The climate in climate economics: Online appendix

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Online Appendix A Summary table of the models and parameters

Name	b_{12}	b_{23}	$M^{EQ}_{AT,UO,LO}$	$M^{INI}_{AT,UO,LO}$	c_1	c_3	c_4	f_{2xco2}	t_{2xco2}	$T^{INI}_{AT,OC}$
CDICE (MMM-MMM)	0.054	0.0082	(607, 489, 1281)	(851, 628, 1323)	0.137	0.73	0.00689	3.45	3.25	(1.1, 0.27)
CDICE-HadGEM2-ES	0.054	0.0082	(607, 489, 1281)	(851, 628, 1323)	0.154	0.55	0.00671	2.95	4.55	(1.1, 0.27)
CDICE-GISS-E2-R	0.054	0.0082	(607, 489, 1281)	(851, 628, 1323)	0.213	1.16	0.00921	3.65	2.15	(1.1, 0.27)
CDICE-MESMO	0.059	0.008	(607, 305, 865)	(851, 403, 894)	0.137	0.73	0.00689	3.45	3.25	(1.1, 0.27)
CDICE-LOVECLIM	0.067	0.0095	(607, 600, 1385)	(850, 770, 1444)	0.137	0.73	0.00689	3.45	3.25	(1.1, 0.27)
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DICE-2016	0.12	0.007	(588, 360, 1720)	(851, 460, 1740)	0.1005	0.088	0.025	3.6813	3.1	(0.85, 0.0068)

Table 1 Labelling of the models, and their respective parameterization. Note that the coefficients for the DICE-2016 model correspond to a five-year time step, whereas all other models are defined for an annual time step.

Online Appendix B Generic DICE formulation and calibration

In section 6, we laid out the DICE model as a whole, that is, we specified the dynamic optimization problem that we solve with a calibration of parameters that are either given by DICE-2016 or by variants of our CDICE. We call this dynamic problem, specified by Eqs. (8) to (12), the generic formulation of the DICE model in terms of its formal structure. The generic formulation allows for an easy switch in parameters between different versions of the DICE model (DICE-2007, DICE-2016) as well as a switch to the CDICE parametrization.[1] The functional forms of DICE-2007 and DICE-2016 are slightly different; however, they do not provide a critical difference in the results. This fact is especially relevant for the laws of motion of exogenous parameters in both models because of the differences in time steps (DICE-2007 operates with a ten-year time step, and DICE-2016 operates with a five-year time step), differences in units of measurement for emissions (GtC in DICE-2007 versus GtCO2 in DICE-2016) and other minor divergences. We believe that it is important to have a unified

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[1]Note that in the online Appendix Section B, we also include DICE-2007, for completeness, as there is a vast number of research papers and policy studies that have used this formulation of the DICE model.

specification for the DICE model, which in turn allows a broader group of researchers to consider updating their parametrization of the model with respect to our proposed CDICE.

Another feature of the generic specification of the DICE model is that we explicitly allow for the time step. The time step enters all relevant equations as the multiplication factor Δ_t with respect to the annual baseline calibration for all the parameters. This applies both for the main equations of the model specified in Sections B.3 and 4 and for exogenously specified parameters of the model in Section B.1.

This online Appendix is organized as follows. We present the generic equations and calibration for all the exogenous parameters of the DICE-2007 (reads Value 2007 in the tables with parameters), DICE-2016 (reads Value 2016), and CDICE first. Then we provide core equations of the model corresponding to the mass of carbon evolution, temperature evolution, and economic growth with calibrations corresponding to DICE-2007, DICE-2016, and CDICE. When necessary, we provide a detailed comment on how the functional forms of certain equations were changed to be the same in the generic formulation and, at the same time, equivalent to the initial formulation of DICE-2007 and DICE-2016 (for example, the law of motion for TFP).

B.1 Exogenous variables

We start by introducing the law of motion for exogenous variables that are time dependent. The law of motion for labor is given in Eq. (B.1) along with labor growth presented in Eq. (B.2):

$$L_t = L_0 + (L_\infty - L_0)\left(1 - \exp\left(-\Delta_t \delta^L t\right)\right), \tag{B.1}$$

$$g_t^L = \frac{\frac{dL_t}{dt}}{L_t} = \frac{\Delta_t \delta^L}{\frac{L_\infty}{L_\infty - L_0}\exp\left(\Delta_t \delta^L t\right) - 1}. \tag{B.2}$$

The numerical values of the parameters for the world population and its growth rate given in Table 2:

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
Annual rate of convergence	δ^L	0.035	0.0268
World population at starting year [millions]	L_0	6514	7403
Asymptotic world population [millions]	L_∞	8600	11500
Time step of a model	Δ_t	10	5/1

Table 2 Generic parameterization for the evolution of labor.

The total factor productivity evolves according to the equation Eq. (B.3) with its growth rate following Eq. (B.4):

$$A_t = A_0 \exp\left(\frac{\Delta_t g_0^A (1 - \exp(-\Delta_t \delta^A t))}{\Delta_t \delta^A}\right). \tag{B.3}$$

$$g_t^A = \frac{\frac{dA_t}{dt}}{A_t} = \Delta_t g_0^A \exp\left(-\Delta_t \delta^A t\right). \tag{B.4}$$

The respective parameters of the total factor productivity evolution and TFP growth rate are given in Table 3:

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
Initial growth rate for TFP per year	g_0^A	0.01314	0.0217
Decline rate of TFP growth per year	δ^A	0.001	0.005
Initial level of TFP	A_0	0.0058	0.010295
Time step of a model	Δ_t	10	5/1

Table 3 Generic parametrization for the evolution of TFP.

The carbon intensity, defined in Eq. (B.5) for DICE-2007 and in Eq. (B.6), characterizes how much anthropogenic carbon inflows the climate system due to production activity:

$$\sigma_t = \sigma_0 \exp\left(\frac{\Delta_t g_0^\sigma (1 - \exp(-\Delta_t \delta^\sigma t))}{\Delta_t \delta^\sigma}\right). \tag{B.5}$$

The carbon intensity in DICE-2016 is given by:

$$\sigma_t = \sigma_0 \exp\left(\frac{\Delta_t g_0^\sigma}{\log(1 + \Delta_t \delta^\sigma)} \left((1 + \Delta_t \delta^\sigma)^t - 1\right)\right). \tag{B.6}$$

The parametrization for carbon intensity processes is given by Table 4:

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
Initial growth of carbon intensity per year	g_0^σ	-0.0073	-0.0152
Decline rate of decarbonization per year	δ^σ	0.003	0.001
Initial carbon intensity (1000GtC)	σ_0	0.00013418	0.00009556
Time step	Δ_t	10	5/1

Table 4 Generic parameterization for the carbon intensity evolution.

DICE-2016 uses a backstop technology capable of mitigating the full amount of industrial emissions that enter the atmosphere. The cost of backstop technology is assumed to be initially high but could be reduced over time, which is reflected in the definition of the coefficient of the abatement cost function $\theta_{1,t}$ as defined in Eq. (B.7) for DICE-2007 and Eq. (B.8) for DICE-2016.[2]
The abatement cost in DICE-2007 is given by:

$$\theta_{1,t} = \frac{p_0^{\text{back}}(1 + \exp\left(-g^{\text{back}} t\right))1000\sigma_t}{\theta_2}. \tag{B.7}$$

The abatement cost in DICE-2016 is given by:

$$\theta_{1,t} = \frac{p_0^{\text{back}} \exp\left(-g^{\text{back}} t\right) 1000 \cdot \text{c2co2} \cdot \sigma_t}{\theta_2}. \tag{B.8}$$

The parameters for the abatement cost are presented in Table 5:

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
Cost of backstop 2005 thUSD per tC 2005	p_0^{back}	0.585	-
Cost of backstop 2010 thUSD per tCO2 2015	p_0^{back}	-	0.55
Initial cost decline backstop cost per year	g^{back}	0.005	0.005
Exponent of control cost function	θ_2	2.8	2.6
Transformation coefficient from C to CO2	c2co2	-	3.666

Table 5 Generic parametrization for the abatement cost.

The non-industrial emissions from land use and deforestation decline over time according to Eq. (B.9), with

[2] The scale parameter 1000 in the equations (B.7) and (B.8) reflects the fact that we use a 1000 GtC unit of measurement; the parameter c2co2 transforms carbon intensity measured in GtC into GtCO2, as the backstop price in DICE-2016 is given for GtCO2 instead of GtC.

parameters are presented in Table 6:

$$E_{\text{Land},t} = E_{\text{Land},0} \exp\left(-\Delta_t \delta^{\text{Land}} t\right). \tag{B.9}$$

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
Emissions from land 2005 (1000GtC per year)	$E_{\text{Land},0}$	0.0011	-
Emissions from land 2015 (1000GtC per year)	$E_{\text{Land},0}$	-	0.000709
Decline rate of land emissions (per year)	δ^{Land}	0.01	0.023
Time step	Δ_t	10	5/1

Table 6 Generic parametrization for the emissions from land.

The exogenous radiative forcings that result from non-CO2 GHG are described in Eq. (B.10):

$$F_t^{EX} = F_0^{EX} + \frac{1}{\mathrm{T}/\Delta_t}(F_1^{EX} - F_0^{EX}) \min(t, \mathrm{T}/\Delta_t). \tag{B.10}$$

The parameters of the exogenous radiative forcings are given in Table 7:

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
2000 forcings of non-CO2 GHG (Wm-2)	F_0^{EX}	-0.06	-
2015 forcings of non-CO2 GHG (Wm-2)	F_0^{EX}	-	0.5
2100 forcings of non-CO2 GHG (Wm-2)	F_1^{EX}	0.3	1.0
Number of years before 2100	T	100	85
Time step	Δ_t	10	5/1

Table 7 Generic parametrization for the exogenous forcing.

B.2 The equations of the climate system

The laws of motion for the mass of carbon in the atmosphere are presented in the main body of the text with Eq. (1). The temperature equations are given in Eqs. (4) and (5). However, we believe it can be helpful to provide the full description of the equations above together with their parametrization that can be used to relate DICE-2007 to DICE-2016 and CDICE.

The evolution for the mass of carbon in all three reservoirs is given by the Eqs. (B.11) to (B.13), carbon emissions are determined with the Eq. (B.14):

$$M_{t+1}^{\text{AT}} = (1 - \Delta_t b_{12}) M_t^{\text{AT}} + \Delta_t b_{12} \frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} M_t^{\text{UO}} + \Delta_t E_t, \tag{B.11}$$

$$M_{t+1}^{\text{UO}} = \Delta_t b_{12} M_t^{\text{AT}} + (1 - \Delta_t b_{12} \frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} - \Delta_t b_{23}) M_t^{\text{UO}} + \Delta_t b_{23} \frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}} M_t^{\text{LO}}, \tag{B.12}$$

$$M_{t+1}^{\text{LO}} = \Delta_t b_{23} M_t^{\text{UO}} + (1 - \Delta_t b_{23} \frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}}) M_t^{\text{LO}}, \tag{B.13}$$

$$E_t = \sigma_t Y_t^{\text{Gross}} (1 - \mu_t) + E_t^{\text{Land}}. \tag{B.14}$$

$$\tag{B.15}$$

The parameters for the laws of motion for the masses of carbon, as well as the starting values and equilibrium values, are given in Table 8.

Calibrated parameter	Symbol	2007	2016	CDICE
Carbon cycle, annual value	b_{12}	0.0189288	0.024	0.054
Carbon cycle, annual value	b_{23}	0.005	0.0014	0.0082
Time step	Δ_t	10	5	1
Equilibrium concentration in atmosphere (1000GtC)	$M_{\text{EQ}}^{\text{AT}}$	0.587473	0.588	0.607
Equilibrium concentration in upper strata (1000GtC)	$M_{\text{EQ}}^{\text{UO}}$	1.143894	0.360	0.489
Equilibrium concentration in lower strata (1000GtC)	$M_{\text{EQ}}^{\text{LO}}$	18.340	1.720	1.281
Concentration in atmosphere 2015 (1000GtC)	$M_{\text{INI}}^{\text{AT}}$	0.8089	0.851	0.851
Concentration in upper strata 2015 (1000GtC)	$M_{\text{INI}}^{\text{UO}}$	1.255	0.460	0.628
Concentration in lower strata 2015 (1000GtC)	$M_{\text{INI}}^{\text{LO}}$	18.365	1.740	1.323

Table 8 Generic parametrization for the mass of carbon.

The temperature evolution is determined by Eqs. (B.16) to (B.18) with parameters and starting values given in Table 9:

$$T_{t+1}^{\text{AT}} = T_t^{\text{AT}} + \Delta_t c_1 F_t - \Delta_t c_1 \frac{F_{2\text{XCO2}}}{T_{2\text{XCO2}}} T_t^{\text{AT}} - \Delta_t c_1 c_3 \left(T_t^{\text{AT}} - T_t^{\text{OC}}\right), \tag{B.16}$$

$$T_{t+1}^{\text{OC}} = T_t^{\text{OC}} + \Delta_t c_4 \left(T_t^{\text{AT}} - T_t^{\text{OC}}\right), \tag{B.17}$$

$$F_t = F_{2\text{XCO2}} \frac{\log(M_t^{\text{AT}}/M_{\text{base}}^{\text{AT}})}{\log(2)} + F_t^{EX}. \tag{B.18}$$

Calibrated parameter	Symbol	2007	2016	CDICE
Temperature coefficient, annual value	c_1	0.022	0.0201	0.137
Temperature coefficient, annual value	c_3	0.3	0.088	0.73
Temperature coefficient, annual value	c_4	0.01	0.005	0.00689
Forcings of equilibrium CO2 doubling (Wm-2)	$F_{2\text{XCO2}}$	3.8	3.6813	3.45
Eq temperature impact (°C per doubling CO2)	$T_{2\text{XCO2}}$	3.0	3.1	3.25
Eq concentration in atmosphere (1000GtC)	$M_{\text{base}}^{\text{AT}}$	0.5964	0.588	0.607
Atmospheric temp change (°C) from 1850	T_0^{AT}	0.7307	0.85	1.1
Lower stratum temp change (°C) from 1850	T_0^{OC}	0.0068	0.0068	0.27
Time step	Δ_t	10	5	1

Table 9 Generic parametrization for the temperature.

B.3 Economy equations

The capital evolution and gross output in both models, DICE-2007 and DICE-2016, are given by:

$$K_{t+1} = (1 - \delta^K)^{\Delta_t} K_t + \Delta_t I_t, \tag{B.19}$$

$$Y_t^{\text{Gross}} = (A_t L_t)^{1-\alpha} K_t^{\alpha}. \tag{B.20}$$

The functional forms of damages differ in DICE-2007 and DICE-2016. In DICE-2007, damages, abatement costs, output net of damages, and output net of damages and abatement costs are given by:

$$\Omega_t = \frac{1}{1 + \psi_1 T_t^{\mathrm{AT}} + \psi_2 (T_t^{\mathrm{AT}})^2}, \tag{B.21}$$

$$\Theta_t = \theta_{1,t} \mu_t^{\theta_2}, \tag{B.22}$$

$$Y_t^{\mathrm{Net}} = Y_t^{\mathrm{Gross}} \cdot \Omega_t = \frac{Y_t^{\mathrm{Gross}}}{1 + \psi_1 T_t^{\mathrm{AT}} + \psi_2 (T_t^{\mathrm{AT}})^2}, \tag{B.23}$$

$$Y_t = Y_t^{\mathrm{Gross}} \cdot \Omega_t \cdot (1 - \Lambda_t) = \frac{Y_t^{\mathrm{Gross}} (1 - \theta_{1,t} \mu_t^{\theta_2})}{1 + \psi_1 T_t^{\mathrm{AT}} + \psi_2 (T_t^{\mathrm{AT}})^2}. \tag{B.24}$$

$$\tag{B.25}$$

The same variables for DICE-2016 are given by the following equations:

$$\Omega_t = \psi_1 T_t^{\mathrm{AT}} + \psi_2 (T_t^{\mathrm{AT}})^2, \tag{B.26}$$

$$\Theta_t = \theta_{1,t} \mu_t^{\theta_2}, \tag{B.27}$$

$$Y_t^{\mathrm{Net}} = Y_t^{\mathrm{Gross}} \cdot (1 - \Omega_t) = Y_t^{\mathrm{Gross}} (1 - \psi_1 T_t^{\mathrm{AT}} - \psi_2 (T_t^{\mathrm{AT}})^2), \tag{B.28}$$

$$Y_t = Y_t^{\mathrm{Gross}} \cdot (1 - \Lambda_t - \Omega_t) = Y_t^{\mathrm{Gross}} (1 - \theta_{1,t} \mu_t^{\theta_2} - \psi_1 T_t^{\mathrm{AT}} - \psi_2 (T_t^{\mathrm{AT}})^2). \tag{B.29}$$

$$\tag{B.30}$$

Consumption in both DICE-2007 and DICE-2016 models is given by:

$$C_t = Y_t - I_t. \tag{B.31}$$

However, the utility function in DICE-2007 differs slightly from the utility used in DICE-2016. Utility in DICE-2007 is the following:

$$U_t = \sum_{t=0}^{T} \beta^t \cdot \Delta_t \cdot \frac{\left(\frac{C_t}{L_t}\right)^{1-1/\psi} - 1}{1 - 1/\psi} L_t. \tag{B.32}$$

And utility in DICE-2016 is given by:

$$U_t = \sum_{t=0}^{T} \beta^t \cdot \Delta_t \cdot \frac{\left(\frac{C_t}{L_t}\right)^{1-1/\psi} - 1}{1 - 1/\psi} L_t. \tag{B.33}$$

The discount rate in both DICE-2007 and DICE-2016 models is the same and determined as:

$$\beta = \frac{1}{(1 + \rho)^{\Delta_t}}. \tag{B.34}$$

The parameters for the economic part of DICE-2007 and DICE-2016 are given in Table 10.

Calibrated parameter	Symbol	Value 2007	Value 2016/CDICE
Capital annual depreciation rate	δ^K	0.1	0.1
Elasticity of capital	α	0.3	0.3
Damage parameter	ψ_1	0.0	0.0
Damage quadratic parameter	ψ_2	0.0028388	0.00236
Exponent of control cost function	θ_2	2.8	2.6
Risk aversion	ψ	0.5	0.69
Time preferences	ρ	0.015	0.015
Initial capital (trillions USD 2015)	K_0	-	223.
Initial capital (trillions USD 2005)	K_0	137.	-
Time step	Δ_t	10	5/1

Table 10 Generic parameterization for the parameters of economy.

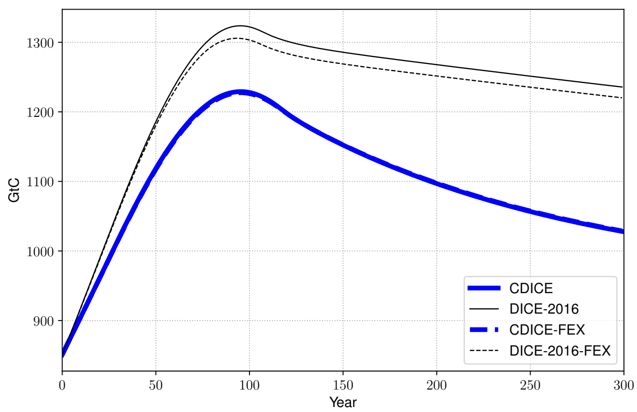
Online Appendix C Sensitivity exercise: change in the exogenous forcing

In section 4.5 we used an alternative evolution equation for exogenous forcings, namely

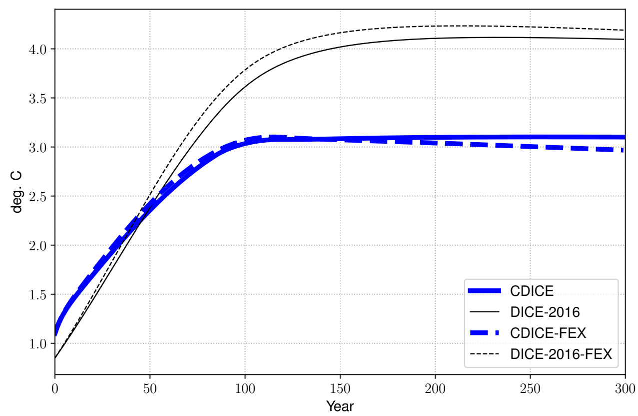
$$F^{\mathrm{EX}} = 0.3 \cdot F_t^{\mathrm{CO2}}. \tag{C.1}$$

This specification was adopted instead of Eq. (B.10) that was used in DICE-2016. As was shown in Figure 7, this switch from the original DICE-2016 exogenous forcings can make quite a difference in the temperature evolution. However, when computing the simulation results in section 6, we used the original exogenous forcings equation Eq. (B.10) to remain consistent with the DICE-2016 model and to focus on analyzing the influence of the re-calibrated climate part on policies, which was the goal of the section. Therefore, in this section of the online Appendix, we want to investigate the consequences of implementing time-dependent exogenous forcings that differ for the DICE-2016 and CDICE models.

Below we present some solution results for the optimization problem Eq. (8) when the agent chooses optimal investment and mitigation paths. Furthermore, we compare DICE-2016 and CDICE with original and alternative exogenous forcings. Overall changing exogenous forcings does not make much of a difference for the modeling results. However, there are a couple of interesting observations that are worth mentioning.



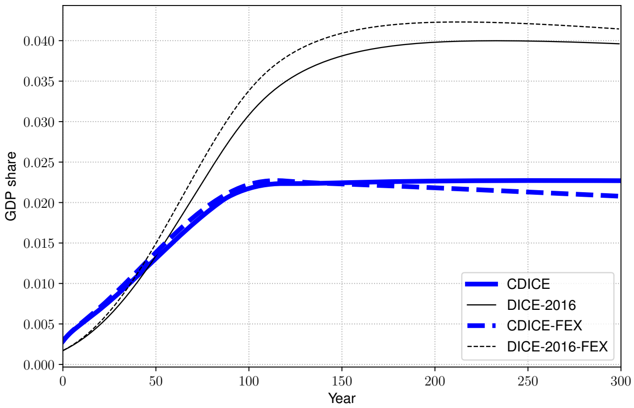
(a) Mass of carbon in the atmosphere



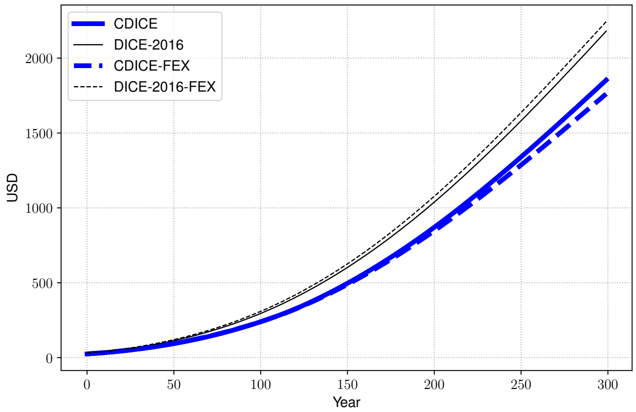
(b) Temperature of the atmosphere

Fig. 1 Mass of carbon in the atmosphere (left) and temperature of the atmosphere (right) for DICE-2016, and CDICE models with different exogenous forcing evolution (DICE-2016-FEX, CDICE-FEX show the model variables under the assumption of time dependent exogenous forcings) for an optimal abatement case. Year zero on the graphs corresponds to a starting year 2015.

From Figure 1 we see that change in the exogenous forcings does not affect the CDICE model much but has some effect on DICE-2016. In DICE-2016 exogenous forcings being time-dependent results in a lower mass of carbon in the atmosphere than in DICE-2016 with the original forcings. However, it leads to an even higher temperature increase. The reason for this increased sensitivity of DICE-2016 is the same as for its excessive sensitivity to the discount rate. The carbon cycle and temperature equations are not in balance. Thus they overreact in transmitting changes in the inputs of the model towards outputs.



(a) Damages



(b) SCC

Fig. 2 Damages (left) and the social cost of carbon (right) for DICE-2016, and CDICE models with different exogenous forcings evolution (DICE-2016-FEX, CDICE-FEX show the model variables under the assumption of time dependent exogenous forcings) for an optimal abatement case. Year zero on the graphs corresponds to a starting year 2015.

Figure 2 reports damages and the social cost of carbon for models under consideration. Both variables are coherently in line with the expectations that one might get analyzing temperature equations. DICE-2016 with time-dependent forcings predicts more damages and thus a higher social cost of carbon. For CDICE, both damages and the social cost of carbon remain roughly the same, with CDICE variables being slightly lower for alternative exogenous forcing formulation.

Online Appendix D Computational details

In this online Appendix, we briefly outline how we numerically solve the re-calibrated DICE-2016 model (cf. Section 6) by applying "Deep Equilibrium Nets"(DEQNs) [Azinovic et al., 2022]. To do so, we proceed in two steps. First, we summarize in Section D.1 the DEQN solution technique in the most general form. Second, we detail in Section D.2 how the CDICE model can be cast in a form amenable to DEQNs. Section D.3 finally reports detailed error statistics for our benchmark CDICE model. For further implementation details, we refer the reader to the repository hosted under the following URL: `https://github.com/ClimateChangeEcon/Climate_in_Climate_Economics` and which contains all replication codes.

D.1 Deep Equilibrium Nets in a Nutshell

From an abstract perspective, the DEQN algorithm is a simulation-based solution method using deep neural networks[3] to compute an approximation of the *optimal policy function* $\mathbf{p} : X \rightarrow Y \subset \mathbb{R}^K$ to a dynamic model under the assumption that the underlying economy can be characterized via discrete-time first-order equilibrium conditions, that is,

$$\mathbf{G}(\mathbf{x}, \mathbf{p}) = \mathbf{0} \; \forall \mathbf{x} \in X, \tag{D.1}$$

[3] See, e.g., Goodfellow et al. [2016] for a textbook treatment of neural networks.

Intuitively, DEQNs work as follows: An unknown policy function is approximated with a neural network, that is, $\mathbf{p}(\mathbf{x}) \approx \mathcal{N}_{\nu}(\mathbf{x})$, and where the ν's are ex-ante unknown coefficients of the neural network that have to be determined based on some suitable loss function measuring the quality of a given approximation at a given state of the economy.

The DEQN algorithm is started by randomly initializing the ν's [Glorot and Bengio, 2010], that is, an arbitrary guess for the ex-ante unknown approximate policy function. Next, we simulate a sequence of $N_{\text{path length}}$ states. Starting from some given state $\mathbf{x}_t$, the next state $\mathbf{x}_{t+1}$ is a result of the policies encoded by the neural network, $\mathcal{N}_{\nu}(\mathbf{x})$, and remaining model-implied dynamics.

If we knew the (approximate) policy function satisfying the equilibrium conditions, expression (D.1) would hold along a simulated path. However, since the neural network is initialized with random coefficients, $\mathbf{G}(\mathbf{x}_t, \mathcal{N}_{\nu}(\mathbf{x_t})) \neq \mathbf{0}$ along the simulated path of length $N_{\text{path length}}$. This fact is now leveraged to improve the quality of the guessed policy function. Specifically, DEQNs use the error in the equilibrium conditions as a loss function, that is,

$$\ell_{\nu} := \frac{1}{N_{\text{path length}}} \sum_{\mathbf{x}_t \text{on sim. path}} \sum_{m=1}^{N_{eq}} \left(G_m(\mathbf{x}_t, \mathcal{N}_{\nu}(\mathbf{x}_t))\right)^2, \tag{D.2}$$

where $G_m(\mathbf{x}_t, \mathcal{N}_{\nu}(\mathbf{x}_t)))$ represent all the N_{eq} first-order equilibrium conditions of a given model. Expression (D.2) can now be used to update the parameters of the network with any variant of gradient descent,[4] namely,

$$\nu_k^{'} = \nu_k - \alpha^{\text{learn}} \frac{\partial \ell(\boldsymbol{\nu})}{\partial \nu_k}, \tag{D.3}$$

where $\nu_k^{'}$ represents the updated $k-$th parameter of the neural network, that is, $\mathcal{N}_{\nu} = \mathcal{N}_{\nu}^{'}$, and where $\alpha^{\text{learn}} \in \mathbb{R}$ denotes the so-called learning rate. The updated neural network-based representation of the policy is subsequently used to simulate a sequence of length $N_{\text{path length}}$ steps, along which the loss function is recorded, and the latter is again used to update the network parameters. This iterative procedure is pursued until $\ell_{\nu} < \epsilon \in \mathbb{R}$, that is, until an approximate equilibrium policy, has been found.

In summary, the DEQN algorithm consists of four building blocks: i) deep neural networks for approximating the equilibrium policies; ii) a suitable loss function measuring the quality of a given approximation at a given state of the economy; iii) an updating mechanism to improve the quality of the approximation; and iv) a sampling method for choosing states for updating and evaluating of the approximation quality. We next outline each of these components for clarity in the context of CDICE.

D.2 Mapping CDICE onto Deep Equilibrium Nets

In this online Appendix, we introduce a formal way of mapping CDICE, but more broadly speaking, non-stationary models, onto the neural network-based DEQN solution framework. To ensure the replicability of our procedure, we next thoroughly list every mathematical manipulation that is required.

We begin by stating the CDICE model (cf. the online Appendix B for the detailed calibration), that is,

$$\max_{\{C_t,\mu_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t}{L_t}\right)^{1-1/\psi} - 1}{1 - 1/\psi} L_t \tag{D.4}$$

$$\text{s.t.} \quad K_{t+1} = \left(1 - \Omega\left(T_{\text{AT},t}\right) - \Theta\left(\mu_t\right)\right) K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} + (1-\delta)K_t - C_t \quad (\lambda_t) \tag{D.5}$$

$$M_{t+1}^{\text{AT}} = (1 - b_{12}) M_t^{\text{AT}} + b_{12} \frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} M_t^{\text{UO}} + \sigma_t (1 - \mu_t) K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} + E_t^{\text{Land}} \quad \left(\nu_t^{\text{AT}}\right) \tag{D.6}$$

$$M_{t+1}^{\text{UO}} = b_{12} M_t^{\text{AT}} + \left(1 - b_{12} \frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} - b_{23}\right) M_t^{\text{UO}} + b_{23} \frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}} M_t^{\text{LO}} \quad \left(\nu_t^{\text{UO}}\right) \tag{D.7}$$

[4] In our practical applications, we use "Adam" [Kingma and Ba, 2014].

$$M_{t+1}^{\mathrm{LO}} = b_{23} M_t^{\mathrm{UO}} + \left(1 - b_{23} \frac{M_{\mathrm{EQ}}^{\mathrm{UO}}}{M_{\mathrm{EQ}}^{\mathrm{LO}}}\right) M_t^{\mathrm{LO}} \quad \left(\nu_t^{\mathrm{LO}}\right) \tag{D.8}$$

$$T_{t+1}^{\mathrm{AT}} = T_t^{\mathrm{AT}} + c_1 \left(F_{2\mathrm{XCO2}} \frac{\log(M_t^{\mathrm{AT}} / M_{\mathrm{base}}^{\mathrm{AT}})}{\log(2)} + F_t^{EX}\right) - c_1 \frac{F_{2\mathrm{XCO2}}}{T_{2\mathrm{XCO2}}} T_t^{\mathrm{AT}} - c_1 c_3 \left(T_t^{\mathrm{AT}} - T_t^{\mathrm{OC}}\right) \quad \left(\eta_t^{\mathrm{AT}}\right) \tag{D.9}$$

$$T_{t+1}^{\mathrm{OC}} = T_t^{\mathrm{OC}} + c_4 \left(T_t^{\mathrm{AT}} - T_t^{\mathrm{OC}}\right) \quad \left(\eta_t^{\mathrm{OC}}\right) \tag{D.10}$$

$$0 \leq \mu_t \leq 1 \quad (\lambda_t^{\mu}), \tag{D.11}$$

where the Lagrange and KKT multipliers we will employ below have been added in parentheses for completeness.

The state of the economy at time t is given by

$$\mathbf{x}_t \in \mathbb{R}^6 := \left(k_t, M_t^{\mathrm{AT}}, M_t^{\mathrm{UO}}, M_t^{\mathrm{LO}}, T_t^{\mathrm{AT}}, T_t^{\mathrm{OC}}, t\right)^T. \tag{D.12}$$

Note that we take time t as an exogenous state to account for the non-stationary nature of the IAM, whereas all other states are endogenously determined. Furthermore, to ensure computational tractability, we follow Traeger [2014] and map the unbounded physical time $t \in [0, \infty)$ via the strictly monotonic transformation

$$\tau = 1 - \exp(-\vartheta t). \tag{D.13}$$

into the unit interval, $\tau \in (0, 1]$.[5] To scale back from the auxiliary time τ to the physical time, the inverse transformation of equation (D.13) can be applied, that is,

$$t = -\frac{\ln(1 - \tau)}{\vartheta}. \tag{D.14}$$

Next, we re-scale consumption and capital as

$$c_t := \frac{C_t}{A_t L_t}, k_t := \frac{K_t}{A_t L_t}. \tag{D.15}$$

The following normalization of the Lagrangian and the KKT multipliers helps to stabilize the solution process, that is:

$$\begin{aligned}
\hat{\lambda}_t &:= \frac{\lambda_t}{A_t^{1-\frac{1}{\psi}} L_t}, \hat{\lambda}_t^{\mu} := \frac{\lambda_t^{\mu}}{A_t^{1-\frac{1}{\psi}} L_t}, \hat{\nu}_t^{\mathrm{AT}} := \frac{\nu_t^{\mathrm{AT}}}{A_t^{1-\frac{1}{\psi}} L_t}, \hat{\nu}_t^{\mathrm{UO}} := \frac{\nu_t^{\mathrm{UO}}}{A_t^{1-\frac{1}{\psi}} L_t}, \hat{\nu}_t^{\mathrm{LO}} := \frac{\nu_t^{\mathrm{LO}}}{A_t^{1-\frac{1}{\psi}} L_t}, \\
\hat{\eta}_t^{\mathrm{AT}} &:= \frac{\eta_t^{\mathrm{AT}}}{A_t^{1-\frac{1}{\psi}} L_t}, \hat{\eta}_t^{\mathrm{OC}} := \frac{\eta_t^{\mathrm{OC}}}{A_t^{1-\frac{1}{\psi}} L_t}.
\end{aligned} \tag{D.16}$$

Furthermore, we introduce a quantity called the “effective discount factor”,

$$\hat{\beta}_t := \exp\left(-\rho + \left(1 - \frac{1}{\psi}\right) g_t^A + g_t^L\right). \tag{D.17}$$

The policy function $\mathbf{p}$ we intend to approximate with the aid of deep neural networks is given by

$$\mathcal{N}_{\nu}(\mathbf{x}_t) \in \mathbb{R}^9 := \left(k_{t+1}, \mu_t, \hat{\lambda}_t, \hat{\lambda}_t^{\mu}, \hat{\nu}_t^{\mathrm{AT}}, \hat{\nu}_t^{\mathrm{UO}}, \hat{\nu}_t^{\mathrm{LO}}, \hat{\eta}_t^{\mathrm{AT}}, \hat{\eta}_t^{\mathrm{OC}}\right). \tag{D.18}$$

and consists of the choice variables (k_{t+1}, μ_t)[6] as well as the Lagrange and KKT multipliers.

[5] An alternative way to deal with transitory dynamics is to use so-called “shooting methods” (see, e.g., Arellano et al. [2020], and references therein).

[6] In our computations, we follow the literature and employ k_{t+1} instead of c_t as a choice variable, as it allows for a simpler update of the state variable k_t. Nevertheless, we still keep the first-order conditions with respect to k_{t+1} and c_t. In practical applications, the

Next, we derive the first-order conditions in order to form a loss function for the CDICE model that is suitable for DEQNs (cf. equation (D.2)). To do so, we start by formulating the Lagrangian of the model, which reads,

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \hat{\beta}_t \Bigg[& \frac{c_t^{1-1/\psi} - A_t^{1/\psi-1}}{1-1/\psi} \\
& + \hat{\lambda}_t \left\{ \left(1 - \Omega\left(T_{\mathrm{AT},t}\right) - \Theta\left(\mu_t\right)\right) k_t^{\alpha} + (1-\delta) k_t - c_t - \exp\left(g_t^A + g_t^L\right) k_{t+1} \right\} \\
& + \hat{\lambda}_t^{\mu} \left\{ 1 - \mu_t \right\} \\
& + \hat{\nu}_t^{\mathrm{AT}} \left\{ (1-b_{12}) M_t^{\mathrm{AT}} + b_{12} \frac{M_{\mathrm{EQ}}^{\mathrm{AT}}}{M_{\mathrm{EQ}}^{\mathrm{UO}}} M_t^{\mathrm{UO}} + \sigma_t (1-\mu_t) A_t L_t k_t^{\alpha} + E_t^{\mathrm{Land}} - M_{t+1}^{\mathrm{AT}} \right\} \\
& + \hat{\nu}_t^{\mathrm{UO}} \left\{ b_{12} M_t^{\mathrm{AT}} + \left(1 - b_{12} \frac{M_{\mathrm{EQ}}^{\mathrm{AT}}}{M_{\mathrm{EQ}}^{\mathrm{UO}}} - b_{23}\right) M_t^{\mathrm{UO}} + b_{23} \frac{M_{\mathrm{EQ}}^{\mathrm{UO}}}{M_{\mathrm{EQ}}^{\mathrm{LO}}} M_t^{\mathrm{LO}} - M_{t+1}^{\mathrm{UO}} \right\} \\
& + \hat{\nu}_t^{\mathrm{LO}} \left\{ b_{23} M_t^{\mathrm{UO}} + \left(1 - b_{23} \frac{M_{\mathrm{EQ}}^{\mathrm{UO}}}{M_{\mathrm{EQ}}^{\mathrm{LO}}}\right) M_t^{\mathrm{LO}} - M_{t+1}^{\mathrm{LO}} \right\} \\
& + \hat{\eta}_t^{\mathrm{AT}} \left\{ T_t^{\mathrm{AT}} + c_1 \left(F_{\mathrm{2XCO2}} \frac{\log(M_t^{\mathrm{AT}}/M_{\mathrm{base}}^{\mathrm{AT}})}{\log(2)} + F_t^{EX} \right) - c_1 \frac{F_{\mathrm{2XCO2}}}{T_{\mathrm{2XCO2}}} T_t^{\mathrm{AT}} - c_1 c_3 \left(T_t^{\mathrm{AT}} - T_t^{\mathrm{OC}}\right) - T_{t+1}^{\mathrm{AT}} \right\} \\
& + \hat{\eta}_t^{\mathrm{OC}} \left\{ T_t^{\mathrm{OC}} + c_4 \left(T_t^{\mathrm{AT}} - T_t^{\mathrm{OC}}\right) - T_{t+1}^{\mathrm{OC}} \right\} \Bigg]
\end{aligned} \tag{D.19}
$$

The Lagrangian is now used to compute the first-order conditions, that is,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \Leftrightarrow \ & \exp\left(g_t^A + g_t^L\right) \hat{\lambda}_t - \hat{\beta}_t \Big[\hat{\lambda}_{t+1} \left(\left(1 - \Omega\left(T_{\mathrm{AT},t+1}\right) - \Theta\left(\mu_t\right)\right) \alpha k_{t+1}^{\alpha-1} + (1-\delta) \right) \\
& + \hat{\nu}_{t+1}^{\mathrm{AT}} \sigma_{t+1} (1-\mu_{t+1}) A_{t+1} L_{t+1} \alpha k_{t+1}^{\alpha-1} \Big] = 0
\end{aligned} \tag{D.20}
$$

$$
\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Leftrightarrow c_t^{-1/\psi} A_t^{1-1/\psi} L_t - \hat{\lambda}_t = 0 \tag{D.21}
$$

$$
\frac{\partial \mathcal{L}}{\partial \mu_t} = 0 \Leftrightarrow \hat{\lambda}_t \Theta'\left(\mu_t\right) k_t^{\alpha} + \lambda_t^{\mu} + \hat{\nu}_t^{\mathrm{AT}} \sigma_t A_t L_t k_t^{\alpha} = 0 \tag{D.22}
$$

$$
\frac{\partial \mathcal{L}}{\partial M_{\mathrm{AT},t+1}} = 0 \Leftrightarrow \hat{\nu}_t^{\mathrm{AT}} - \hat{\beta}_t \left[\hat{\nu}_{t+1}^{\mathrm{AT}} (1-b_{12}) + \hat{\nu}_{t+1}^{\mathrm{UO}} b_{12} + \hat{\eta}_{t+1}^{\mathrm{AT}} c_1 F_{\mathrm{2XCO2}} \frac{1}{\ln 2 M_{\mathrm{AT},t+1}} \right] = 0 \tag{D.23}
$$

$$
\frac{\partial \mathcal{L}}{\partial M_{\mathrm{UO},t+1}} = 0 \Leftrightarrow \hat{\nu}_t^{\mathrm{UO}} - \hat{\beta}_t \left[\hat{\nu}_{t+1}^{\mathrm{AT}} b_{12} \frac{M_{\mathrm{EQ}}^{\mathrm{AT}}}{M_{\mathrm{EQ}}^{\mathrm{UO}}} + \hat{\nu}_{t+1}^{\mathrm{UO}} \left(1 - b_{12} \frac{M_{\mathrm{EQ}}^{\mathrm{AT}}}{M_{\mathrm{EQ}}^{\mathrm{UO}}} - b_{23}\right) + \hat{\nu}_{t+1}^{\mathrm{LO}} b_{23} \right] = 0 \tag{D.24}
$$

$$
\frac{\partial \mathcal{L}}{\partial M_{\mathrm{LO},t+1}} = 0 \Leftrightarrow \hat{\nu}_t^{\mathrm{LO}} - \hat{\beta}_t \left[\hat{\nu}_{t+1}^{\mathrm{UO}} b_{23} \frac{M_{\mathrm{EQ}}^{\mathrm{UO}}}{M_{\mathrm{EQ}}^{\mathrm{LO}}} + \hat{\nu}_{t+1}^{\mathrm{LO}} \left(1 - b_{23} \frac{M_{\mathrm{EQ}}^{\mathrm{UO}}}{M_{\mathrm{EQ}}^{\mathrm{LO}}}\right) \right] = 0 \tag{D.25}
$$

$$
\frac{\partial \mathcal{L}}{\partial T_{\mathrm{AT},t+1}} = 0 \Leftrightarrow \hat{\eta}_t^{\mathrm{AT}} - \hat{\beta}_t \left[-\lambda_{t+1} \Omega'(T_{\mathrm{AT},t+1}) k_{t+1}^{\alpha} + \hat{\eta}_{t+1}^{\mathrm{AT}} (1 - c_1 \frac{F_{\mathrm{2XCO2}}}{\mathrm{T2xco2}} - c_1 c_3) + \hat{\eta}_{t+1}^{\mathrm{OC}} c_4 \right] = 0 \tag{D.26}
$$

$$
\frac{\partial \mathcal{L}}{\partial T_{\mathrm{OC},t+1}} = 0 \Leftrightarrow \hat{\eta}_t^{\mathrm{OC}} - \hat{\beta}_t \left[\hat{\eta}_{t+1}^{\mathrm{AT}} c_1 c_3 + \hat{\eta}_{t+1}^{\mathrm{OC}} (1-c_4) \right] = 0 \tag{D.27}
$$

$$
\frac{\partial \mathcal{L}}{\partial \hat{\lambda}_t} = 0 \Leftrightarrow \left(1 - \Omega\left(T_{\mathrm{AT},t}\right) - \Theta\left(\mu_t\right)\right) k_t^{\alpha} + (1-\delta) k_t - c_t - \exp\left(g_t^A + g_t^L\right) k_{t+1} = 0 \tag{D.28}
$$

$$
\frac{\partial \mathcal{L}}{\partial \hat{\nu}_t^{\mathrm{AT}}} = 0 \Leftrightarrow (1-b_{12}) M_t^{\mathrm{AT}} + b_{12} \frac{M_{\mathrm{EQ}}^{\mathrm{AT}}}{M_{\mathrm{EQ}}^{\mathrm{UO}}} M_t^{\mathrm{UO}} + \sigma_t (1-\mu_t) A_t L_t k_t^{\alpha} + E_t^{\mathrm{Land}} - M_{t+1}^{\mathrm{AT}} = 0 \tag{D.29}
$$

performance of neural networks can, as in our case, often benefit from redundant information (see, e.g., Azinovic et al. [2022] for more details).

$$\frac{\partial \mathcal{L}}{\partial \hat{\nu}_t^{\text{UO}}} = 0 \Leftrightarrow b_{12} M_t^{\text{AT}} + \left(1 - b_{12}\frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} - b_{23}\right) M_t^{\text{UO}} + b_{23}\frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}} M_t^{\text{LO}} - M_{t+1}^{\text{UO}} = 0 \tag{D.30}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\nu}_t^{\text{LO}}} = 0 \Leftrightarrow b_{23} M_t^{\text{UO}} + \left(1 - b_{23}\frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}}\right) M_t^{\text{LO}} - M_{t+1}^{\text{LO}} = 0 \tag{D.31}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\eta}_t^{\text{AT}}} = 0 \Leftrightarrow T_t^{\text{AT}} + c_1\left(F_{2\text{XCO2}}\frac{\log(M_t^{\text{AT}}/M_{\text{base}}^{\text{AT}})}{\log(2)} + F_t^{EX}\right) - c_1\frac{F_{2\text{XCO2}}}{T_{2\text{XCO2}}} T_t^{\text{AT}} - c_1 c_3\left(T_t^{\text{AT}} - T_t^{\text{OC}}\right) - T_{t+1}^{\text{AT}} = 0 \tag{D.32}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\eta}_t^{\text{OC}}} = 0 \Leftrightarrow T_t^{\text{OC}} + c_4\left(T_t^{\text{AT}} - T_t^{\text{OC}}\right) - T_{t+1}^{\text{OC}} = 0, \tag{D.33}$$

as well as the KKT condition, that is,

$$1 - \mu_t \geq 0 \quad \perp \quad \lambda_t^{\mu} \geq 0. \tag{D.34}$$

In our practical applications, though, we replace the KKT condition Eq. (D.34) above with the Fischer-Burmeister function (see, e.g., Maliar et al. [2021], and references therein), that is,

$$\Psi^{\text{FB}}\left(\hat{\lambda}_t^{\mu}, 1 - \mu_t\right) = \hat{\lambda}_t^{\mu} + (1 - \mu_t) - \sqrt{\left(\hat{\lambda}_t^{\mu}\right)^2 + (1 - \mu_t)^2}, \tag{D.35}$$

where we replaced $\hat{\lambda}_t^{\mu}$ such that

$$\hat{\lambda}_t^{\mu} := -\hat{\lambda}_t \Theta'(\mu_t)\, k_t^{\alpha} - \hat{\nu}_t^{\text{AT}} \sigma_t A_t L_t k_t^{\alpha} \tag{D.36}$$

holds. We also use expression (D.21) to replace consumption c_t in the budget constraint (D.28).

One feature that appears when working with first-order conditions of an IAM is the need to compute them not only with respect to the economic choice variables such as μ_t and c_t, but also with respect to the climate variables, despite the fact that they are not choice variables. The reason for this is that one needs to assess the marginal effects of the change in choice variables that propagate through the climate system. Those effects cannot be computed analytically here, which is why we need Lagrange multipliers associated with every single climate equation (cf. expressions (D.6)-(D.10)) to estimate the shadow price of a marginal change in a respective constraint.

Using all the above definitions, the eight individual components that enter the loss function amendable for the DEQN algorithm read as

$$\begin{aligned} l_1 := \exp\left(g_t^A + g_t^L\right)\hat{\lambda}_t - \hat{\beta}\Big[\hat{\lambda}_{t+1}\left(\left(1 - \Omega\left(T_{\text{AT},t+1}\right) - \Theta\left(\mu_t\right)\right)\alpha k_{t+1}^{\alpha-1} + (1-\delta)\right) + \\ \hat{\nu}_{t+1}^{\text{AT}}(1 - \mu_{t+1})\sigma_{t+1}A_{t+1}L_{t+1}\alpha k_{t+1}^{\alpha-1}\Big] = 0 \end{aligned} \tag{D.37}$$

$$l_2 := \left(1 - \Omega\left(T_{\text{AT},t}\right) - \Theta\left(\mu_t\right)\right) k_t^{\alpha} + (1-\delta)\, k_t - c_t - \exp\left(g_t^A + g_t^L\right) k_{t+1} = 0 \tag{D.38}$$

$$l_3 := \hat{\nu}_t^{\text{AT}} - \beta\left[\hat{\nu}_{t+1}^{\text{AT}}(1 - b_{12}) + \hat{\nu}_{t+1}^{\text{UO}} b_{12} + \hat{\eta}_{t+1}^{\text{AT}} c_1 F_{2\text{XCO2}}\frac{1}{\ln 2 M_{\text{AT},t+1}}\right] = 0 \tag{D.39}$$

$$l_4 := \hat{\nu}_t^{\text{UO}} - \beta\left[\hat{\nu}_{t+1}^{\text{AT}} b_{12}\frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} + \hat{\nu}_{t+1}^{\text{UO}}\left(1 - b_{12}\frac{M_{\text{EQ}}^{\text{AT}}}{M_{\text{EQ}}^{\text{UO}}} - b_{23}\right) + \hat{\nu}_{t+1}^{\text{LO}} b_{23}\right] = 0 \tag{D.40}$$

$$l_5 := \hat{\nu}_t^{\text{LO}} - \beta\left[\hat{\nu}_{t+1}^{\text{UO}} b_{23}\frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}} + \hat{\nu}_{t+1}^{\text{LO}}\left(1 - b_{23}\frac{M_{\text{EQ}}^{\text{UO}}}{M_{\text{EQ}}^{\text{LO}}}\right)\right] = 0 \tag{D.41}$$

$$l_6 := \hat{\eta}_t^{\text{AT}} - \beta\left[-\hat{\lambda}_{t+1}\Omega'(T_{\text{AT},t+1})k_{t+1}^{\alpha} + \hat{\eta}_{t+1}^{\text{AT}}(1 - c_1\frac{F_{2\text{XCO2}}}{\text{T2xco2}} - c_1 c_3) + \hat{\eta}_{t+1}^{\text{OC}} c_4\right] = 0 \tag{D.42}$$

$$l_7 := \hat{\eta}_t^{\text{OC}} - \beta\left[\hat{\eta}_{t+1}^{\text{AT}} c_1 c_3 + \hat{\eta}_{t+1}^{\text{OC}}(1 - c_4)\right] = 0 \tag{D.43}$$

$$l_8 := \hat{\lambda}_t^{\mu} + (1-\mu_t) - \sqrt{\left(\hat{\lambda}_t^{\mu}\right)^2 + (1-\mu_t)^2} = 0, \tag{D.44}$$

and result in the total loss function given by

$$\ell_{\nu} := \frac{1}{N_{\text{path length}}} \sum_{\mathbf{x}_t \text{ on sim. path}} \sum_{m=1}^{N_{eq}=8} \left(l_m(\mathbf{x}_t, \mathcal{N}_{\nu}(\mathbf{x}_t))\right)^2 . \tag{D.45}$$

The final ingredient we need for the DEQN algorithm is the evolution of the state $\mathbf{x}_t$ one period forward such that the loss function (D.45) can be evaluated along a simulated path. In CDICE, $\mathbf{x}_{t+1}$ is given by

$$\mathbf{x}_{t+1} = \left(k_{t+1}, M_{t+1}^{\text{AT}}, M_{t+1}^{\text{UO}}, M_{t+1}^{\text{LO}}, T_{t+1}^{\text{AT}}, T_{t+1}^{\text{OC}}, t+1,\right)^T , \tag{D.46}$$

where k_{t+1} is updated as a choice variable from the policy function Eq. (D.18) and the climate variables $M_{t+1}^{\text{AT}}, M_{t+1}^{\text{UO}}, M_{t+1}^{\text{LO}}, T_{t+1}^{\text{AT}}$, and T_{t+1}^{OC} can be updated via equations (D.29)-(D.33), whereas time t is simply incremented by one unit.

D.3 Error Statistics

Table 11 provides detailed error statistics for the benchmark CDICE model at convergence (cf. all the normalized expressions (D.37) - (D.44) that contribute to the loss function (D.45)). All other models presented in the article reach a similar level of accuracy.

Stats.	l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8
mean	$1.13e-04$	$5.32e-04$	$2.29e-04$	$3.76e-04$	$1.15e-04$	$1.82e-05$	$3.07e-05$	$5.04e-04$
std	$6.35e-05$	$1.23e-04$	$1.15e-04$	$7.75e-05$	$6.31e-05$	$1.49e-05$	$2.42e-05$	$3.17e-04$
min	$1.19e-07$	$1.90e-05$	$5.36e-07$	$5.39e-05$	$1.18e-05$	$5.96e-08$	$< 1.0e-8$	$< 1.0e-8$
0.1%	$1.19e-07$	$1.91e-05$	$5.36e-07$	$5.39e-05$	$1.18e-05$	$5.96e-08$	$< 1.0e-8$	$< 1.0e-8$
25%	$5.82e-05$	$4.28e-04$	$1.37e-04$	$3.23e-04$	$6.71e-05$	$8.15e-06$	$1.09e-05$	$1.89e-04$
50%	$1.15e-04$	$5.28e-04$	$2.58e-04$	$3.62e-04$	$9.70e-05$	$1.33e-05$	$2.60e-05$	$5.06e-04$
75%	$1.70e-04$	$6.34e-04$	$3.16e-04$	$4.11e-04$	$1.58e-04$	$2.32e-05$	$4.71e-05$	$7.83e-04$
max.	$3.26e-04$	$9.83e-04$	$5.36e-04$	$7.40e-04$	$3.13e-04$	$8.54e-05$	$1.26e-04$	$1.30e-03$

Table 11 Summary statistics of the loss function components along the simulation path for the optimal solution of the CDICE model. Following the literature (see, e.g., Brumm and Scheidegger [2017]), we define the maximum error as the 99.9 percent quantile of the error distribution.

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