

Probability and Random Variable

Assignment-1

1

Indupalli Sishir Anand
BT22BTECH11009

Introduction

Question: The coin is tossed 2 times. Find the probability of getting atmost one head.

Solution: Let X be the random variable representing the number of heads obtained in two coin tosses. The given information is summarised in Table 0

| Parameters | Value | Description |
|------------|-------|-----------------------------------|
| n | 2 | Number of trials in an Experiment |
| p | 1/2 | Probability of Success |
| q | 1/2 | Probability of Failure |

TABLE 0

where n is the number of trials, p is the probability of getting heads in a fair coin and q is the probability of not getting a head in a fair coin. The probability mass function (PMF) of X is given by:1

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (1)$$

The probability of getting at most one head is the same as finding $\Pr(X \leq 1)$. To do this using the CDF method, we need to calculate the cumulative distribution function (CDF) of X :

$$F_X(k) = \Pr(X \leq k) = \sum_{k=-\infty}^k p_X(k) = \sum_{k=-\infty}^k \binom{n}{k} p^k (1-p)^{n-k} \quad (2)$$

\therefore

$$F_X(1) = \Pr(X \leq 1) = p_X(0) + p_X(1) \quad (3)$$

Now, by using 1

\therefore
for $k=0$,

$$p_X(0) = \Pr(X = 0) = \binom{2}{0} \left(\frac{1}{2}\right)^0 \left(1 - \left(\frac{1}{2}\right)\right)^2 = \frac{1}{4} \quad (4)$$

for $k=1$

$$p_X(1) = \Pr(X = 1) = \binom{2}{1} \left(\frac{1}{2}\right)^1 \left(1 - \left(\frac{1}{2}\right)\right)^1 = \frac{1}{2} \quad (5)$$

From the Cumulative distribution function(CDF) of X given in equation 2,

$$F_x(1) = \Pr(X \leq 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \quad (6)$$

\therefore probability of getting at most one head is $\frac{3}{4}$.