

Today: My summer work & future research



- Motivation
- The PNP problem
 - Least-Squares
 - Fully Bayesian
 - MCMC

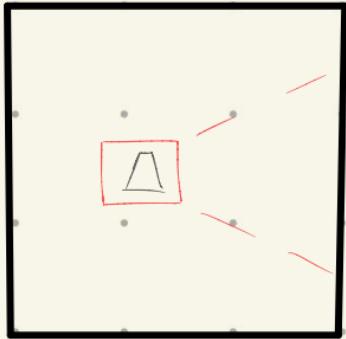


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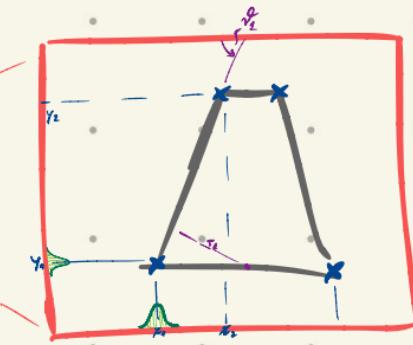
- Software Framework
 - Automatic Diff.
 - Unitful type system
- Faster VR through SoftArgMax



The VLS Problem



Stage 1:
Find runway



Stage 2:
Extract runway
parametrization



Step 3:
Solve position

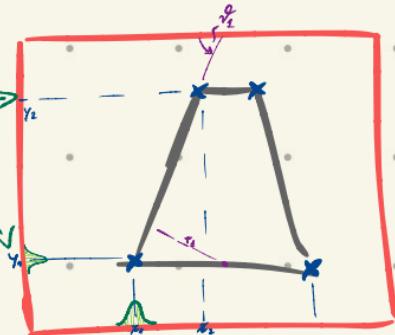
Why camera?

Why uncertainty?

Useful features?

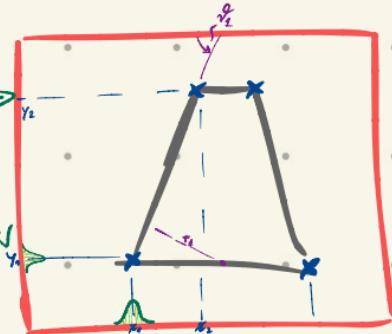
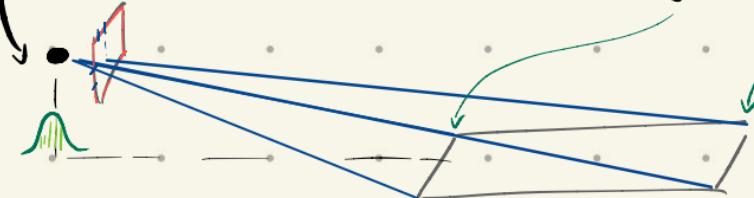
Sensitivity to
errors?

How to solve for this given



First approach:

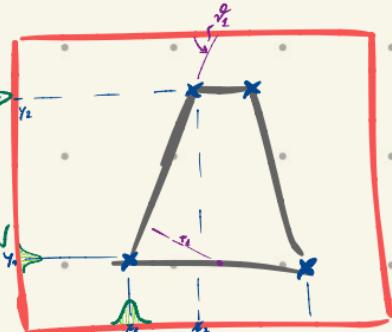
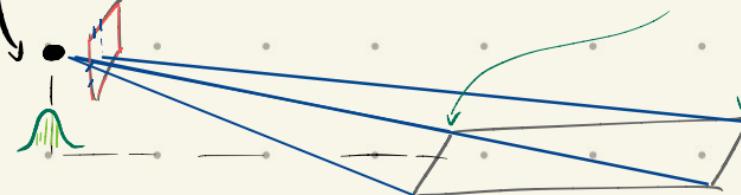
How to solve for this given



First approach:

$$\begin{aligned} \operatorname{argmin}_{\text{of}} & [\text{project}(\text{corner}_1) - \text{measured}(\text{corner}_1)]^2 \\ & + [\text{project}(\text{corner}_2) - \text{measured}(\text{corner}_2)]^2 \\ & + \dots \\ & + [\text{angle}_{\text{rhs}} - \text{angle}_{\text{measured}}]^2 \\ & + \dots \end{aligned}$$

How to solve for this given



First approach:

$$\underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_i [f_i(\boldsymbol{\theta}) - o_i]^2$$

↑
location
↓
observations
from stage 2

Problems:

- sensitivity of minimum?
- $[1m]^2 + [1^{\circ}]^2$?!
- Using uncertainties?
- producing uncertainties?
- weighting terms by "influence"?
- correlated errors?

Task: Study usefulness of sideline angles.

Approach 1: OpenCV PnP solver.

18:50 Tue 14 Nov



4.8.0-dev

Open Source Computer Vision

Main Page Related Pages Modules Namespaces ▾ Classes ▾ Files ▾ Examples Java documentation

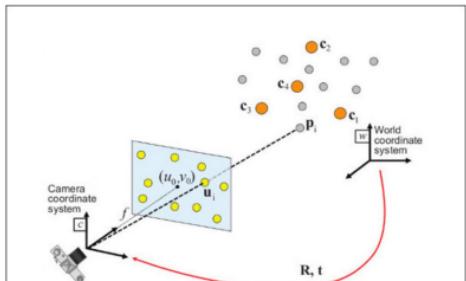
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Perspective-n-Point (PnP) pose computation

Pose computation overview

The pose computation problem [178] consists in solving for the rotation and translation that minimizes the reprojection error from 3D-2D point correspondences.

The `solvePnP` and related functions estimate the object pose given a set of object points, their corresponding image projections, as well as the camera intrinsic matrix and the distortion coefficients, see the figure below (more precisely, the X-axis of the camera frame is pointing to the right, the Y-axis downward and the Z-axis forward).



Essentially:

$$\arg \min \sum_i (\text{project}(\text{corner}_i) - \text{measured}(\text{corner}_i))^2$$

- hand-computed gradients
- no weights

→ no angles?
→ points only



18:51 Tue 14 Nov

$$\begin{bmatrix} \mathbf{R} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{Z}_w \\ 1 \end{bmatrix}$$

Pose computation methods

Refer to the `cv::SolvePnPMethod` enum documentation for the list of possible values. Some details about each method are as follows:

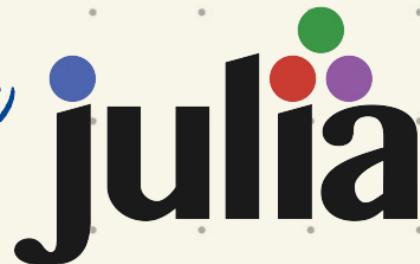
- `cv::SOLVEPNP_ITERATIVE`: Iterative method based on a Levenberg-Marquardt optimization. In this case the function minimizes reprojection error, that is the sum of squared distances between the observed projections `"imagePoints"` and `cv::projectPoints` `"objectPoints"`. Initial solution for non-planar `"objectPoints"` needs at least 6 points and uses the DL planar `"objectPoints"` needs at least 4 points and uses pose from homography decomposition.
- `cv::SOLVEPNP_P3P`: Method is based on the paper of X. S. Gao, X.-R. Hou, J. Tang, H.-F. Chang "Complete Solution of the Three-Point Problem" ([96]). In this case the function requires exactly four object and image points.
- `cv::SOLVEPNP_AP3P`: Method is based on the paper of T. Ke, S. Roumeliotis "An Efficient Algebraic Solution to the Perspective-Three-Point Problem" ([141]). In this case the function requires exactly four object and image points.
- `cv::SOLVEPNP_EPNP`: Method has been introduced by F. Moreno-Noguer, V. Lepetit and P. Fua in the paper "EPnP: Efficient Camera Pose Estimation" ([153]).
- `cv::SOLVEPNP_DLS`: Broken implementation. Using this flag will fallback to EPnP. Method is based on the paper of J. Hesch and S. Roumeliotis "A Direct Least-Squares (DLS) Method for PnP" ([122]).
- `cv::SOLVEPNP_UPNP`: Broken implementation. Using this flag will fallback to EPnP. Method is based on the paper of A. Penate-Sánchez, J. Andrade-Cetto, F. Moreno-Noguer "Exhaustive Linearization for Focal Length Estimation" ([209]). In this case the function also estimates the parameters f_x and f_y , assuming that both cameraMatrix is updated with the estimated focal length.
- `cv::SOLVEPNP_IPPE`: Method is based on the paper of T. Collins and A. Bartoli. "Infinitesimal Plane-Based Pose Estimation requires coplanar object points".
- `cv::SOLVEPNP_IPPE_SQUARE`: Method is based on the paper of Toby Collins and Adrien Bartoli. "Infinitesimal Plane-Based Pose Estimation for marker pose estimation. It requires 4 coplanar object points defined in the following order:
 - point 0: [squareLength / 2, squareLength / 2, 0]
 - point 1: [squareLength / 2, squareLength / 2, 0]
 - point 2: [squareLength / 2, squareLength / 2, 0]
 - point 3: [squareLength / 2, squareLength / 2, 0]
- `cv::SOLVEPNP_SQPNP`: Method is based on the paper "A Consistently Fast and Globally Optimal Solution to the Perspective-Terakis and M. Loukakis ([263]). It requires 3 or more points.

P3P

The `cv::solveP3P()` estimates an object pose from exactly 3 3D-2D point correspondences. A P3P problem has up to 4 solutions.

Approach 2: Write own (flexible) PuP solver.

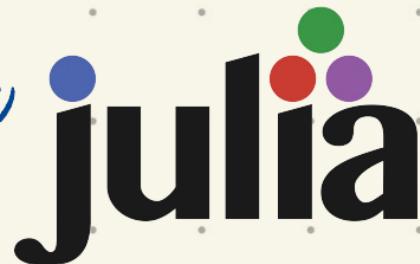
- projection code
- automatic differentiation, 2nd order (?) or third!
- trust-region Newton method
- or Levenberg–Marquardt method



→ recent funding by
Boeing, Chan-Zuckerberg
foundation, Williams F1 team,
& others

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-



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Two challenges left

- 1) How to add $[px]$, $[m]$, $[^o]$, etc... ?
- 2) How much error do we typically make ?



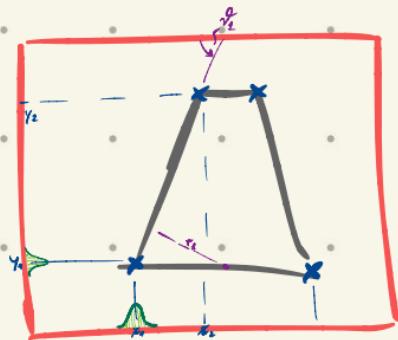
"simulation"

$$\arg \min_{\theta} \sum_i [f_i(\theta) - o_i]^2$$

location
observations
from stage 2

$$\arg \min_{\theta} \sum_i \left[\frac{f_i(\theta) - o_i}{\sigma} \right]^2$$

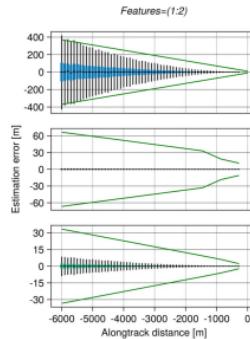
typical error
standard deviation



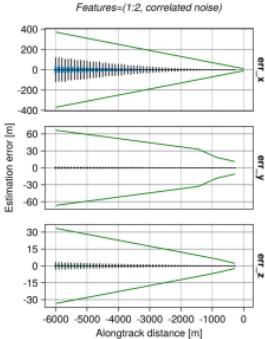
Demo time

Some results:

Estimation errors over alongtrack distance



Estimation errors over alongtrack distance



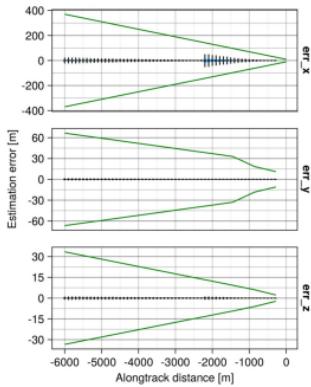
correlated
noise

multiple
runways

adding
angles

no improvement
for $\sigma_{\text{angle}} > 1^\circ$

Estimation errors over alongtrack distance (•)



σ_{angle}	0.0°	0.01°	0.1°	0.3°	0.5°	1.0°	no angles
σ_x	101.22	104.61	105.89	110.58	122.37	159.51	156.54
σ_y	0.3863	0.3967	0.3986	0.4267	0.4869	0.5968	0.5678
σ_z	2.1189	2.1929	2.2021	2.3073	2.5945	3.4132	3.3251

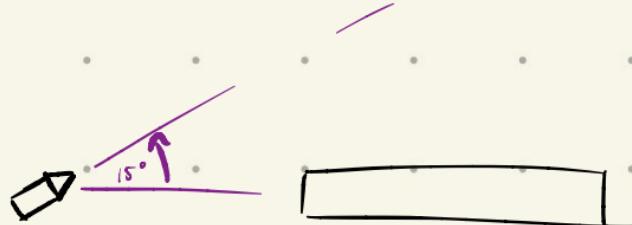
✓33% for $\sigma_{\text{angle}} < 0.1^\circ$

More results:

Influence of misalignment

σ_{angle}	0.0°	5.0°	10.0°	15.0°	20.0°	25.0°
σ_x	3.622	4.034	15.41	43.16	62.25	71.97
σ_y	0.319	0.350	4.641	32.24	46.76	55.61
σ_z	0.327	0.333	1.242	14.29	21.11	27.26

$\sim 10x$ at 15°



Influence of attitude error

σ_{angle}	0.0°	0.01°	0.1°	0.3°	0.5°	1.0°
σ_x	146.39	148.52	144.93	149.27	150.55	150.22
σ_y	0.5724	0.8271	6.1943	17.536	30.030	58.797
σ_z	3.1291	3.1956	6.8751	16.831	29.688	57.163

$10-60x$
error increase if
attitude is wrong 0.5°

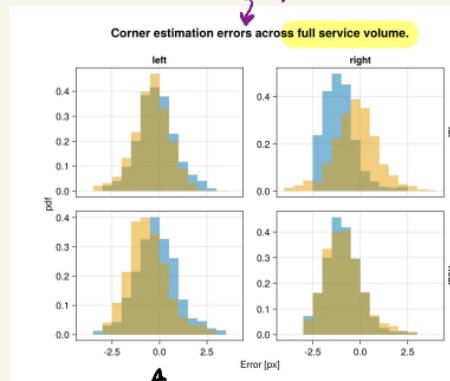


POV you're launching
in Australia.

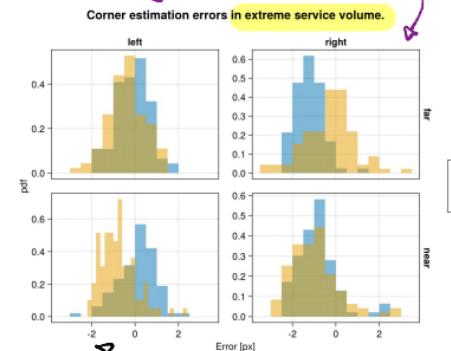


Interlude: Empirical error distribution

~1300 samples



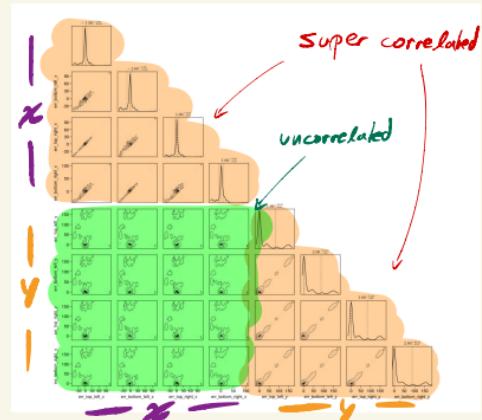
only ~130 samples?



Approx. 1pxl of std

Takeaways:

- More hand-labeled data in extreme service vol
- huge discrepancy between auto-labels & hand-labels
- 1 pxl of std error is pretty good
- huge error correlation is noteworthy



Other topics & continuing research

- analytic approach to sensitivity
- more principled loss
- fully Bayesian "one-shot" pose uncertainty
- pose uncertainty "ground truth" through MCMC
- Stage 2 NN design w/ big speedup & easy uncertainties.

