Antenna Impedance

The complex antenna impedance is defined in terms of resistive (real) and reactive (imaginary) components.

$$Z_A = R_A + jX_A$$

R_A - Antenna resistance
[(dissipation) ohmic losses + radiation]

X_A - Antenna reactance
[(energy storage) antenna near field]

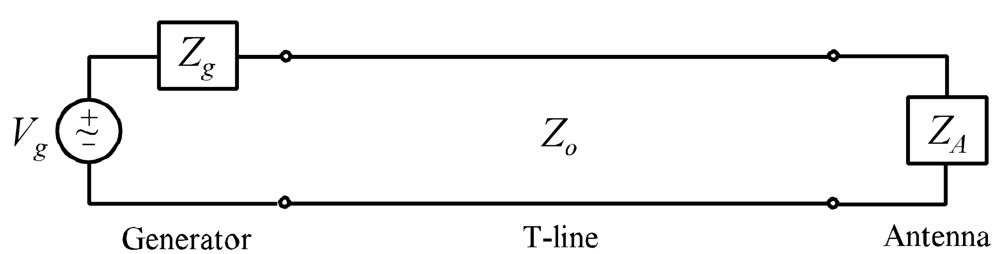
We may define the antenna resistance as the sum of two resistances which separately represent the ohmic losses and the radiation.

$$R_A = R_r + R_L$$

R_r - Antenna radiation resistance (radiation)

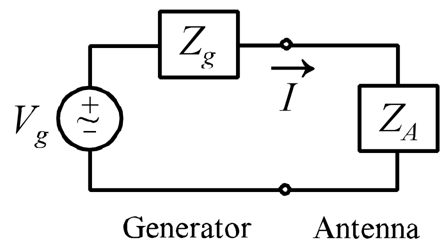
R_L - Antenna loss resistance (ohmic loss)

The typical transmitting system can be defined by a generator, transmission line and transmitting antenna as shown below.

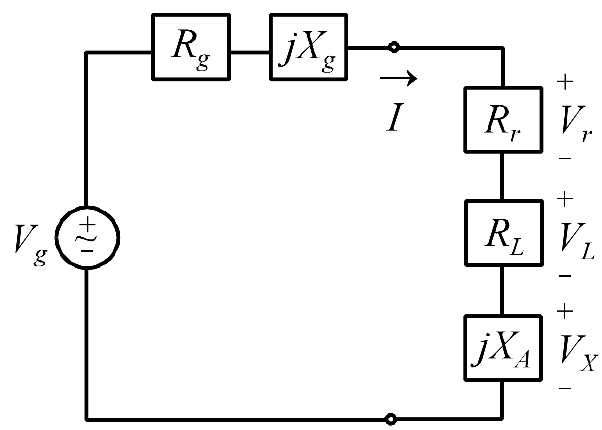


The generator is modeled by a complex source voltage V_g and a complex source impedance Z_g.

In some cases, the generator may be connected directly to the antenna.



Inserting the complete source and antenna impedances yields



The complex power associated with any element in the equivalent circuit is given by

$$P = V_{rms} I_{rms}^* = \frac{1}{2} V_{peak} I_{peak}^*$$

where the * denotes the complex conjugate. We will assume peak values for all voltages and currents in expressing the radiated power, the power associated with ohmic losses, and the reactive power in terms of specific components of the antenna impedance. The peak current for the simple series circuit shown above is

$$I = \frac{V_g}{Z_g + Z_A} = \frac{V_g}{(R_g + R_r + R_L) + j(X_g + X_A)}$$

The power radiated by the antenna (P_r) may be written as

$$P_r = \frac{1}{2} V_r I^* = \frac{1}{2}(I R_r) I^* = \frac{1}{2} |I|^2 R_r$$

$$|I| = \frac{|V_g|}{\sqrt{(R_g + R_r + R_L)^2 + (X_g + X_A)^2}}$$

$$P_r = \frac{|V_g|^2 R_r}{2\,[(R_g + R_r + R_L)^2 + (X_g + X_A)^2]}$$

The power dissipated as heat (P_L) may be written

$$P_L = \frac{1}{2} V_L I^* = \frac{1}{2}(I R_L) I^* = \frac{1}{2} |I|^2 R_L$$

$$P_L = \frac{|V_g|^2 R_L}{2\,[(R_g + R_r + R_L)^2 + (X_g + X_A)^2]}$$

The reactive power (imaginary component of the complex power) stored in the antenna near field (P_X) is

$$P_X = \frac{1}{2} V_X I^* = \frac{1}{2}(j I X_A) I^* = \frac{j}{2} |I|^2 X_A$$

$$P_X = \frac{j\,|V_g|^2 X_A}{2\,[(R_g + R_r + R_L)^2 + (X_g + X_A)^2]}$$

From the equivalent circuit for the generator/antenna system, we see that maximum power transfer occurs when

$$Z_A = Z_g^*$$

$$R_A = R_r + R_L = R_g$$

$$X_A = -X_g$$

The circuit current in this case is

$$I = \frac{V_g}{Z_g + Z_A} = \frac{V_g}{2(R_r + R_L)}$$

The power radiated by the antenna is

$$P_r = \frac{1}{2} V_r I^* = \frac{1}{2}(I R_r) I^* = \frac{1}{2} |I|^2 R_r$$

$$P_r = \frac{|V_g|^2 R_r}{8(R_r + R_L)^2}$$

The power dissipated in heat is

$$P_L = \frac{1}{2} V_L I^* = \frac{1}{2}(I R_L) I^* = \frac{1}{2} |I|^2 R_L$$

$$P_L = \frac{|V_g|^2 R_L}{8(R_r + R_L)^2}$$

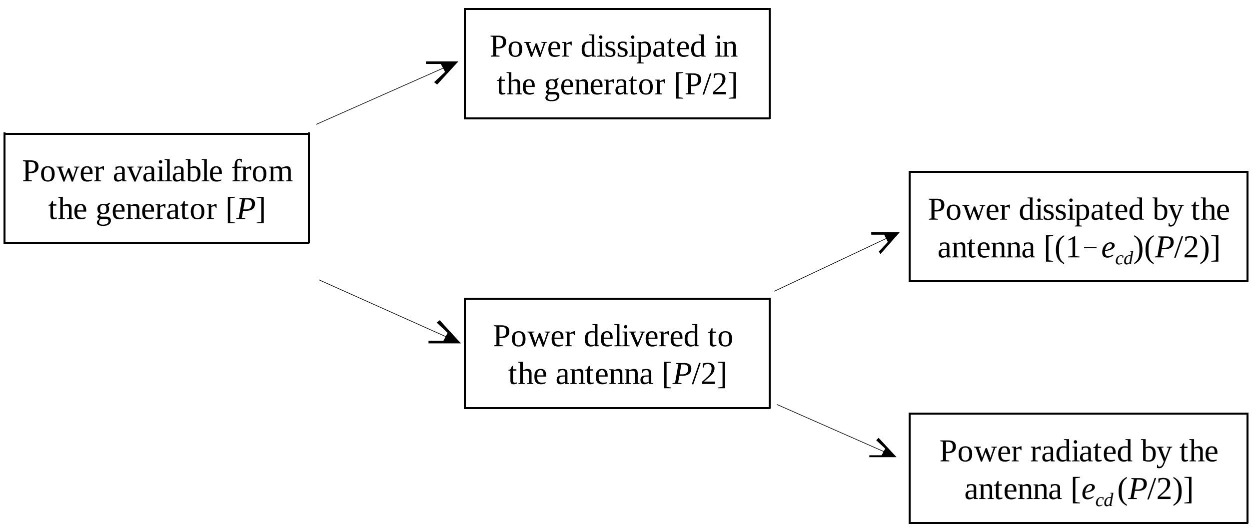
The power available from the generator source is

$$P = \frac{1}{2} V_g I^* = \frac{|V_g|^2}{4(R_r + R_L)}$$

The power dissipated in the generator resistance is

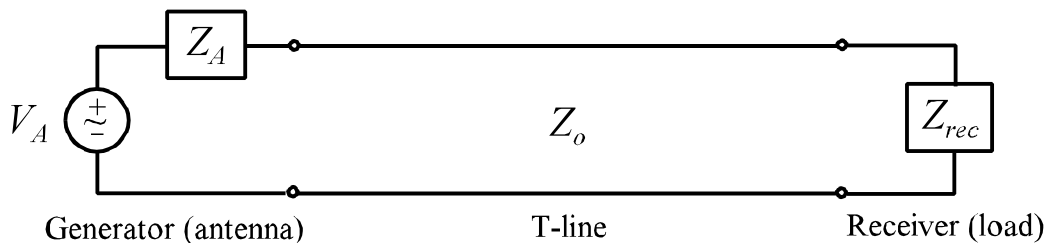
$$P_g = \frac{1}{2}(IR_g)I^* = \frac{1}{2}|I|^2 R_g = \frac{|V_g|^2 R_g}{8(R_r + R_L)^2} = \frac{|V_g|^2}{8(R_r + R_L)} = \frac{1}{2}P$$

<u>Transmitting antenna system summary</u> (maximum power transfer)

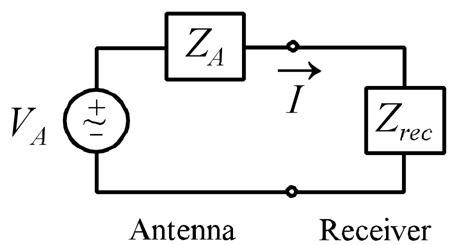


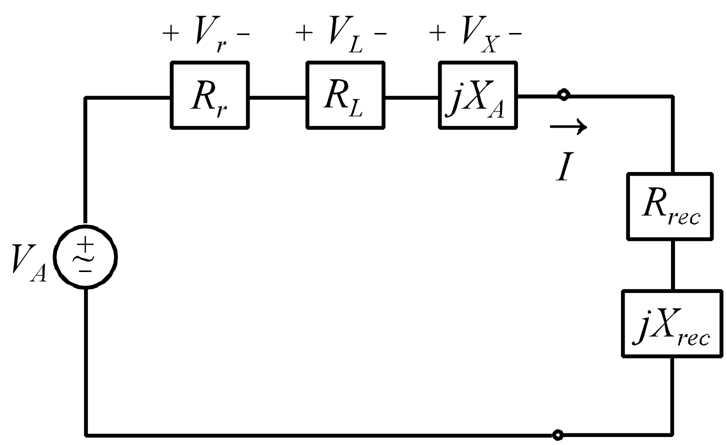
With an ideal transmitting antenna (e_{cd} = 1) given maximum power transfer, one-half of the power available from the generator is radiated by the antenna.

The typical receiving system can be defined by a generator (receiving antenna), transmission line and load (receiver) as shown below.



Assuming the receiving antenna is connected directly to the receiver





For the receiving system, maximum power transfer occurs when

$$Z_A = Z_{rec}^*$$

$$R_A = R_r + R_L = R_{rec}$$

$$X_A = -X_{rec}$$

The circuit current in this case is

$$I = \frac{V_A}{Z_g + Z_A} = \frac{V_A}{2(R_r + R_L)}$$

The power captured by the receiving antenna is

$$P = \frac{1}{2} V_A I^* = \frac{|V_A|^2}{4(R_r + R_L)}$$

Some of the power captured by the receiving antenna is re-radiated (scattered). The power scattered by the antenna (P_{scat}) is

$$P_{scat} = \frac{1}{2} V_r I^* = \frac{|V_A|^2 R_r}{8(R_r + R_L)^2} = e_{cd} \frac{P}{2}$$

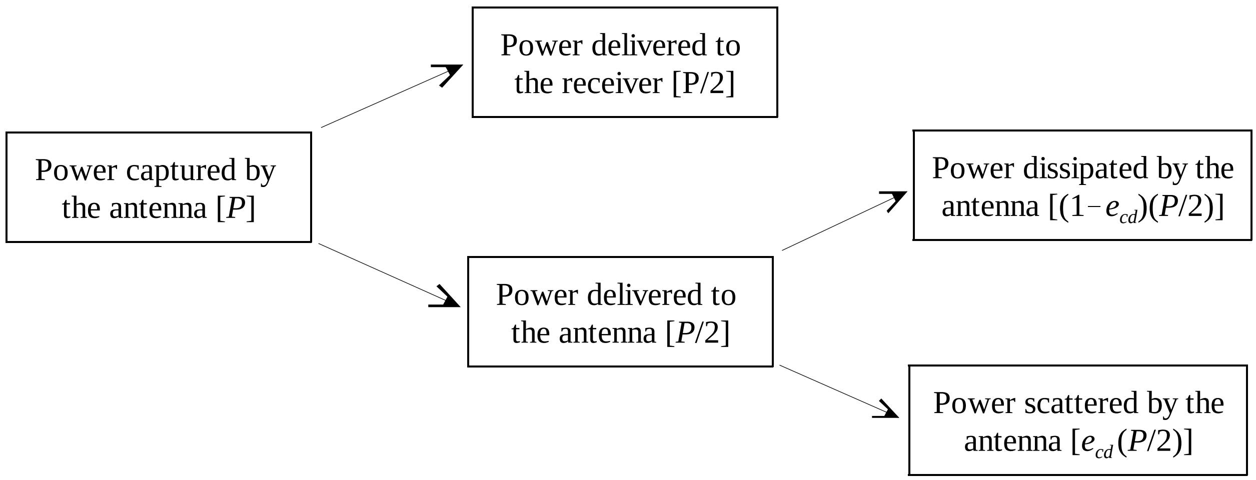
The power dissipated by the receiving antenna in the form of heat is

$$P_L = \frac{1}{2} V_L I^* = \frac{|V_A|^2 R_L}{8(R_r + R_L)^2} = (1 - e_{cd}) \frac{P}{2}$$

The power delivered to the receiver is

$$P_{rec} = \frac{1}{2}(I R_{rec}) I^* = \frac{1}{2} |I|^2 R_{rec} = \frac{|V_A|^2 R_{rec}}{8(R_r + R_L)^2} = \frac{|V_A|^2}{8(R_r + R_L)} = \frac{1}{2} P$$

Receiving antenna system summary (maximum power transfer)



With an ideal receiving antenna (e_{cd} = 1) given maximum power transfer, one-half of the power captured by the antenna is re-radiated (scattered) by the antenna.