Probability and Random Variable Assignment-1

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Introduction

Question: The coin is tossed 2 times. Find the probability of getting atmost one head.

Solution: Let X be the random variable representing the number of heads obtained in two coin tosses. Since there are only two possible outcomes for each coin toss, the distribution of X follows a binomial distribution with parameters n = 2 and p = 1/2, where n is the number of trials and p is the probability of getting a successful head in a fair coin.

Now, The probability mass function (PMF) of K is given by:

$$\Pr(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{1}$$

For this problem, we want to find the probability of getting at most one head, which is the same as finding $\Pr(X \leq 1)$. To do this using the CDF method, we need to calculate the cumulative distribution function (CDF) of X:

$$F(k) = \Pr(X \leq k) = \sum_{k=-\infty}^{k} \Pr(X = k) \tag{2}$$

$\therefore$

$$F(1) = \Pr(X \leq 1) = \Pr(X = 0) + \Pr(X = 1) \tag{3}$$

where F(K) represents the cumulative distribution function of a random variable k.

Now, by using equation (1),

$\therefore$

$$\Pr(X = 0) = \binom{2}{0}\left(\frac{1}{2}\right)^0\left(1 - \left(\frac{1}{2}\right)\right)^2 = \frac{1}{4} \tag{4}$$

$$\Pr(X = 1) = \binom{2}{1}\left(\frac{1}{2}\right)^1\left(1 - \left(\frac{1}{2}\right)\right)^1 = \frac{1}{2} \tag{5}$$

From the Cumulative distribution function(CDF) of X given in equation (2) and substituting equation (4) and (5),

$$F(1) = \Pr(X \leq 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \tag{6}$$

$\therefore$ probability of getting at most one head is $\frac{3}{4}$.