Probability and Random Variable Assignment-1

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Introduction

Question: The coin is tossed 2 times. Find the probability of getting atmost one head.
Solution: Let X be the random variable representing the number of heads obtained in two coin tosses. The given information is summarised in Table 0

Parameters	Value	Description
n	2	Number of trials in an Experiment
p	1/2	Probability of Success
q	1/2	Probability of Failure

TABLE 0

where n is the number of trials, p is the probability of getting heads in a fair coin and q is the probability of not getting a head in a fair coin. The probability mass function (PMF) of X is given by:

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{1}$$

The probability of getting at most one head is the same as finding $\Pr(X \leq 1)$. To do this using the CDF method, we need to calculate the cumulative distribution function (CDF) of X:

$$F_x(k) = \Pr(X \leq k) = \sum_{k=-\infty}^{k} p_X(k) = \sum_{k=-\infty}^{k} \binom{n}{k} p^k (1-p)^{n-k} \tag{2}$$

$\therefore$

$$F_x(1) = \Pr(X \leq 1) = p_X(0) + p_X(1) \tag{3}$$

Now, by using equation (1),

$\therefore$

for k=0,

$$p_X(0) = \Pr(X = 0) = \binom{2}{0} \left(\frac{1}{2}\right)^0 \left(1 - \left(\frac{1}{2}\right)\right)^2 = \frac{1}{4} \tag{4}$$

for k=1

$$p_X(1) = \Pr(X = 1) = \binom{2}{1}\left(\frac{1}{2}\right)^1\left(1 - \left(\frac{1}{2}\right)\right)^1 = \frac{1}{2} \tag{5}$$

From the Cumulative distribution function(CDF) of X given in equation (2),

$$F_x(1) = \Pr(X \leq 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \tag{6}$$

$\therefore$ probability of getting at most one head is $\frac{3}{4}$.