1 Variables

1. The set of endogenous state variables has the following elements:

   K_t

   CODE: Kx (*code variables: states end with 'x', controls with 'y'*)

2. The set of exogenous state variables has the following elements:

   $T^D_{n,t}$, $\phi_{n,t}$, $L_{n,t}$, $d_{ni,t}$, $\chi_{n,t}$

   CODE: ...

3. The set of control variables has the following elements:

   $A^D_{n,t}$, ...

   CODE:

2 Equations

legend: states in green (given), controls in orange (given by policy guess from current states), next period's exogenous states in magenta (to be integrated over), and next period's controls in blue (given by policy guess at next period's states), parameters are black

N equations. Sectoral productivity

$$A^D_{n,t} = (1/\gamma)\left(T^D_{n,t}\right)^{1/\theta} \tag{1}$$

The $A^D_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{A^D_t}$.

N equations. Service sector output

$$0 = Y^S_{n,t} - \omega_n \phi_{n,t} \tag{2}$$

The $Y^S_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{Y^S_t}$.

N equations. Rental rate

$$0 = Y_{n,t} - Y^D_{n,t} - Y^S_{n,t} \tag{3}$$

The $Y_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{Y_t}$ and the $Y^D_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{Y^D_t}$.

N equations. Wage

$$0 = w_{n,t} - \beta^L \frac{Y_{n,t}}{L_{n,t}} \tag{4}$$

The $w_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{w_t}$.

N equations. Rental rate

$$0 = r_{n,t} - \beta^K \frac{Y_{n,t}}{K_{n,t}} \tag{5}$$

The $r_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{r_t}$.

N equations. The cost of a bundle of factors

$$0 = b_{n,t} - {w_{n,t}}^{\beta^L} {r_{n,t}}^{\beta^K} \tag{6}$$

The $b_{n,t}$ are organized in a $N \times 1$ vector $\mathbf{b_t}$.

N equations. The price index of the durable (tradable) sector

$$0 = p^D_{n,t} - \left(\sum_{i=1}^{N} \left(\frac{b_{i,t} d_{ni,t}}{A^D_{i,t}} \right)^{-\theta} \right)^{-\frac{1}{\theta}} \tag{7}$$

N equations per tradable sector and country (including the relation wrt to itself). As we model only one tradable sector per country, this boils down to $N \times N$ equations. The fraction of durable goods that country n obtains as imports from country i

$$0 = \pi^D_{ni,t} - \left(\frac{b_{i,t} d_{ni,t}}{p^D_{n,t} A^D_{i,t}} \right)^{-\theta} \tag{8}$$

These fractions imported form an $N \times N$ matrix $\mathbf{\Pi_t}$.

N equations. The absorption of the durable good in a country

$$0 = \mathbf{\Pi_t}^{-1} \mathbf{Y^D_t} - \mathbf{X^D_t} \tag{9}$$

N equations. The law of motion of capital

$$0 = K_{n,t+1} - \chi_{n,t} \left(\frac{X^D_{n,t}}{p^D_{n,t}} \right) {K_{n,t}}^{1-\alpha} + (1-\delta) K_{n,t} \tag{10}$$

N equations. The Euler equations

$$0 = \frac{p^D_{n,t}}{\alpha \chi_{n,t}} \left(\frac{X^D_{n,t}}{p^D_{n,t} K_{n,t}} \right)^{1-\alpha} - \rho \frac{p^D_{n,t+1}}{\alpha \chi_{n,t+1}} \left(\frac{X^D_{n,t+1}}{p^D_{n,t+1} K_{n,t+1}} \right)^{1-\alpha} \left[\chi_{n,t+1} (1-\alpha) \left(\frac{X^D_{n,t+1}}{p^D_{n,t+1} K_{n,t+1}} \right)^{\alpha} + (1-\delta) \right] + \rho r_{n,t+1} \tag{11}$$

3 Laws of motion

3.1 LoM for exogenous states

N equations

$$\ln T^{D}_{n,t} = \rho_T \ln T^{D}_{n,t-1} + \varepsilon_{T,t} \tag{12}$$

$N-1$ equations

$$\ln \phi_{n,t} = \rho_{\phi_n} \ln \phi_{n,t-1} + \varepsilon_{\phi_n,t} \tag{13}$$

1 equations

$$\phi_{N,t} = 1 - \sum_{i=1}^{N-1} \phi_{i,t} \tag{14}$$

N equations

$$\ln \chi_{n,t} = \rho_{\chi_n} \ln \chi_{n,t-1} + \varepsilon_{\chi_n,t} \tag{15}$$

$N \times (N-1)$ equations

$$\ln d_{ni,t} = \left(1 - \rho_{d_{ni}}\right) \bar{d}_{ni} + \rho_{d_{ni}} \ln d_{ni,t-1} + \varepsilon_{d_{ni},t} \tag{16}$$

N equations

$$\ln d_{nn,t} = 0 \tag{17}$$

N equations

$$\ln L_{n,t} = \left(1 - \rho_{L_n}\right) \bar{L}_n + \rho_{L_n} \ln L_{n,t-1} + \varepsilon_{L_n,t} \tag{18}$$

4 Steady State

Solving for steady state in two steps: First, solve for steady state values of K_n and Y_n via the non-linear system of equations below. The system can be solved by minimizing the sum of the two error terms defined at the end:

N equations. Wage

$$w_n = \beta^L \frac{Y_n}{L_n} \tag{19}$$

N equations. Rental rate

$$r_n = \beta^K \frac{Y_n}{K_n} \tag{20}$$

N equations. Cost of a bundle of factors

$$b_n = \frac{Y_n}{B\,(L_n)^{\beta^L}\,(K_n)^{\beta^K}} \tag{21}$$

where $B = \left(\beta^L\right)^{-\beta^L}\left(\beta^K\right)^{-\beta^K}$.

N equations. The price index of the durable (tradable) sector

$$p_n^D = \left(\sum_{i=1}^{N}\left(\frac{b_i d_{ni}}{A_i^D}\right)^{-\theta}\right)^{-1/\theta} \tag{22}$$

N equations. The fraction of durable goods that country n obtains as imports from country i

$$\pi_{ni} = \left(\frac{b_i d_{ni}}{p_n^D A_i^D}\right)^{-\theta} \tag{23}$$

N equations. The absorption of the durable good in a country

$$\mathbf{X_t^D} = \mathbf{\Pi_t}^{-1}\mathbf{Y_t^D} \tag{24}$$

N equations $\longrightarrow N$ error terms

$$\left|\frac{\frac{X_n^D}{p_n^D K_n}}{\left(\frac{\delta}{\chi_n}\right)^{1/\alpha}} - 1\right| = error\ terms\ \ (= 0 \text{ in theory}) \tag{25}$$

N equations $\longrightarrow N$ error terms

$$\left|\frac{X_n^D}{Y_n} - \beta^K\frac{\alpha\delta\rho}{1-\rho(1-\alpha\delta)}\right| = error\ terms\ \ (= 0 \text{ in theory}) \tag{26}$$

Second, use the steady state values of K_n and Y_n to calculate w_n, r_n, b_n, p_n^D, π_{ni}, and X_n^D.

5 Parameters

$$\gamma = \left[\Gamma\left(\frac{\theta-\sigma+1}{\theta}\right)\right]^{-1/(\sigma-1)|}$$

where Γ is the Gamma function.

Also note that $\theta > \sigma - 1$ and $\sum_{n=1}^{N}\omega_n = 1$ must hold.

6 Parameters

Symbol (Code)	Parameter	Value
θ ()		2
σ ()		2.5
$\forall n$: ω_n ()		0.5
β^L ()		2/3
β^K ()		1 - β^L
δ ()		0.1
α ()		0.55
ρ ()		0.95
$\forall n$: ρ_{T_n} ()		0.85
$for\ n-1\ countries$: ρ_{ϕ_n} ()		0.85
$\forall n$: ρ_{χ_n} ()		0.85
$\forall n$ & i: $\bar{d}_{ni}$ ()		0.5
$\forall n$: d_{nn} ()		1
$\forall n$: $\bar{L}_n$ ()		0