the interaction between scholastic pedagogy and the teaching of music.[7] His studies

[2] Although this study focuses on Jehan as a music theorist, my hypothesis regarding the possible connection between Jehan de Meur and the Mertonian mathematicians sheds new light on Jehan as a mathematician, who so far has not been associated with the novel trends of fourteenth-century mathematics. On the one hand, Jehan's *Notitia* of 1320 antecedes the earliest known sources of the Mertonian Calculators and may have been a possible source of early quantification in physics. On the other hand, it could be the case that both Jehan and the Calculators, independently, drew upon early medical theories of quantification and/or Continental theories of kinematics developed already in the thirteenth century. See Marshall Clagett, *Science of Mechanics in the Middle Ages* (Wisconsin, 1959), pp. 163-167. My concern here is to show a material and methodological relationship between Jehan de Meur and the Oxford Calculators, and not to prove any empirical-historical connection.

[3] Lawrence Gushee, "Questions of Genre in Medieval Treatises on Music," In *Gattungen der Musik in Einzeldarstellungen. Gedenkschrift Leo Schrade* (Bern, 1973), pp. 365-433.

[4] Max Haas, "Die Musiklehre im 13. Jahrhundert von Johannes de Garlandia bis Franco," in *Die Mittelalterliche Lehre von der Mehrstimmigkeit. Geschichte der Musiktheorie*, Vol. 5 (Darmstadt, 1984), pp. 89-160; and "Eine Übersicht über die Musiklehre im Kontext der Philosophie der 13. und frühen 14. Jahrhunderts", *Forum Musicologicum*, Basler Beiträge zur Musikgeschichte. Band 3 (Schweiz, 1982), pp. 353-381.

[5] The Aristotelian notion of hierarchy, for example, is reflected in Franco's argumentation from the simple to the composite, from the general to the particular, and from essence to substance (Lawrence Gushee, "Questions of Genre in Medieval Treatises on Music," in *Gattungen der Musik in Einzeldarstellungen. Gedenkschrift Leo Schrade* (Bern, 1973), pp. 426-427. In addition, Gushee notes that Aristotelian notions such as *genus* and *species, natura* and *materia, essentia* and *accidentia* often appear in thirteenth century treatises, the use of which, he maintains, adds an element of complexity so pronounced that it cannot be explained satisfactorily even by referring back to Aristotle (Ibid. 425).

[6] Max Haas, "Eine Übersicht über die Musiklehre im Kontext der Philosopie der 13. und frühen 14. Jahrhunderts," *Forum Musicologicum*, Basler Beiträge zur Musikgeschichte. Band 3 (Schweiz, 1982), pp. 349-371.

[7] Max Haas, "Die Musiklehre im 13. Jahrhundert von Johannes de Garlandia bis Franco," in *Die Mittelalterliche Lehre von der Mehrstimmigkeit. Geschichte der Musiktheorie*, Vol. 5 (Darmstadt, 1984), pp. 115-131.

provide a new perspective on the problem of weighing the impact of Aristotle on musical thought in the age of Scholasticism.[8]

There were, however, obstacles to the integration of Aristotle's teachings with mensural theory: musical mensuration itself proved to be inconsistent with Aristotelian logic, which prohibited the mixture of entities of different *genera* and even of different species.[9] Aristotle's injunction ruled out transportation of methods from one discipline to another. In Aristotelian philosophy, physics—which deals with real, sensible, corruptible, and moving entities—could not be studied by mathematics which concerns eternal, abstract and absolutely motionless entities. Just as mathematics and physics were considered categorically incommensurable, so also the different species within mathematics. For example, arithmetic, which concerns discrete quantities, could not be applied to geometry, which consists of continuous magnitudes.[10] Time could be represented by lines because both share the character of a continuum, but straight lines and circles are of different species and are therefore not comparable.[11] Mensural theories were concerned with the quantification of different note-values, that is, the quantification of physical motions-in-time. As such, they entailed category-mistakes associated with the vice of crossing the boundaries between mathematics

[8] While the said researches demonstrate the general presence of an Aristotelian mode of thought in music theories, a closer look into the doctrinal body of Aristotelianism will reveal certain tensions and incompatibilities between basic postulates of Aristotle's categories, on the one hand, and the very exigencies of measuring rhythmic motion against the continuum of time, on the other. In other words, although the very influence of Aristotelianism on musical theories has been solidly corroborated, attention should be drawn to the immanent difficulties of integrating Aristotelianism with the contents of mensural theories. It is the nature of musical mensuration, hence the basic implicit postulates of mensural theories, which proves to be inconsistent with Aristotelian thought.

[9] The absolute autonomy of each discipline reflects Aristotle's world view, according to which each natural entity has its unique and unequivocal place in the universe and is identified by an exhaustive definition according to its closest genus and specific difference. Aristotle assumed that no specific difference can appear in more than one genus, and that the different essences are incommensurable and mutually exclusive. Thus, nature as a whole was absolutely heterogeneous: the world consists of a well-ordered hierarchy of different essences. Just as each such class of an essence is governed by its unique principles, so are the different classes or branches of science.

[10] Aristotle, *Posterior Analytics*, I.7. 75a38-75b6.

[11] Aristotle, *Physics*, VII.4. 248a19-248b11.

and physics, and between the discrete and the continuous.[12] The relationship of *longa* and *brevis* in modal rhythms of the thirteenth century suggested, if not demanded, a quantitative numerical analysis, such as has been provided by Augustine for poetry.[13] But Aristotelian methods prescribed a qualitative analysis. How did thirteenth-century theorists reconcile these demands? The accommodation of Aristotelian methods proceeded in stages. The first stage is reflected in the musical thought of the pre-Franconians, the second stage in the theories of Franco of Cologne and Lambert.

In pre-Franconian theories, the basic quantitative values of long and breve acquire a conceptual refinement through the shift of focus towards the Aristotelian logical category of opposite predicates. The basic components of meters—abstracted into the general notion of the long vs. the short—come to be conceived of in terms of quality rather than quantity, namely, durations conceptualized as "predicates" of a "subject" which is sound itself. Once reasoned into the status of predicates, and abstracted into the only two options of duration, the

[12] There seems to be, then, an overlooked conceptual clash between the subject-matter of the rhythmic treatises and the Aristotelian tools applied to the doctrinal framework of theorizing rhythm. Yet conflicts and tensions generally characterize the reception of Aristotle in philosophy and theology. Aristotelianism appeared on the scene after Christian philosophy steeped in Platonism was already well accepted, and some of its basic notions could hardly be smoothly dovetailed into the Christian dogma. In the domain of Christian philosophy, Aristotle was adjusted to the traditional Christian-Platonic creed, so well exemplified by the "Thomistic Compromise." See Charles Lohr, "The Medieval Interpretation of Aristotle." In *The Cambridge History of Later Medieval Philosophy*, ed. N. Kretzmann, A. Kenny, and J. Pinborg. (Cambridge, 1982), pp. 92-94. To mention a few examples, Christian philosophers, as well as theologians, disdained Aristotle's claim that his cosmology was the only possible explanation of the way things are in nature, because this "necessitarianism" infringed on most fundamental Christians' belief in God's omnipotence and absolute free will. Aristotle, by contrast, did not believe in the eternity of the soul, neither in the eternity of the world, which obviously conflicts with the notion of creation in Genesis. (Ibid, pp. 87-92)

[13] I follow and complement Leo Treitler's view on the much-discussed question of whether or not the modal system was modeled after the Augustinian theory of quantitative poetical meters. Treitler shows that the crystallization of the modal patterns in practice and the conceptualization of the modal system in theory were two different processes. Accordingly, he argues that the undeniable fact that "the explanations of the modal system bear the conceptual stamp of the tradition of metrics does not prove the assumption of an a priori system, nor that modal practice was theory-induced." (Leo Treitler, "Regarding Meter and Rhythm in the Ars Antiqua," *Musical Quarterly* 65 [1979], 542.) A further examination of rhythmic treatises of the *ars antiqua* shows that theorists could not rely exclusively on the traditional literature of quantitative poetical meters. The quantitative theories had to be reconciled with the qualitative Aristotelian system of reasoning. In order to accommodate qualitative Aristotelian thought, the old rhythmic quantities were detached from their traditional philosophical setting (as for example, the *via Augustinis*) to be merged with and expounded within the context of Aristotelian notions and principles.

long and the breve become qualitatively distinct. Hence, long and breve are considered as categorically differentiated to the point of being mutually exclusive. They are thus considered as discontinuous, as oppositions. This is the fundamental framework where the Aristotelian logic of predicates applied to musical durations is superimposed on the quantitative durational notions.[14] The author of the *Introductio Musicæ Secundum Magistrum Johannem de Garlandia* conceived the addition of rhythmic values to the older non-measured music as an act analogous to the acquisition of a predicate by a subject: "A *subject* in music is the conjunction of tones or rests in the right and properly observed manner. A *predicate* is the legitimate art of fitting this music in the right proportion and by observing diligently all the modes; the part of philosophy providing it is the art of metrics."[15]

This summary statement of the addition of rhythmic values to the older non-measured music is cast in terms of the Aristotelian logic of predicates. This logic restricted propositions to one of the two forms: (1) "S" is "A," or (2) "A" can be a predicate of "S." The rhythmic component of a sound is considered an attribute of that sound. Thus, the "subject" is the configuration of pitches, the "predicate" is the mensuration. "This note is long," "that note is short."

Subsequently, the pre-Franconian theorists classified these predicates, not according to quantity but according to quality, and observed that they were opposites. Anonymous VII (of C.S I) says,

> "Any figure or note, which is the same thing, is either a *brevis* or a *longa*."[16]

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[14] It should be noted that qualitative language adapted to musical mensuration does not agree with the very core of mathematical thinking. Mathematically, long and breve are two points on a continuum of values; they relate to each other as whole to part. In mathematics, there are no opposites or contraries; a number can be smaller, bigger, a multiple or a certain part of another number, but cannot ever be the opposite of any other value.

[15] "Subjectum in musica est aliquarum vocum seu pausationum conjunctio modo debito ac proprie observato. Predicatum est ipsius musice ars legitima proportionate omnibus suis modis diligenter observatis, cui partem philosophie supponatur ars metrice." (*Introductio musicae secundum magistrum Johannem de Garlandia*. Ed. E. de Coussemaker, in *Scriptorum de musica medii aevi*, Vol. 1 (Paris, 1864), p. 158. (Hereafter CS 1).

[16] "Omnis figura sive omnis nota, quod idem est, aut est brevis aut longa." (Anonymous VII, *De musica libellus*, CS 1 (Paris, 1864), p. 379. [New edition by G. Reaney in CSM 36.]

He is making a categorical differentiation between long and short note values. The distinction between long and short, essential to the new measured music, involved the *longa* being exactly twice as long as the *brevis*—a quantitative differentiation, specifically the ratio 2:1, or a unit and its double. Anonymous VII, however, insisted on the discontinuity between them as reflected in his division of each of the two distinct classes of long and short notes into their sub-classes:

> "There are three kinds (differentia) of longs. One is a long note of two, one of three, one of six time units. There are three kinds (differentia) of short notes. One is a *recta brevis* containing one unit of time, and it is written in this way: [...] Or a *semibrevis* that contains one unit of time and is written like that [...]. Or a *plica brevis* descending (written) like that [...] Or a *plica brevis* ascending (written) like that [...]"[17]

This division (which certainly reflects the scholastic method of classification) demonstrates that, according to the pre-Franconians, breve and long do not belong to a larger whole in the same species, which contains both, and which would have made possible a quantitative comparison between them. Instead, the pre-Franconians considered the relation between *longa* and *brevis* as a relation between opposites. In Aristotelian logic the category of relation (the first species in which one can pose things as A versus B) is not mathematical, but one of opposition.

> "Pairs of opposites which fall under (the category of) relation are explained by a reference of the one to the other. (The reference being indicated by the preposition "of" or by some

[17] "De longis triplex est differentia. Quedam est longa duorum, quedam trium, quedam sex temporum.... De brevibus triplex est differentia: quedam est recta brevis continens unum tempus et scribitur tali modo: [...] Quedam semibrevis continens in se dimidium unum tempus et scribitur, sic: [...] Quedam plica brevis descendendo, ut hic: [...] Quedam plica brevis ascendendo, ut hic: [...]" (Ibid, pp. 379-380).

other preposition.) Thus, double is a relative term, for that which is double is explained as a double of something."[18]

The interpretation of long and breve, not as relative quantities but as opposing qualities, invokes Aristotle's insistence on the absolute indivisibility and eternity of all essential and accidental qualities. Such qualities do not contain within themselves the several distinct degrees that would permit a quantification from less to more, in the same way that a certain whole contains its various constituent parts. For Aristotle, change in intensity, as in acceleration/deceleration, or hotter/colder, cannot be represented, because for him the very notion of a rate of change was impossible: "there cannot be motion of motion or becoming of becoming or in general change of change."[19]

In thinking of long and short in these qualitative terms, the pre-Franconians avoided a mathematical mode of thought and instead followed Aristotelian logic, ultimately Aristotelian ontology. This gave pre-Franconian theory a very distinctive character.

Franco, working within the same framework of Aristotelian philosophy, used a new set of attributes as predicates of note values, involving the pair of opposites perfection/imperfection. He applied this specifically to the *longa*, which could be perfect, containing three tempora, or imperfect containing only two tempora. Now although Franconian notation is often spoken of as being more precise and less ambiguous than previous mensural notation, in fact it is not less ambiguous simply because it is more thrifty; a single note form, the *longa*, is used to represent two different values (and the difference can

[18] Aristotle, *Categories*, 10, 11b, 24-26. The words enclosed in parentheses do not appear in the Latin version of Aristotle's *Categoriae*. The Latin version reads: "Quecumque quidem igitur ut ad aliquid opponuntur, ipsa quod sunt oppositorum dicuntur vel quomodocumque aliter ad ipsa, velut duplum dimidii, ipsum quod est, alterius dicitur duplum; alicuius enim duplum dicitur" (Minio-Paluello, Laurentius ed., *Aristotle's Latinus*, I, 1-5 *Categoriae vel Praedicamenta*, in *Corpus Philosophorum Medii Aevi Academiarum Consociatarum Auspiciis et Consilio Editum* (Leiden, 1961), p. 108. In all the other references to Aristotle's works, the English translation corresponds to the Latin texts.

[19] Aristotle, *Physics*, Book V. 2, 225b 15. Aristotle stressed that qualities are always equal to themselves in all their actual instances and insisted on the genuine autonomy of each mathematical discipline. His view found its expression in the thirteenth-century distinction between *quantitas virtualis*—the intensity of a given quality—and *quantitas corporalis*—the number of the bodies or subjects carrying this form. While accepting the general view that the quantity of bodies is measured by counting the number of bodies sharing the form, philosophers of the thirteenth century still adhered to the Aristotelian principle of *participatio*, according to which different degrees of intensity reflect different degrees of participation of the bodies in the immutable, indivisible quality.

be determined only by context). And while Franco's imperfect *longa* of two tempora has the same quantity of duration as his altered *brevis* (two *tempora*), it is categorically differentiated, because of their different opposite predicates: the *longa* is a *longa* and the *brevis* a *brevis*. The mathematical identity between them seems to be irrelevant: the important fact is that imperfection is a non-essential or accidental quality, or a special case of an essentially perfect *longa*, while alteration is an accidental quality of a *brevis*. And it is by accidental circumstance that the *longa* is made mathematically identical with the accidental circumstance of the altered *brevis*; as such, then, the quantitative aspect of duration has no effect on the essential categorical nature of rhythmic values. This is to say that Franconian mensural theory is qualitative rather than quantitative.

Franco's pair of opposites perfection/imperfection reflects Aristotle's central notion of "being," or perfection and "privation." Franco stated that the opposition between an actual sound and its privation—a rest—is the principle which governs measured music. Continuing faithful to Aristotle, Franco assumed that being takes precedence over privation. Thus, he started with actual sounds, because "being" or "actuality" is prior to "potency."

> "Since discant itself is governed both by actual sound and by the opposite, that is, by its omission, and these are different, their signs will also be different. And since actual sound precedes its omission, just as "habit" precedes "privation" let us speak of figures which represent actual sound before speaking of rests which represent its omission."[20]

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[20] "Sed cum ipse discantus tam voce recta quam eius contrario, hoc est voce amissa, reguletur, et ista sint diversa, horum erunt diversa signa. Sed cum prius sit vox recta quam amissa, quoniam habitus precedit privationem, prius dicendum est de figuris que vocem rectam significant, quam de pausis que amissam." (Franco de Colonia, *Ars cantus mensurabilis*, ed. G. Reaney and A. Gilles, *Corpus Scriptorum de Musica* 18 (1974), pp. 28-29. (All volumes of the *Corpus Scriptorum de Musica* will hereafter be cited as CSM plus the volume number.) Franco commits himself overtly to the philosophical-theological notion of perfection. While a comprehensive analysis of the philosophical notion of perfection is certainly beyond the scope of this paper, it should be noted that Aristotle identified perfection with a complete realization or actualization of the final cause (*causa finalis*), namely, that for the sake of which a thing comes to be. Imperfection in this context denotes a state of privation or incomplete realization of one's potentiality. The repercussion of this understanding in the Franconian style is clear. Imperfection occurs owing to a process of subtraction through which an originally or

Franco's use of the Aristotelian notion of perfection led directly to far-reaching ideas of Aristotle on metaphysics, as well as ideas of Thomas of Aquinas on theology. Aristotle's notion of the relation between the "highest good" and the world is demonstrated in the parable of the army, often used by thirteenth-century philosophers as the starting-point of discussions of the relation between God and the world; Nature was conceived to signify, to some degree, God's presence in the essence of real things. Aristotle says:

> "We must consider also in which of two ways the nature of the universe contains the good and the highest good, whether as something separate and by itself or as the order of the parts. Probably in both ways, as an army does; for its good is found both in its order and in its leader, and more in the latter: for he does not depend on the order but it depends on him. And all things are ordered together somehow, but not all alike—both fish and fowl and plants; and the world is not such that one thing has nothing to do with another but they are connected. For all are ordered together to one end, but it is as in a house, where the free men are least at liberty to act at random, but all things or most things are already ordained for them, while the slaves and the animals do little for the common good, and for the most part live at random: for this is the sort of principle that constitutes the nature of each. I mean, for instance, that all must at least come to be dissolved into their elements, and there are other functions similarly in which all share for the good of the whole."[21]

The free men represent the sun and planets, which move in an eternal, perfect, circular, and absolutely uniform motion. The slaves and the animals stand for the inferior and corruptible concrete earthly entities. The form of the circle has been recognized since Antiquity as

essentially perfect value is deprived of, or "loses" one-third of itself if, and only if, followed or preceded by a breve. The ontological dependence of imperfection on perfection is evident in Franconian notational practice.

[21] Aristotle, *Metaphysics*, XII, 10, 1075a 11-24.

an equivalent to perfection and beauty. Circular motion is therefore superior to all other possible motion, representing simplicity, symmetry, and perfection. Furthermore, compared to other types of change or motion, circularity is a motion, but at the same time is also static. It transcends or sublates motion while conserving it. In other words, circular motion represents the closest possible image of God's *equilibrium* and self-sufficiency.

In spite of their manifest imperfection and random behaviour, sensible sublunar entities, represented in our parable by the slaves, share for the good of the whole by being subjected to the principle of imitation and the principle of repetition or recurrence. Inferior worldly beings *imitate* celestial bodies through the eternal cycles of all natural phenomena, which inevitably recur through the same process. Hence, cyclic motion is the principle of becoming in nature, manifesting order, regularity, and uniformity.

These ideas were crystallized by Thomas Aquinas in his doctrine of *analogia entis*. The world, according to Thomas, is an *imago Dei*. God, Thomas argues, is the only being whose essence necessitates His existence, for God's essence and existence are one. Other things in the world resemble God due to their very existence, for existence is God's essential property. Furthermore, things in the world resemble God as far as they have goodness, beauty and perfection. Perfection (*integritas sive perfectio*) is one of the three formal criteria of beauty, and means the complete actualization of that for the sake of which a thing comes to be, so that none of it is missing.[22] The more perfect the being, the closer it is to God. That is, the relation between God—the creator of the world—and the world, is like that between the creator and his creation, or the relation between a cause and its effect; the cause imparts traces to the effect, and the effect therefore resembles its cause.

For Franco, as for Aristotle and Thomas, perfection is transcendent, but in other ways it is immanent in music, for perfection is found in the system as one of the rhythmic modes—the traditional

[22] "The first type of perfection is present when the thing has all that makes up its substance. The whole object's form is its perfection and arises out of the integrity of its parts." (Umberto Eco, *The Aesthetics of Thomas Aquinas*, translated by H. Bredin [Massachusetts, 1988], p. 99).

fifth mode. Franco, however, renumbered this fifth mode as the first mode, giving it a superior position as the logical origin of the other modes, thus placing it above the system in accord with the divine attributes of perfection. In this respect, Franco's concept of rhythm is different from his predecessors, who were concerned only with the immanent, intrinsic order of each of the six rhythmic modes, rather than with the higher, abstract level of metric organization.

The Franconian system, with its central notion of perfection, guaranteed regularity and uniformity in rhythmic motion. Franco gave, thus, definitive expression to a general notion of thirteenth-century mensural theories, namely the uniform circular and cyclic flow of perfections. The uniform flow and the resulting rhythmic periodicity of musical motion in the *ars antiqua* can be understood as a deliberate attempt to subject music to the same principle which governed the world order according to the Aristotelian-Thomistic school of philosophy. Music, like all other natural events, imitates the immobile God and perfect circular, periodic cosmic motion.[23]

Lambert, explaining why the perfect long constitutes the first mode, used the phrase "*regit, non regitur*." This calls to mind Aristotle's notion of "*movens non movetur*" which he used to describe his first principle of order, the Prime Mover. Lambert used "*regit, non regitur*" to characterize the perfect long.

> "It is said to be the first mode, for it is composed only of perfect figures, (and) this figure is never combined in a ligature, but appears independently, and being alone it never suffers diminution; it rules and is not ruled, commands without being used for the sake of others."[24]

[23] The presence of this principle of periodicity, known better as the principle of repetition or recurrence, rounds off my claim that the concept of music during the Franconian phase was founded on three major principles that guarantee the order and rationality of the Aristotelian universe, namely, the principle of materialization, imitation (*mimesis*), and recurrence.

[24] "Primus modus dicitur, qui tantum componitur perfectis figuris, . . . Et hoc patet igitur quod nunquam comprimitur in ligaturis, sed liber excipitur, et solus non patitur unquam a pressuris; regit et non regitur, imperans non utitur aliorum curis" (Lambertus, *Tractatus de musica*, CS 1, p. 279. It is interesting to note that not only Aristotelian ontology is at work here but also his ethics. Lambert defines the perfect long as that kind of being that is not subordinate to another. This brings to mind the Aristotelian dichotomy between arts whose ends serve higher arts, and arts whose ends are to be pursued for their own sake. See Aristotle, *Nicomachean Ethics*, Book I. 1.1094a-1094b.

A direct echo of Thomas' conception of the world as an image of God is found in Lambert's text:

> "Therefore, it refers to the highest Trinity, and justly so; because every natural entity is found to consist of three (aspects) in resemblance to the divine nature. In voices and sounds and all other things the consonance consists of three, namely *diatessaron*, *diapente* and *diapason*. All things formed by nature follow the Trinity, and indeed even the ignorant must believe necessarily that everything that reaches artistic perfection bears the seed of this perfection in itself from the outset."[25]

The Philosophical and Mathematical Foundation of Jehan's Theory: The Scientific Revolution of the Fourteenth Century

In contrast to the Franconian ideal of regular, uniform, perfect motion, Jehan de Meurs (writing around 1320) attempted to account for rhythmic variability and imperfection. Focusing on changes from one mensuration to another, Jehan laid the foundation for conceiving music as an imitation of an earthly, variable, imperfect process, rather than of divine perfection. Jehan's rhythmic theory does not seem, at first glance, to carry any metaphysical import; nonetheless, it is tied to new concepts and theories of mathematics and philosophy that eroded the Aristotelian injunction against crossing disciplines, or mixing species, and undermined the Aristotelian-Thomistic notion of immutable, indivisible, therefore unquantifiable qualities. These new theories led immediately to revolutionary mathematical analyses of physical variability and natural changes.

It was the philosopher Duns Scotus (1265-1308) who broke with the Aristotelian tradition. In contrast to the Aristotelian-Thomistic

[25] "Et ideo non immerito ad summam refertur trinitatem, quia res quelibet naturalis ad similitudinem divine nature ex tribus constare invenitur. In vocibus et sonis, et rebus omnibus trina tantum consistit consonantia, scilicet diatessaron, diapente et diapason. Hanc igitur trinitatem omnia naturaliter formata consequuntur, quoniam rebus omnibus ab origine prima naturaliter inherentem in summo et primo artifice finisse imperitos necessario credere oportet" (Lambertus, CS 1, p. 270.)

doctrine of qualities, Scotus posited that accidental qualities have a latitude (*latitudo formarum*), which means that they are divisible into infinite "parts," that is, infinitely possible degrees of intensity; if the degree is changed, the individual quality also changes. In other words, in this new ontology things are individuated no longer by their substantial form but rather by all their accidental forms that are particular to each individual. Thus, when accidental qualities are contracted into an individual thing, they are represented as concrete instances at given degrees of intensity. The range of a quality in its individual instances could then be interpreted as an increase in the number of parts of that quality. Such an account of intensification lends itself almost naturally to a geometrical or arithmetical representation: since it is the quality itself that has parts, variation in the intensity of a quality can be explained as the addition or subtraction of parts of that quality.[26]

The history of, as well as the driving forces behind the Scotist revolution, have been extensively studied and documented.[27] What concerns us here, however, is its scientific applications, the most important of which is the new fourteenth-century mathematics of motion. In mathematics, Scotus' philosophical refinements found immediate repercussions in the works of a group of English mathematicians, who flourished in the Merton School of Science at Oxford during the first half of the fourteenth century. These mathematicians, known as the Oxford Calculators, began to analyze the process of change (*intensio et remissio formarum*), such as changes in the velocity of motion—acceleration, or changes in acceleration.

The Oxford Calculators were divided in their conception of a mathematical model which explains variability, and in its application,

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[26] Edith Sylla, "Medieval Concepts of the Latitude of Forms: The Oxford Calculators," *Archives d'histoire doctrinale et littéraire du moyen âge* 40 (1973), pp. 251-257. See also Amos Funkenstein, *Theology and the Scientific Imagination from the Middle Ages to the Seventeenth Century* (Princeton, 1986), pp. 308-309.

[27] For the history of *metabasis* in the Middle Ages, see Steven Livesey, "Metabasis: The Interrelationship of Sciences in Antiquity and the Middle Ages," Ph.D. Dissertation, UCLA, 1982. For the theological background of the dissolution of the Aristotelian world's view and the resulting early experimentations with mixing *genera* or quantification of qualities, see Edith Sylla, "Medieval Concepts of the Latitude of Forms: The Oxford Calculators," pp. 223-283, and also Edith Sylla, "Medieval Quantifications of Qualities: The Merton School," *Archive for the History of Exact Sciences* 8 (1971), pp. 9-39.

specifically in how they understood the relation between latitudes and degrees. For example, some measured intensity by proximity to the maximal degree of the latitude, and remission by the distance from this maximal degree, while others measured intensity by the distance from the zero degree of the latitude, and remission by the proximity to the zero degree.[28] But either way, this approach marks a revolutionary departure from both the classic and medieval notion of absolute incompatibility between opposite qualities, such as hot and cold, discrete and continuous, order and disorder, and eventually perfection and imperfection.

The Calculator John Dumbleton conceived degrees to be divisible into their constituent parts and argued that "latitudes" are physically identifiable as the intensity of a quality at given points.[29] In other words, he identified the latitude of a quality with the degree (*gradus*) of intensity in its concrete instances. This enabled him to develop a one-dimensional coordinate system to represent this latitude of forms. The increase of intensity was seen as analogous to a geometrical line, in which the parts, however minute, represent degrees of intensity. Intensification in this context entails passing through all the intensities between the given degree and the degree attained. This understanding made it possible to measure the variability of real natural processes by comparing different lines representing different intensities, that is, by adding or subtracting parts of intensities.[30] Of crucial significance is the fact that this notion of a latitude of qualities

[28] Marshall Clagett, "Richard Swineshead and the Late Medieval Physics," *Osiris* 9 (1950): 132-138.

[29] Sylla, "Medieval Concepts of the Latitude of Forms," pp. 251-252.

[30] "Subiectum qualitatis intenditur et remittitur per adquisitionem et deperditionem realem qualitatum sicut quantitas maioratur et minoratur per appositionem partium et amotionem earundem . . . Item ponit quod qualitas quam ponimus intensibilem et remissibilem componitur ex qualitatibus eiusdem speciei coextensis simul, que sunt partes generate in motu alterationis, et quelibet manet in fine, et omnes sic coextense faciunt gradum sicut partes quantitative faciunt quantitatem . . . ita tamen quod quelibet qualitas est divisibilis in infinitum qualitative in qualitates" (Sylla, "Medieval Concepts of the Latitude of Forms," p. 255, n. 83). Attention should be called to one of the earlier pre-fourteenth century developments, which may have influenced Dumbleton's theory. It is well known that one of the major sources of quantification of qualities were the Arabic and other medical studies. Galen's "*De simplicibus medicinis*" and Constantine's "*Liber graduum*" explain the range of variation in human health in terms of the latitude of the human temperament. This latitude, they argue, contains four degrees, each having three parts. Curiously enough, Jehan likewise divided the latitude of duration into four degrees, each subdivided into three parts. See below, p. 18.

makes apparently opposing qualities, such as hot and cold or fast and slow, the two extremes of a continuum that extends from the minimum to the maximum degree of intensity.[31]

As a result, mathematics was no longer divorced from physics; arithmetical computations of intensifications mixed the two different mathematical species, the discrete quantities of arithmetics and the continuous quantities of geometry. A new type of mathematical physics emerged, replacing the Aristotelian physics of definitions and classifications according to essential predicates, with an attempt to classify all the possible distributions of qualities throughout space and time, using terms such as *minima*, *maxima*, *latitudo*, and *gradus*.

In a similar way, Jehan dispensed with the qualitative approach of the thirteenth-century theoreticians, who regarded each of the rhythmic values as a different species:

> "As for times, one is more, the other is less. More, when the motion prolongs (throughout longer time), less when (the motion) is shorter, measured by the same dimension. But these do not differ in species, since more or less does not vary the species."[32]

It is clear that here Jehan is taking part in the scholastic debate over the possibility of comparing qualities in terms of more and less, in general, and over the nature of the intensity and remission of forms, in particular. He recalls the position of Duns Scotus and the Calculators, holding that "more" or "less" of the same quality does not change the

[31] In the terminology of the Calculators, the minimal degree was often named "non gradus," that is, zero degree of intensity. The maximal degree was usually referred to as "gradus summus," or "gradus intensissimus."

[32] "Temporis aliud maius aliud minus: maius, quod motum prolixiorem, minus, quod breviorem habet ceteris eisdem, secundum unam dimensionem metitur. Haec autem specie non differunt, nam maius et minus speciem non variant" (CSM 17, p. 66). U. Michels (1970) suggests that Jehan speaks in this passage about the general notion of time (as distinct from musical time): "auf den Zeitbegriff im allgemeinen, nicht im musikalisch speziellen Sinn bezieht sich die Erklärung zum tempus maius und minus bei Muris" (Michels, *Die Musiktraktate des Johannes de Muris*, p. 3). I interpret the passage as referring to musical time. Therefore, the term *specie* in the above quotation refers to "short" and "long" prolongations ("maius, quod motum prolixiorem, minus, quod breviorem habet ceteris eisdem, . . .) and not, as implied by Michels's commentary, to "perfect and "imperfect" rhythmical values.

species. Jehan's reasoning for endorsing quantitative comparison between shorter and longer rhythmic values may be construed as a reflection of Dumbleton's theory, which stressed the distinction between essential non-variable qualities, and accidental variable ones.[33] And the very core of Jehan's theory is the insistence on the continuous dimension of time in which both types of value (long and breve) participate. Hence, his theory echoes the Calculator's insistence on the existence of a larger whole in the same species or dimension, without which comparison by quantification is impossible; for a continuum, by definition, cannot be composed of things of different species.[34]

Jehan did not merely reflect the new mathematical trends of quantifying intensive magnitudes; he adopted them and spoke their analytical language.[35] For Jehan the physical quality to be quantified

[33] Accidental qualities, Dumbleton argues, naturally possess some latitude within themselves and can therefore undergo intensive changes. This new approach was diametrically opposed to Thomas' distinction between the pair of concepts "small" and "big" (*parva vel magna*) on the one hand, and the pair "more" and "less" (*magis et minus*) on the other. "Small" and "big," he argued, refer to the number of bodies carrying a given quality, say the quality of "whiteness;" therefore, they represent an arithmetical sum. But "more" and "less" signify the participation of the body in the immutable and indivisible quality. See Clagett, "Richard Swineshead and the Late Medieval Physics," pp. 132-134. Dumbleton's famous comparison between man and ass illustrates the necessity of common species to make quantitative relation possible. Even if the perfection of an ass were multiplied an infinite number of times, he claims, the resulting sum of perfections would not be greater than the perfection of one such ass. Consequently, there is no proportion between the perfection of an ass and the perfection of a man. "Sed infinite tales perfectiones sicut perfectio azini non maiorem una facient. Ideo non est proportio in quo homo est magis ens quam azinus" (Sylla, "Medieval Concepts of the Latitudes of Form," p. 253 n. 79).

[34] Dumbleton's argument reads as follows: "Item non proprie fit distantia ex illis que sunt diverse speciei et non communicant . . . Pro isto est notandum quod omnis talis comparatio que fit in abstracto ut aliquid est magis substantia alia semper est respectu illorum que sunt diverse speciei non tamen ut sit unum alio in aliqua proportione magis. Proprie dicitur quod omnis proportio sive fuerit arsmetrica, geometrica, semper minus se habet respectu maioris tamquam pars aliquota eius que aliquotiens sumpta finite vel infinite reddit suum totum maius" (Sylla, "Medieval Concepts of the Latitude of Forms," p. 253 n. 79). Jehan says: "As has been shown in the first book, voice is generated with motion, since it is of the kind of successive things. Therefore it is, while being made, and is not, when having been made . . . Therefore voice must of necessity be measured by time. Time is the measure of motion." "Ut in primo <libro> ostensum est, vox generatur cum motu, cum sit de genere successivorum. Ideo quando fit, est, sed cum facta est, non est . . . Igitur vocem necessario oportet tempore mensurari. Est autem tempus mensura motus. Sed hic tempus est mensura vocis prolatae cum motu continuo" (CSM 17, p. 65).

[35] In a philosophical vein, Jehan grasped the revolutionary principle of quantifying physics, albeit the structural incompatibility between mathematics and physics. His conceptual *substratum* does not necessitate a qualitative unity between the measure and the measured. Like Dumbleton, Jehan gave separate reality to the latitude of given qualities and identified the latitude with the degree of intensity in a given instance.

was the duration of a note value. Considered as a physical entity (*forma naturalis*), the duration of a musical sound has its specific latitude, limited by minima and maxima. For in nature there are neither infinitely big nor infinitely small magnitudes; rhythmical divisions are empirically limited by the nature of sound and of the human voice.[36]

> "Therefore the voice (or musical sound) measured by time constitutes a union of two forms: the natural and the mathematical. Even though according to the one (the mathematical) the division is endless, according to the other (the natural) its division must end somewhere. Just as for all things by nature there is a limit in magnitude and augmentation, so also in smallness and diminution. Natural things prove that nature is limited by a maximum and a minimum. The voice, being in itself a natural form to which quantity is joined by accident, must have limits for its division, the latitude of which cannot be surpassed by any sound, however fleeting. We wish to understand these limits by reason."[37]

Jehan demonstrates the latitude and the limits (*termini*) of a musical sound by the following table, in which the latitude of a prolonged sound was divided into four degrees of perfection (*gradus perfectionis*). Each degree was further subdivided into three parts in the relation 3:2:1.[38]

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[36] Here we see Jehan's sensitivity to the distinction between physics and mathematics. The fact that a sound can be measured by mathematics does not entail any identical structure between the sound and mathematics. While in the abstract, mathematical measure can be applied to a physical being, rhythmic divisions depend on physical constraints.

[37] "Quoniam ergo vox tempore mensurata unionem duarum formarum, naturalis scilicet et mathematicae, comprehendit, licet quod ratione alterius fractio non cessaret, tamen propter aliam vocis divisionem necessarium est alicubi terminari. Nam sicut omnium natura constantium positus est terminus et ratio magnitudinis et augmenti sic parvitatis et diminuti. Demonstrant enim naturales, quod natura ad maximum et minimum terminatur. Vox autem est per se forma naturalis iuncta per accidens quantitati. Igitur oportet eam habere terminos fractionis, quorum latitudinem nulla vox quantacumque frangibilis valeat praeterire. Hos autem terminos volumus comprehendere ratione" (CSM 17, p. 69).

[38] Jehan's table calls to mind the old medical method of dividing human temperament or health into four grades, each having three possible variants. On the medical contribution to early fourteenth-century quantifications, see Sylla, "Medieval Concepts of the Latitude of Forms," p. 252.

Figure 1

	3	81	longissima		
	2	54	longior		primus gradus
	1	27	longa	idem	
	3	27	perfecta		
	2	18	imperfecta		secundus gradus
	1	9	brevis	idem	
	3	9	brevis		
	2	6	brevior		tertius gradus
	1	3	brevissima	idem	
	3	3	parva		
	2	2	minor		quartus gradus
	1	1	minima		

Jehan's degrees of perfection certainly reflect Dumbleton's notion of latitudes and his "addition" theory of the intension and remission of forms.[39] Not only at the extreme low end of the continuum, now called in music "minima"—the specific term from the new mathematics, but also the larger degrees are measured by the rhythm of the minim, and any degree representing a note-value is the sum of its constituent parts.

> "Beginning with the unity, which is the third part of a triple value which is perfect, up to 81, which is similarly perfect, these are said to be the limits from the maximum to the minimum of any sound, and the entire length of such sound is included between these limits."[40]

[39] See above, pp. 13-15.

[40] "Ab unitate igitur, quae tertia pars est ternarii qui perfectus est, usque ad 81, qui similiter est perfectus, dicuntur esse termini de maximo ad minimum cuiuslibet vocis, totaque eius longitudo inter hos terminos est inclusa" (CSM 17, p. 72).

So far we have seen that in both the new musical theory and the Mertonian mathematics, qualitative relations were quantified by redefining the relationship between physical objects and their qualities in mathematical terms. Music theorists and mathematicians discussed quantification in the same language. It now remains to be seen whether these terms were applied in the two disciplines to the same kinds of problems or objects.

Jehan's Four Mensurations: The Distribution of Various Qualities throughout Time and Space

Having laid the philosophical ground for quantifying qualities, the Calculators were preoccupied with the application of latitudes and degrees to actual physical distribution of qualities in given subjects. First, they sorted out and classified the possible types of variability as different distributions of qualities throughout space and time. Their principal distinction was between uniform (*uniformis*) and difform (*difformis*) distribution. In other words, they distinguished between subjects manifesting a quality in uniform intensity or a movement in uniform velocity (distribution of a quality in time) and subjects manifesting several intensities of a quality. Examples would be (a) a subject that starts with a given degree which increases gradually until a much higher degree is reached, or (b) an accelerated or decelerated motion.[41]

This large class of difformly qualified subjects was further divided into different types of non-regular distributions. In discussing velocity the most common distinctions were between *motus uniformis* (uniform motion), *motus difformis* (difform motion), *motus uniformiter difformis* (motion in which the velocity increases uniformly), and *motus uniformiter difformiter difformis* (motion in which the acceleration increases uniformly). The following definitions of these four types of motions are from John of Holland's *De motu*, ca. 1360:

[41] John Murdoch and Edith Sylla, "The Science of Motion." In *Science in the Middle Ages*, ed. D.C. Lindberg (Chicago, 1978), pp. 233-234.

"Of local motions some are uniform, some are difform (non-uniform=*difformis*)...Motion uniform as to time is the motion of some moving body whereby the body traverses an equal space in every equal part of the time in which it is moved.

Movement difform as to time is the motion of a mobile body whereby the body traverses more space in one part of the time than in some other equal part of time. Motion difform as to time is twofold: some uniformly difform, some difformly difform.

Local motion uniformly difform is a difform movement in any designated part of which the middle speed of that part exceeds the minimum terminal speed by the same increment (latitudo) as the middle exceeded by the maximum terminal speed. Motion difformly difform occurs when uniformly difform movement is nonexistent.

To be moved as to time is twofold, either uniformly or difformly. To be moved uniformly as to time is to traverse an equal space in every equal period of time.... To be moved difformly as to time is twofold, to increase speed or to decrease speed. To increase speed is twofold, uniformly or difformly. To increase speed uniformly is to acquire an equal increment of speed in every equal part of the time."[42]

[42] "Motuum localium quidam est uniformis, quidam est difformis . . . Motus uniformis quo ad tempus est motus alicuius mobilis quo ipsum mobile in omni parte equali temporis pro quo illud movetur pertransit spatium equale.... Motus difformis quo ad tempus est motus mobilis quo ipsum mobile in una parte temporis plus pertransit quam in alia parte temporis sibi equali. Motus difformis quo ad tempus est duplex; nam quidam est uniformiter difformis, quidam difformiter difformis. Motus localis uniformiter difformis describitur a calculatoribus sic: Motus localis uniformiter difformis est motus difformis, cuius quacunque parte signata medius gradus illius partis per equalem latitudinem excedit extremum remissius eiusdem sicut ipse ab extremo intensiori illius partis exceditur. Motus difformiter difformis est motus difformis non existens uniformiter difformis. Moveri quo ad tempus est duplex, vel uniformiter vel difformiter. Uniformiter moveri quo ad tempus est in omni parte temporis equali equale spatium pertransire . . . Difformiter moveri quo ad tempus est duplex, vel est intendere motum vel est remittere motum. Intendere motum est duplex, vel uniformiter intendere vel difformiter intendere. Uniformiter intendere motum est in omni parte temporis equali equalem latitudinem motus acquirere," Marshall Clagett, *Science of Mechanics in the Middle Ages* (Wisconsin, 1959), pp. 247-248.

In his *Notitia artis musicae*, Jehan treats a similar problem. Like the Calculators, he sorts out types of rhythmic variability by analyzing all the possible divisions of the *longa*. Hence, he classifies the possible distributions of shorter rhythmic values throughout a definite extension (the *longa*). Jehan's classifications are based on the distinction between perfect division (a division into three equal parts) and imperfect division (division into two equal parts). His four distributions are as follows: *Longa perfecte perfecta* is a long that contains nine semibreves. *Longa perfecte imperfecta* is a long that is imperfected by a perfect breve, and thus contains six breves. *Longa imperfecte perfecta* is a long imperfected by one or two semibreves, and contains eight or seven semibreves. *Longa imperfecte imperfecta* is that which is imperfected by both a perfect breve and one or two semibreves, and contains four or five semibreves.[43] A resemblance of terms between the distinctions in mathematics and music is certainly evident, as is, it seems to me, a resemblance of content.

Perfect and Imperfect Divisions: The Translation of Disorder into Order by Quantification of the Relation between Different Distributions

Jehan did not permit duple meter as such. This has already been noticed by U. Michels,[44] and is clearly reflected in Jehan's four divisions of the long, which show that duple values result from the imperfection of perfect values. To understand Jehan's approach to imperfection in music, we need to consider it in the broader context of vigorous attempts made in the fourteenth century to mediate between disorder and order, infinity and finiteness, regularity and irregularity, perfection and imperfection. Mathematicians searched for ways of understanding, rationalizing, and quantifying the non-uniform distributions of qualities. Difformly qualified subjects, or difformly qualified motion, that is, motion with constantly changing velocity, such as the velocity in a uniformly accelerated motion, represented

[43] "Alia perfecte perfecta, alia perfecte imperfecta, alia imperfecte perfecta, alia imperfecte imperfecta" (CSM 17, p. 95).
[44] See above, n. 1.

disorder to ancient and medieval thinkers. Following the old Pythagorean table of contraries, philosophers identified order with perfection, finiteness, and absolute regularity or uniformity. Disorder was synonymous with imperfection, infinity, irregularity, and difformity. Conceiving latitudes as geometrical lines, the Calculators understood such latitude of forms to be continuous, that is, divisible into infinitely smaller segments. Thus, all difform distributions involved the notion of infinity; for example, uniformly accelerated motion is a motion composed of an infinite number of different velocities.

The central idea behind such measurement and quantitative analyses of different difform distributions was to translate disorder into order. Such translations were based on the possibility of finding a simple uniform state or motion equivalent to the initial difform or irregular motion. By quantifying the relationship between uniform motion and difform motion, mathematicians demonstrated a continuity between opposites, that in antiquity and up to the fourteenth century had been considered absolutely incommensurable.

Perhaps the most famous result of these endeavours to mediate between opposites was the formulation of the rule of "mean speed" which translated a uniformly difform motion to a simple uniform motion: A body moving uniformly difformly (in uniform acceleration) will in a given time cover the same distance it would while moving uniformly with its mean speed. In the terminology of the Calculator Richard Swineshead, the rule reads:

> "Since local motion uniformly difform corresponds to its mean degree (of velocity) in regard to effect, so it is evident that at the same time so much is traversed by means of (a uniform movement) at the mean degree as by means of the uniformly difform movement. Furthermore, any difform motion—or any other quality—corresponds to some degree."[45]

[45] "Cum motus localis uniformiter difformis correspondet quo ad effectum suo medio gradui, sic patet quod tantum per idem tempus ponitur pertransiti per medium [gradum] sicut per illum motum localem uniformiter difformem. Unde quiscunque motus difformis, et etiam quecunque alia qualitas, alicui gradui correspondet." (Clagett 1959, pp. 245-6 for the Latin, and p. 244 for the English translation). A shorter version of this rule reads: "Omnis latitudo motus uniformiter acquisita vel deperdita suo gradui medio correspondet."

This basic rule engendered a list of variations, as the following one: "If a subject has a given degree at one end and increases rapidly in temperature at first as one moves away from this end of the subject, and then increases more and more slowly with the distance until the degree of the other end is reached, it will correspond to a greater degree than a subject whose temperature increases uniformly from one end to the other."[46] Other equations could involve two different types of motion, circular and linear. The following rule concerns the problem of rotation in which the speed of each point of the rotating body is in direct proportion to the distance of the point from the center of the body: In a uniform motion of a rotating wheel, where each point is moving at a different velocity, the velocity of the wheel as a whole is measured by the linear path traversed by the point which is in most rapid motion.[47] Here again, a motion composed of infinite velocities, represented by the infinite number of points on the radius of a rotating wheel, is translated into a simple uniform motion.

Jehan used analogous procedures to deal with the problem of imperfection, which, as a principle, he was not ready to accept. He did not allow duple meters as such, and he continued to speak of perfection in theological symbolism.[48] Accordingly, he argued against "modern" composers who claimed in vain that their new style introduced imperfection into music.

[46] Sylla, "Medieval Concepts of the Latitude of Forms," p. 257.

[47] Marshall Clagett, *Science of Mechanics in the Middle Ages*, pp. 235-237.

[48] Time and again Jehan repeats the old Christian belief that the Trinity—the principle of the world order—is reflected in all "perfections" throughout the whole chain of being: "That all perfection rests in the ternary number is made clear by several analogous conjectures. In God, the most perfect being, there is unity of substance, a trinity of persons, threefold as one, one as threefold. There is a total correspondence of unity to trinity. After God, in the separate intelligences, in the being and essence and in a composition of both (in one substance), the ternary number appears again. In the first heaven [there are]: the mover, the moving, and time. Three (attributes) appear in stars and sun: heat, ray, splendor; in elements: action, passion, matter; in individuals: generation, corruption, dissolution; in all finite time: beginning, middle, end; in all curable diseases: increase, stability, decline." "Quod autem in ternario quiescat omnis perfectio, patet ex multis veresimilibus coniecturis. In Deo enim, qui perfectissimus est, unitas est in substantia, trinitas in personis; est igitur trinus unus et unus trinus. Maxima ergo convenientia est unitatis ad trinitatem. In intelligentia post Deum esse et essentia et compositum ex hiis sub numero ternario reperitur. In primo corporum caelo: movens, mobile, tempus. Tria sunt in stellis et sole: calor, radius, splendor; in elementis: actio, passio, materia; in individuis: generatio, corruptio, subiectum; in omni tempore finibili: principium, medium, finis; in omni morbo curabili: augmentum, status, declinatio" (CSM 17, p. 67).

> "To every time interval which measures sound, our predecessors reasonably assigned a certain mode of perfection prescribing this sort of time interval in order that it might support a ternary division, for they believed all perfection to be in the ternary number. For this reason they prescribed perfect time as the measure of all music, knowing that it is unsuitable for the imperfect to be found in art. Yet certain moderns believe themselves to have discovered the opposite of this, which is to be rejected."[49]

Jehan had to account for the use in practice of even, imperfect rhythmic values, which ever since the days of Pythagoras had been associated with indefinite, unlimited, chaotic matter. In order to legitimize the use of imperfect rhythmic values, Jehan believed that he needed to demonstrate that the imperfection or disorder could be translated into terms of perfection and order. He did this by pointing out that the three imperfect numbers (being multiples of 2) which appear in his four grades of perfection, namely, 6, 18, 54, are also multiples of 3, hence resemble ternary numbers and participate in the Trinity.

$$6 = 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$54 = 2 \times 3 \times 3 \times 3$$

Therefore, they can be conceived as perfect numbers.

> "Since the ternary number is found in all things in one form or another, its perfection here should not be doubted any more. And conversely the binary number, owing to its being shorter,

[49] "In omni vero tempore vocem mensurante quemdam modum perfectionis priores rationabiliter assignaverunt, illud tempus tale ponentes, quod per ternarium posset suscipere sectionem, opinantes in ternatio omnem esse perfectionem. Et propter hoc tempus perfectum pro mensura cantus cuiuslibet posuerunt, scientes quod in arte imperfectum non convenit reperiri, quamvis huius oppositum aliqui moderni, quod abest, se crediderunt invenisse" (CSM 17, p. 66).

> remains imperfect, for the binary number is infamous. But any composite number can be justly considered as perfect due to its resemblance and agreement with the ternary number."[50]

In other words, order and disorder were no longer mutually exclusive, disjunctive concepts; they could be compared and equated. One could translate musical disorder into terms of order.

At the end of his short treatise Jehan expressed this idea by saying that motion in imperfect time (*tempus imperfectum*) can be equal to motion in perfect time (*tempus perfectum*), for three binary values are equal to two ternary values.

> "It should be noted that singing can be done with perfect notes relating to imperfect time as three *breviores*, and with imperfect notes relating to perfect time as two breves. Three binary values are made equal to two ternary values in (the numbers) 6, 12, 18, 24, etc. Thus three perfect binaries in imperfect time are as two imperfect ternaries in perfect time. When they alternate one with the other, they finally become equal by equal proportion. And the singing is in perfect (time) with perfect values and in imperfect (time) with imperfect values, whichever is fitting."[51]

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[50] "Cum igitur ternarius omnibus se ingerat quodammodo, hunc esse perfectum non debet amplius dubitari. Per cuius oppositum, cum ab ipso recedat binarius, relinquitur imperfectus, cum etiam binarius numerus sit infamis. Sed unum compositus, sic quilibet numerus convenientiaque, quam habet ad ternarium, perfectum potest merito reputari" (CSM 17, 68-69). The proposed reading of the above-quoted passage differs from that of U. Michels; according to Michels the sentence 'Sed unum compositus sic quilibet numerus convenientiaque, quam habet ad ternatium, perfectum potest merito reputari' refers to the unit, i.e.: 'compositus' is (for Michels) the unit of each '*gradus perfectionis*.' This interpretation conflicts with Jehan's theory and also with Michels' own commentary. For Jehan, all the values functioning as the unit of each grade had a double meaning, being both the neutral unit of the next greater ternary or perfect value, and the perfect value of the next smaller value, as is indicated clearly in his table (CSM 17, p. 79). Actually, this double function is advocated by Michels' own commentary (Michels, *Die Musiktraktate des Johannes de Muris*, p. 76). The point to be stressed is that Jehan did not need any proof for the perfection of these unit-values, because they are at one and the same time ternary and unitary, therefore perfect numbers. His main goal was to account for the use of imperfect values in the new art of the *Moderni*.

[51] "In fine huius opusculi notandum est, quod contingit fieri cantum ex perfectis notulis de tempore imperfecto ut tres breviores, et ex imperfectis de tempore perfecto ut duo breves. Adaequantur enim tres binarii et duo ternarii in 6, 12, 18, 24 et sic addendo 6. Sunt autem tres binarii perfecti de 'tempore' imperfecto, sed duo ternarii imperfecti de perfecto, et ad invicem revolvuntur et aequa proportione finaliter adaequantur. Et ex perfectis de perfecto et imperfectis de imperfecto sicut convenit decantatur" (CSM 17, p. 84).

By the end of the fourteenth century, the mathematical techniques of dealing with simultaneous variable rates would account for the extended syncopation of the *ars subtilior*. And Jehan's discussion of equivalence was the means of dealing with the innovative use of proportions.

By translating rhythmic imperfection or disorder into terms of perfection and order, and focusing on transitions from one kind of measurement into another, Jehan made a first attempt to sort out and explain the disconcerting problem of variability in music.

Through a close reading of Jehan's *Notitia*, there seems to transpire an earthly concept of music, one which eventually allowed rhythms to imitate worldly motions, to manifest variety of intensities, instead of imitating the immutable and perfect spheres. And Jehan's mediation between opposites may also be construed as preparing the conceptual environment for later, fifteenth-century mediation between consonances and dissonances. But that is another story.

In his own time, we can understand Jehan's rhythmic theory as providing a radically new environment for fourteenth-century music.

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