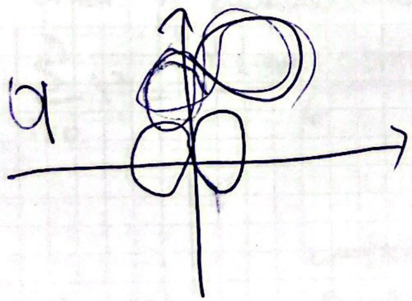
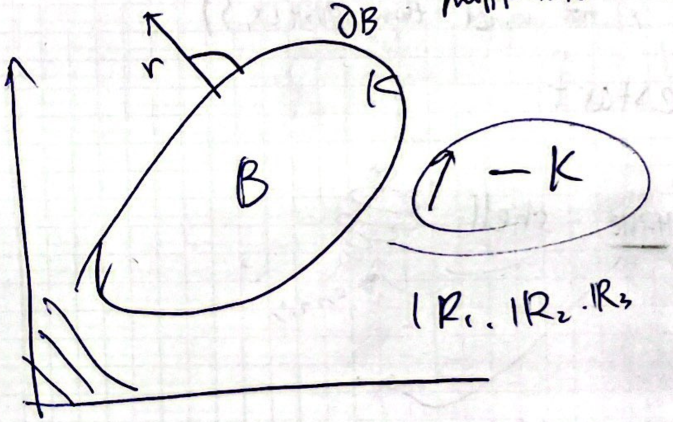
W

M

$\gamma = \partial B : [a,b] \to \mathbb{R}^2$

Geschwindigkeit feld





$\mathbb{R}_1 . \mathbb{R}_2 . \mathbb{R}_3 = \mathbb{R}.$

hoengu

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SCS super computing systems

M

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$\mathbb{R}^3 \rightarrow \mathbb{R}$ $\quad \frac{\partial K_1}{\partial x}, \frac{\partial K_2}{\partial y} \;\&\; \frac{\partial K_3}{\partial z}.$

$K(x,y) = \begin{pmatrix} 3y - e^{\sin(x)} \\ 7x + \sqrt{y^4+1} \end{pmatrix}$

$\mathrm{div}(K) = \frac{\partial K_1}{\partial x} + \frac{\partial K_2}{\partial y} + \frac{\partial K_3}{\partial z}.$

Code Bei ~~Groten~~ Green

$\mathrm{div}(K)(x,y) = \dots$

$K = \begin{pmatrix} P \\ Q \end{pmatrix} \rightarrow$ Ebene $\quad \mathrm{div}(K) = P_x + Q_y$

$\iint_B (Q_x - P_y)\,dA$

- 4
- 10
- $\frac{2y^3}{\sqrt{y^4+1}} - \cos(x)e^{\sin(x)}$

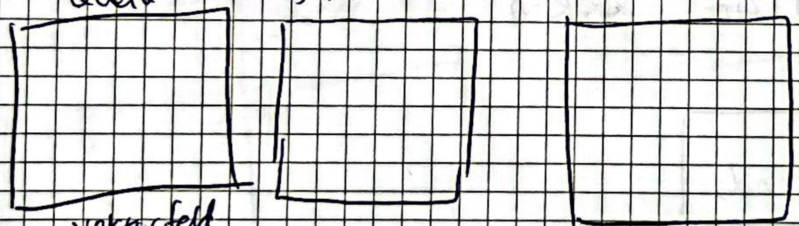
Negativ

Skalarprodukt Vectors $\boxed{\nabla \cdot K} \rightarrow$ Notation Anwendung!

* Divergenz
- Quellen
- Senken
- Keinen Quelle

$(\alpha(x) = e\sin(x))$

Quelle Senke



Vektorfeld

$\frac{2y^3}{\sqrt{y^4+1}} \times \frac{\sqrt{y^4+1}}{2y^3}$

$Q(x,y) = 7x + \sqrt{y^4+1} \Rightarrow$

$Q_y(x,y) = 0 + \frac{1}{2}\frac{4y^3}{\sqrt{y^4+1}}$

positiv

negativ

verschwinding → weg

verschwinden

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Flüssigkeitsmenge / Randkurve $\boxed{\gamma = \partial B}$

S6! $\mathbb{R}^2 =$

Δ_{L_i} in γ kleine Geradenstücke L_i

S7! $\Delta_{L_i} \approx |\gamma'(t_i)| \Delta t_i$

$\underbrace{K}_{\text{Vektorfeld}} \approx K(\gamma(t_i))$ & $n \approx n(\gamma(t_i))$ auf L_i ungefähr konstant.

Pro Zeiteinheit Δt_i

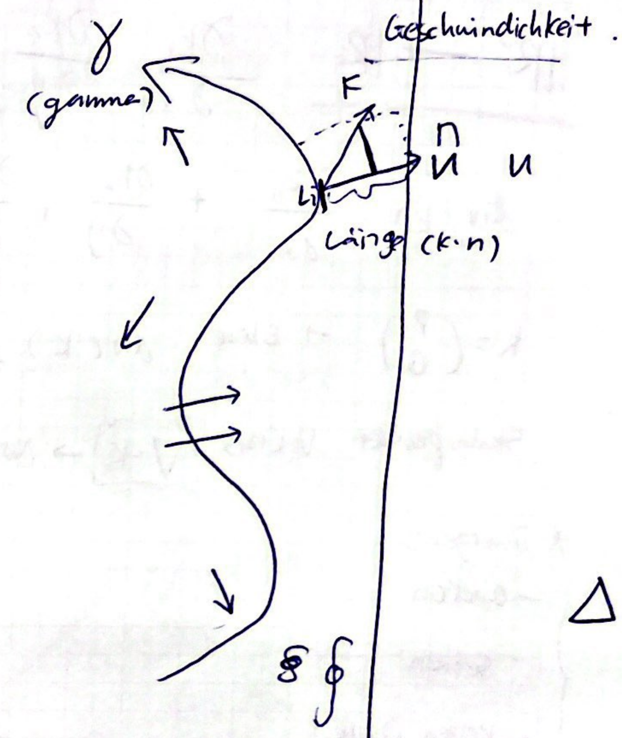
SKP $K \cdot n$ = Anteil der Flüssigkeit / L_i in Rich

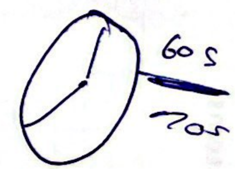
Gesamtmenge durch $L_i = (K \cdot n) \Delta L_i = (K \cdot n) |\gamma'(t_i)| \Delta t_i$

= Fläche des Rechtecks mit Kanten $\Delta L_i = |\gamma'(t_i)| \Delta t_i$ & $K \cdot n$

Gesamtmenge / $\gamma \approx$ $\Leftrightarrow$

$i \to \infty$ $\int_a^b (K \cdot n) |\gamma'(t_i)| \, dt \overset{\text{Def}}{=} \boxed{\oint_\gamma K \cdot n \, ds}$





* Gesamtmenge Flüssigkeit pro Zeiteinheit / $\boxed{\gamma = \oint_\gamma K \cdot n \, ds}$

Kurvenintegral für ______ $K \cdot n : \mathbb{R}^2 \to \mathbb{R}$

$(x, y) \mapsto K(x, y) \cdot n(x, y)$ SKP

Flüssigkeit

$\oint_\gamma K \cdot n\, ds$

$\oint_\gamma K \cdot n\, ds > 0$

$\oint_\gamma K \cdot n\, ds < 0$

$\oint$

Satz von Gauss

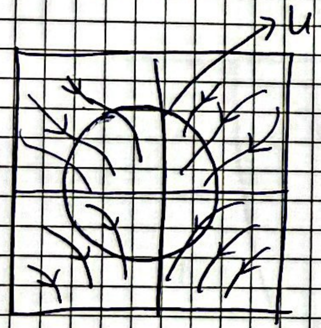
$K = (P, Q)$

$$\oint_{\gamma = \partial B} K \cdot n\, ds = \iint_B \operatorname{div}(K)\, dA$$

Clicker

hinein

hinaus



$\oint_\gamma K \cdot n < 0$

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Divergenz (Clicker)

Gegeben sei das Vektorfeld K mit $K(x,y) = \begin{pmatrix} 3y - e^{\sin(x)} \\ 7x + \sqrt{y^4+1} \end{pmatrix}$.

Mit Definition und Kettenregel sind

$$P(x,y) = 3y - e^{\sin(x)} \implies P_x(x,y) = 0 - \cos(x)e^{\sin(x)}$$

und

$$Q(x,y) = 7x + \sqrt{y^4+1} \implies Q_y(x,y) = 0 + \frac{1}{2}\frac{4y^3}{\sqrt{y^4+1}}$$

und

$$\begin{aligned} \operatorname{div}(K)(x,y) &= P_x(x,y) + Q_y(x,y) = -\cos(x)e^{\sin(x)} + \frac{2y^3}{\sqrt{y^4+1}} \\ &= \frac{2y^3}{\sqrt{y^4+1}} - \cos(x)e^{\sin(x)}. \end{aligned}$$

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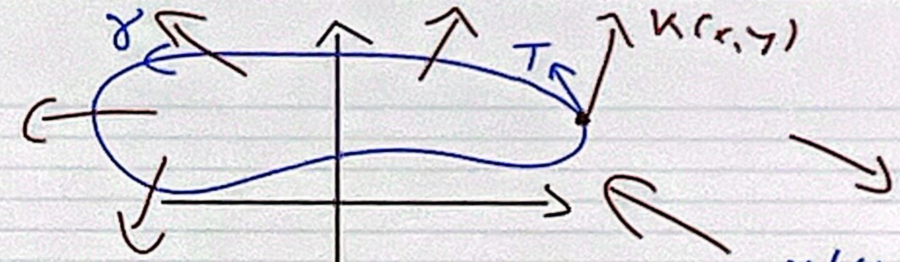
Bisher Arbeit

„K = Kraftfeld"



$$W = \oint_\gamma K \cdot d\gamma \underset{\text{Def}}{=} \int_\gamma (K \cdot T)\, ds$$

$$\underbrace{K \cdot T}_{\mathbb{R}^2 \to \mathbb{R}}$$

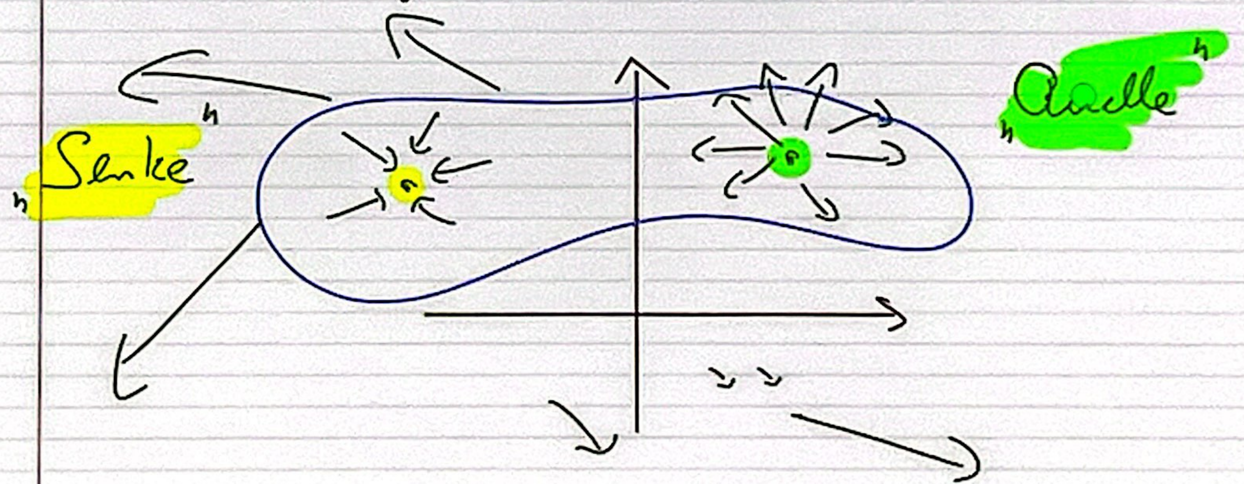
$$(x,y) \mapsto K(x,y) \cdot T(x,y)$$

Skalar

$$T = T(x,y) = \frac{\gamma'(t)}{|\gamma'(t)|}$$

$$\text{„} \; T(\gamma(t))$$

Jetzt: „K = Geschwindigkeitsfeld einer Flüssigkeit"



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