

Probability and Random Variable

Assignment-1

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Introduction

Question: The coin is tossed 2 times. Find the probability of getting atmost one head.

solution: Let K be the random variable representing the number of heads obtained in two coin tosses. Since there are only two possible outcomes for each coin toss, the distribution of K follows a binomial distribution with parameters $n = 2$ and $p = 1/2$, where n is the number of trials and p is the probability of getting a successful head in a fair coin.

Now, The probability mass function (PMF) of K is given by:

$$\Pr(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

For this problem, we want to find the probability of getting at most one head, which is the same as finding $\Pr(K \leq 1)$. To do this using the CDF method, we need to calculate the cumulative distribution function (CDF) of K:

$$F(k) = \Pr(K \leq k) = \sum_{k=-\infty}^x \Pr(K = k)$$

\therefore

$$F(1) = \Pr(K \leq 1) = \Pr(K = 0) + \Pr(K = 1)$$

where F(x) represents the cumulative distribution function of a random variable X.

\therefore

$$\Pr(K = 0) = \binom{2}{0} \left(\frac{1}{2}\right)^0 \left(1 - \left(\frac{1}{2}\right)\right)^2 = \frac{1}{4}$$

$$\Pr(K = 1) = \binom{2}{1} \left(\frac{1}{2}\right)^1 \left(1 - \left(\frac{1}{2}\right)\right)^1 = \frac{1}{2}$$

From the Cumulative distribution function(CDF) of K given above

$$\Pr(K \leq 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

\therefore probability of getting at most one head is $\frac{3}{4}$.