Teaching Economics to the Machines

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How to Integrate Economics and AI

- Teach AI about economic objectives
  - ↪ The goal is not to build the most accurate model by some arbitrary metric, but one that best serves the economic objectives.
- Teach AI about incentives
  - ↪ How to deal with strategic agents, moral hazard, adversarial attacks?
- Teach AI about economic theory
  - ↪ Look-up tables in the age of AI
  - ↪ Economic theory can guide our learning from the data.

Outline

1. Teaching AI about Economic Theory

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1. Teaching AI about Economic Theory
   - Look-up Tables in the Age of AI
   - Economic Theory to Guide Machine Learning
   - An application in option pricing

A Motivating Example

Bates Model

$$\frac{dS_t}{S_{t^-}} = (r - d - \lambda m)\,dt + \sqrt{v_t}\,dW_1 + dZ_t,$$
$$dv_t = \kappa(\theta - v_t)\,dt + \sigma\sqrt{v}\,dW_2, \quad \mathrm{E}[W_{1,t}W_{2,t}] = \rho\,dt,$$

where Z is a pure jump process with intensity λ and double-exponential log jump size J:

$$\omega(J) = p\frac{1}{\nu_u}e^{-\frac{1}{\nu_u}J}1_{\{J>0\}} + (1-p)\frac{1}{\nu_d}e^{\frac{1}{\nu_d}J}1_{\{J<0\}}, \text{ with } p \geq 0.$$

Pricing a European call option:

$$C(S_0, v_0, T) = \mathrm{E}_0\left[e^{-rT}(S_T - K)^+\right]$$

A Motivating Example

- Bates model:
  - ↪ Extension of the Black-Scholes model with stochastic volatility and double-exponential jumps.
  - ↪ 4 observable states ($m_t, T-t, r_t, d_t$), 1 hidden state (v_t), and 8 parameters ($\kappa, \theta, \sigma, \rho, \lambda, p, \nu_u, \nu_d$)
  - ↪ Solution technique: Fourier inversion of conditional characteristic function.

- Potential applications:
  - ↪ Can option-implied jump risk measure predict returns or crashes?
  - ↪ What is the impact of parameter instability for option pricing? Market making?
  - ↪ How to evaluate the model's out-of-sample (pricing or hedging) performance?
  - ↪ How well does the model match the empirical distribution of option returns?
  - ↪ How to compute value-at-risk for an option portfolio?

How costly is it?

Chen, Didisheim and Scheidegger (2025)

- SPX: 4000 option prices on a typical day
- Horse race:
  - ↪ Modern FFT implementation vs. deep surrogate
  - ↪ Single core vs. GPU

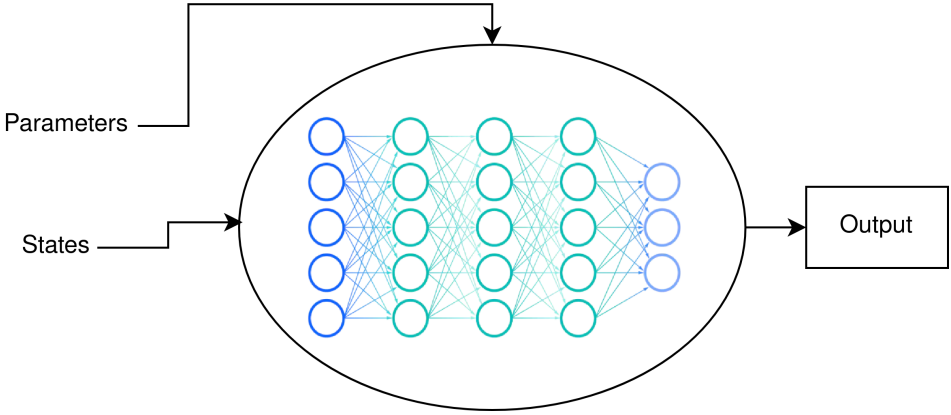
	FFT	Deep Surrogate	Deep Surrogate + GPU
pricing, 1-day	10s	0.6s	0.06s
estimation, 1-day	180s per iter	3.2s per iter	0.3s per iter
estimation, 1-year	125h	2.2h	.2h

A standard solution in science and engineering: Look-up tables

$$\Pr(z \le z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2} dz$$

z_1	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857

Deep Surrogate: “Look-up Table” in the age of AI



$$\phi(\mathbf{x}_t|\theta_{NN}) = y_t$$

Problem formulation

- Let an economic model be represented by a set of restrictions between observable states $s \in \mathbb{R}^n$, hidden states $h \in \mathbb{R}^h$, and endogenous quantities $y \in \mathbb{R}^k$, with parameters $\theta \in \mathbb{R}^p$,

$$y = F(s, h|\theta)$$

- By treating θ as pseudo-states, we can focus on the augmented state $x \equiv (s, h, \theta) \in \mathbb{R}^d$ with $d = n + h + p$, and

$$y = f(x) = F(s, h|\theta)$$

- By imposing a hierarchical prior on θ, we can derive a model-implied joint probability distribution of x and y, $\mathbb{P}_{(\mathcal{X},\mathcal{Y})}$.

Deep Surrogate

- Ideally, for a given a loss function $\mathscr{L}$, we search in the set of all possible functions to mimize the expected loss of approximation:

$$\hat{f} = \arg\min \left\{ E\left[\mathscr{L}(f(x), y)\right] : f \in \mathscr{Y}^{\mathscr{X}} \right\}$$

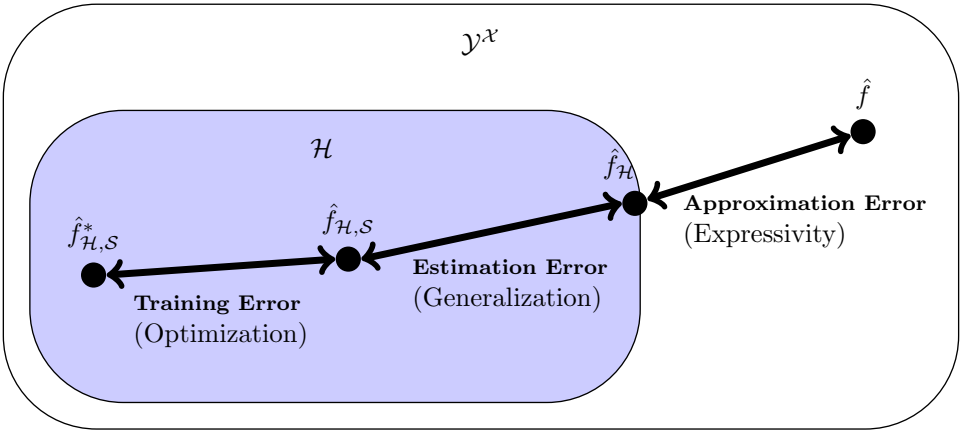
  ↪ But we already know $f \Rightarrow \hat{f} = f$

- Goal of deep surrogate: Generate a training set $S = ((x_i, y_i))_{i=1}^{m}$ of iid samples drawn from $\mathscr{X} \times \mathscr{Y}$; find a function from the set $\mathscr{H}$ of deep neural networks to minimize the empirical loss of approximation:

$$\hat{f}_{\mathscr{H},S} = \arg\min \left\{ \frac{1}{m} \sum_{i=1}^{m} \mathscr{L}(f(x_i), y_i) : f \in \mathscr{H} \right\}$$

  ↪ A neural network is characterized by activation function σ, architecture $a = (a_0 = d, a_1, \ldots, a_{L-1}, a_L = k)$, and parameterization $\theta_{NN} = \left((W_l, b_l)\right)_{l=1}^{L}$

Sources of approximation errors



Deep Surrogate – Expressivity

Universal approximation theorem

For every $f \in C$ and every $\epsilon > 0$, there exists a shallow neural network $\Phi_{f,\epsilon}$ with width N such that $\|\Phi_{f,\epsilon} - f\|_\infty \leq \epsilon$.

- Funahashi (1989), Hornik, Stinchcombe, and White (1989), Cybenko (1989)

UAT also applies to the "dual problem:" NNs with fixed width and unrestricted depth

- Hanin and Sellke (2017), Lu et al. (2017), Hanin (2019)
- Requires network width $\geq d$

Deep Surrogate – Curse of dimensionality

- Approximation for general class of smooth functions typically require the number of parameters $P(\mathbf{a}_{d,\epsilon})$ of the networks grows exponentially in d, as does the size of the training sample.
  - ↪ For reference, approximating a d-dimensional function on a Cartesian grid requires $\mathcal{O}(n^d)$ evaluations.

- With additional regularity conditions and/or special structure, one can train an accurate surrogate with lower network complexity and fewer observations.

- Berner et al. (2020): A deep neural network can overcome the curse of dimensionality when approximating the solution of a d-dim Kolmogorov equation with affine drift and diffusion.

$$\max\{m_{d,\epsilon}, P(\mathbf{a}_{d,\epsilon})\} \leq \text{poly}(d, 1/\epsilon)$$

  - ↪ Solution to the PDE can be written as expectation over terminal condition (Feynman-Kac).
  - ↪ Terminal values can be readily approximated by network with certain activation function.

Additional guidance from learning theory

- **Activation**: ReLU struggles with the approximation of certain functions (e.g., $x \to x^2$) relative to activation functions that are C^2.
  - ↪ Rolnick and Tegmark (2018)

- **Depth**: Advantage of deep neural networks over shallow ones.
  - ↪ Far more complex shallow networks might be needed to obtain the same approximation accuracy as a deep network.
  - ↪ Yarotsky (2017), Petersen and Voigtlaender (2018), Rolnick and Tegmark (2018)

Why Surrogates?

Deep surrogate is different from standard ML:

- Compared to other methods, deep neural networks are more hungry for data.
- The cost of producing a large training sample should be an important consideration.
- Unlike in standard ML, we know the true model $\Rightarrow$ unlimited data (only limited by computational resources); essentially no errors.
- Use a large number of epochs and aggressive learning rate (low risk of over-fitting).

Once trained, the deep surrogate

- is highly accurate;
- is cheaper to use by orders of magnitude; makes the gradients readily available;
- is easy to store (for 10^6 parameters - 20 MB vs. $\sim 10^6$GB when using Cartesian grid).

Pay the cost upfront; use it for free later.

- High quality surrogates can be shared with and build on by a community.
- Deep surrogates for workhorse quantitative economic and financial models.

Comparison of approximation methods

	Polynomials	Splines	(Adaptive) sparse grids	Gaussian processes	Deep neural networks
High-dimensional input	✓	✗	✓	✓	✓
Capturing local features	✗	✓	✓	✓	✓
Irregularly-shaped domain	✓	✗	✗	✓	✓
Large amount of data	✓	✓	✓	✗	✓

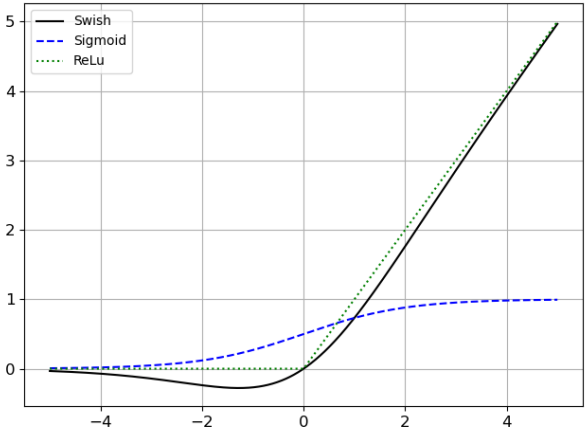
Application: Option pricing

- We build deep surrogates for the Heston model (HM) and Bates model (BDJM), and analyze their empirical performances in several new dimensions.
- Key findings:
  1. Out of sample, RF outperforms parametric counterparts for short horizons (< 7 days) but underperforms at long horizons.
  2. Structural models are much better than RF in hedging performance.
  3. Significant evidence of time-varying parameter instability for HM and BDJM.
  4. Parameter instability linked to market liquidity.
  5. Tail risk estimated through BDJM is informative of future market returns.

Application: Option pricing

- Objective of a surrogate: Replace $f(\cdot)$ by a surrogate model $\widehat{f}(\cdot)$, which
  - ↪ predicts the output of $f(\cdot)$ with high accuracy;
  - ↪ is cheap to evaluate.
- How do we know when we are done? Examine approximation errors out of sample.
- Active learning: Sample more from regions where approximation errors are large $\Rightarrow$ retrain $\Phi(\cdot)$.
- Theory-guided sampling: Sample strategically depending on the shape of f.

Swish: Continuous derivatives



$$Sigmoid: \frac{1}{1+e^{-x}}, \quad ReLu: max(x,0), \quad Swish: \frac{x}{1+\exp(-\gamma x)},$$

Building the surrogate

- Suppose the set of admissible inputs $\mathscr{X}$ is a hypercube, $[\underline{x}, \overline{x}]^d$.

$$\underline{x} = [\underline{x}^{(1)}, \underline{x}^{(2)}, ..., \underline{x}^{(m)}], \quad \overline{x} = [\bar{x}^{(1)}, \bar{x}^{(2)}, ..., \bar{x}^{(m)}]$$

- Draw x_i uniformly from $[\underline{x}, \overline{x}]^d$ and compute $y_i = f(x_i)$.
- For a given training sample of size m, we train the neural network $\phi(x|\theta_{NN})$ by minimizing the average ℓ_1-norm of the difference between the actual outcome and DNN prediction,

$$\theta^*_{NN} = \arg\min_{\theta_{NN}} \frac{1}{m} \sum_{i=1}^{m} |\phi(x_i|\theta_{NN}) - y_i|.$$

Building the surrogate

- To check the quality of the surrogate model, we generate a new random sample $\left(\mathbf{x}_j^o, \mathbf{y}_j^o\right)$ of size J and compute the validation error.
- A convergence criterion could be

$$\sup_j \|\phi(x_j^o|\theta_{NN}^*) - y_j^o\|_1 \leq \varepsilon \qquad (\dagger)$$

  for some $\varepsilon > 0$.
- If (†) is not satisfied, retrain ϕ with (potentially) a deeper layer and more training data (especially from regions where approximation error is large).

Architecture

HM (9-d)

- 6 fully connected layers of 400 neurones with *swish* activations.
- 806,001 trainable parameters.
- Custom input layer.
- Surrogate's training size: 10^8.

BDJM (13-d)

- 7 fully connected layers of 400 neurones with *swish* activations.
- 967,201 trainable parameters.
- Custom input layer.
- Surrogate's training size: 10^9.

Structural estimation using deep surrogate

- Once we have the deep surrogate, it is very friendly to structural estimation.
- For example, suppose we have a set of moment conditions

$$\mathrm{E}[f(\Omega_t|\theta)] = 0$$

- In finite sample, we have

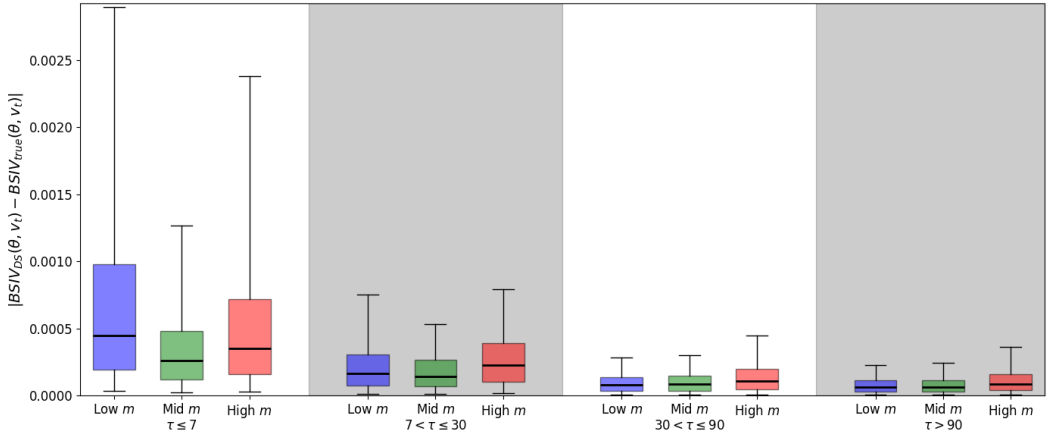
$$g_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} f(\Omega_t, \theta)$$

- GMM estimator:

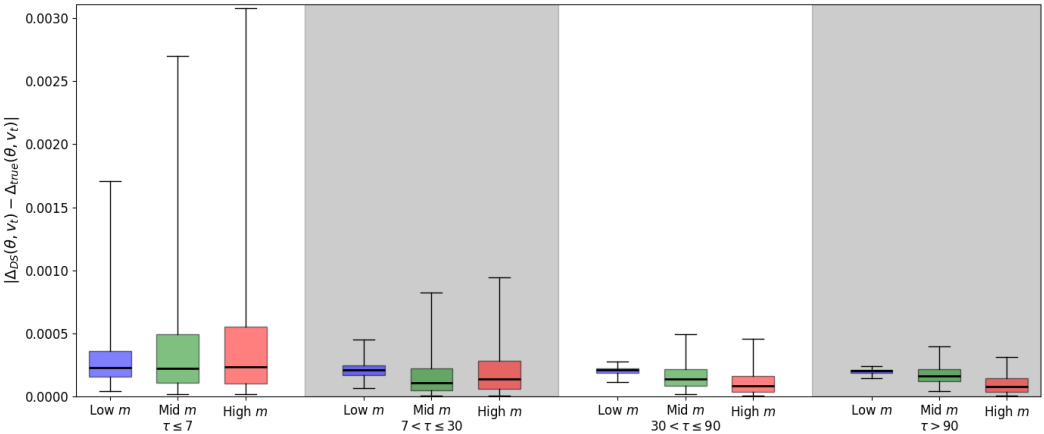
$$\hat{\theta}_{GMM} = \arg\min_{\theta} g_n'(\theta)\, W\, g_n(\theta)$$

- We can use the deep surrogate $\phi()$ to replace $f()$. Moreover, through automatic differentiation the surrogate provides the gradients of $\phi(\cdot)$ analytically, which translate the GMM FOCs into a system of nonlinear equations.

Accuracy of the deep surrogate: IV



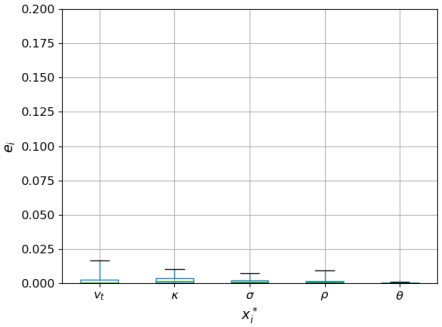
Accuracy of the deep surrogate: Delta



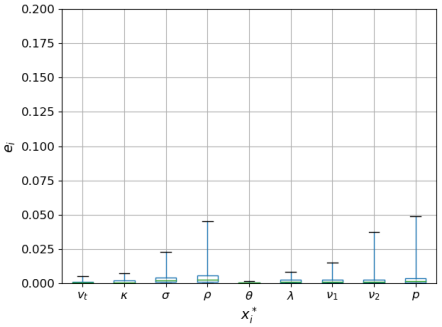
Identifying Parameters

To test our surrogate we,

- simulate N=1000 options with same parameters and hidden states, but randomly selected maturity and moneyness;
- use surrogate to estimate the parameters and hidden state;
- measure performance using relative estimation error: $e_i = \frac{|x_i^{true} - x_i^*|}{|\bar{x}_i - \underline{x}_i|}$.

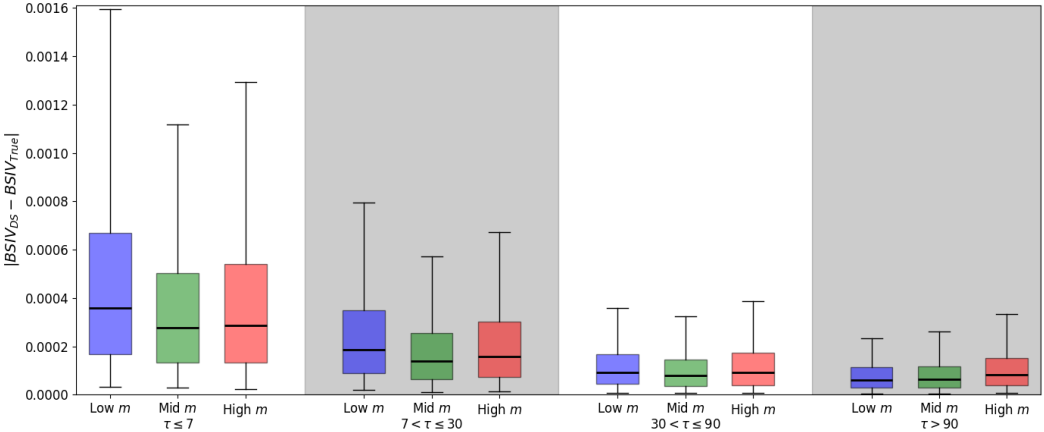


(a) HM

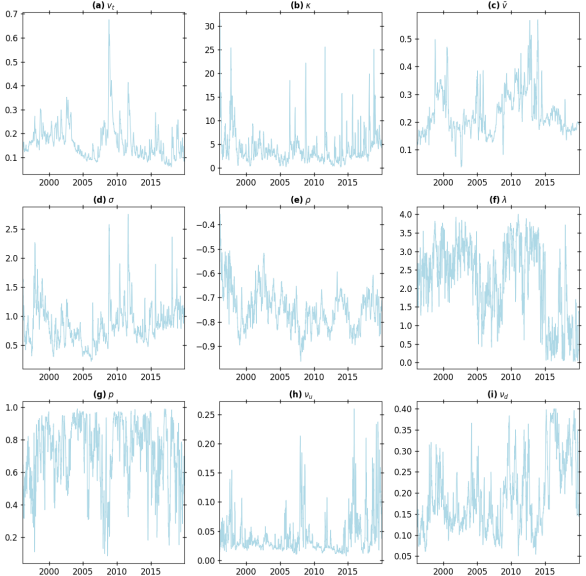


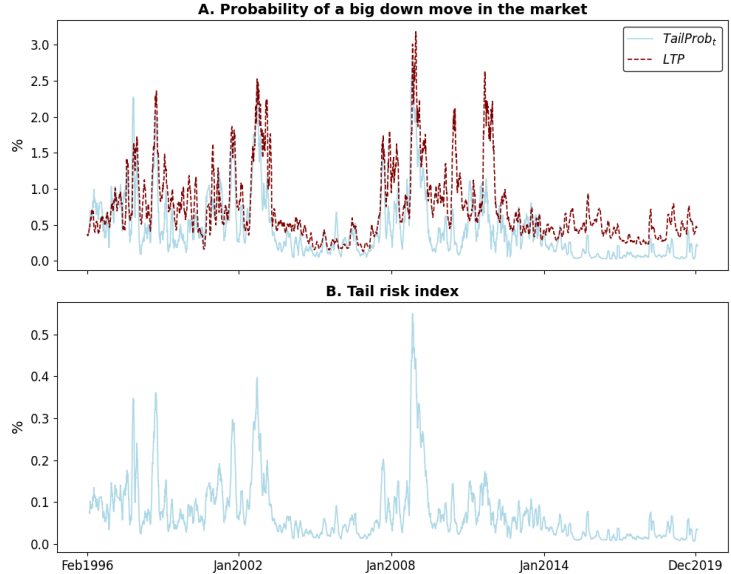
(b) BDJM

OOS pricing errors due to parameter estimation errors



Parameter Estimates




A. Probability of a big down move in the market
$TailProb_t$
LTP
%
3.0
2.5
2.0
1.5
1.0
0.5
0.0
B. Tail risk index
%
0.5
0.4
0.3
0.2
0.1
0.0
Feb1996
Jan2002
Jan2008
Jan2014
Dec2019

Predicting Market Crashes

	10% Weekly Down Move				5% Daily Drop within a Week			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LTP	1.468***			0.052	1.917***			-0.364
	(0.365)			(0.777)	(0.423)			(0.694)
TailProb		1.258***		-0.935		1.601***		-3.272***
		(0.188)		(0.812)		(0.218)		(0.716)
TailRisk			8.131***	12.917**			11.177***	29.903***
			(1.027)	(5.643)			(1.263)	(5.453)
Constant	-7.190***	-6.776***	-6.907***	-6.854***	-6.569***	-5.980***	-6.383***	-5.992***
	(0.606)	(0.405)	(0.407)	(0.599)	(0.677)	(0.410)	(0.461)	(0.539)
Pseudo-R^2	0.080	0.119	0.152	0.159	0.173	0.230	0.331	0.386

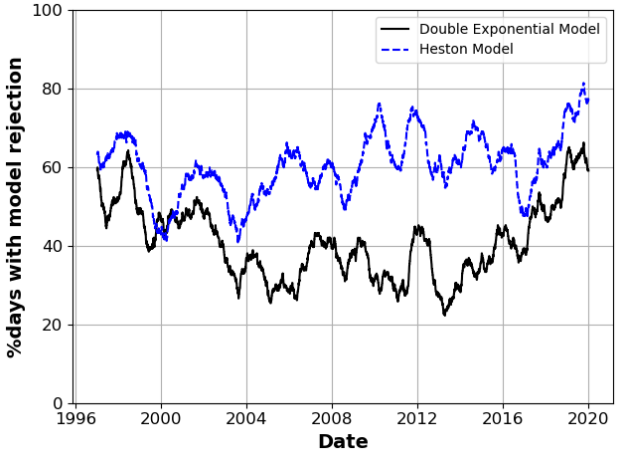
Parameter Stability

- We can test whether the models' parameters are stable across time.
- We use the test developed by Andersen, Fusari, and Todorov (2015):

$$(\Theta_t - \Theta_{t+1})' \left(\widehat{\text{Avar}}(\Theta_t) + \widehat{\text{Avar}}(\Theta_{t+1})\right)^{-1} (\Theta_t - \Theta_{t+1}) \xrightarrow{\mathscr{L}-s} \chi^2(q)$$

  - Θ_t, and Θ_{t+1} denote the estimated parameters in the time periods t and $t+1$, respectively.
  - $\widehat{\text{Avar}}(\Theta_t)$ and $\widehat{\text{Avar}}(\Theta_{t+1})$ denote consistent estimates of the asymptotic variances of these parameters.

Parameter Stability



(c) $\alpha = 1\%$

Parameter Stability and Market Liquidity

- Parameter instability increases the difficulty for hedging, which raises inventory risks for market makers.
- We examine whether parameter instability translates directly into larger bid-ask spreads. To do so, we estimate:

$$\frac{Ask_{t,i} - Bid_{t,i}}{Bid_{t,i}} = \alpha + \beta_1 \log \chi^2(q)_{i,t,BDJM} + \beta X_{i,t} + \epsilon_{i,t}$$

Parameter Stability and Market Liquidity

$\log \chi^2(q)_{BDJM}$		0.0012***		0.001***
		(0.0)		(0.0)
$\log \chi^2(q)_{HM}$			0.0012***	0.001***
			(0.0)	(0.0)
C_i	-0.1466***	-0.1462***	-0.1462***	-0.1459***
	(0.0004)	(0.0004)	(0.0004)	(0.0004)
$C_i * T_{i,t}$	0.0002***	0.0002***	0.0002***	0.0002***
	(0.0)	(0.0)	(0.0)	(0.0)
$C_i * K_{i,t}$	0.0001***	0.0001***	0.0001***	0.0001***
	(0.0)	(0.0)	(0.0)	(0.0)
$T_{i,t}$	-0.0004***	-0.0004***	-0.0004***	-0.0004***
	(0.0)	(0.0)	(0.0)	(0.0)
$JUMP_t$	-0.0089***	-0.0065***	-0.0076***	-0.0059***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)
one	0.4073***	0.4076***	0.4056***	0.4062***
	(0.0003)	(0.0003)	(0.0003)	(0.0003)
$K_{i,t}$	-0.0001***	-0.0001***	-0.0001***	-0.0001***
	(0.0)	(0.0)	(0.0)	(0.0)
VIX_t	-0.0021***	-0.0023***	-0.0022***	-0.0023***
	(0.0)	(0.0)	(0.0)	(0.0)
$Volume_{i,t}$	-0.0***	-0.0***	-0.0***	-0.0***
	(0.0)	(0.0)	(0.0)	(0.0)
Observations	4,948,103	4,948,103	4,948,103	4,948,103
R^2	0.2176	0.218	0.218	0.2183

Summary

- Surrogate models are functions which replicate a structural-model at an exponentially lower computational cost.
- We treat states and parameters as pseudo-states and train a single neural network to build deep-surrogates of complex option pricing models.
- Thanks to those surrogate we can
  - ↪ compare the performance of these models with simple non-parametric and highlight shapes of the implied volatility surface where parametric models underperform;
  - ↪ assess parameter stability;
  - ↪ construct model-implied tail risk measures;
  - ↪ characterize conditional distribution of option returns.

Deep Surrogate Library

Git Repository

https://github.com/DeepSurrogate/

- Please give it a try!
- Next steps: Expand the offering of models in option pricing and other fields.
- Further explore the economic consequences of parameter instability and information content of tail risk measures.

Outline

1. Teaching AI about Economic Theory
   - Look-up Tables in the Age of AI
   - Economic Theory to Guide Machine Learning
   - An application in option pricing

The End of Theory?

- Chris Anderson, Wired (2008):

  "All models are wrong, but some are useful." So proclaimed statistician George Box 30 years ago, and he was right. But what choice did we have? ... Today companies like Google, which have grown up in an era of massively abundant data, don't have to settle for wrong models. Indeed, they don't have to settle for models at all.

- Ilya Sutskever, Reuters (2024):

  "The 2010s were the age of scaling, now we're back in the age of wonder and discovery once again ... Scaling the right thing matters more now than ever."

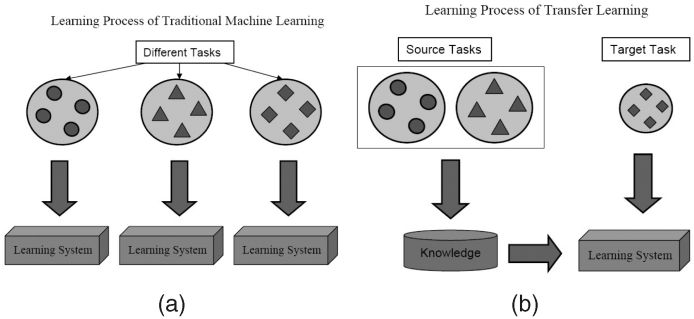
How should we teach a robot to play pool?

How should we teach a robot to play pool?

- Data-driven approach:
  - ↪ x: initial velocity, angle, point of contact ...
  - ↪ y: scattering angles, final velocities
  - ↪ However, labeled data are expensive to collect.
- Theory-driven approach:
  - ↪ Model of elastic collision
  - ↪ Conservation of linear momentum and kinetic energy ⇒ complete predictions
  - ↪ However, the model is likely misspecified.
- Why not combine the knowledge from theory and data?
  - ↪ Our answer: transfer learning

Transfer learning

- Using pre-trained models on large datasets (source domain) to accelerate learning on a smaller dataset (target domain) for a specific task.



  - ↪ Computer vision: pre-train CNNs on ImageNet ⇒ medical image analysis
  - ↪ Natural language processing: pre-trained LLMs ⇒ sentiment analysis

- In our setting: Source domain = economic model; target domain = empirical data.

Teaching economics to the machines

- Training an ML model on the simulated data from a theoretical model helps it inherit the information about structural restrictions.
- TL: Theory-informed starting point + fine-tuning allow the structural restrictions to regulate learning on real data.
- Why might this be helpful?
  - ↪ Bias-variance tradeoff: Valid restrictions help with learning. Misspecified models also add bias.
  - ↪ Theory-informed initialization can mitigate issues in model training, such as avoiding local minima and saddle points.
- Potential benefits:
  - ↪ Variance reduction
  - ↪ Speed up training on real data; less demand for real data
  - ↪ Improve generalizability beyond training data boundaries

The setup

- $\mathscr{X}$ and $\mathscr{Y}$: feature and target space, with unknown probability distribution $\mathbb{P}_{(\mathscr{X},\mathscr{Y})}$.
- Denote a neural network with L layers by $F(L;\sigma_1,\cdots,\sigma_L;W_1,\cdots,W_L)$.
  - ↪ σ_i: activation function
  - ↪ $W=[W_1,\cdots,W_L]$: network weights
- Standard approach to train a neural network: Find a training set $S=((x_i,y_i))_{i=1}^m$ and a loss function $\mathscr{L}(f(x),y)$. Minimize the empirical loss:

$$W^* = \arg\min_W \frac{1}{m}\sum_{i=1}^m \mathscr{L}(F(x_i;W),y_i)$$

Economic model

- An economic model imposes certain restrictions between x and y; corresponding distribution: $\mathbb{Q}_{(\mathscr{X},\mathscr{Y})}$.
- It implies a conditional distribution for y given x, or at a minimum, the conditional expectation of y:

$$\mathbb{E}^{\mathbb{Q}}[y|x] = g(x)$$

- The model can be misspecified: $\mathbb{Q} \neq \mathbb{P}$

Feature augmentation

- Heterogeneous transfer learning: Features in the source domain (x^S) and target domain (x) are often different.
  - ↪ Might want to include features beyond those in a (parsimonious) structural model.
  - ↪ Latent states and structural parameters are not directly observable.

- Feature augmentation:

$$\Phi_S(x_i^S) = \langle P_C x_i^S,\ P_S x_i^S,\quad e_i^S \rangle$$
$$\Phi_T(x_i) = \langle Q_C x_i,\quad e_i^T,\ Q_T x_i \rangle$$

  - ↪ Source domain: Supplement synthetic data with random draws of additional features.
  - ↪ Target domain: Filtering or conditioning down to deal with hidden states, structural parameters.

    - ★ The network trained in the source domain can be used to do filtering efficiently (Chen, Didisheim and Scheidegger 2025).

Source domain

- Generate synthetic data $\widetilde{S} = \left((\tilde{x}_i, \tilde{y}_i)\right)_{i=1}^{M}$:
  1. Draw $\tilde{x}_i$ from multivariate uniform distribution.
  2. Draw $\tilde{y}_i$ from theory-implied conditional distribution of y given x, or from $\mathbb{E}^{\mathbb{Q}}[y|x]$.
- In the source domain, we train the neural networks to learn from the synthetic dataset:

$$\widetilde{W}^* = \arg\min_{\widetilde{W}} \left(\lambda_0 \widetilde{\mathscr{L}}_0 + \sum_{j=1}^{K} \lambda_j \widetilde{\mathscr{L}}_j \right)$$

- Loss based on prediction errors:

$$\widetilde{\mathscr{L}}_0 = \sum_{i=1}^{M} w_{0i} \left| F(\tilde{x}_i | \widetilde{W}) - \tilde{y}_i \right|$$

- Economics-informed losses: Utilize other restrictions from theory.

$$\widetilde{\mathscr{L}}_j = \sum_{i=1}^{M} w_{ji} \left| \partial_j F(\tilde{x}_i | \widetilde{W}) - \partial_j g(\tilde{x}_i) \right|, \quad \forall j > 0$$

  ↪ Physics-informed losses: Raissi, Perdikaris, and Karniadakis (2019)

Target domain

- In the target domain, train with real data $S = \left((x_i, y_i)\right)_{i=1}^{m}$.
- Identical network architecture as in the source domain; use the weights from the source domain for initialization:

$$\widehat{W}^* = \arg\min_{\widehat{W}} \sum_{i=1}^{m} w_i \left| F(x_i|\widehat{W}) - y_i \right|,$$

  with initial weights $\widetilde{W}^*$.
- Fine-tuning: Small learning rate and early stopping (controlled by patience parameter).
- Alternative approaches:
  - ↪ Augment the network from the source domain with new layers.
  - ↪ Partial fine-tuning: fixing bottom part of the network (frozen layers)

Comparison with alternative approaches

- Imposing structural restrictions in ML model:
  - ↪ Long tradition in rational expectations econometrics
  - ↪ Physics-informed Neural Networks (Raissi, Perdikaris, and Karniadakis 2019); Using NA conditions in SDF estimation (Chen, Pelger, and Zhu 2023)
  - ↪ In TL, structural restrictions help with learning but are not treated as hard constraints, since misspecified restrictions likely add more bias.

- Bayesian approach:
  - ↪ Bayesian VAR: Deriving informative priors from an economic model to aid VAR estimation.
  - ↪ Examples: Random walk (Doan, Litterman, and Sims 1984); DSGE (DeJong, Ingram, and Whiteman 1993; Del Negro and Schorfheide 2004)
  - ↪ TL: easier to implement, significantly more flexible at capturing nonlinearity, and controls the "strength of prior" differently.

- Boosting:
  - ↪ Train an ML model on the prediction errors from a structural model.
  - ↪ Example: Boosting parametric option pricing models (Almeida, Fan, Freire, and Tang 2023)
  - ↪ The structural model restrictions are not used to guide the learning in the boosting step.

The Bias-Variance Trade-off for TL

- DGP:

$$y_i = x_{1i} + x_{2i} + x_{1i}x_{2i} + \epsilon_i, \quad \epsilon \sim N(0, \sigma^2)$$

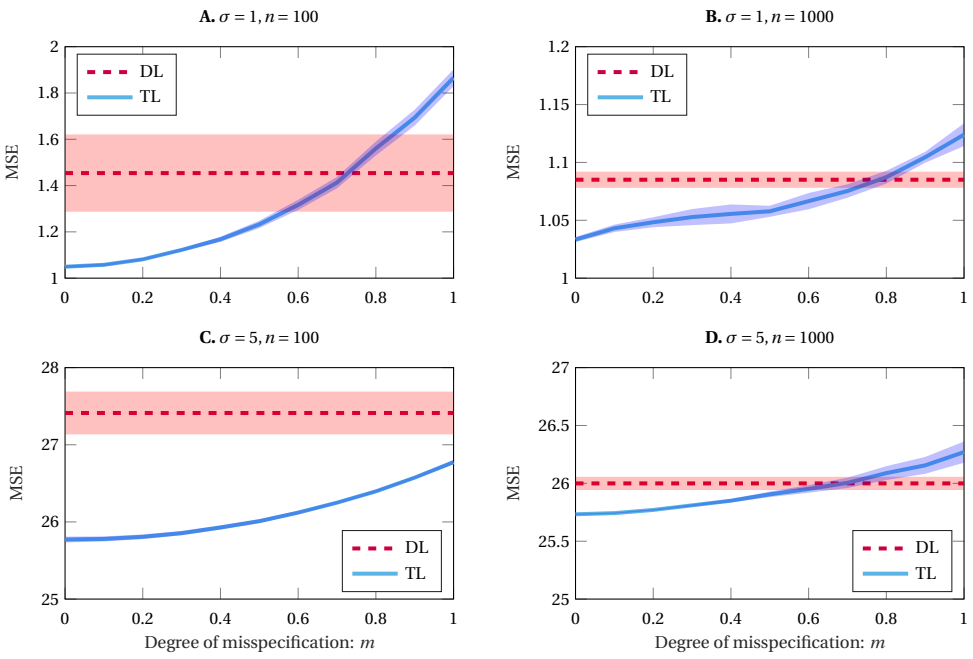
  ↪ x_{1i} and x_{2i}: IID standard normal

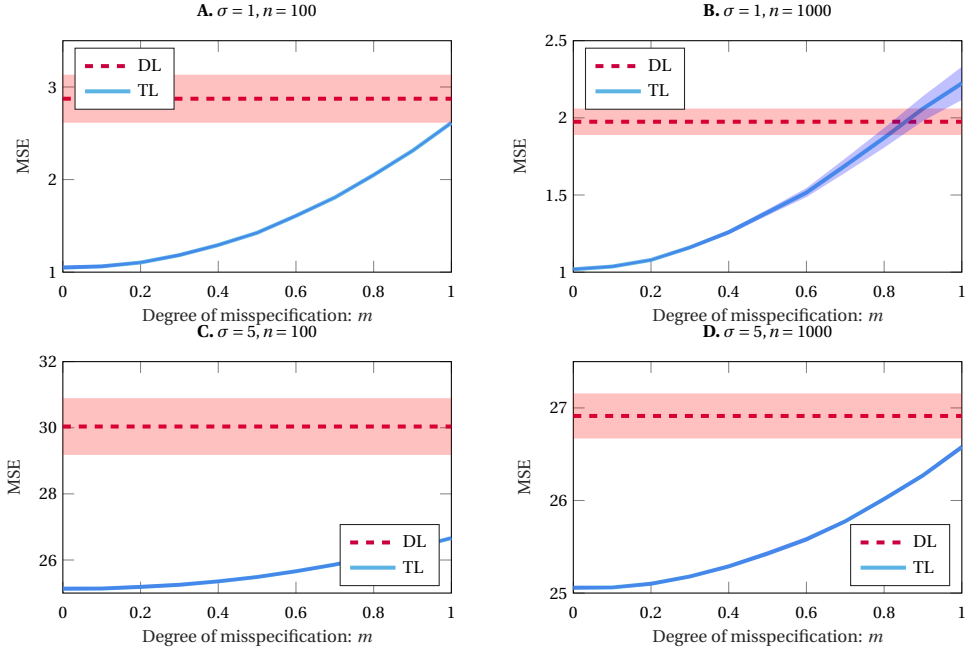
- Misspecified "theory":

$$y_i = x_{1i} + x_{2i} + (1 - m)x_{1i}x_{2i} + \epsilon_i, \quad m \in [0, 1]$$

- TL with theory as the source domain vs. DL, both trained on a sample of size n.
  ↪ MLP with 2 hidden layers, 64 neurons in each layer, ReLU activation

- Examine the impact of model misspecification by varying m, σ, and n:
  ↪ m: degree of misspecification
  ↪ σ: high vs. low signal-to-noise ratio
  ↪ n: data-rich vs. data-scarce environment


A. $\sigma = 1, n = 100$
B. $\sigma = 1, n = 1000$
C. $\sigma = 5, n = 100$
D. $\sigma = 5, n = 1000$
DL
TL
MSE
Degree of misspecification: m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MSE
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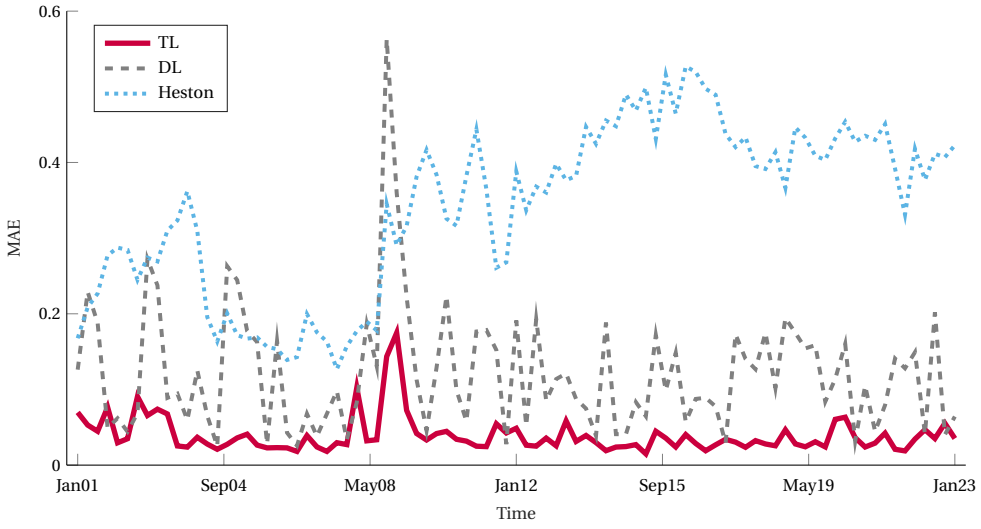
Outline

1. Teaching AI about Economic Theory
   - Look-up Tables in the Age of AI
   - Economic Theory to Guide Machine Learning
   - An application in option pricing

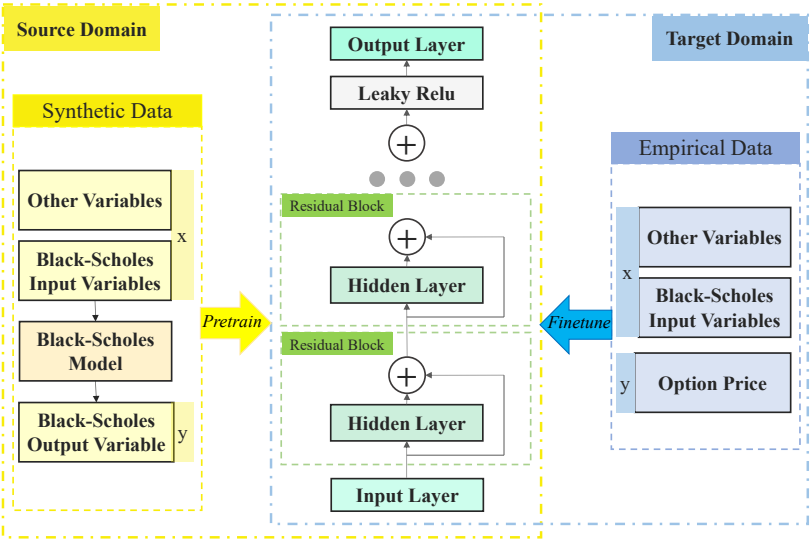
Example application

- A horse race between theory-based transfer learning (TL) and standard deep neural networks (DL) for option pricing.
  - ↪ Use the Black-Scholes model in the source domain.
  - ↪ Augment the set of features with past returns, bid-ask spreads, put-call ratios ... (total of $d = 16$ features)
- Architecture: 16 hidden-layer ResNet, 22 neurons in each layer ($> 7,800$ trainable parameters), LeakyRelu activation
- Rolling window: At the end of each quarter, use data from the past 3 months to train the models; use the models to predict option prices in the next 3 months.
  - ↪ Mimicking a setting with limited labeled data in the target domain.
- Other candidate models: Heston model, DL with BS penalty, boosted BS

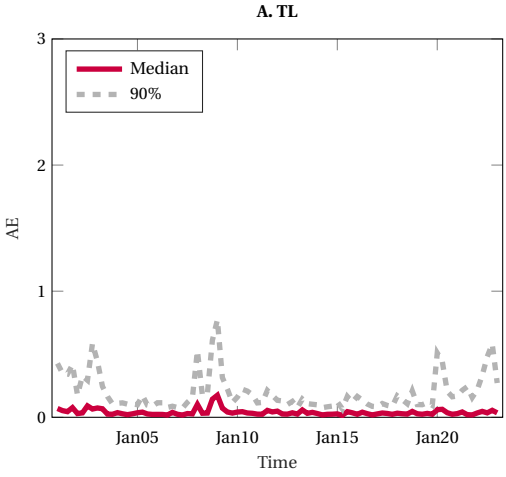
TL outperforms DL and Heston out of sample (based on IV-MAE)

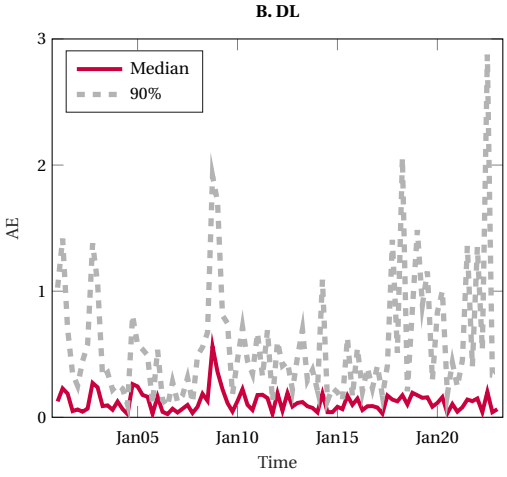


TL Framework



TL has less extreme outliers

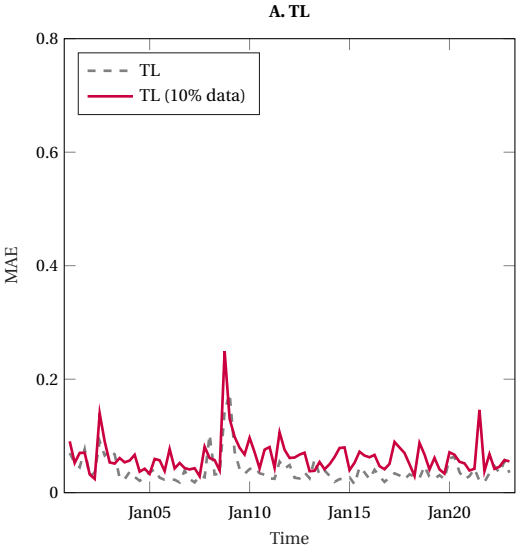


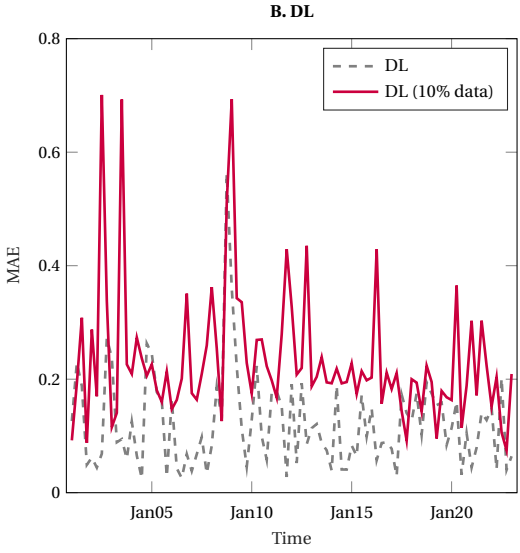


Performance attribution

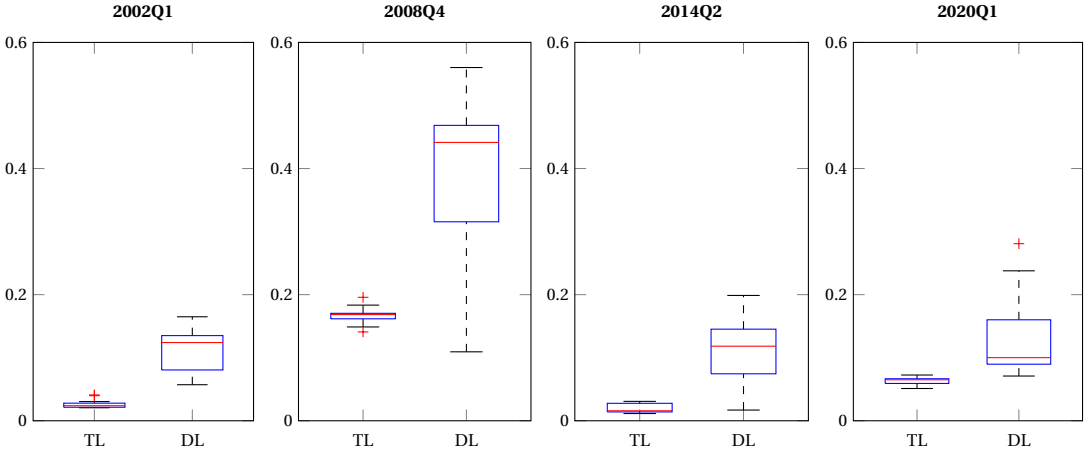
- Regressing the difference in absolute pricing errors between the DL and TL model at contract-day level on a variety of attributes.
- TL has a bigger advantage relative to DL:
  - ↪ for options with median and long maturity (>14 days), OTM (especially DOTM), with better liquidity (high volume);
  - ↪ when the market is more volatile;
  - ↪ when volatility spikes;
  - ↪ when inputs are “uncommon” relative to training sample (Mahalanobis distance).

TL is better at learning from a smaller sample





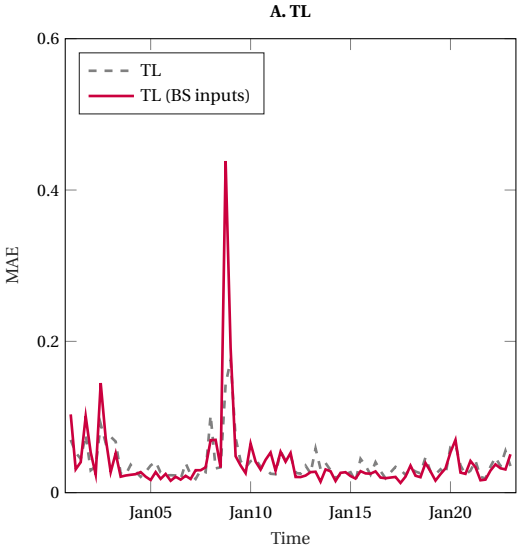
TL is less sensitive to the randomness of training

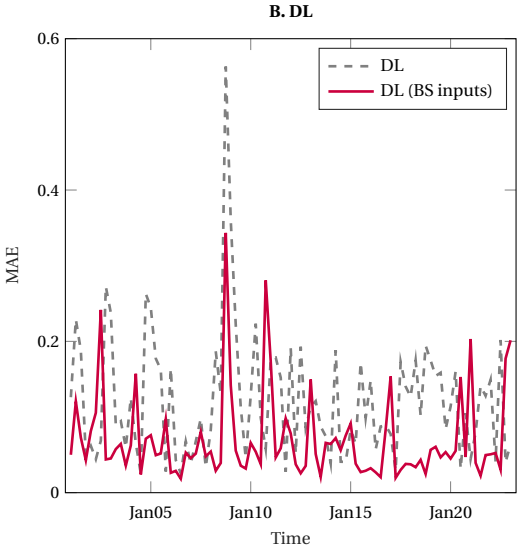


What is source of TL out-performance?

- Theory helps identify relevant inputs?
  - ↪ Compare to DL with the same inputs
- More data?
  - ↪ Rolling-window vs. expanding-window
  - ↪ Pooling real data with simulated data
- Other ways to bring in structural model information
  - ↪ Introducing a penalty in the loss function based on the violation of BS restrictions.
  - ↪ Boosting BS with OLS.
- Would any restrictions work?
  - ↪ Use no-arbitrage restrictions in the source domain

Theory does more than identifying inputs





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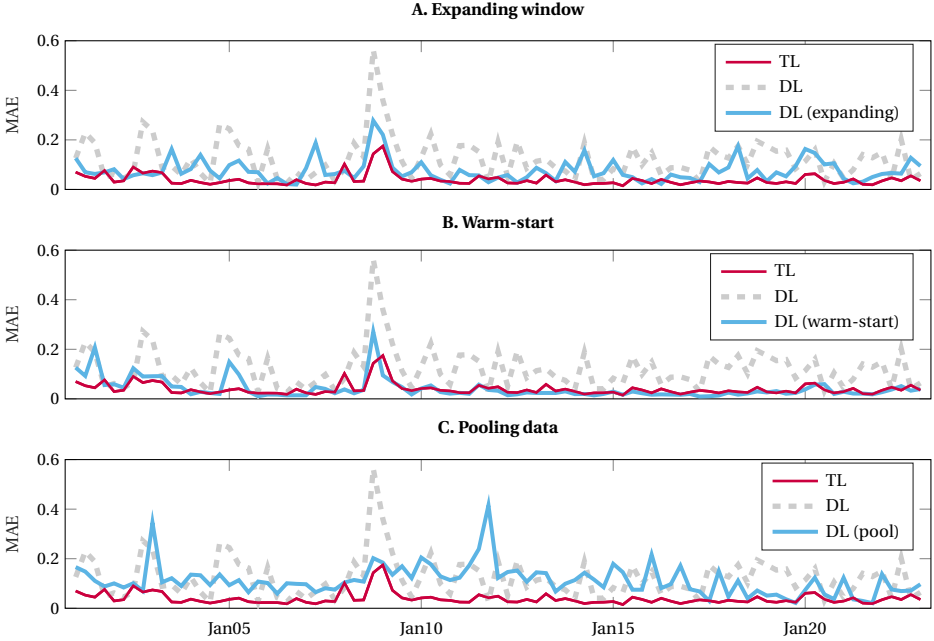
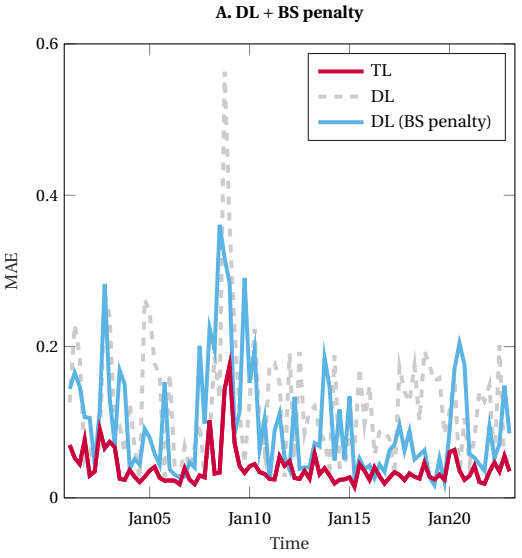
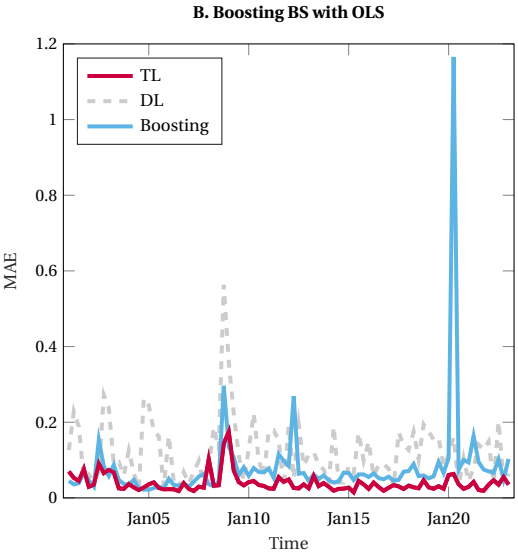

A. Expanding window
MAE
0.6
0.4
0.2
0
TL
DL
DL (expanding)
B. Warm-start
MAE
0.6
0.4
0.2
0
TL
DL
DL (warm-start)
C. Pooling data
MAE
0.6
0.4
0.2
0
TL
DL
DL (pool)
Jan05
Jan10
Jan15
Jan20

What is source of TL out-performance?

- Theory helps identify relevant inputs?
  - ↪ Compare to DL with the same inputs
- More data?
  - ↪ Rolling-window vs. expanding-window
  - ↪ Pooling real data with simulated data
- Other ways to bring in structural model information
  - ↪ Introducing a penalty in the loss function based on the violation of BS restrictions.
  - ↪ Boosting BS with OLS.
- Would any restrictions work?
  - ↪ Use no-arbitrage restrictions in the source domain

Alternative ways to incorporate model information

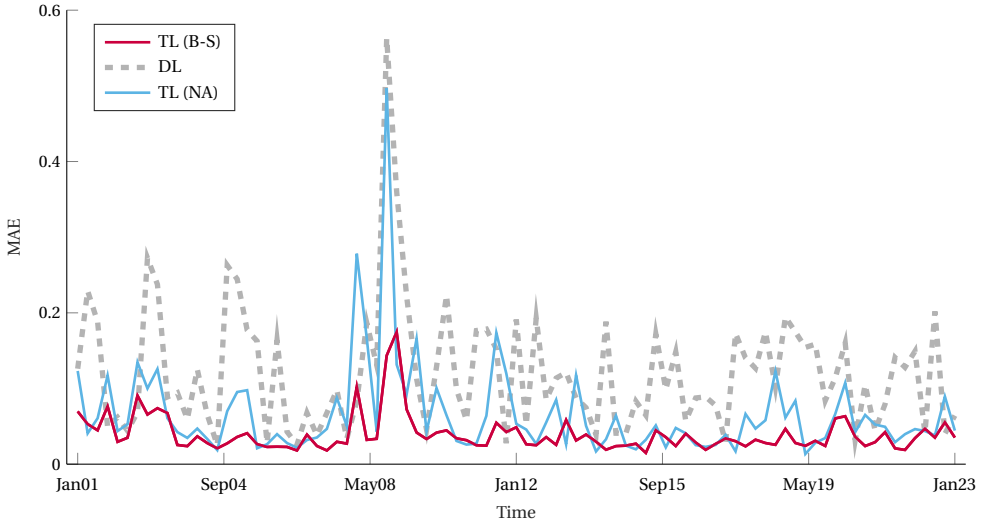




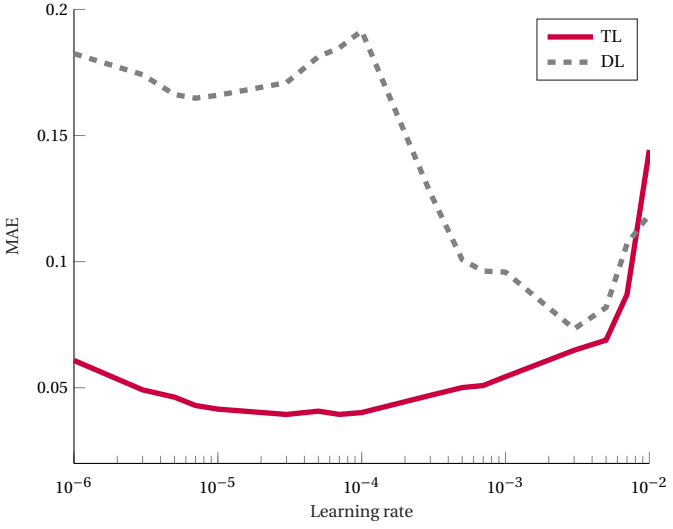
What is source of TL out-performance?

- Theory helps identify relevant inputs?
  - ↪ Compare to DL with the same inputs
- More data?
  - ↪ Rolling-window vs. expanding-window
  - ↪ Pooling real data with simulated data
- Regularization by imposing structural restrictions directly?
  - ↪ Introducing a penalty in the loss function based on the violation of the structural model restrictions.
  - ↪ Boosting the structural model with OLS.
- Would any restrictions work?
  - ↪ Use no-arbitrage restrictions in the source domain

Using no-arbitrage restrictions in the source domain



The importance of fine-tuning

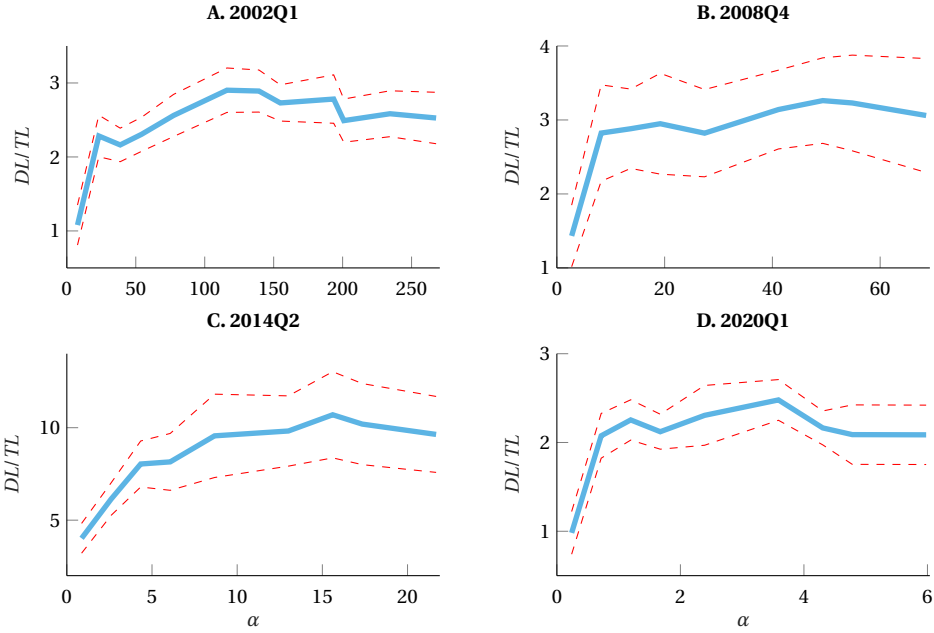


TL vs. Informative Prior

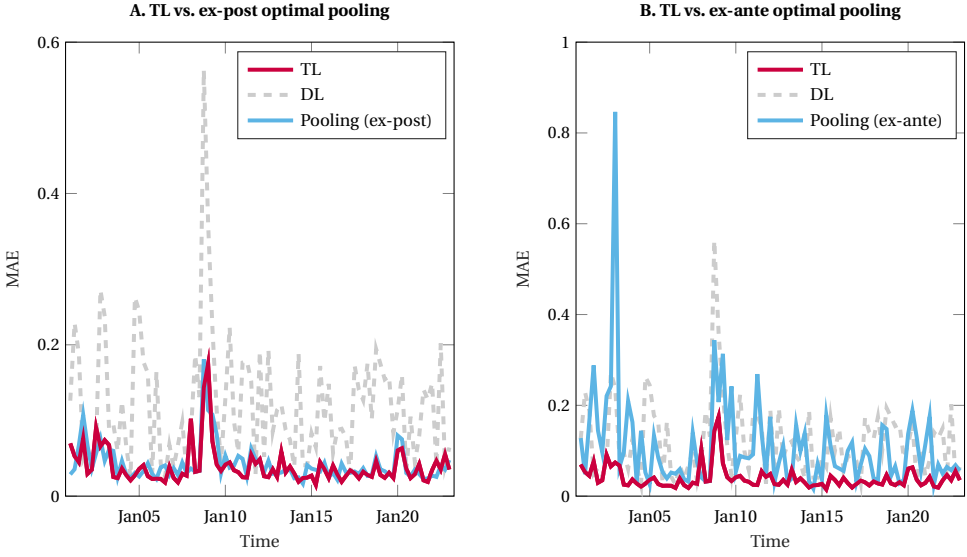
- Consider relative sample size

$$\lambda = \frac{\text{sample size of synthetic data}}{\text{sample size of real data}}$$

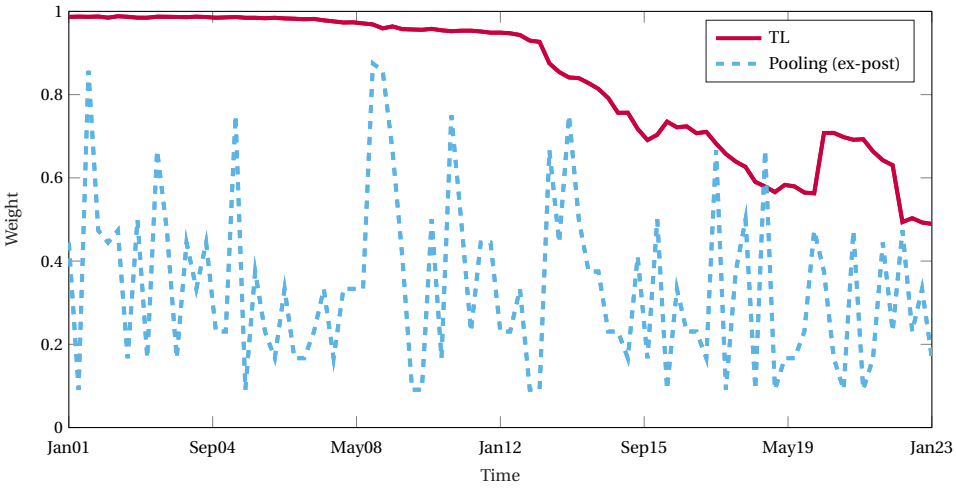
- In Bayesian VAR, λ effectively controls the prior precision.
- Del Negro and Schorfheide (2004): An inverse U-shaped relation between forecasting accuracy and λ.
  - ↪ Intuition: Too little or too much weight on the DSGE model can both hurt performance.
- TL: More synthetic data will improve the accuracy of the structural model. But the learned weights in the source domain are still only the initial values for the next stage.
- How much the final weights will devaite depends on:
  - ↪ Gap between the structural model and true DGP
  - ↪ Hyperparameters of learning in the target domain: learning rate, epochs


A. 2002Q1
B. 2008Q4
C. 2014Q2
D. 2020Q1
DL/TL
α

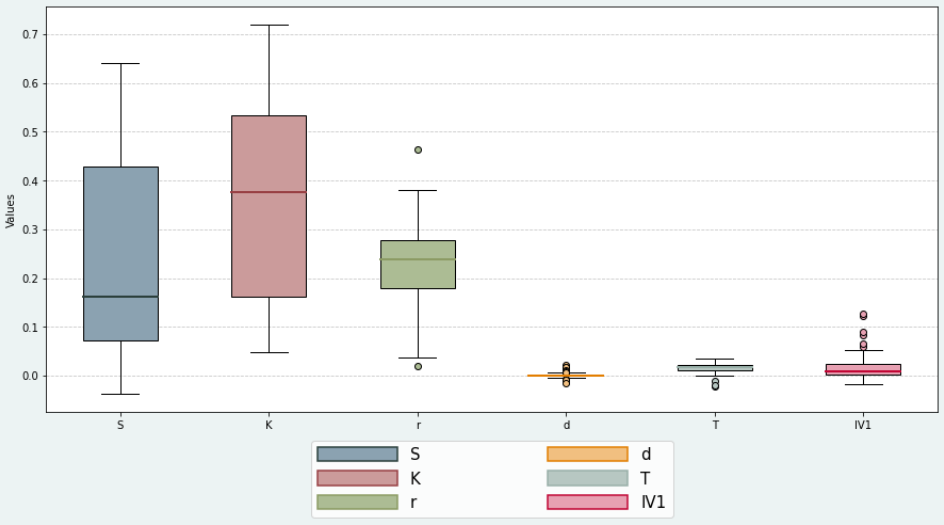
TL vs. Informative Prior



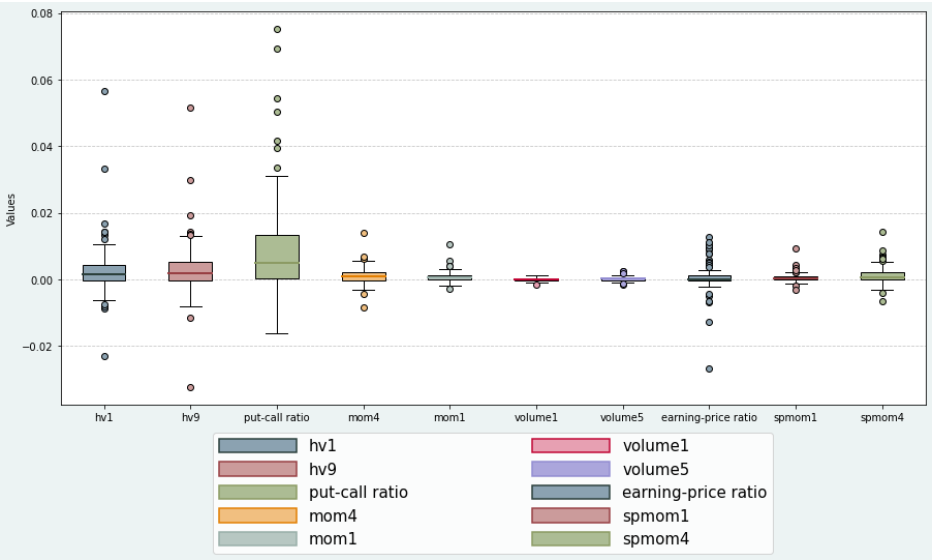
TL vs. Informative Prior



Feature importance



Feature importance



Summary

- "All models are wrong, but some are useful."
  - ↪ Guiding ML with economic theory through TL helps train better models with less data.
  - ↪ TL also helps us evaluate the usefulness of a model based on its complementarity with actual data.
- TL can also guide us to improve theories.
  - ↪ Use feature importance to identify relevant features missing from a structural model.
  - ↪ Relative performance analysis can reveal states where model misspecification is severe.
  - ↪ Ex-post relative sample size can be used to quantify value of theory (for prediction).
- Not specific to neural networks. The framework easily applies to other ML models.

Theories are dead? Long live theories!

The Future of Finance and Economics?

Virtual playgrounds for (economic) AI agents