

Solving the (non-stationary) DICE model with Deep Equilibrium Nets

The climate in climate economics (Folini et. al (2024))

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- The main policy instrument for dealing with climate change is to price carbon (in the form of carbon tax or quotas)
- To determine carbon price (in a form of the social cost of carbon) we need integrated assessment models (IAMs) that link together economy and climate
- Fully-fledged climate models are too big (months of supercomputer node-hours to compute), so economists use climate emulators - models, that are supposed to mimic fully-fledged climate science
- Climate part of most prominent IAMs (DICE, PAGE, FUND etc) in economics was criticised by Dietz et al. (2021) for being misaligned with climate physics (CMIP5). ¹

¹CMIP5 - Coupled Model Intercomparison Project in 5th phase. CMIP provides a framework for coordinated climate change experiments and contributes climate science knowledge for IPCC reports.

- **How can we test a climate part of any IAMs against climate science benchmark (CMIP5)?**
 - We cannot research economic side of the climate change problem without being sure that climate part is reliable
- **Can we provide a reliable yet simple climate emulator for economic research that is in accord with CMIP5?**
 - We need rather simple climate emulator to allow for complications on economic side (uncertainties, heterogeneity etc).

1. Develop a standardized suite of tests that is necessary and sufficient and allows to test climate part of any IAM against climate science benchmarks and validate it
2. Re-calibrate the most widely used economy-climate model DICE Nordhaus (2018) according to the developed testing framework and in line with state-of-the-art climate science (CMIP5)
3. Allow for a transparent arbitrary time step (1y, 5y, 10y) in the newly calibrated DICE model (CDICE)
4. The social cost of carbon that comes from DICE-2016 is underestimated
5. CDICE climate calibration reduces sensitivity of the social cost of carbon with respect to discount rate.

- Literature on IAM: standardized necessary and sufficient suite of tests for climate part of any IAM
- Literature using DICE as a building block: no need to change functional forms in climate part, it is enough to modify coefficients to get the climate aligned with climate science
- Literature re-calibrating DICE for an arbitrary time step as it allows to study the models in different time scales
- Policy:
 1. reconciliation of DICE and IPCC AR5 IPCC (2014) with respect to the long-run temperature evolution
 2. alleviation of the debate about high sensitivity of the social cost of carbon produced by DICE to the discount factor.

Outline of the presentation

Set of tests for climate module

- Overview of tests

- Calibration of Carbon Cycle

- Calibration of Energy Balance

- Gold standard climate test

Results

- The social cost of carbon

- Sensitivity to discount rate

Set of tests for climate module

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Overview of tests

Calibration of Carbon Cycle

Calibration of Energy Balance

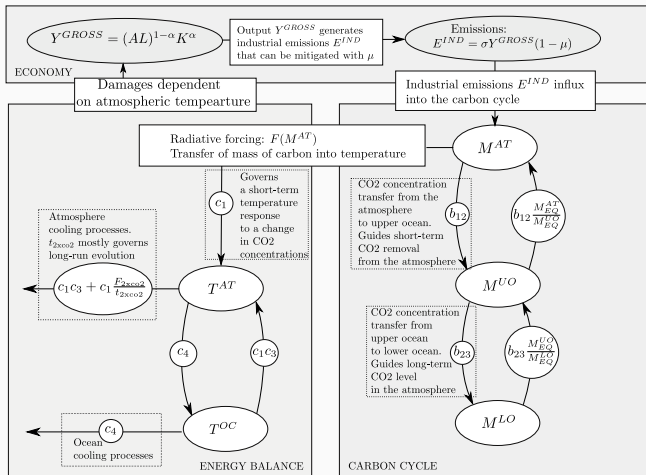
Gold standard climate test

Results

The social cost of carbon

Sensitivity to discount rate

Schematic version of an IAM



1. Calibration of Carbon Cycle: 100 GtC pulse to the atmosphere
2. Calibration of Energy Balance: instantaneous CO₂ quadrupling
3. Calibration of Energy Balance: atmospheric CO₂ increases at 1% per year
4. Gold standard climate test: CMIP5 historical data and RCP evolution

Set of tests for climate module

- Overview of tests

- Calibration of Carbon Cycle**

- Calibration of Energy Balance

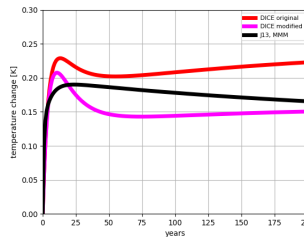
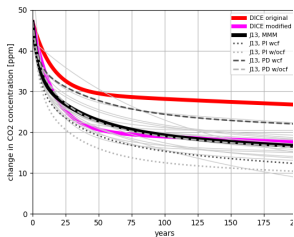
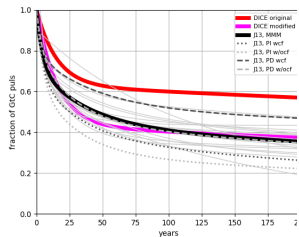
- Gold standard climate test

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- The social cost of carbon

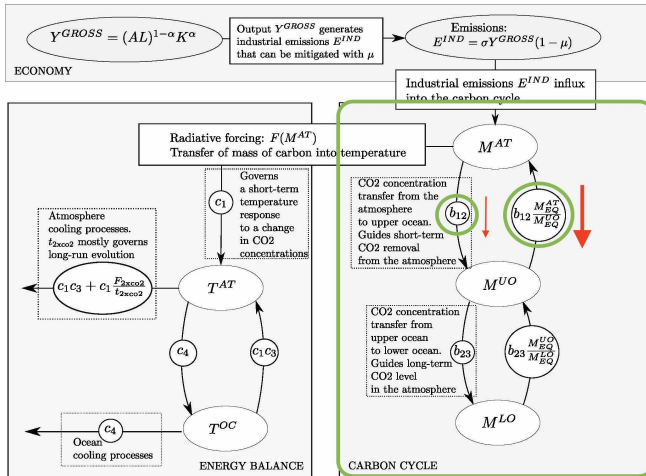
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Carbon cycle test: 100 GtC pulse to the atmosphere



| Model | b_{12} | b_{23} | M_{eq} | $b_{12} M_{EQ}^{AT} / M_{EQ}^{UO}$ |
|---------------|----------|----------|------------------|------------------------------------|
| DICE2016, 5yr | 0.12 | 0.007 | (588, 360, 1720) | 0.196 |
| CDICE | 0.053 | 0.0042 | (607, 600, 1772) | 0.054 |

Carbon cycle: modifications



Modification of carbon cycle

Feedback of CO₂ from upper ocean to the atmosphere decreased which allows quicker carbon removal from the atmosphere.

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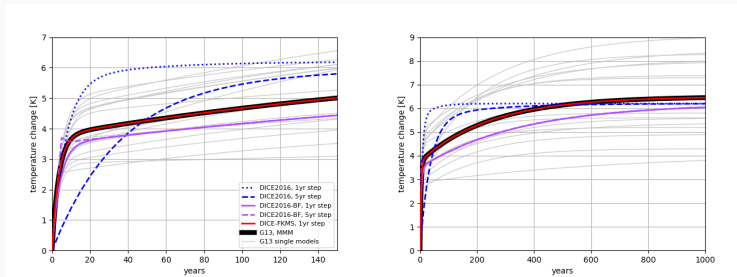
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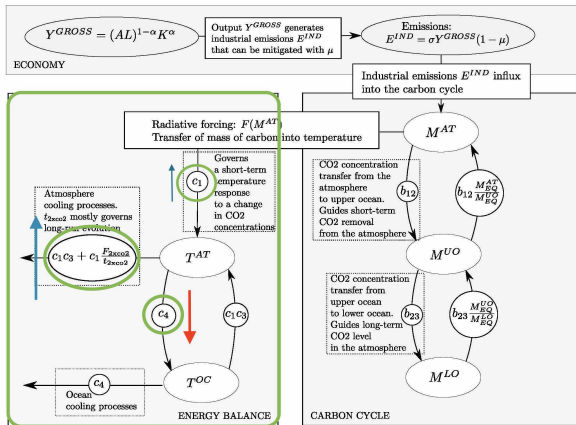
- Sensitivity to discount rate

Temperature test 1: instantaneous CO2 quadrupling

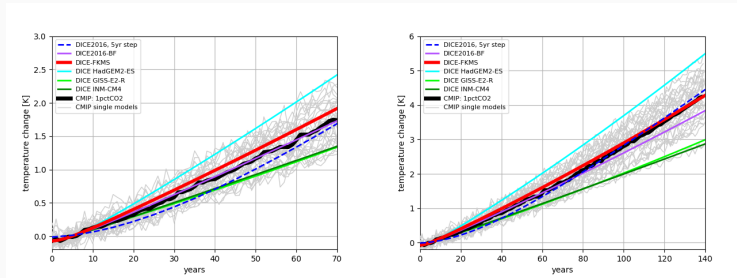


| Model | c_1 | c_3 | c_4 | $t_{2\times\text{CO}_2}$ | $F_{2\times\text{CO}_2}$ | $\frac{F_{2\times\text{CO}_2}}{t_{2\times\text{CO}_2}}$ | $c_1 c_3$ |
|---------------|--------|-------|---------|--------------------------|--------------------------|---|-----------|
| DICE2016, 5yr | 0.1005 | 0.088 | 0.025 | 3.1 | 3.6813 | 1.19 | 0.0088 |
| CDICE | 0.137 | 0.73 | 0.00689 | 3.25 | 3.45 | 1.06 | 0.1 |

Temperature: modifications



Temperature test 2: atmospheric CO2 increases at 1% per year



| Model | c_1 | c_3 | c_4 | $t_{2\times\text{CO}_2}$ | $F_{2\times\text{CO}_2}$ | $\frac{F_{2\times\text{CO}_2}}{t_{2\times\text{CO}_2}}$ | $c_1 c_3$ |
|---------------|--------|-------|---------|--------------------------|--------------------------|---|-----------|
| DICE2016, 5yr | 0.1005 | 0.088 | 0.025 | 3.1 | 3.6813 | 1.19 | 0.0088 |
| CDICE | 0.137 | 0.73 | 0.00689 | 3.25 | 3.45 | 1.06 | 0.1 |

Modification of energy balance

- Decrease in speed of long-term warming effect of the atmosphere leads to lower long-term temperature of the atmosphere
- Increase in short-term warming parameter and decrease in feedback from atmospheric to ocean allows for quicker response of temperature with respect to growth in CO₂ concentrations.

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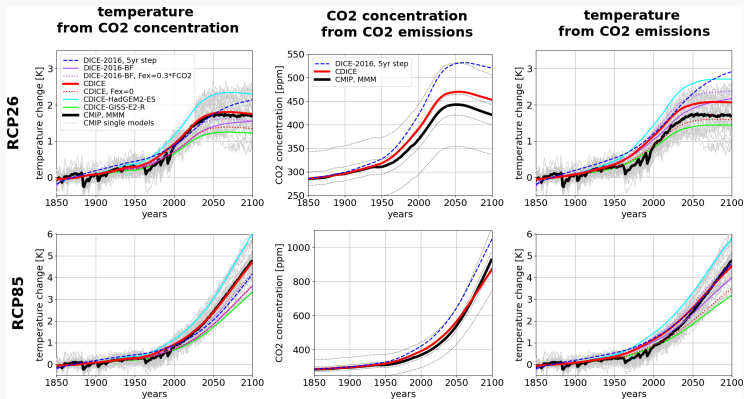
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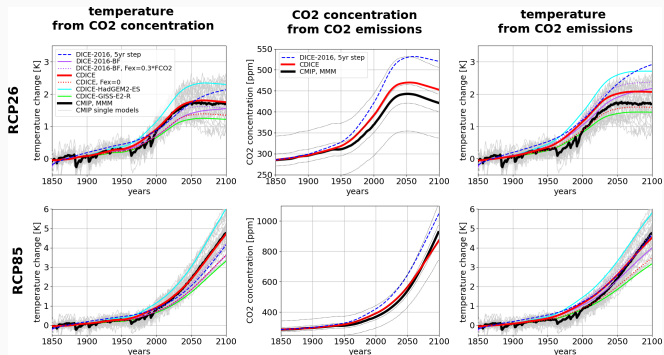
- Sensitivity to discount rate

CMIP5 historical data and RCP² evolution



²Representative Concentration Pathways describe different trajectories of future emission concentrations. RCP2.6, RCP4.5, RCP6, and RCP8.5 are named after a possible range of radiative forcing values in the year 2100

CMIP5 historical data and RCP evolution



| Model | c_1 | c_3 | c_4 | $t_{2 \times CO_2}$ | $F_{2 \times CO_2}$ | $\frac{F_{2 \times CO_2}}{t_{2 \times CO_2}}$ | b_{12} | b_{23} | M_{eq} |
|---------------|--------------|-------------|----------------|---------------------|---------------------|---|--------------|---------------|-------------------------|
| DICE2016, 5yr | 0.1005 | 0.088 | 0.025 | 3.1 | 3.6813 | 1.19 | 0.12 | 0.007 | (588, 360, 1720) |
| CDICE | 0.137 | 0.73 | 0.00689 | 3.25 | 3.45 | 1.06 | 0.053 | 0.0042 | (607, 600, 1772) |
| HadGEM2-ES | 0.154 | 0.55 | 0.00671 | 4.55 | 2.95 | 0.65 | 0.072 | 0.0042 | (607, 600, 1772) |
| GISS-E2-R | 0.213 | 1.16 | 0.00921 | 2.15 | 3.65 | 1.70 | 0.072 | 0.0042 | (607, 600, 1772) |

Table 1: Values of free parameters of bug-fixed DICE-BF and re-calibrated temperature equations DICE-G13

Modification of temperature and carbon cycle

- Functional form of equations for temperature comes from climate science emulators [Geoffrey et al.2013]
- We can match historical data and RCP projections from CO₂ concentrations to temperature quite precise
- Carbon cycle functional forms of DICE are slightly different from the climate emulator [Joos et al. 2013]
- Matching from CO₂ concentrations to CO₂ emissions is less precise but which nevertheless gives a benchmark for the 100GtC pulse.

- Carbon cycle: upper ocean feedback to the atmosphere reduced, quicker carbon removal from the atmosphere
- Energy balance (1): long-term speed of warming decreased, thus lower long-term temperature
- Energy balance (2): short-term warming increased, thus quicker temperature reaction to the CO₂ concentration increase
- Calibrating climate parts separately allows to avoid potential danger of compensating errors between carbon cycle and temperature.

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$$V_0 = \max_{\{K_{t+1}, \mu_t\}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t}{1000L_t}\right)^{1-1/\psi} - 1}{1 - 1/\psi} L_t \quad (1)$$

$$\text{s.t. } K_{t+1} = (1 - \Theta(\mu_t) - \Omega(T_{AT,t})) A_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta) K_t - C_t \quad (2)$$

$$M_{t+1}^{AT} = (1 - b_{12}) M_t^{AT} + b_{12} \frac{M_{EQ}^{AT}}{M_{EQ}^{UO}} M_t^{UO} + \sigma_t Y_t^{\text{Gross}} (1 - \mu_t) + E_t^{\text{Land}} \quad (3)$$

$$M_{t+1}^{UO} = b_{12} M_t^{AT} + (1 - b_{12} \frac{M_{EQ}^{AT}}{M_{EQ}^{UO}} - b_{23}) M_t^{UO} + b_{23} \frac{M_{EQ}^{UO}}{M_{EQ}^{LO}} M_t^{LO} \quad (4)$$

$$M_{t+1}^{LO} = b_{23} M_t^{UO} + (1 - b_{23} \frac{M_{EQ}^{UO}}{M_{EQ}^{LO}}) M_t^{LO} \quad (5)$$

$$T_{t+1}^{AT} = T_t^{AT} + c_1 \left(F_{2XCO2} \frac{\log(M_t^{AT}/M_{EQ}^{AT})}{\log(2)} + F_t^{EX} \right) - c_1 \frac{F_{2XCO2}}{t_{2XCO2}} T_t^{AT} - c_1 c_3 (T_t^{AT} - T_t^{OC}) \quad (6)$$

$$T_{t+1}^{OC} = T_t^{OC} + c_4 (T_t^{AT} - T_t^{OC}) \quad (7)$$

$$\text{with } \Omega(T_{AT,t}) = \psi_1 T_t^{AT} + \psi_2 (T_t^{AT})^2, \quad (8)$$

$$\Theta(\mu_t) = \theta_{1,t} \mu_t^{\theta_2} \quad (9)$$

Social Cost of Carbon definition

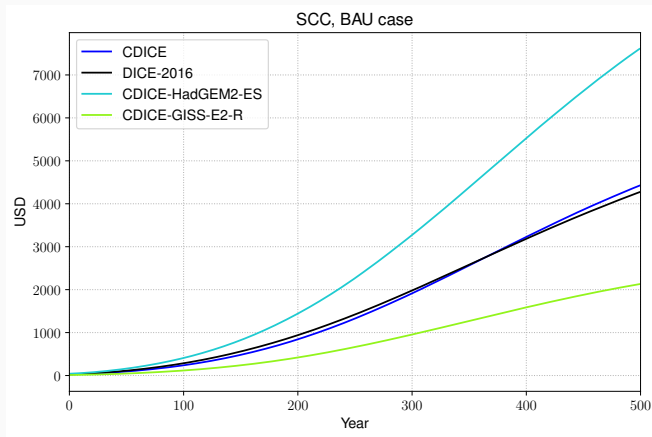
"Social cost of carbon is the present value of future damages associated with an incremental increase (by convention, 1 metric tonne) in CO2 emissions in a particular year expressed in consumption equivalent terms" [Fiscal Policy to Mitigate Climate Change. A Guide for Policymakers, 2012].

Social Cost of Carbon computation

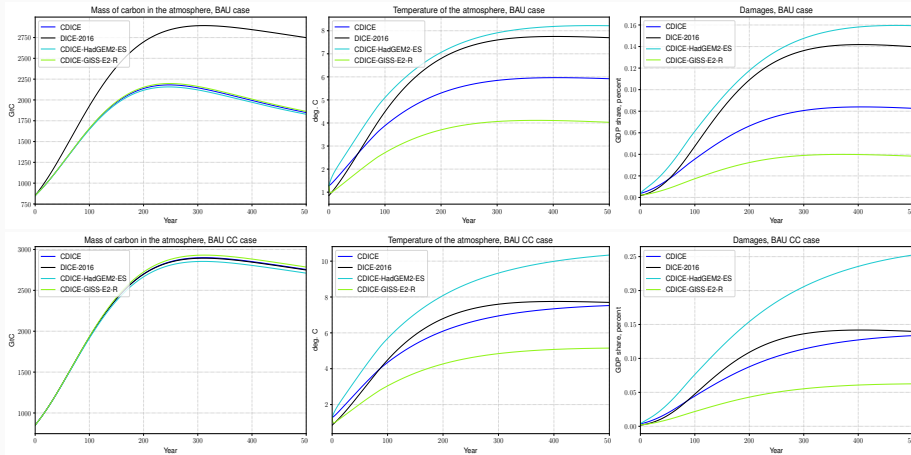
Social cost of carbon is the marginal loss associated with an emission of one extra ton of CO2 in consumption equivalent terms.

$$SCC_t = -\frac{\partial V_t / \partial E_t}{\partial V_t / \partial C_t} = -\frac{\partial V_t / \partial M_{AT,t}}{\partial V_t / \partial K_t}. \quad (10)$$

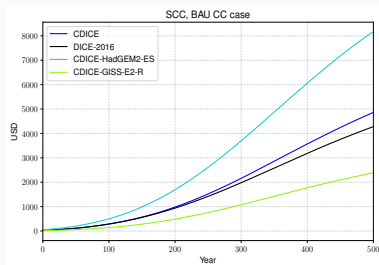
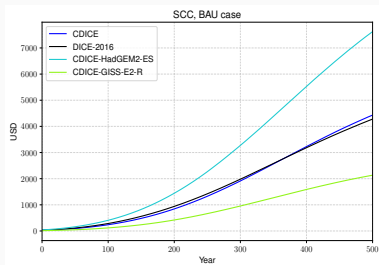
Business-as-usual case: Social Cost of Carbon



Effect of higher mass of carbon



Effect of higher mass of carbon on SCC



- Extra ton of carbon affects the value function through temperature and damages
- Two flaws of the DICE: too small and too slow carbon removal from the atmosphere and too slow in short-run but too strong in a long-run warming
- If we emit additional ton of CO₂ in a short run temperature won't respond enough, damages won't increase enough
- When this additional ton emitted starts to have an effect on temperature, first it's too far away (discounting), second, temperature reacts too strong anyway, so not a big difference

Set of tests for climate module

Overview of tests

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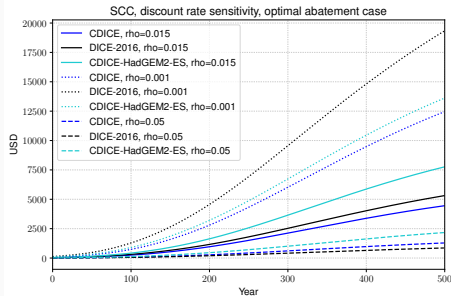
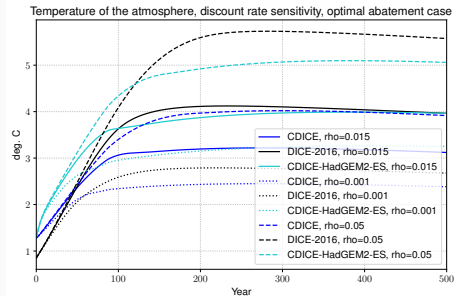
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Results

The social cost of carbon

Sensitivity to discount rate

Sensitivity to discount rate



- Long-run temperature in DICE-2016 is too high
- When discount rate is also high (0.05) the damages arising from this high temperature are discounted away, so we allow for higher temperature and too low SCC
- When discount rate is low (0.001) it gives low temperature and high SCC
- We reduced long-term temperature evolution thus got stabilization of the SCC with respect to the discount rate.

- DICE specification of climate part with proper calibration delivers simple yet tractable climate emulator that can be used for answering economic questions
- CDICE adds clarity to the computation of SCC and discussion around sensitivity of SCC to the discount rate
- If there is a need to modify climate part in IAM it should pass four tests: 2 stylised tests for carbon cycle and temperature separately, one realistic test for temperature and joint test for carbon cycle and temperature against with CMIP5 data
- "Naive" calibration against CMIP5 data won't work as there are a lot of endogenous processes underlying this data which are well known and should be included in modeling process as idealized tests.

References

- Dietz, S., Van Der Ploeg, F., Rezai, A., and Venmans, F. (2021). Department of Economics Discussion Paper Series Are economists getting climate dynamics right and does it matter? Are economists getting climate dynamics right and does it matter? (900).
- IPCC (2014). Climate Change 2014. *Climate Change 2014: Synthesis Report*, pages 1–169.
- Nordhaus, W. (2018). Projections and uncertainties about climate change in an era of minimal climate policies. *American Economic Journal: Economic Policy*, 10(3):333–360.