Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET4 147: Signal Processing for Communications

Spring 2016

Homework (deadline 8 June)

In this homework, we will study channel and symbol estimation. Make a short report (pdf format prefered) containing the required Matlab files, plots, explanations, and answers, and turn it in on the deadline (hard copy or by e-mail). In case something is unclear, assistance is given by Sundeep Chepuri, room HB17.070 (during office hours), e-mail: `s.p.chepuri@tudelft.nl`.

You may work in groups of two, unless you prefer to work alone.

As in HW2, consider the function `function x = gen_data1(h,s,P,N,SNR)` and construct the data matrix

$$\mathbf{X} = \begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \\ x(\frac{1}{P}) & x(1+\frac{1}{P}) & \cdots & x(N-1+\frac{1}{P}) \\ \vdots & \vdots & & \vdots \\ x(\frac{P-1}{P}) & x(1+\frac{P-1}{P}) & \cdots & x(N-1+\frac{P-1}{P}) \end{bmatrix} : \; P \times N.$$

Assume that $\boldsymbol{\tau} = [0.1 \; 0.6]^{\mathrm{T}}$ and $\boldsymbol{\beta} = [1e^{j\phi_1} \; 0.7e^{j\phi_2}]^{\mathrm{T}}$ with random ϕ_1 and ϕ_2. Further, take an oversampling factor of $P = 5$, a burst length of $N = 100$, and an SNR of 30 dB.

Pilot-based channel estimation

1. In the previous homework, we assumed that the channel $\mathbf{h}$ is known and that we have to estimate the symbols $\mathbf{s}$. Let us now assume that $\mathbf{s}$ is known. How can we estimate the channel $\mathbf{h}$ from $\mathbf{X}$ and $\mathbf{s}$? Make a Matlab function to implement this pilot-based channel estimation

   ```
   function h = pilot(X,s)
   ```

   Hint: First derive how $\mathbf{X}$ can be decomposed in $\mathbf{X} = \mathbf{H}\mathcal{S}_L + \mathbf{N}$. Then construct the matrix $\mathcal{S}_L$ from the data sequence $\mathbf{s}$ and compute an estimate of $\mathbf{H}$ from $\mathbf{X}$ and $\mathcal{S}_L$

2. Plot the real and imaginary part of the channel estimate and compare it to the real and imaginary part of the true channel. Do they match?

Blind channel and symbol estimation

3. Compute the SVD of $\mathbf{X}$ and determine the null-subspaces of the columns and rows of $\mathbf{X}$ (these null-subspaces are the subspaces orthogonal to the columns and rows of $\mathbf{X}$ in the absence of noise).

4. Can we estimate the channel $\mathbf{h}$ from the column null-subspace $\mathbf{U}_n$ of $\mathbf{X}$? If so, derive the Matlab function

   ```
   function h = blind_channel(X)
   ```

   Plot the real and imaginary part of the channel estimate and compare them to the real and imaginary part of the true channel. Do they match? Note that there is at least a (complex) scalar ambiguity between the true and estimated channel.

5. Can we estimate the symbols $\mathbf{s}$ from the row null-subspace $\mathbf{V}_n$ of $\mathbf{X}$? If so, derive the Matlab function

   ```
   function s = blind_symbol(X)
   ```

   Plot the estimated symbols in the complex plane and compare them to the true symbols. Do they match? Again, note there is at least a (complex) scalar ambiguity between the true and estimated symbols.