Simulation of a Simplified DICE Model with Neural Networks in a Stationary Environment

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1 Introduction

Integrated Assessment Models are nowadays the most used method to evaluate the global interaction between human activity and environmental changes. It allows to evaluate how human activity drives climate change and what is the optimal path to mitigate it.
The landmark for such class of models is Nordhaus' DICE that was first published in 1992. The aim of this model is not only to theoretically understand the interaction between economy and climate, but also to give numerical estimates of the future environmental parameters mostly temperature and carbon mass. Thus, DICE has been updated throughout the decades to provide better estimates.

2 Presentation of the Model

A simplified approach of DICE-2016s[1]. Furthermore, many of the variable are considered as stationary such as abatement costs, labor supply or productivity. We have the following equation :

1. Utility is CRRA :$U(C,L) = \frac{(1000\frac{C}{L})^{1-1/\psi}-1}{1-1/\psi}L$
2. Production is a Cobb-Douglas function: $Y_t(K_t, L, A) = AK_t^{\alpha}L^{1-\alpha}$
3. The law of motion of capital is:$K_{t+1} = (1-\delta)K_t + I_t$
4. Emissions given a mitigation path m_t with $0 \leq m_t \leq 1$ are $E_t = \eta_0(1-m_t)Y_t + \eta_1$
5. Carbon Mass follow the equation: $M_{t+1} =_t +E_t$
6. Temperature of the atmosphere follows the law: $T_{t+1} = \mu_0 + \mu_1 * T_t + \mu_2 log(M_t)$

[1] see Ikefuji, Laeven, Magnus Muris (2021)

7. The damage function is given by :$D_t = \frac{1-\xi_0 m_t^{\theta}}{1+_t^2}$

8. The use of production is given by :$D_t Y_t = C_t + I_t$

The problem is the following: $max \sum^{+\infty}(C_t, L)$ given the constraint above.

The numerical specifications that we used are :

L	7.403
A	5.115
β	$\frac{1}{1+0.15}$
ψ	0.69
δ	0.0
η_0	0.3
η_1	2.6
ρ	0.9942
μ_0	-2.8672
μ_1	0.8954
μ_2	0.4622
ξ_0	0.052
ξ_1	0.00265
θ	2.6

3 Algorithm

We build a Deep Equilibrium Net Algorithm method for dynamix stocastic problems[2]. We build a two layers neural neural network with 100 and 200 hidden nodes and set softmax activation function. We noticed that softmax was the best activation function for minimizing loss. The network predicts 3 values: m_t, I_t and the Value function evaluated at the current state V_t. We pass 7 inputs which are K_t, T_t, M_t, A, L, Y_t and $r_t = \alpha D_t A K_t^{\alpha-1} L^{1-\alpha} + (1-\delta)$. We define the following losses:

1. $l(\hat{c_t}, \hat{c_{t+1}}) = \frac{\beta \hat{r_{t+1}} U'(\hat{c_{t+1}})}{U'(\hat{c_t})} - 1$ (Euler Condition)

2. $l(\hat{V_t}, \hat{V_{t+1}}) = \frac{U(\hat{c_{t+1}} + \beta \hat{V_{t+1}}}{\hat{V_t}} - 1$ (Bellman Equation)

3. Punish For Negative Investment,Consumption Temperature and wrong mitigation.

4. We compute a unique loss function using MSE.

We use an Adam Optimizer with a decaying learning rate to avoid increasing loss during training. The algorithm is the following.

1. Start with a random valid state

[2]see Azinovic,Gaegauf Scheidegger (2022)

2. Simulate n state given the neural network

3. Compute the loss function

4. Update the weights of the neural network

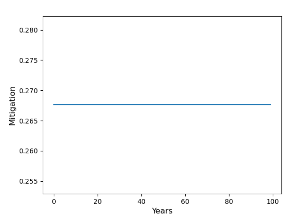
5. Re-simulate data with the last episode simulated and repeat the steps above until the loss is small as decided.

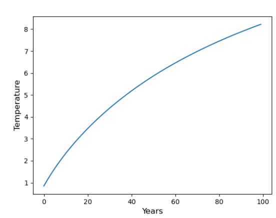
We use python to compute it especially the library *tensorflow* for implementing the neural network model

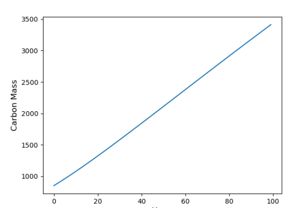
4 Results

We plot the results after 500 iterations of the algorithm that took approximately 30 minutes. The average error is $3\text{x}10^{-3}$ .The convergence is fast. It is due that we have only 3 parameters to estimate: Investment mitigation pathway and Value function. We start simulating the pathway with an initial capital $K_1 = 223$, an initial temperature $T_1 = 0.85$ and initial carbon mass $M_1 = 851$. For this we predict the value of the next period of the Bellman equation and find the consumption, mitigation pathway and investment for the current period. We plot the results over 100 years.

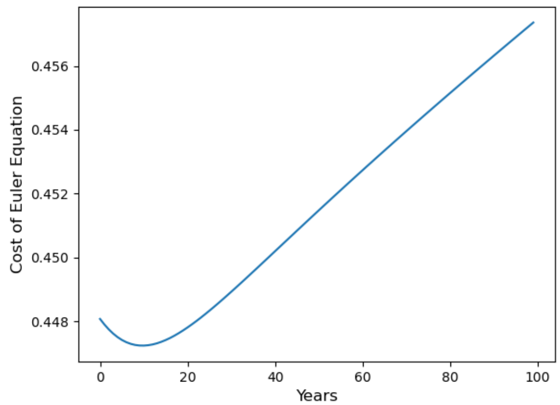
We find that mitigation is constant over time. In a stationary state, it is constant because choose how to reduce their emissions given some parameters such as labor supply, productivity and emissions parameters, that varies in the real DICE model.







Temperature and Carbon mass tends to increase lower but not stop increasing. It is not optimal for agent to mitigate fully their emissions.



We plot here the Euler Equation Cost. It is very small when we compare it to the values of the outputs such as consumption but still it is increasing linearly over the year which shows that the neural network needs to be better fine-tuned.

5 Conclusion

With this simple example, we saw how Deep Equilibrium is an generalizable method to estimate numerical macroeconomic method that we could make more complex. Neural networks are able to estimate functions with highly non-linear functions. This method could very useful for IAMs because it could estimate more complex model in a much lower time as neural networks are freed from the curse of the dimensionality.