Supplementary Information for

LDR: Linearized Dimensionality Reduction for Localizing Magnetic Capsule Endoscopes

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APPENDIX A.

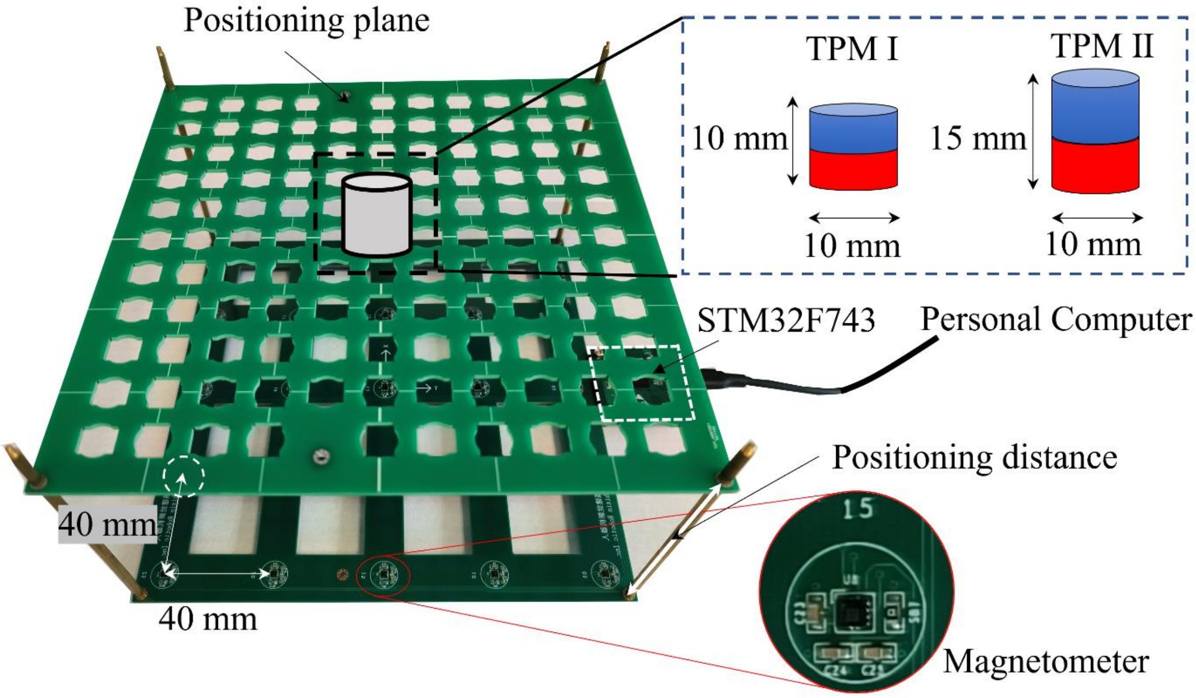

Fig. S1. The experimental setup for verifying the proposed algorithm. The sensor array is a 5×5 3-axis magnetometer array with $\pm 1.2\times10^{-3}$ T measurement range and 10 Hz sampling rate.

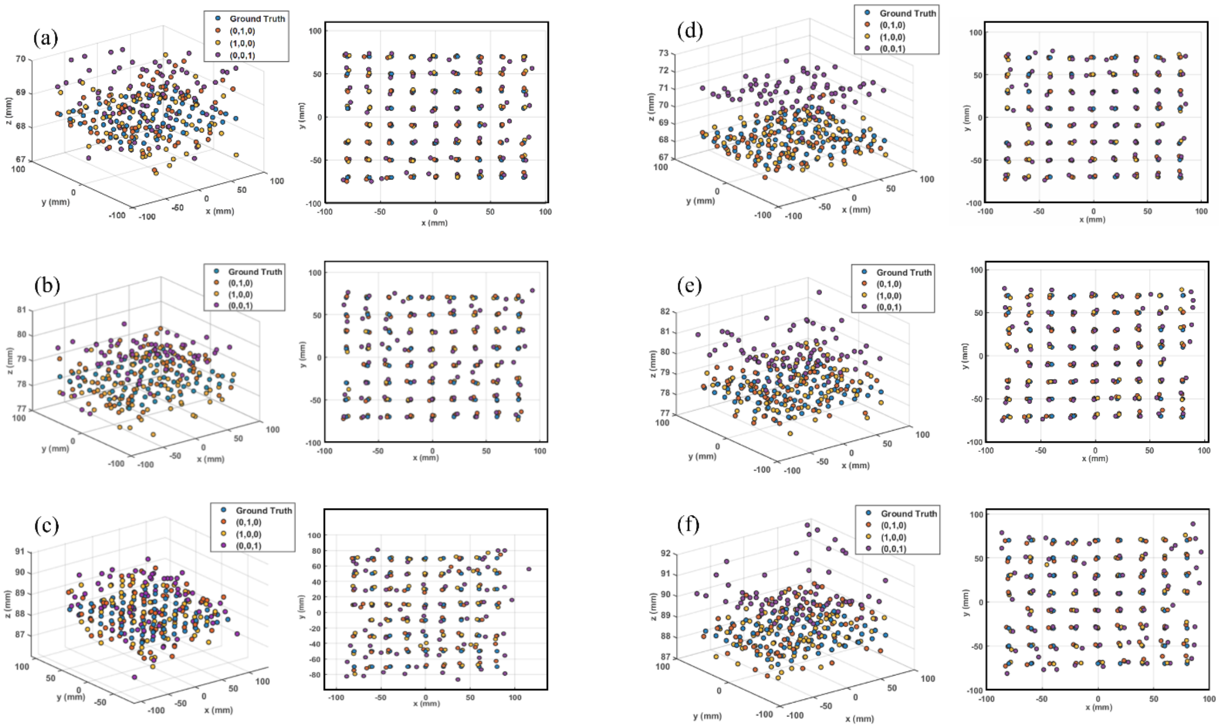

Fig. S2. The visualization of the discrete positioning results, total 18 situations classified by target magnet's volume and orientation, and height of the positioning plane. Target Magnet I: (a) Height 6.85cm; (b) Height 7.85cm; (c) Height 8.85cm; Target Magnet II: (d) Height 6.85cm; (e) Height 7.85cm; (f) Height 8.85cm. Blue indicates the ground truth; Orange indicates orientation (0,1,0); Yellow indicates orientation (1,0,0); Purple indicates orientation (0,0,1).

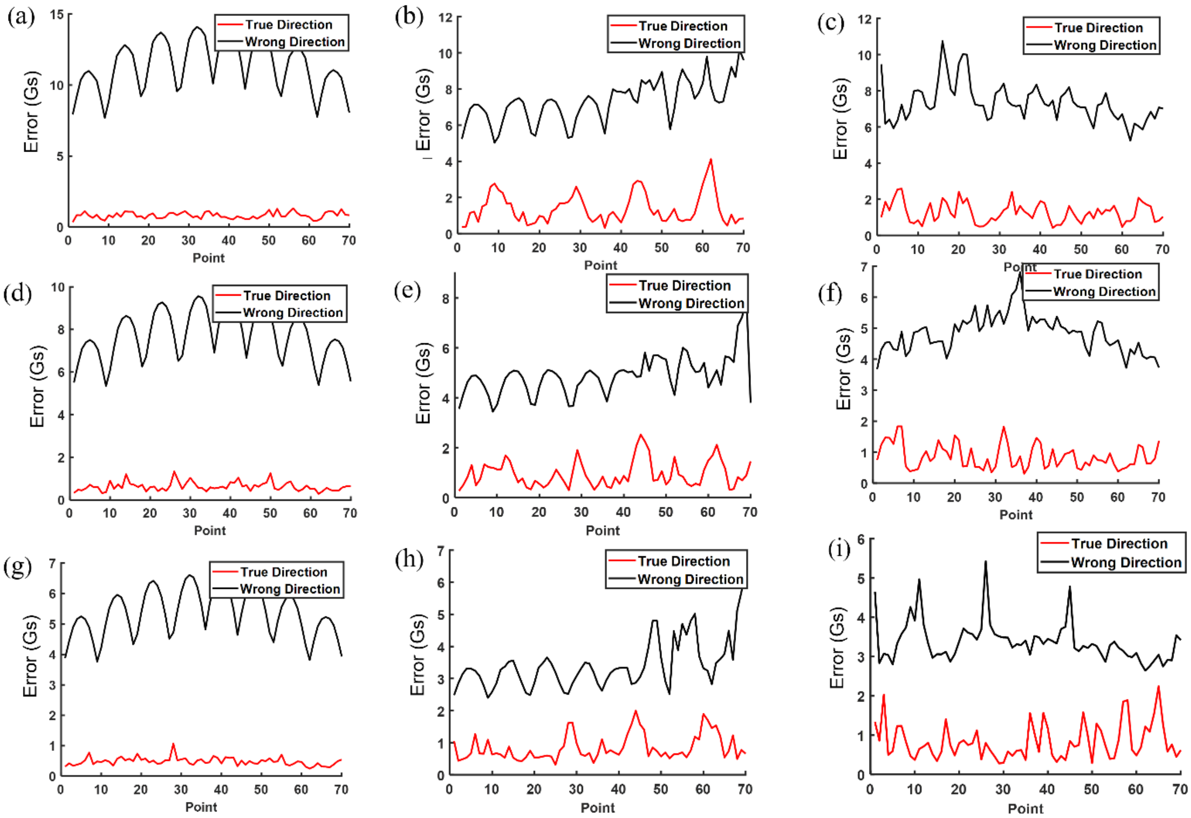

Fig. S3. Verification of $\mathbf{H}_{+}$ and $\mathbf{H}_{-}$: the comparison result of each direction $\boldsymbol{o_i}$ with different h_i. (a) Height: 6.85 cm, Orientation: (0,1,0); (b) Height: 6.85 cm, Orientation: (1,0,0); (c) Height: 6.85 cm, Orientation: (0,0,1); (d) Height: 7.85 cm, Orientation: (0,1,0); (e) Height: 7.85 cm, Orientation: (1,0,0); (f) Height: 7.85 cm, Orientation: (0,0,1); (g) Height: 8.85 cm, Orientation: (0,1,0); (h) Height: 8.85 cm, Orientation: (1,0,0); (i) Height: 8.85 cm, Orientation: (0,0,1).

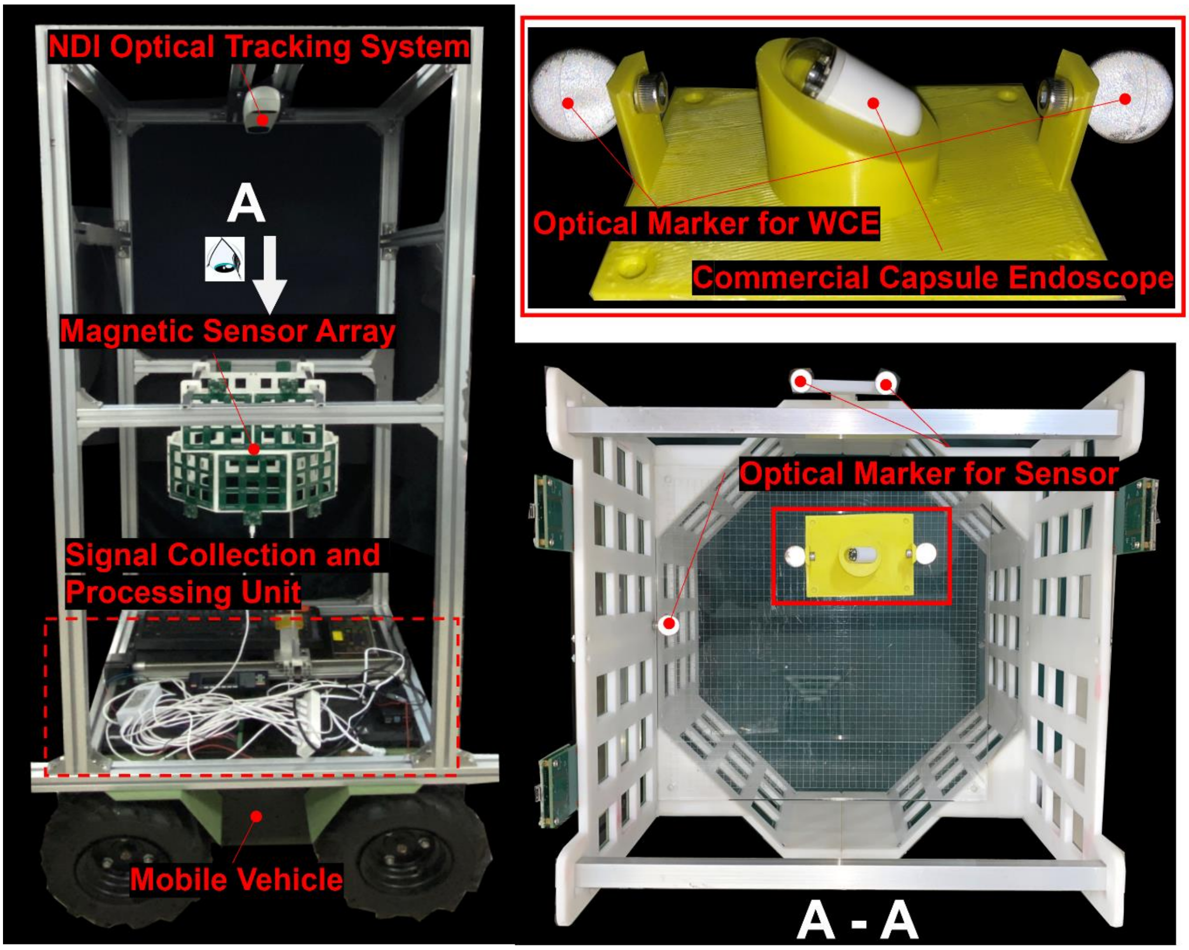

Fig. S4. Dynamic experiment setup. A commercial capsule endoscope (ANKON Technologies Co., Ltd) is used as the tracking target. A mobile vehicle is used to simulate a walking patient. A wearable magnetic sensor array is fixed upon the vehicle. An optical tracking system (NDI Polaris) is mounted on the upper bracket of the mobile vehicle to provide the TPM's ground truth.

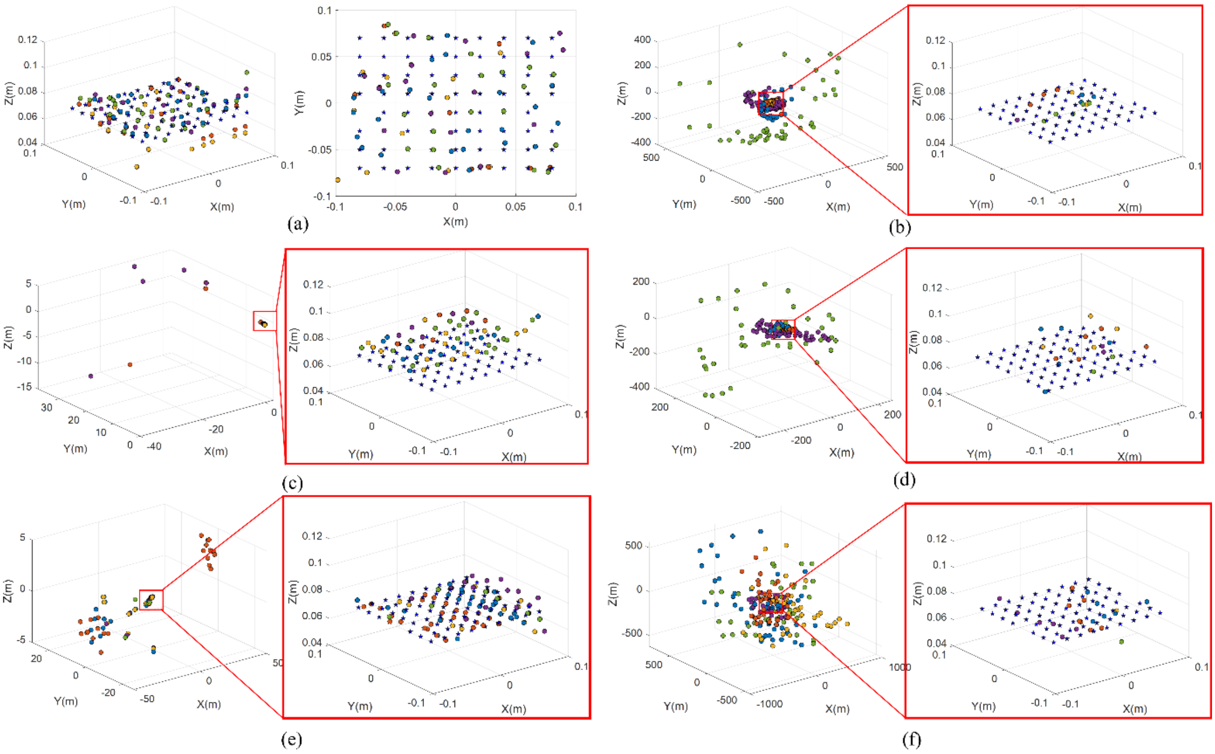

Fig. S5. Visualization of comparison results for iterative initial value sensitivity. Without loss of generality, we performed a total of 5 rounds of testing, with 70 sites in each round. There are 350 test points for each case, and we have selected three cases: (a) LDR: height (6.85cm) and orientation (0,0,1); (b) LM: height(6.85cm) and orientation (0,0,1); (c) LDR: height (7.85cm) and orientation (0,1,0); (d) LM: height (7.85cm) and orientation (0,1,0); (e) LDR: height(6.85cm) and orientation (1,0,0);(f) LDR: height(6.85cm) and orientation (1,0,0). For (a), the number of points calculated in the setting range is 312. However, the number is 29 in (b). For (c) and (d), the number is 212 and 26, respectively. And it turns to 125 in (e) and 29 in (f). Different colors indicate five rounds of positioning results, some of which overlap because they are the same and only one of the colors is displayed.

TABLE SI: Positioning Error of Target Magnet I

	Position Error (mm)				Orientation Error (°)			
Height(Ori.) /cm(-)	Mean	STD	Max	Min	Mean	STD	Max	Min
6.85(0;0;1)	4.8	2.1	9.9	1.8	9.1	5.1	22.3	1.0
6.85(0;1;0)	3.3	1.9	11.0	0.9	4.4	2.7	11.7	0.2
6.85(1;0;0)	2.2	1.0	5.7	0.8	3.5	2.6	29.7	0.1
7.85(0;0;1)	5.5	2.9	16.2	1.0	10.2	5.8	29.7	0.7
7.85(0;1;0)	3.9	2.3	11.8	1.0	5.0	3.3	16.4	0.8
7.85(1;0;0)	2.9	1.5	9.0	0.6	4.2	2.7	13.1	0.8
8.85(0;0;1)	9.0	4.3	20.0	1.2	17.5	9.0	35.1	1.7
8.85(0;1;0)	5.1	2.3	12.2	1.0	8.0	4.2	22.9	0.3
8.85(1;0;0)	3.6	2.3	13.8	0.8	8.2	6.5	31.4	0.2

TABLE SII: Positioning Error of Target Magnet II

	Position Error (mm)				Orientation Error (°)			
Height(Ori.) /cm(-)	Mean	STD	Max	Min	Mean	STD	Max	Min
6.85(0;0;1)	3.7	2.6	8.4	0.4	7.5	4.7	20.7	1.2
6.85(0;1;0)	3.3	1.8	8.8	0.5	4.3	2.2	10.2	0.8
6.85(1;0;0)	1.9	0.8	4.2	0.6	3.7	1.6	8.1	0.8
7.85(0;0;1)	4.8	2.8	11.6	1.3	8.9	5.8	29.4	1.5
7.85(0;1;0)	4.3	1.9	9.8	0.8	4.7	2.7	13.3	0.3
7.85(1;0;0)	2.5	1.2	7.8	0.9	4.7	2.3	13.5	0.4
8.85(0;0;1)	6.1	3.8	16.8	1.2	11.8	7.7	32.7	0.5
8.85(0;1;0)	4.8	2.2	11.4	1.1	6.5	4.3	19.2	0.2
8.85(1;0;0)	2.8	1.5	7.3	0.8	4.5	2.6	14.2	0.3

APPENDIX A

$$\mathbb{M}_k = \begin{bmatrix} m_{11k} & m_{12k} & m_{13k} & m_{14k} & m_{15k} & m_{16k} \\ m_{21k} & m_{22k} & m_{23k} & m_{24k} & m_{25k} & m_{26k} \end{bmatrix} \tag{A.1}$$

where

$m_{11k} = \Delta\mathbf{B}_{12kx} + \Delta\mathbf{B}_{34kx}$;

$m_{12k} = \Delta\mathbf{B}_{12ky} + \Delta\mathbf{B}_{34ky}$;

$m_{13k} = \Delta\mathbf{B}_{12kz} + \Delta\mathbf{B}_{34kz}$;

$m_{14k} = \Delta\mathbf{B}_{12kz} \cdot y_1 - \Delta\mathbf{B}_{12ky} \cdot z_1 + \Delta\mathbf{B}_{34kz} \cdot y_3 - \Delta\mathbf{B}_{12ky} \cdot z_3 + \Delta\mathbf{B}_{42ky} \cdot z_1 - \Delta\mathbf{B}_{42kz} \cdot y_1 + \Delta\mathbf{B}_{42kz} \cdot y_2 - \Delta\mathbf{B}_{42ky} \cdot z_2$;

$m_{15k} = \Delta\mathbf{B}_{12kx} \cdot z_1 - \Delta\mathbf{B}_{12kz} \cdot x_1 + \Delta\mathbf{B}_{34kx} \cdot z_3 - \Delta\mathbf{B}_{34kz} \cdot x_3 + \Delta\mathbf{B}_{42kz} \cdot x_1 - \Delta\mathbf{B}_{42kx} \cdot z_1 + \Delta\mathbf{B}_{42kx} \cdot z_2 - \Delta\mathbf{B}_{42kz} \cdot x_2$;

$m_{16k} = \Delta\mathbf{B}_{12ky} \cdot x_1 - \Delta\mathbf{B}_{12kx} \cdot y_1 + \Delta\mathbf{B}_{34ky} \cdot x_3 - \Delta\mathbf{B}_{34kx} \cdot y_3 + \Delta\mathbf{B}_{42kx} \cdot y_1 - \Delta\mathbf{B}_{42ky} \cdot x_1 + \Delta\mathbf{B}_{42ky} \cdot x_2 - \Delta\mathbf{B}_{42kx} \cdot y_2$;

$m_{21k} = \Delta\mathbf{B}_{23kx} + \Delta\mathbf{B}_{41kx}$;

$m_{22k} = \Delta\mathbf{B}_{23ky} + \Delta\mathbf{B}_{41ky}$;

$m_{23k} = \Delta\mathbf{B}_{23kz} + \Delta\mathbf{B}_{41kz}$;

$m_{24k} = \Delta\mathbf{B}_{23kz} \cdot y_2 - \Delta\mathbf{B}_{23ky} \cdot z_2 + \Delta\mathbf{B}_{41kz} \cdot y_4 - \Delta\mathbf{B}_{41ky} \cdot z_4 + \Delta\mathbf{B}_{13ky} \cdot z_2 - \Delta\mathbf{B}_{13kz} \cdot y_2 + \Delta\mathbf{B}_{13kz} \cdot y_3 - \Delta\mathbf{B}_{13ky} \cdot z_3$;

$m_{25k} = \Delta\mathbf{B}_{23kx} \cdot z_2 - \Delta\mathbf{B}_{23kz} \cdot x_2 + \Delta\mathbf{B}_{41kx} \cdot z_4 - \Delta\mathbf{B}_{41kz} \cdot x_4 + \Delta\mathbf{B}_{13kz} \cdot x_2 - \Delta\mathbf{B}_{13kx} \cdot z_2 + \Delta\mathbf{B}_{13kx} \cdot z_3 - \Delta\mathbf{B}_{13kz} \cdot x_3$;

$m_{26k} = \Delta\mathbf{B}_{23ky} \cdot x_2 - \Delta\mathbf{B}_{23kx} \cdot y_2 + \Delta\mathbf{B}_{41ky} \cdot x_4 - \Delta\mathbf{B}_{41kx} \cdot y_4 + \Delta\mathbf{B}_{13kx} \cdot y_2 - \Delta\mathbf{B}_{13ky} \cdot x_2 + \Delta\mathbf{B}_{13ky} \cdot x_3 - \Delta\mathbf{B}_{13kx} \cdot y_3$;