

# Probability and Random Variable

## Assignment-1

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### Introduction

**Question:** The coin is tossed 2 times. Find the probability of getting atmost one head.

**Solution:** Let  $X$  be the random variable representing the number of heads obtained in two coin tosses. The given information is summarised in Table 0

Parameters	Value	Description
n	2	Number of trials in an Experiment
p	1/2	Probability of Success
q	1/2	Probability of Failure

TABLE 0

where  $n$  is the number of trials,  $p$  is the probability of getting heads in a fair coin and  $q$  is the probability of not getting a head in a fair coin. The probability mass function (PMF) of  $X$  is given by:

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (1)$$

The probability of getting at most one head is the same as finding  $\Pr(X \leq 1)$ . To do this using the CDF method, we need to calculate the cumulative distribution function (CDF) of  $X$ :

$$F(k) = \Pr(X \leq k) = \sum_{k=-\infty}^k p_X(k) \quad (2)$$

$\therefore$

$$F(1) = \Pr(X \leq 1) = p_X(0) + p_X(1) \quad (3)$$

Now, by using equation (1),

$\therefore$

$$p_X(0) = \Pr(X = 0) = \binom{2}{0} \left(\frac{1}{2}\right)^0 \left(1 - \left(\frac{1}{2}\right)\right)^2 = \frac{1}{4}$$

$$p_X(1) = \Pr(X = 1) = \binom{2}{1} \left(\frac{1}{2}\right)^1 \left(1 - \left(\frac{1}{2}\right)\right)^1 = \frac{1}{2}$$

From the Cumulative distribution function(CDF) of X given in equation (2),

$$F(1) = \Pr(X \leq 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \quad (4)$$

$\therefore$  probability of getting at most one head is  $\frac{3}{4}$ .