1 Model

The economy has $n = 1, \ldots, N$ countries and $j = 1, \ldots, J$ different goods.

1.1 Household

In each country n, we have a representative household with preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{n,t}, l_{n,t}\right)$$

where $c_{n,t}$ is consumption aggregate and $l_{n,t}$ is labor supply. Here $\beta < 1$ is the discount factor and $\mathbb{E}_0$ the conditional expectation operator.

1.2 Technology

In each country a representative firm can produce the good j with technology:

$$y_{n,j,t} = z_{n,j,t} k_{n,j,t}^{\alpha} l_{n,j,t}^{1-\alpha}$$

by renting capital at price $rc_{n,t}$ and labor at wage $w_{n,t}$.

The efficiency $z_{n,j,t}$ comes from a Fréchet probability distribution:

$$F_n\left(z\right) = e^{-T_{n,t} z^{-\theta}},\ T_{n,t} > 0 \text{ and } \theta > 1$$

that is independent across countries. We can think about $T_{n,t}$ as the absolute advantage of country n and $\theta > 0$ as the dispersion of technology. We assume that $T_{n,t}$ evolves as an autoregressive process:

$$T_{n,t} = \rho T_{n,t-1} + \sigma \varepsilon_{n,t}, \text{ where } \varepsilon_{n,t} \sim \mathcal{N}\left(0, 1\right)$$

1.3 Uses of Goods

The output of good j can be used for consumption, $c_{n,j,t}$, investment, $i_{n,j,t}$, or net exports, $net_{n,j,t}$:

$$y_{n,j,t} = c_{n,j,t} + i_{n,j,t} + net_{n,j,t}$$

Notice that consumption and investment must be (weakly) positive but net exports can be positive or negative.

The consumption aggregate is given by a CES aggregator:

$$c_{n,t} = \left(\sum_{j=1}^{J} c_{n,j,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

Analogously, the investment aggregate is given by another CES aggregator:

$$i_{n,t} = \left(\sum_{j=1}^{J} i_{n,j,t}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$$

and total capital evolves as

$$k_{n,t+1} = \left(1 - \delta\right) k_{n,t} + i_{n,t}$$

1.4 Market Clearing

The market clearing conditions for labor and capital are:

$$l_{n,t} = \sum_{j=1}^{J} l_{n,j,t}$$

$$k_{n,t} = \sum_{j=1}^{J} k_{n,j,t}$$

1.5 International Trade

There is an iceberg cost of delivering goods from country n into country i:

$$\begin{aligned} d_{nn} &= 1 \\ d_{ni} &> 1 \text{ for } n \neq i \end{aligned}$$

with the triangular inequality

$$d_{ni} \leq d_{nk} d_{ki}$$

that ensures that using an intermediate country to trade does not save money with respect to direct delivery.

1.6 Prices

We assume perfect competition and, therefore, the (potential) price in country i of good j produced in country n equals its marginal costs $mc_{n,j,t}$ times the iceberg cost:

$$p_{i,j,n,t} = d_{ni} mc_{n,j,t}$$

In equilibrium, we must have that the (actual) price in country i of good j is the lowest among all the prices of potential producers of the good:

$$p_{i,j,n,t} = \min \{p_{i,j,n,t} : n = 1, \ldots, N\}$$

I need to complete the budget constraints and the global market clearing conditions.

State variables: $T_{n,t}$ and $k_{n,t}$. Let's imagine we have 25 countries: 50 state variables. Easy to add a few more state variables per country.