Smith Chart
&
Matching Network

James Lu

Non-matched Impedance ($\Gamma \neq 0$)

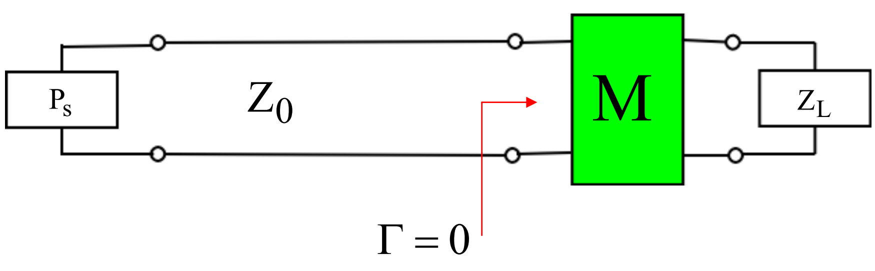
- Reflections lead to $\boldsymbol{Z_{in}}$ variations with line length and frequency
- Power is wasted because of reactive power, which can also damage equipment during short circuit (for example)
- Only partial power is delivered to the load.
- SWR > 1: there will be voltage maxima on the line, voltage breakdown at high power levels
- Noises (bounces or echoes)

Benefits of Matching (Γ=0)

- $Z_{in} = Z_{O,}$ independent of line length, and frequency (over the bandwidth of the matching network)
- Maximum power transfer to the load is achieved
- SWR = 1: no voltage peaks on the line
- No bounces (echoes)

Load Matching

What if the load cannot be made equal to Z_o for some other reasons? Then, we need to build a matching network so that the source effectively sees a match load.



Typically we only want to use **lossless devices** such as capacitors, inductors, transmission lines, in our matching network so that we do not dissipate any power in the network and deliver all the available power to the load.

Normalized Impedance

It will be easier if we normalize the load impedance to the characteristic impedance of the transmission line attached to the load.

$$z = \frac{Z}{Z_o} = r + jx$$

$$z = \frac{1+\Gamma}{1-\Gamma}$$

Since the impedance is a complex number, the reflection coefficient will be a complex number

$$\Gamma = u + jv$$

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \qquad x = \frac{2v}{(1-u)^2+v^2}$$

Smith Charts

The impedance as a function of reflection coefficient can be re-written in the form:

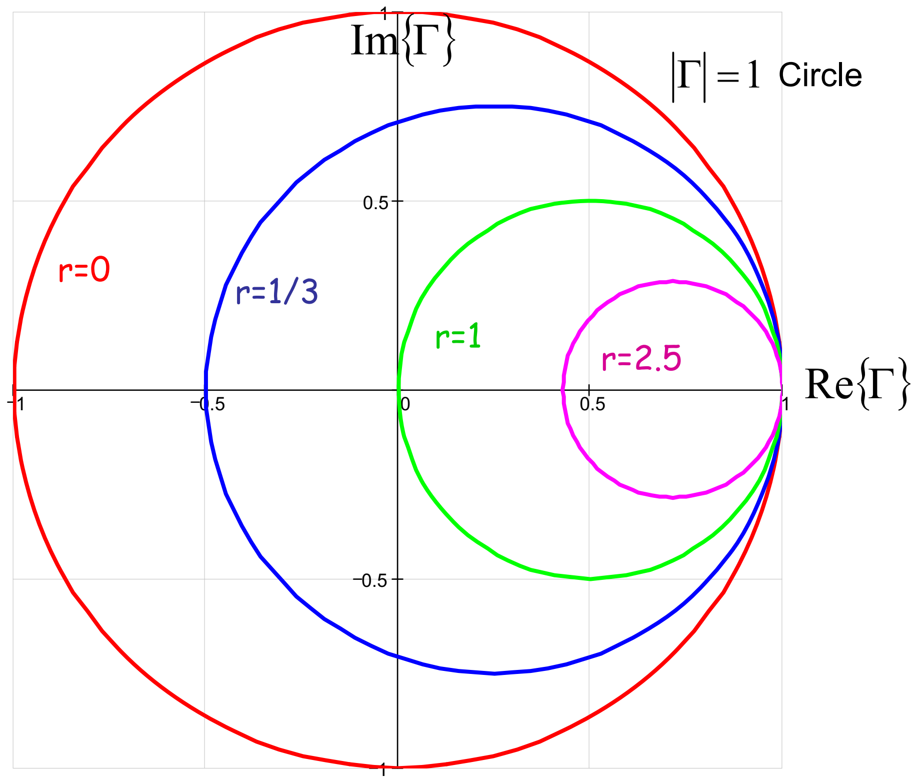
$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \Longrightarrow \left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2}$$

$$x = \frac{2v}{(1-u)^2+v^2} \Longrightarrow (u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2}$$

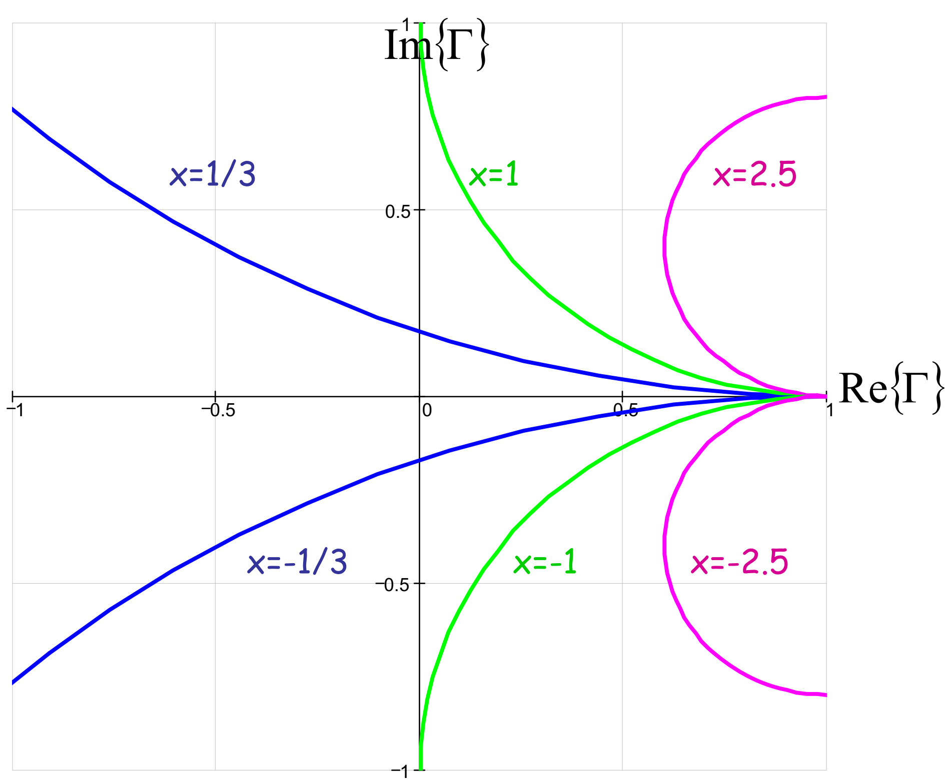
These are equations for circles on the (u,v) plane

$$(x-x_o)^2 + (y-y_o)^2 = a^2$$

Smith Chart – Real Circles



Smith Chart – Imaginary Circles



Smith Chart

Impedances, voltages, currents, etc. all repeat every half wavelength

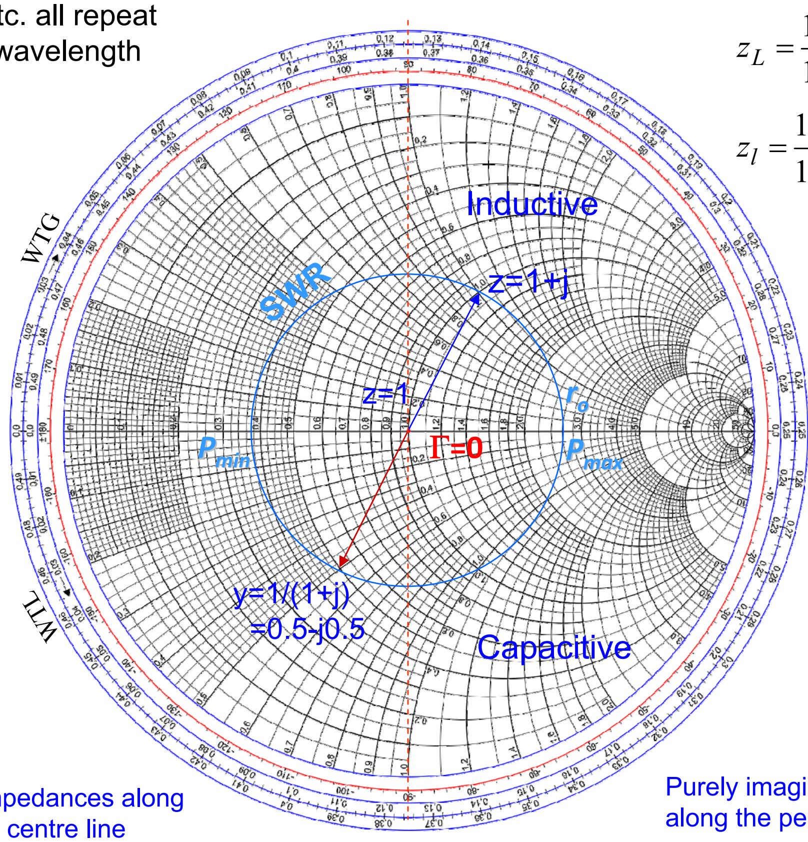
$$\Gamma = \Gamma_L = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = r_L + jx_L$$

$$z_l = \frac{1+\Gamma_l}{1-\Gamma_l} \qquad \Gamma_l = \Gamma e^{-j2\beta l} = \Gamma e^{-j\frac{4\pi}{\lambda}l}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = r_o\,(\text{at}\,P\max)$$

Short
(z=0)
Γ=-1



Open
(z=∞)
Γ=1

Purely real impedances along the horizontal centre line

Purely imaginary impedances along the periphery

Smith Chart Example 1

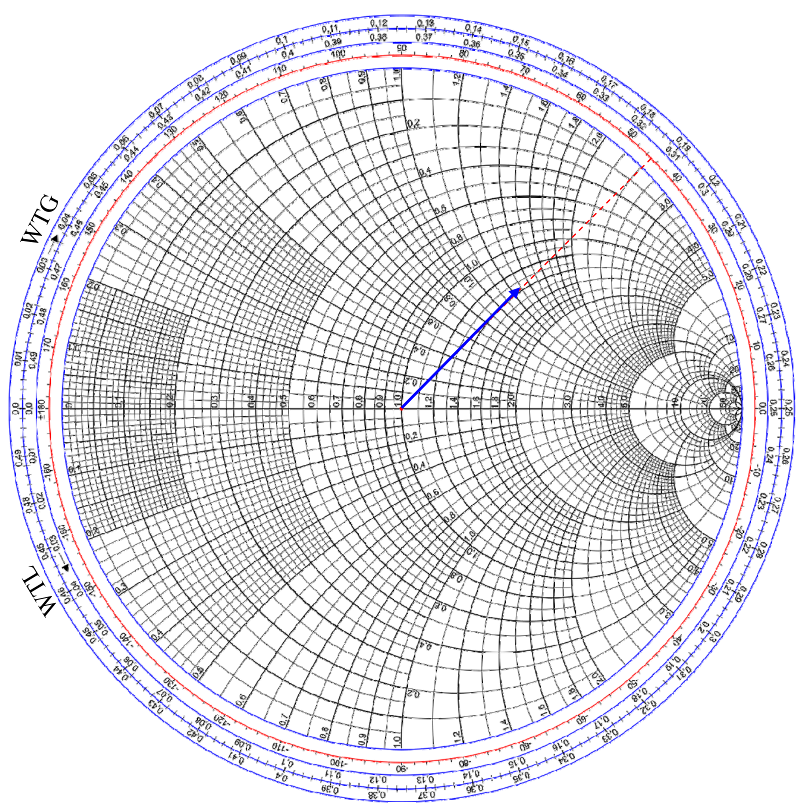
Given:

$$\Gamma_L = 0.5\angle 45^\circ$$

$$Z_o = 50\Omega$$

What is Z_L?

$$Z_L = 50\Omega(1.39 + j1.35) = 69.5\Omega + j67.5\Omega$$



Smith Chart Example 2

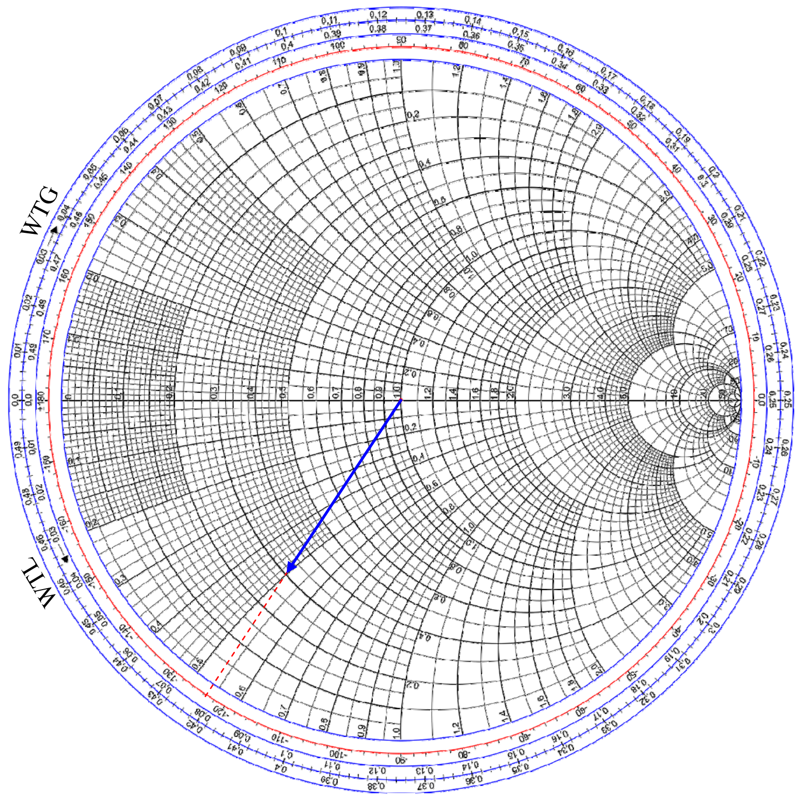
Given:

$$Z_L = 15\Omega - j25\Omega$$
$$Z_o = 50\Omega$$

What is Γ_L?

$$z_L = \frac{15\Omega - j25\Omega}{50\Omega}$$
$$= 0.3 - j0.5$$
$$\Gamma_L = 0.6\angle -123°$$

z1 = 2 + j
z2 = 1.5 -j2
z3 = j4
z4 = 3
z5 = infinity
z6 = 0
z7 = 1
z8 = 3.68 -j18



Smith Chart Example 3

Given:

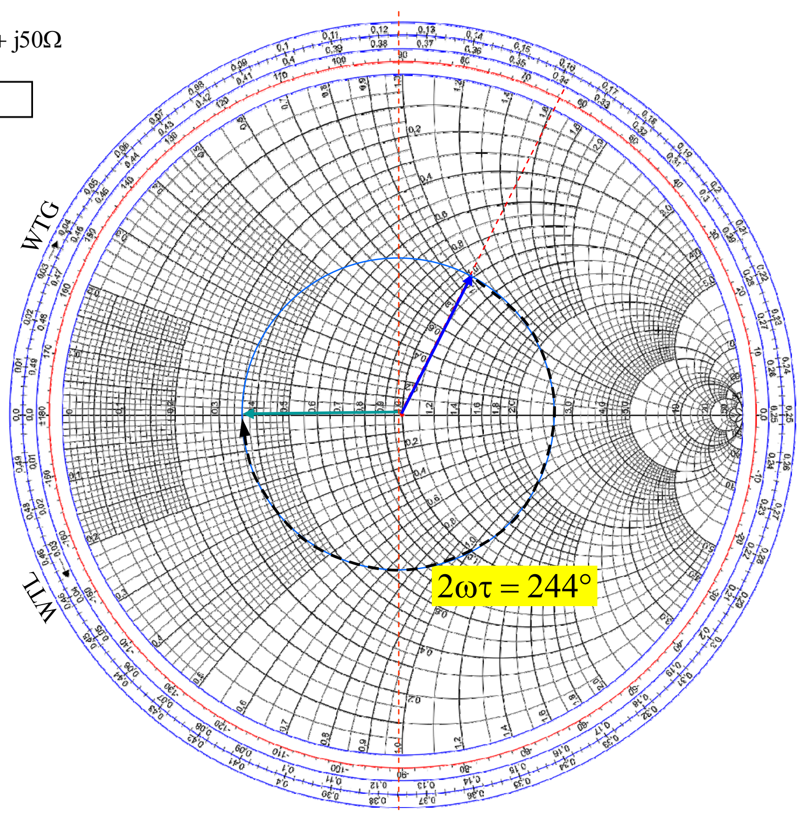
$Z_L = 50\Omega + j50\Omega$

$Z_o = 50\Omega$

$\tau = 6.78ns$

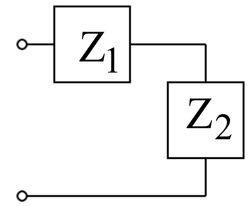
What is Z_{in} at 50 MHz?

$$z_L = \frac{50\Omega + j50\Omega}{50\Omega}$$
$$= 1.0 + j1.0$$
$$\Gamma_L = 0.445\angle 64°$$
$$\Gamma_{in} = \Gamma_L e^{-j2\beta l} = \Gamma_L e^{-j4\pi l/\lambda} = \Gamma_L e^{-j2\omega\tau}$$
$$l = f\lambda\tau = 50 \cdot 10^6 \cdot 6.78 \cdot 10^{-9}\lambda = 0.339\lambda$$
$$\vartheta_{in} = 180°$$
$$\Gamma_{in} = 0.445\angle 180°$$
$$Z_{in} = 50\Omega(0.38 + j0.0) = 19\Omega$$



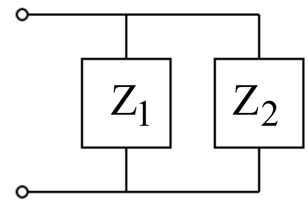
Admittance

A matching network is going to be a combination of elements connected in series AND parallel.



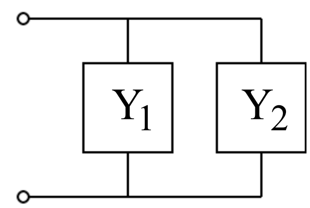
Impedance is well suited when working with series configurations. For example:

$$V = ZI \qquad Z_L = Z_1 + Z_2$$



Impedance is NOT well suited when working with parallel configurations.

$$Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$$



For parallel loads it is better to work with admittance.

$$I = YV \qquad Y_1 = \frac{1}{Z_1} \qquad Y_L = Y_1 + Y_2$$

Normalized Admittance

$$y = \frac{Y}{Y_o} = YZ_o = g + jb$$

$$y = \frac{1-\Gamma}{1+\Gamma}$$

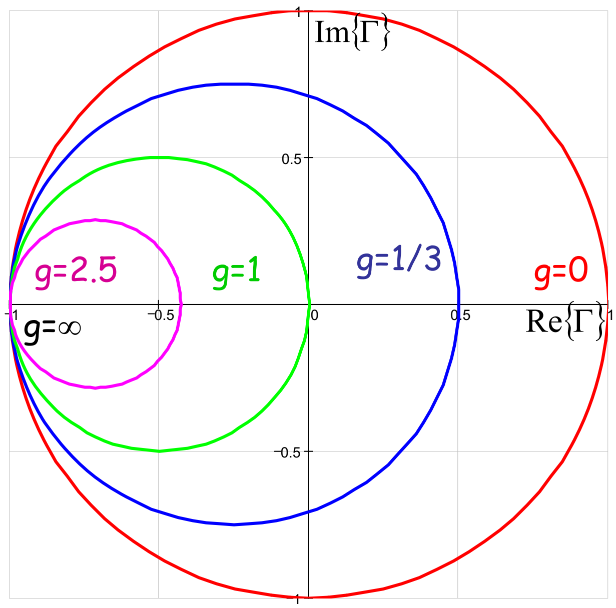
$$g = \frac{1-u^2-v^2}{(1+u)^2+v^2} \quad \Longrightarrow \quad \left(u + \frac{g}{1+g}\right)^2 + v^2 = \frac{1}{(1+g)^2}$$

$$b = \frac{-2v}{(1+u)^2+v^2} \quad \Longrightarrow \quad (u+1)^2 + \left(v + \frac{1}{b}\right)^2 = \frac{1}{b^2}$$

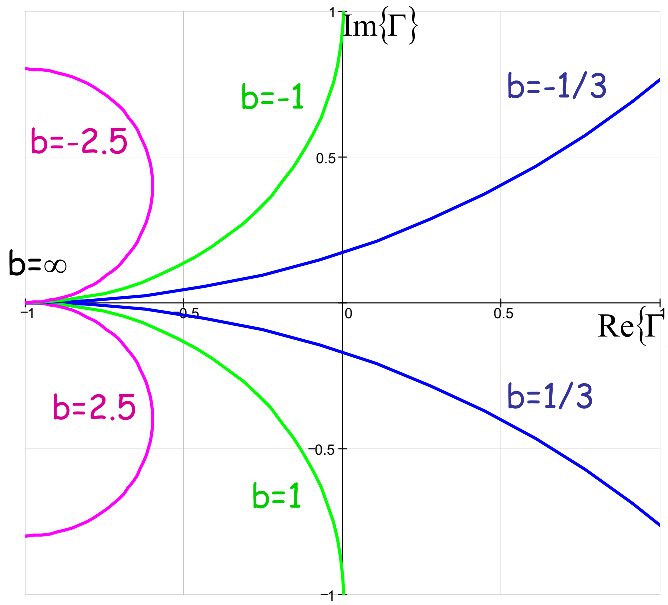
These are equations for circles on the (u,v) plane

Admittance Smith Chart

Conductance Circles



Susceptance Circles



Impedance and Admittance Smith Charts

- For a matching network that contains elements connected in series and parallel, we will need two types of Smith charts
  - impedance Smith chart
  - admittance Smith Chart
- The admittance Smith chart is the impedance Smith chart rotated 180 degrees.
  - We could use one Smith chart and flip the reflection coefficient vector 180 degrees when switching between a series configuration to a parallel configuration.

Admittance Smith Chart Example 1

Given:

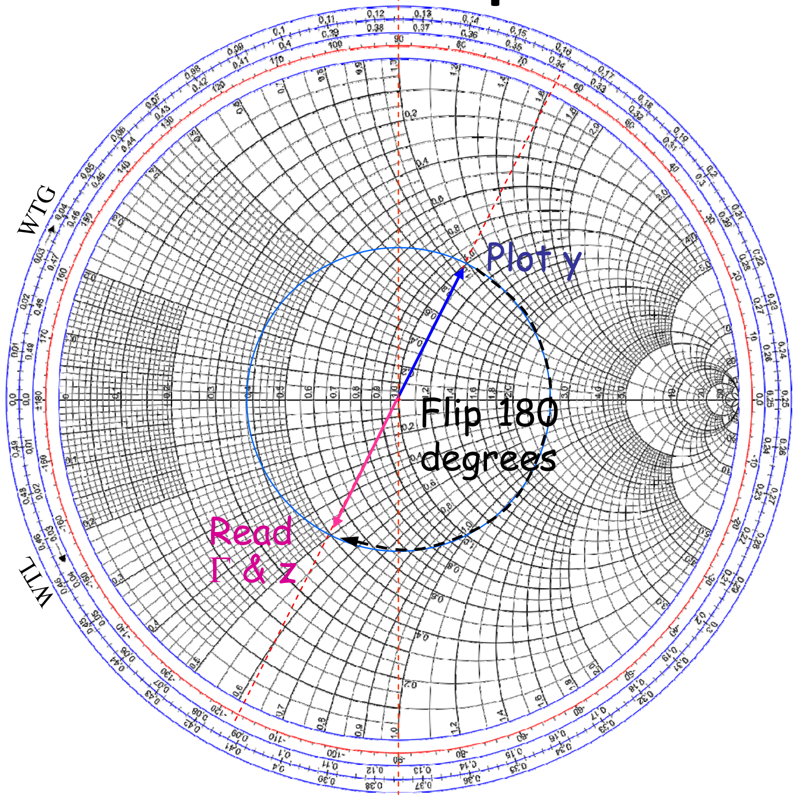
$y = 1 + j1$

Find Γ & z

- Procedure:
  - Plot 1+j1 on chart
    - vector $= 0.445\angle 64°$
  - Flip vector 180 degrees

$\Gamma = 0.445\angle -116°$

$z = 0.5 - j0.5$



Admittance Smith Chart Example 2

Given:

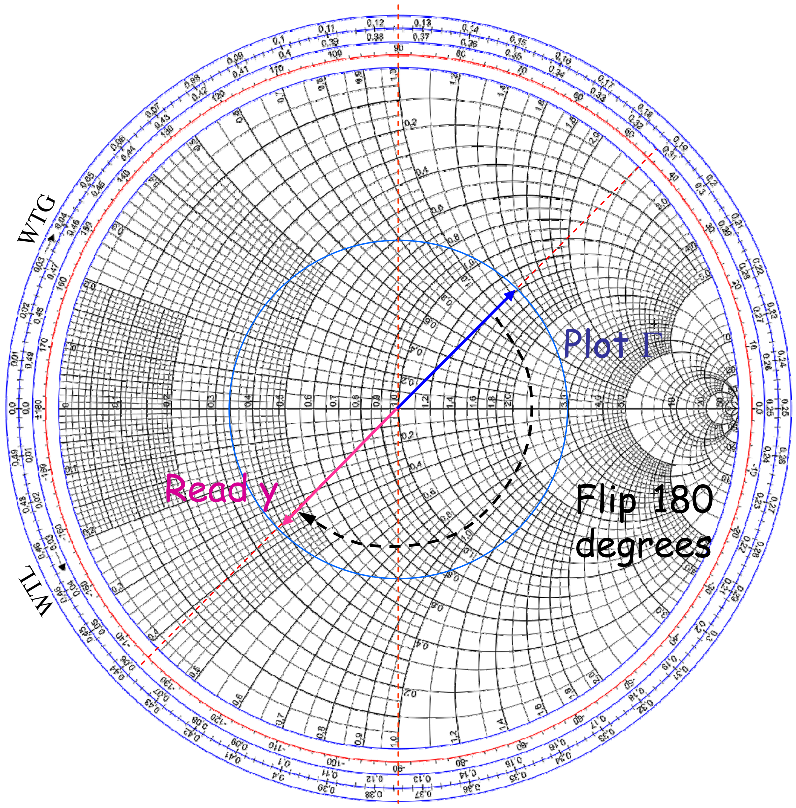
$\Gamma = 0.5\angle +45° \quad Z_o = 50\Omega$

Find Y

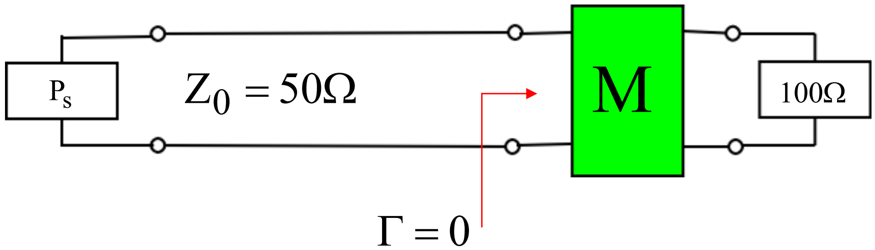
- Procedure:
  - Plot Γ
  - Flip vector by 180 degrees
  - Read coordinate

$y = 0.38 - j0.36$

$$Y = \frac{1}{50\Omega}(0.38 - j0.36)$$
$$= (7.6 - j7.2)\cdot 10^{-3} S$$



Matching Example

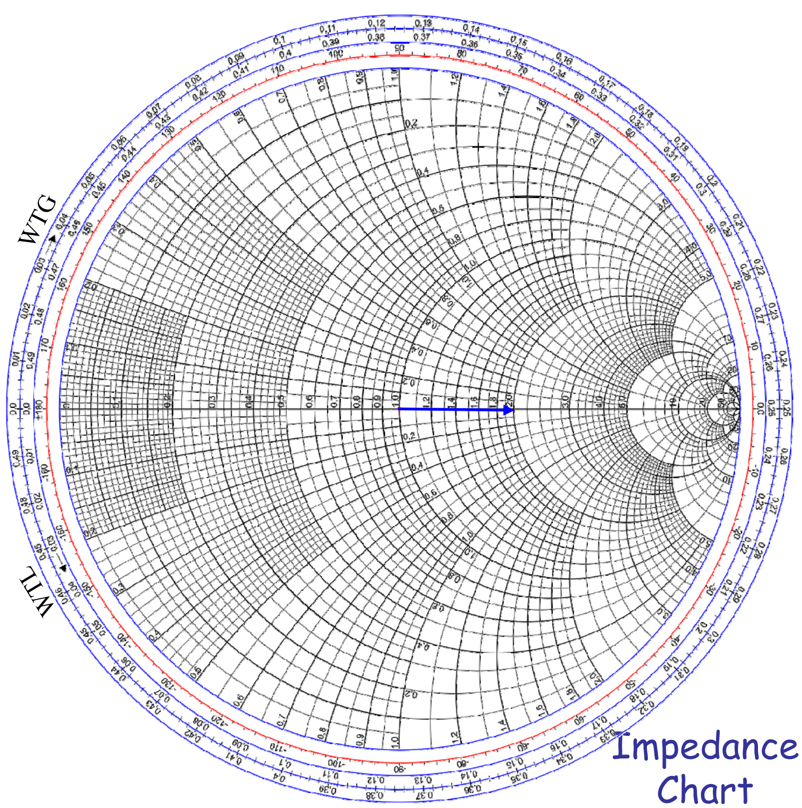


Match 100Ω load to a 50Ω system at 100MHz

A 100Ω resistor in parallel would do the trick, but $\frac{1}{2}$ of the power would be dissipated in the matching network. We want to use only lossless elements such as inductors and capacitors so we don't dissipate any power in the matching network

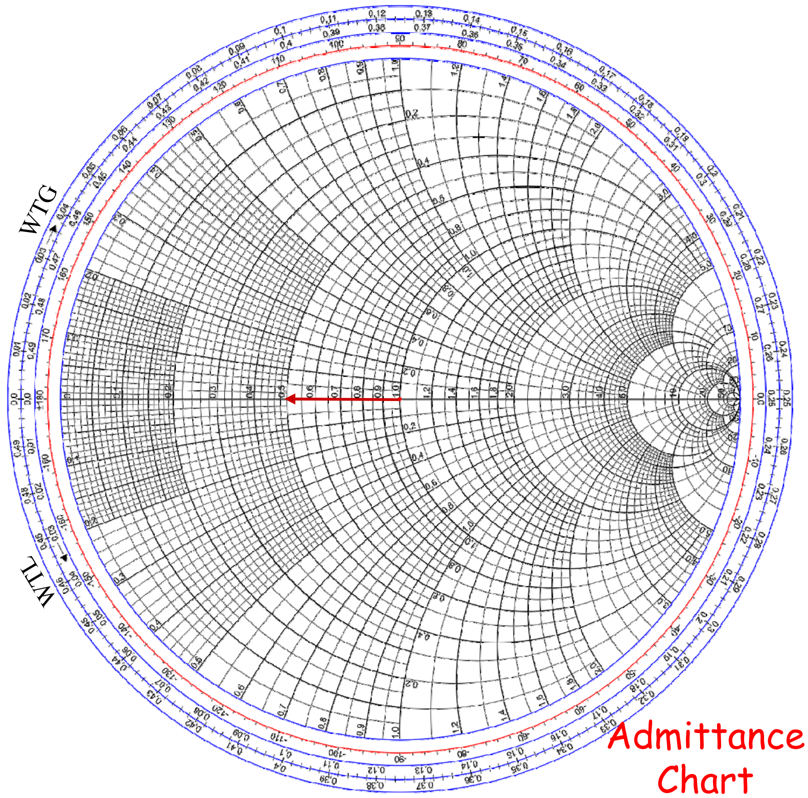
Matching Example

- We need to go from z=2+j0 to z=1+j0 on the Smith chart
- We won't get any closer by adding series impedance so we will need to add something in parallel.
- We need to flip over to the admittance chart



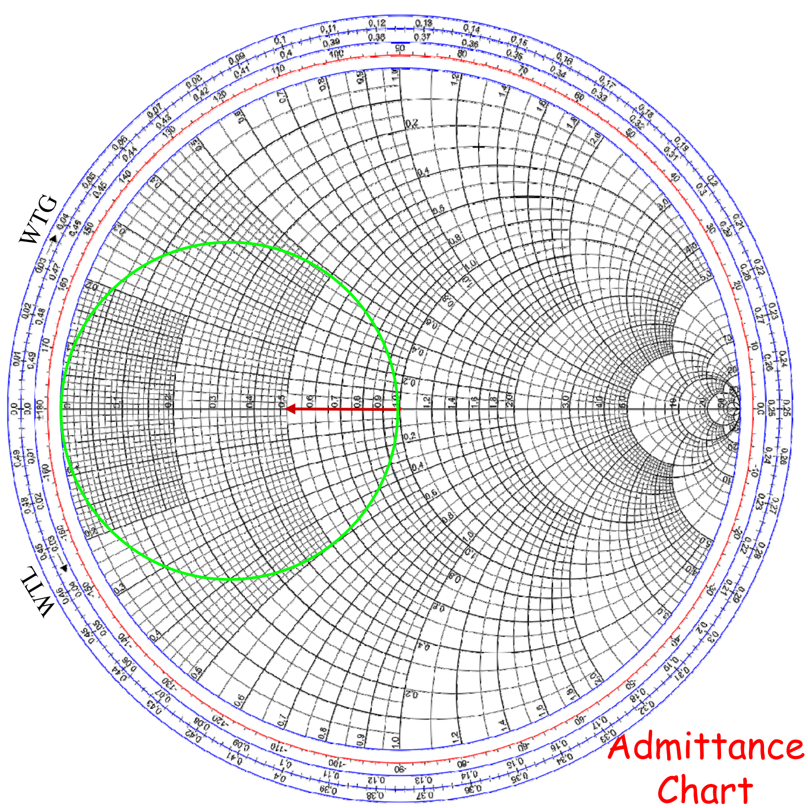
Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide.



Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance.



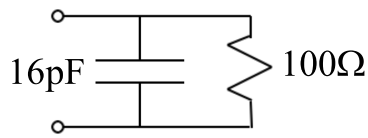
Matching Example

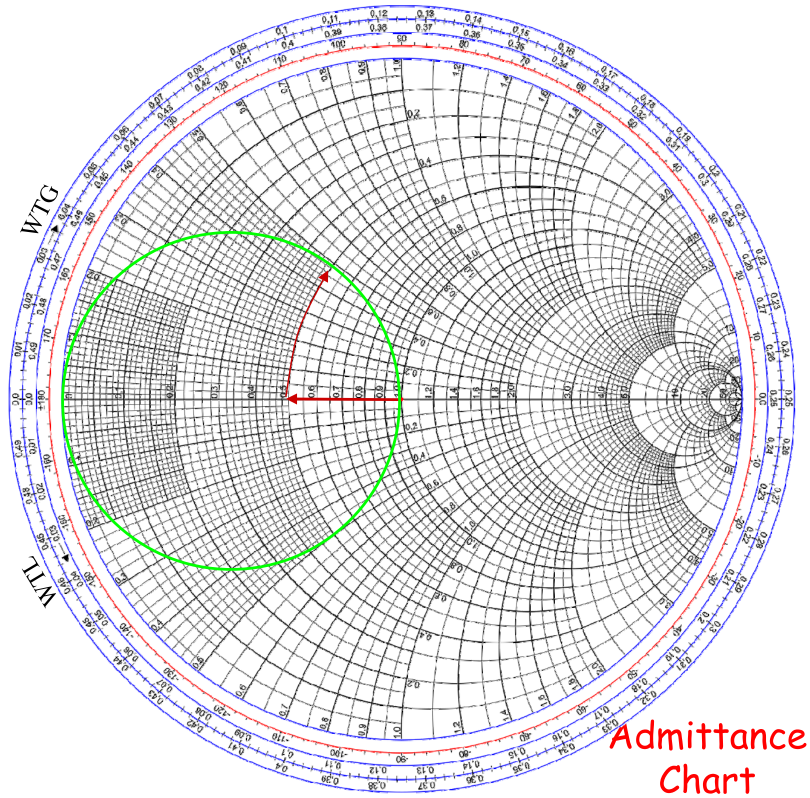
- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance jb = j0.5

$$jb = j0.5$$

$$\frac{j0.5}{50\Omega} = j2\pi(100\text{MHz})C$$

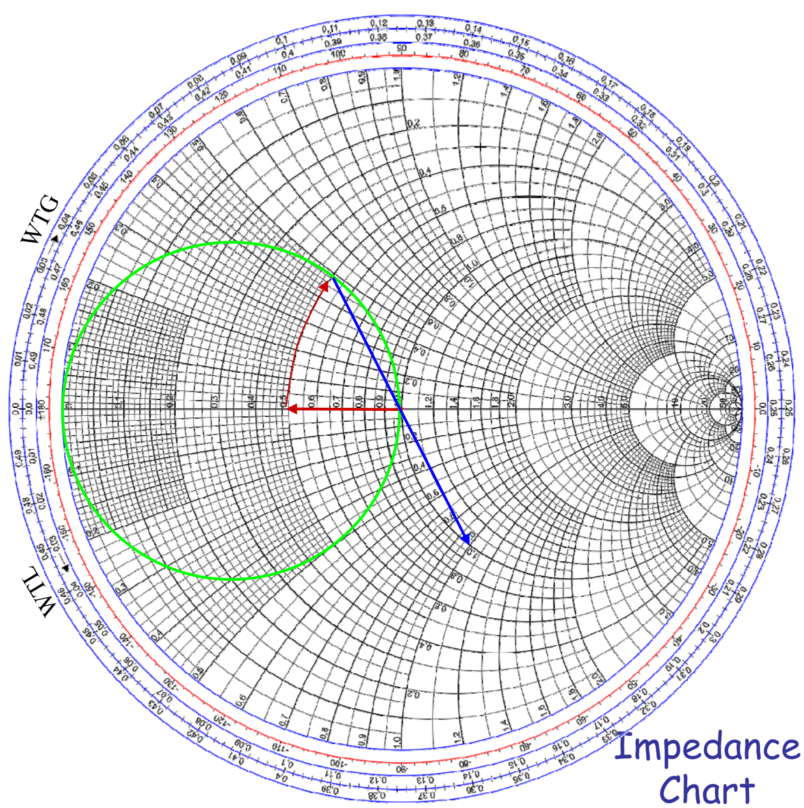
$$C = 16\text{pF}$$





Matching Example

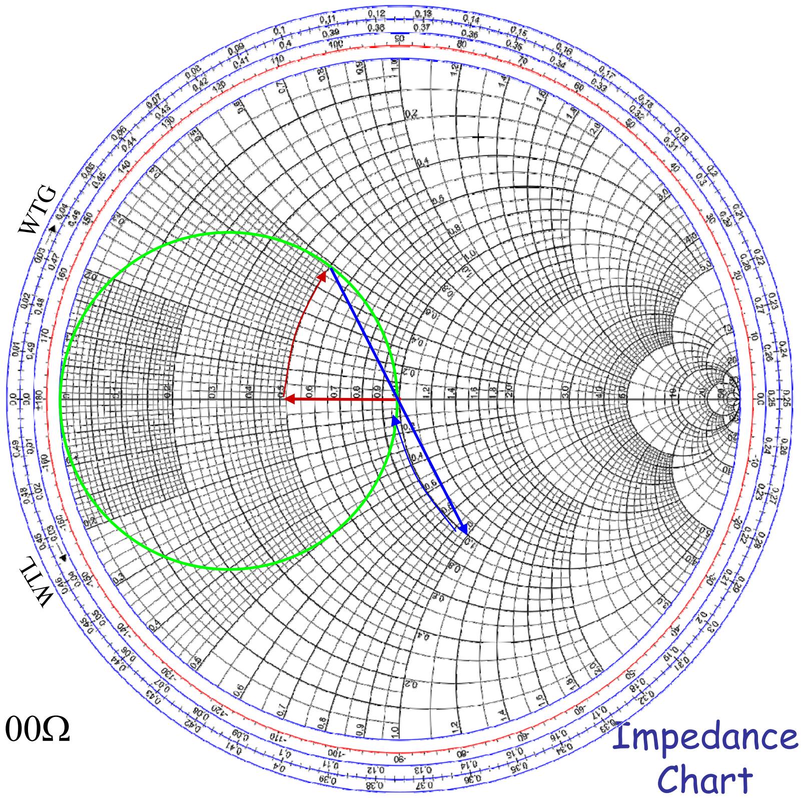
- We will now add series impedance
- Flip to the impedance Smith Chart
- We land at on the r=1 circle at x=-1, i.e. z = 1 – j1

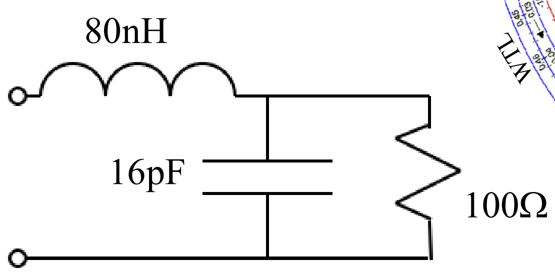


Matching Example

- Add positive imaginary admittance to get to z=1+j0

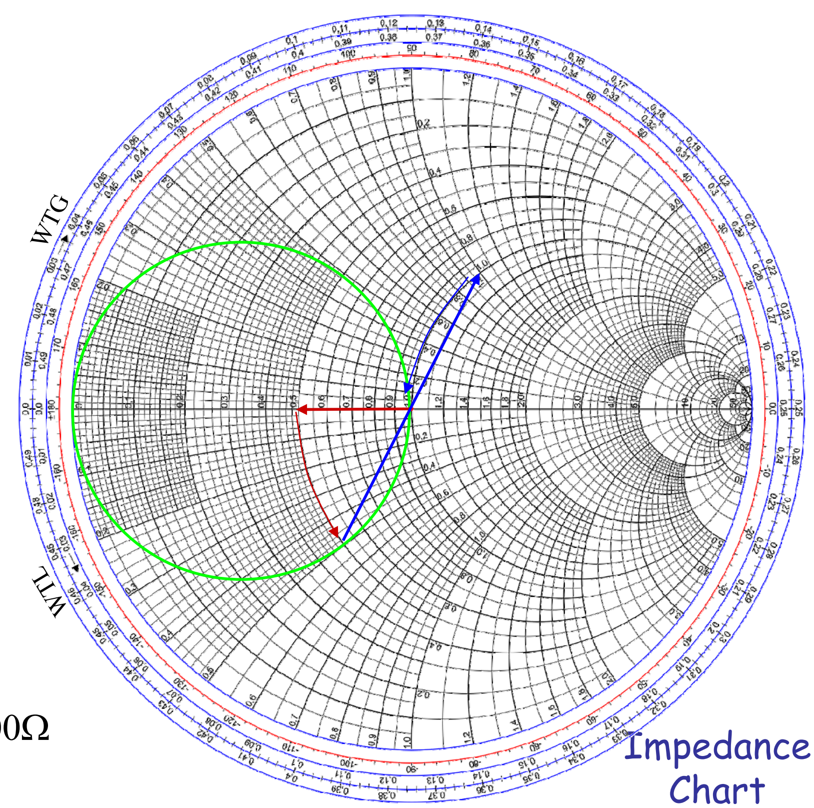
$$jx = j1.0$$
$$(j1.0)50\Omega = j2\pi(100\text{MHz})L$$
$$L = 80\text{nH}$$

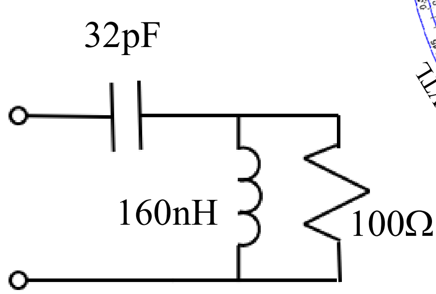




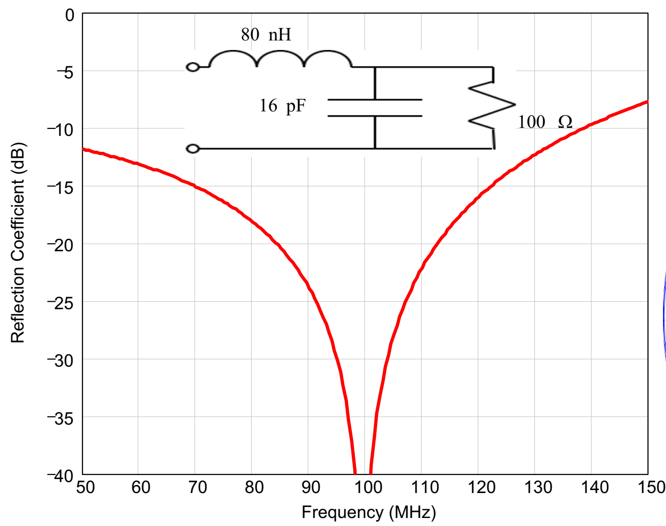
Matching Example

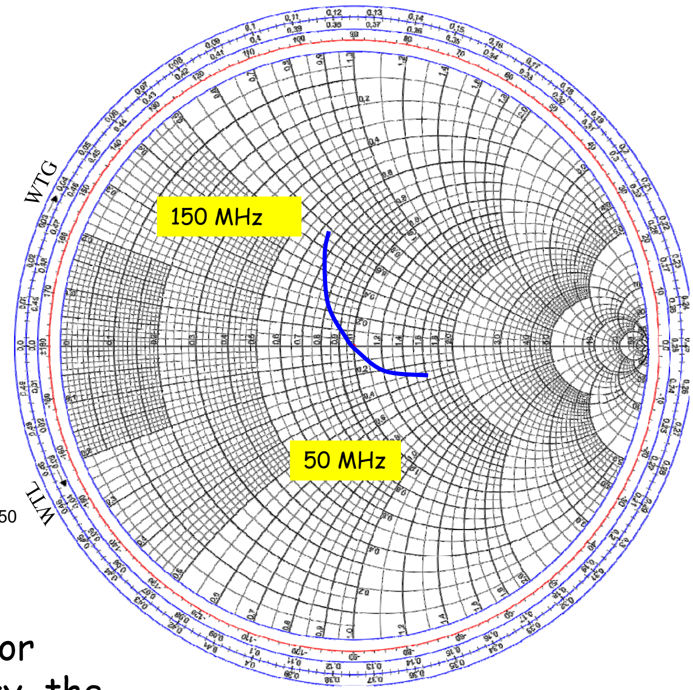
- This solution would have also worked





Matching Bandwidth

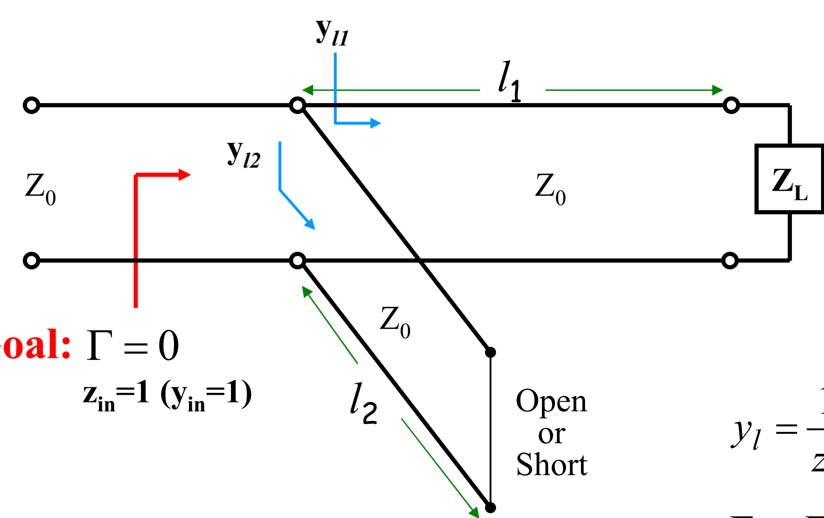




Because the inductor and capacitor impedances change with frequency, the match works over a narrow frequency range

Impedance Chart

Single Stub Tuner



Goal: $\Gamma = 0$

$z_{in}=1$ ($y_{in}=1$)

$$y_l = \frac{1}{z_l} = \frac{1-\Gamma_l}{1+\Gamma_l}$$

$$\Gamma_l = \Gamma e^{-2\beta l}$$

$$y_{in} = y_{l1} + y_{l2} = 1$$
$$= (g_{l1} + jb_{l1}) + jb_{l2}$$

$g_{l1} = 1$ (real-part condition)
$b_{l1} = -b_{l2}$ (imaginary-part condition)
(2 Degrees of freedom)

Stub length $l = \tau u_p = f\lambda\tau$
Phase shift: $\theta_\gamma = 2\beta l = 2(2\pi/\lambda)l = 4\pi(l/\lambda)$

Open: $Z_{in} = -jZ_o\cot\beta l$, or $z_{in} = -j\cot\beta l$
Short: $Z_{in} = jZ_o\tan\beta l$, or $z_{in} = j\tan\beta l$

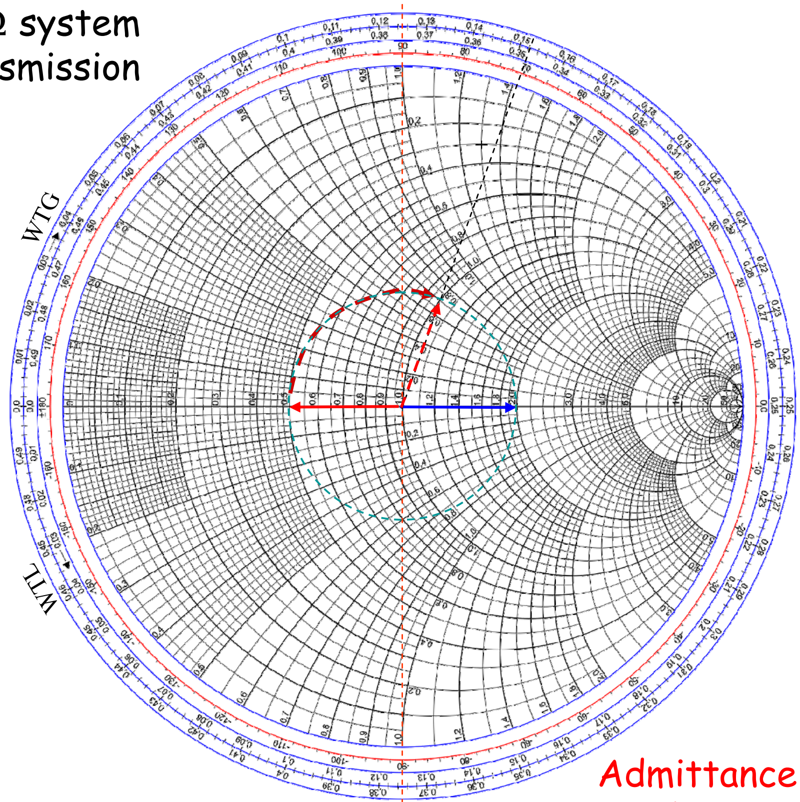
Single Stub Tuner

Match 100Ω load to a 50Ω system at 100MHz using two transmission lines connected in parallel

- Flip to Admittance chart
- y=0.5+j0
- Adding length to Cable 1 rotates the reflection coefficient clockwise to g=1.

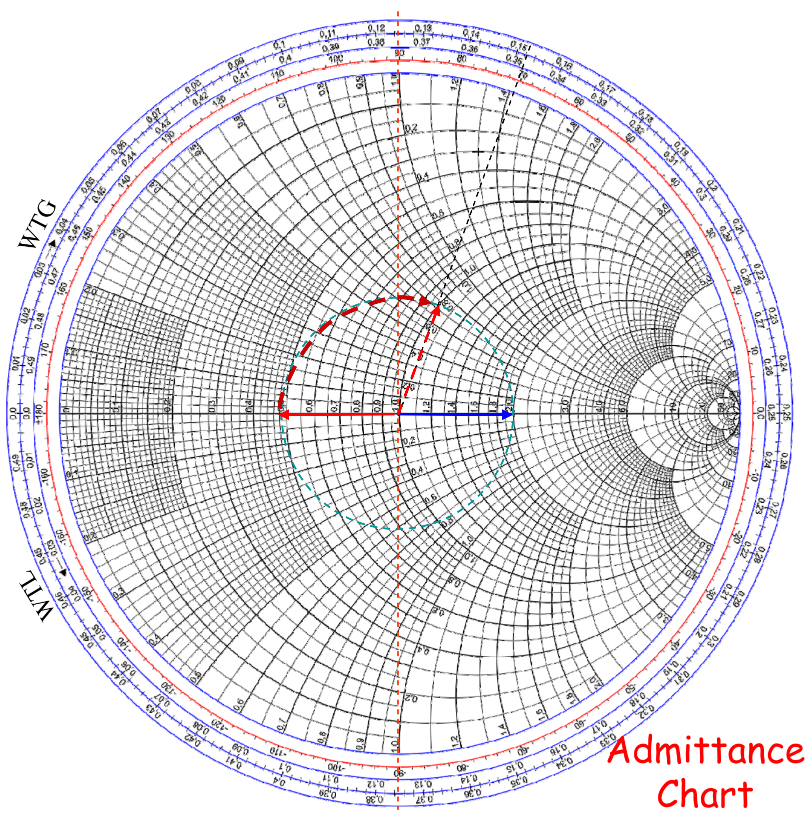
$$l_1 = 0.152\lambda$$

- y_{1l}=1+j0.72



Single Stub Tuner

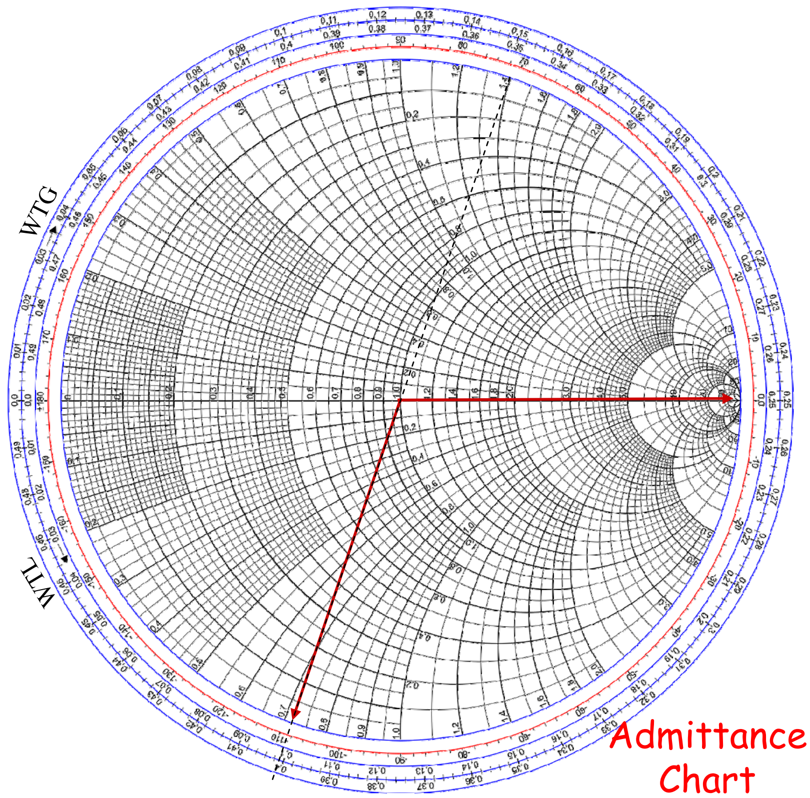
- The stub has to add a normalized admittance of -0.72 to bring the trajectory to the center of the Smith Chart



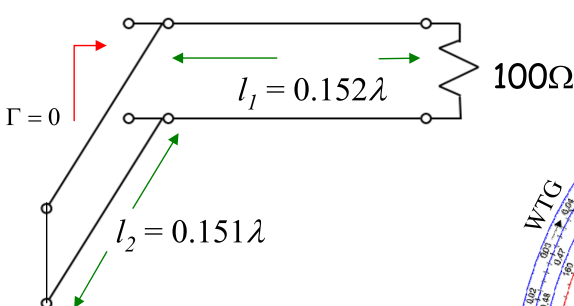
Single Stub Tuner

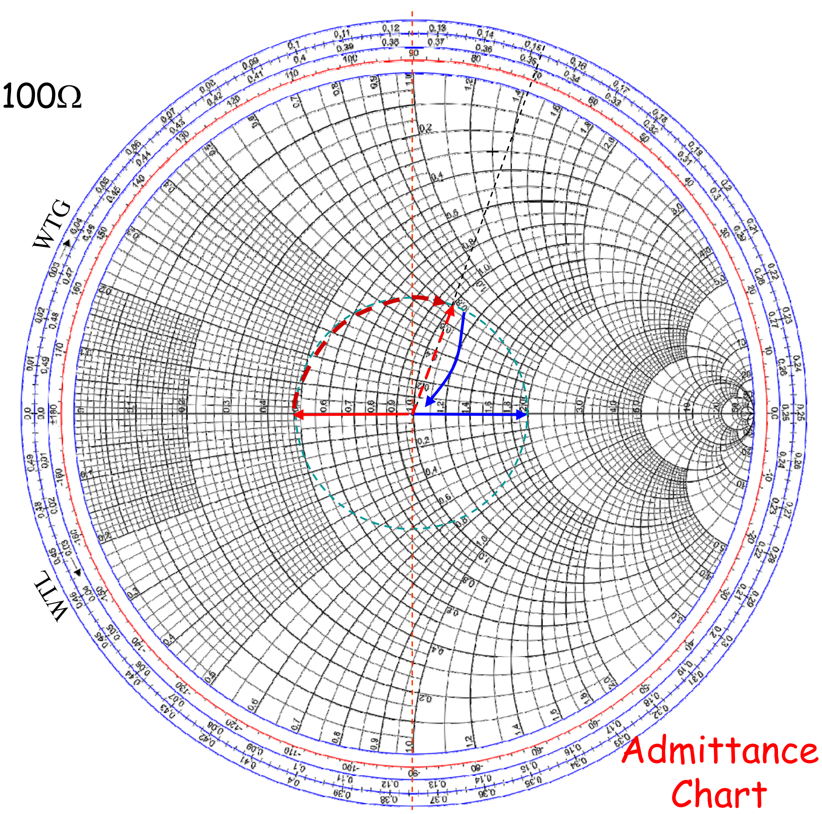
- An short stub of zero length has an admittance=j∞
- By adding enough cable to the short stub, the admittance of the stub will reach to -0.72

$l_2 = (0.401\text{-}0.25)\lambda = 0.151\lambda$

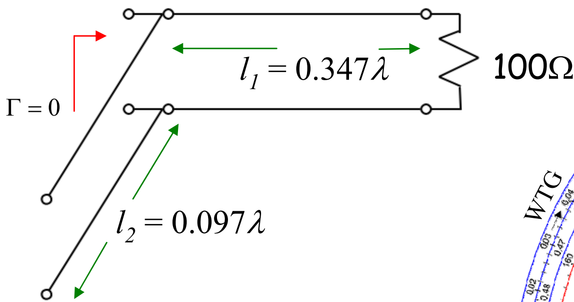


Single Stub Tuner



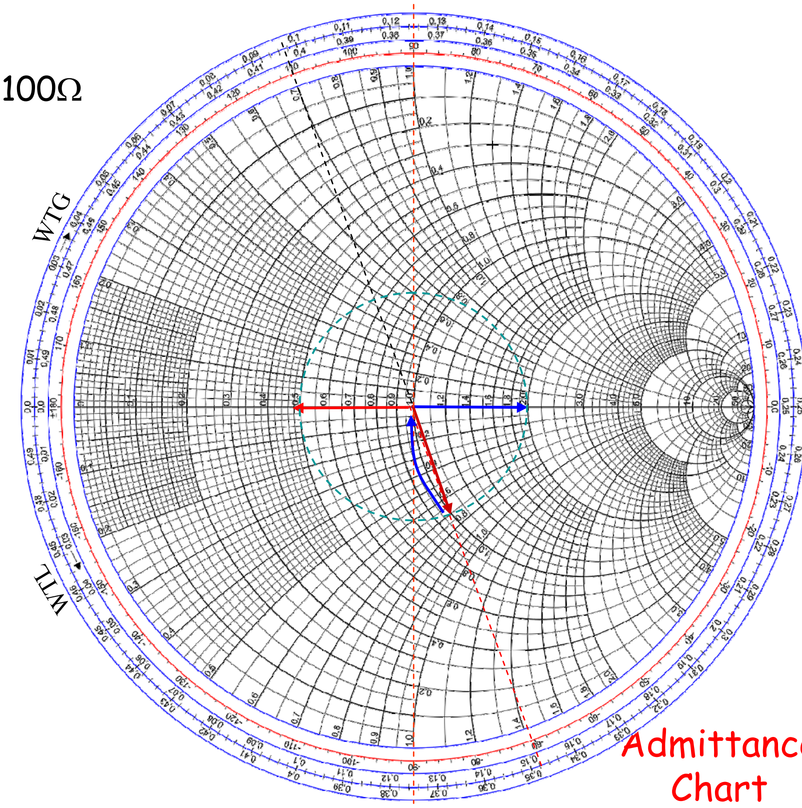


Single Stub Tuner



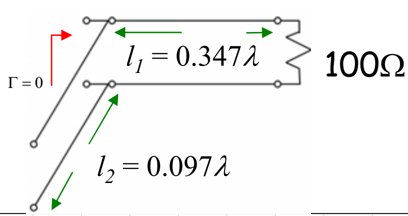
This solution would have worked as well.

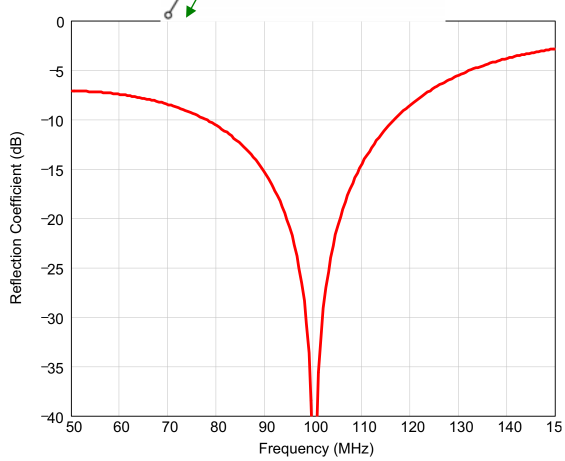
An open stub of zero length has an admittance=j0

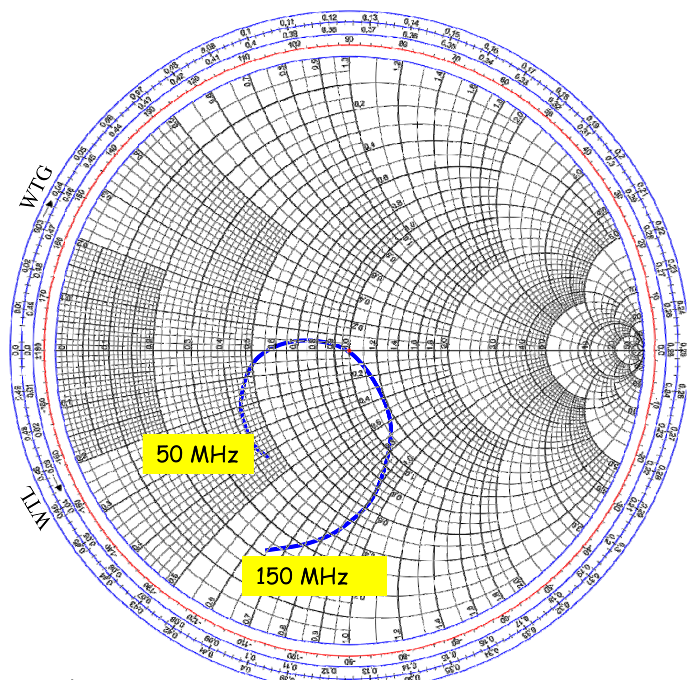


Admittance Chart

Single Stub Tuner Matching Bandwidth







Because the cable phase changes linearly with frequency, the match works over a narrow frequency range

Impedance Chart

Summary

- Impedance matching is necessary to:
  - reduce VSWR
  - obtain maximum power transfer
- Lump reactive elements and a single stub can be used.
- A quarter-wave line can also be used to transform resistance values, and act as an impedance inverter.
- These matching network types are narrow-band: they are designed to operate at a single frequency only.