# Sogang University

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# Graph Algorithm

## Dijkstra’s Shortest Path

#include <queue>

#include <vector>

#include <algorithm>

**using** **namespace** std;

**typedef** pair<**int**,**int**> ii;

vector<vector<ii> > v;

vector<**int**> d;

**const** **int** inf=0x7FFFFFFF;

/\*

 \* v.resize(V+!),d.resize(V+1);

 \*/

**int** dijkstra(**int** s,**int** e) {

    priority\_queue<ii,vector<ii>,greater<ii> > pq;

    fill(d.begin(),d.end(),inf);

    d[s]=0;

    pq.push(ii(d[s],s));

**while** ( !pq.empty() ){

        ii now=pq.top();pq.pop();

**int** cur=now.second;

**if** ( d[cur] < now.first ) **continue**;

**for** ( **int** i = 0 ; i < v[cur].size() ; i++ ) {

            ii next=v[cur][i];

**if** ( d[next.first] > d[cur]+next.second ) {

                d[next.first] = d[cur]+next.second;

                pq.push(ii(d[next.first],next.first));

            }

        }

    }

**return** d[e];

}

## Strongly Connected Component & Bi-connected Component

cc::graph[x].push\_back(y); // 정점 x와 y가 연결됨

result = cc::scc(size); // Strongly Connected Component의 개수

f = (connected[i] == connected[j]); // 정점 i와 j가 같은 SCC에 속하는가?

cc::bcc(size);

n = cc::cut\_vertex\_num; // 절점의 개수

b = cc::cut\_vertex[i]; // 정점 i가 절점인가?

n = cc::cut\_edge\_num; // 절선의 개수

p = cc::cut\_edge[i][0], q = cc::cut\_edge[i][1]; // i번째 절선 p-q

#include <cstdlib>

#include <vector>

**using** **namespace** std;

**namespace** cc

{

**const** **int** SIZE = 10000;

    vector<**int**> graph[SIZE];

**int** connected[SIZE];

**int** cut\_vertex\_num;

**bool** cut\_vertex[SIZE];

**int** cut\_edge\_num, cut\_edge[SIZE][2];

**int** order[SIZE];

**int** visit\_time[SIZE], finish[SIZE], back[SIZE];

**int** stack[SIZE], seen[SIZE];

#define MIN(a,b) (a) = ((a)<(b))?(a):(b)

**int** dfs(**int** size) {

**int** top, cnt, cnt2, cnt3;

**int** i;

        cnt = cnt2 = cnt3 = 0;

        stack[0] = 0;

**for** (i = 0 ; i < size ; i++) visit\_time[i] = -1;

**for** (i = 0 ; i < size ; i++) cut\_vertex[i] = **false**; // CUT VERTEX

        cut\_edge\_num = 0; // CUT\_EDGE

**for** (i = 0 ; i < size ; i++) {

**if** (visit\_time[order[i]] == -1) {

                top = 1;

                stack[top] = order[i];

                seen[top] = 0;

                visit\_time[order[i]] = cnt++;

                connected[order[i]] = cnt3++;

**int** root\_child = 0; // CUT VERTEX

**while** (top > 0) {

**int** j, now = stack[top];

**if** (seen[top] == 0) back[now] = visit\_time[now]; // NOT FOR SCC

**for** (j = seen[top] ; j < graph[now].size() ; j++) {

**int** next = graph[now][j];

**if** (visit\_time[next] == -1) {

**if** (top == 1) root\_child++; // CUT VERTEX

                            seen[top] = j + 1;

                            stack[++top] = next;

                            seen[top] = 0;

                            visit\_time[next] = cnt++;

                            connected[next] = connected[now];

**break**;

                        }

**else** **if** (top == 1 || next != stack[top - 1]) // NOT FOR SCC

                            MIN(back[now], visit\_time[next]); // NOT FOR SCC

                    }

**if** (j == graph[now].size()) {

                        finish[cnt2++] = now; // NOT FOR BCC

                        top--;

**if** (top > 1) {

                            MIN(back[stack[top]], back[now]); // NOT FOR SCC

**if** (back[now] >= visit\_time[stack[top]]) { // CUT VERTEX

                                cut\_vertex[stack[top]] = **true**;

                                cut\_vertex\_num++;

                            }

                        }

                        // CUT EDGE

**if** (top > 0 && visit\_time[stack[top]] < back[now]) {

                            cut\_edge[cut\_edge\_num][0] = stack[top];

                            cut\_edge[cut\_edge\_num][1] = now;

                            cut\_edge\_num++;

                        }

                    }

                }

**if** (root\_child > 1) { // CUT VERTEX

                    cut\_vertex[order[i]] = **true**;

                    cut\_vertex\_num++;

                }

            }

        }

**return** cnt3; // number of connected component

    }

#undef MIN

    vector<**int**> graph\_rev[SIZE];

**void** graph\_reverse(**int** size) {

**for** (**int** i = 0 ; i < size ; i++) graph\_rev[i].clear();

**for** (**int** i = 0 ; i < size ; i++)

**for** (**int** j = 0 ; j < graph[i].size() ; j++)

                graph\_rev[graph[i][j]].push\_back(i);

**for** (**int** i = 0 ; i < size ; i++) graph[i] = graph\_rev[i];

    }

**int** scc(**int** size) {

**int** n;

**for** (**int** i = 0 ; i < size ; i++) order[i] = i;

        dfs(size);

        graph\_reverse(size);

**for** (**int** i = 0 ; i < size ; i++) order[i] = finish[size - i - 1];

        n = dfs(size);

        graph\_reverse(size);

**return** n;

    }

**void** bcc(**int** size) {

**for** (**int** i = 0 ; i < size ; i++) order [ i ] = i;

        dfs(size);

        cut\_vertex\_num = 0;

**for** (**int** i = 0 ; i < size ; i++)

**if** (cut\_vertex[i])

                cut\_vertex\_num++;

    }

} // namespace cc

## Min-cost Max-flow using bellman-ford algorithm

mcmf::init(graph, size); // 그래프 초기화

result = mcmf::maximum\_flow(source, sink); // 최대 매칭, 최소 비용 pair

#include <cstring>

#include <vector>

#include <algorithm>

**using** **namespace** std;

**struct** edge {

**int** target;

**int** capacity; // cap\_t

**int** cost; // cost\_t

};

**namespace** mcmf

{

**typedef** **int** cap\_t; // capacity type

**typedef** **int** cost\_t; // cost type

**const** **int** SIZE = 300;

**const** cap\_t CAP\_INF = 0x7fFFffFF;

**const** cost\_t COST\_INF = 0x7fFFffFF;

**int** n;

    vector<pair<pair<**int**, edge>, **int**> > g;

**int** p[SIZE];

    cost\_t dist[SIZE];

    cap\_t mincap[SIZE];

**int** pth[SIZE];

**void** init(**const** vector<edge> graph[], **int** size) {

**int** i, j;

        n = size;

**memset**(p, -1, **sizeof**(p));

        g.clear();

**for** (i = 0 ; i < size ; i++) {

**for** (j = 0 ; j < graph[i].size() ; j++) {

**int** next = graph[i][j].target;

                edge tmp = graph[i][j];

                g.push\_back(make\_pair(make\_pair(i, tmp), p[i]));

                p[i] = g.size() - 1;

                tmp.target = i;

                tmp.capacity = 0;

                tmp.cost = -tmp.cost;

                g.push\_back(make\_pair(make\_pair(next, tmp), p[next]));

                p[next] = g.size() - 1;

            }

        }

    }

**int** bellman(**int** s, **int** t) {

**int** i, j;

**for** (i = 0 ; i < n ; i++) {

            dist[i] = COST\_INF;

            mincap[i] = 0;

        }

        dist[s] = 0;

        mincap[s] = CAP\_INF;

**bool** flg = **false**;

**for** (i = 0 ; i < n ; i++) {

            flg = **false**;

**for** (j = 0 ; j < g.size() ; j++) {

**int** now, next;

**if** (g[j].first.second.capacity == 0) **continue**;

                now = g[j].first.first;

                next = g[j].first.second.target;

**if** (dist[now] == COST\_INF) **continue**;

**if** (dist[now] + g[j].first.second.cost < dist[next]) {

                    dist[next] = dist[now] + g[j].first.second.cost;

                    pth[next] = j;

                    mincap[next] = min(mincap[now], g[j].first.second.capacity);

                    flg = **true**;

                }

            }

**if** (!flg) **break**;

        }

**if** (flg) **return** -1;

**return** dist[t] != COST\_INF ? 1 : 0;

    }

    pair<cap\_t, cost\_t> maximum\_flow(**int** source, **int** sink) {

        cap\_t total\_flow = 0;

        cost\_t total\_cost = 0;

**int** state;

**while** ((state = bellman(source,sink)) > 0) {

            cap\_t f = mincap[sink];

            total\_flow += f;

            total\_cost += f \* dist[sink];

**for** (**int** i = sink ; i != source; i = g[pth[i]].first.first) {

                g[pth[i]].first.second.capacity -= f;

                g[pth[i] ^ 1].first.second.capacity += f;

            }

        }

**if** (state == -1) **while** (**true**); // it's NP-Hard

**return** make\_pair(total\_flow, total\_cost);

    }

} // namespace mcmf

## Min-cost Max-flow using dijkstra algorithm

mcmf::init(graph, size); // 그래프 초기화

result = mcmf::maximum\_flow(source, sink); // 최대 매칭, 최소 비용 pair

#include <cstring>

#include <queue>

#include <vector>

#include <algorithm>

#include <functional>

**using** **namespace** std;

**struct** edge {

**int** target;

**int** capacity; // cap\_t

**int** cost; // cost\_t

};

**namespace** mcmf

{

**typedef** **int** cap\_t; // capacity type

**typedef** **int** cost\_t; // cost type

**const** **int** SIZE = 5000;

**const** cap\_t CAP\_INF = 0x7fFFffFF;

**const** cost\_t COST\_INF = 0x7fFFffFF;

**int** n;

    vector<pair<edge, **int**> > g;

**int** p[SIZE];

    cost\_t dist[SIZE];

    cap\_t mincap[SIZE];

    cost\_t pi[SIZE];

**int** pth[SIZE];

**int** from[SIZE];

**bool** v[SIZE];

**void** init(**const** vector<edge> graph[], **int** size){

**int** i, j;

        n = size;

**memset**(p, -1, **sizeof**(p));

        g.clear();

**for** (i = 0 ; i < size ; i++) {

**for** (j = 0 ; j < graph[i].size() ; j++) {

**int** next = graph[i][j].target;

                edge tmp = graph[i][j];

                g.push\_back(make\_pair(tmp, p[i]));

                p[i] = g.size() - 1;

                tmp.target = i;

                tmp.capacity = 0;

                tmp.cost = -tmp.cost;

                g.push\_back(make\_pair(tmp, p[next]));

                p[next] = g.size() - 1;

            }

        }

    }

**int** dijkstra(**int** s, **int** t) {

**typedef** pair<cost\_t, **int**> pq\_t;

        priority\_queue<pq\_t, vector<pq\_t>, greater<pq\_t> > pq;

**int** i;

**for** (i = 0 ; i < n ; i++) {

            dist[i] = COST\_INF;

            mincap[i] = 0;

            v[i] = **false**;

        }

        dist[s] = 0;

        mincap[s] = CAP\_INF;

        pq.push(make\_pair(0, s));

**while** (!pq.empty()) {

**int** now = pq.top().second;

            pq.pop();

**if** (v[now]) **continue**;

            v[now] = **true**;

**for** (i = p[now] ; i != -1 ; i = g[i].second) {

**int** next = g[i].first.target;

**if** (v[next]) **continue**;

**if** (g[i].first.capacity == 0) **continue**;

                cost\_t pot = dist[now] + pi[now] - pi[next] + g[i].first.cost;

**if** (dist[next] > pot) {

                    dist[next] = pot;

                    mincap[next] = min(mincap[now], g[i].first.capacity);

                    pth[next] = i;

                    from[next] = now;

                    pq.push(make\_pair(dist[next], next));

                }

            }

        }

**for** (i = 0 ; i < n ; i++) pi[i] += dist[i];

**return** dist[t] != COST\_INF;

    }

    pair<cap\_t, cost\_t> maximum\_flow(**int** source, **int** sink) {

**memset**(pi, 0, **sizeof**(pi));

        cap\_t total\_flow = 0;

        cost\_t total\_cost = 0;

**while** (dijkstra(source, sink)) {

            cap\_t f = mincap[sink];

            total\_flow += f;

**for** (**int** i = sink ; i != source ; i = from[i]) {

                g[pth[i]].first.capacity -= f;

                g[pth[i] ^ 1].first.capacity += f;

                total\_cost += g[pth[i]].first.cost \* f;

            }

        }

**return** make\_pair(total\_flow, total\_cost);

    }

} // namespace mcmf

## Network Flow

netflow::n = XX; // 정점 개수

netflow::capacity[i][j] = XX; // 정점 i에서 j로의 용량

result = netflow::maximum\_flow(source, sink);

f = netflow::flow[i][j]; // 정점 i에서 j로 흐르는 유량

#include <cstring>

#include <queue>

**using** **namespace** std;

**namespace** netflow

{

**typedef** **int** val\_t;

**const** **int** SIZE = 1000;

**const** val\_t INF = 0x7fFFffFF;

**int** n;

    val\_t capacity[SIZE][SIZE];

    val\_t total\_flow;

    val\_t flow[SIZE][SIZE];

**int** back[SIZE];

**inline** val\_t res(**int** a, **int** b) {

**return** capacity[a][b] - flow[a][b];

    }

    val\_t push\_flow(**int** source, **int** sink) {

**memset**(back, -1, **sizeof**(back));

        queue<**int**> q;

        q.push(source);

        back[source] = source;

**while** (!q.empty() && back[sink] == -1) {

**int** now = q.front();

            q.pop();

**for** (**int** i = 0 ; i < n ; i++) {

**if** (res(now, i) > 0 && back[i] == -1) {

                    back[i] = now;

                    q.push(i);

                }

            }

        }

**if** (back[sink] == -1) **return** 0;

**int** now, bef;

        val\_t f = INF;

**for** (now = sink ; back[now] != -1 ; now = back[now])

            f = min(f, res(back[now], now));

**for** (now = sink ; back[now] != -1 ; now = back[now]) {

            bef = back[now];

            flow[bef][now] += f;

            flow[now][bef] = -flow[bef][now];

        }

        total\_flow += f;

**return** f;

    }

    val\_t maximum\_flow(**int** source, **int** sink) {

**memset**(flow, 0, **sizeof**(flow));

        total\_flow = 0;

**while** (push\_flow(source, sink));

**return** total\_flow;

    }

} // namespace netflow

## Network-flow using DINIC algorithm

#include <cstdio>

#include <vector>

#include <limits>

#include <iostream>

#include <queue>

#pragma warning(disable:4996)

**using** **namespace** std;

**struct** NetworkFlow

{

**typedef** **long** **long** Weight;

**struct** Edge {

**int** to; unsigned next;

        Weight cap, flow;

        Edge(**int** to, Weight cap, unsigned next = ~0) : to(to), cap(cap), flow(0), next(next) {}

**inline** Weight res() **const** { **return** cap - flow; }

    };

**int** V;

    Weight totalFlow;

    vector<Edge> edges;

    vector<unsigned> G;

    NetworkFlow(**int** V) : V(V), G(V, ~0), totalFlow(0) {}

    // DINIC Algorithm

    vector<**int**> d;

    vector<unsigned> p;

**void** addEdge(**int** a, **int** b, Weight cab, Weight cba = 0) {

        edges.push\_back( Edge(b, cab, G[a]) );

        G[a] = edges.size() - 1;

        edges.push\_back( Edge(a, cba, G[b]) );

        G[b] = edges.size() - 1;

    }

**bool** levelGraph(**int** S, **int** T) {

        queue<**int**> q; q.push(S);

        d = vector<**int**>(V, -1);

        d[S] = 0;

**while**(!q.empty() && d[T] == -1) {

**int** u = q.front(); q.pop();

**for**(unsigned i = G[u]; i != ~0; i = edges[i].next) {

                Edge &e = edges[i];

**int** v = e.to;

**if**(e.res() > 0 && d[v] == -1) { d[v] = d[u] + 1; q.push(v); }

            }

        }

**return** d[T] != -1;

    }

**int** pushFlow(**int** u, **int** T, Weight amt) {

**if**(!amt || u == T) **return** amt;

**for**(unsigned &i = p[u]; i != ~0; i = edges[i].next) {

            Edge &e = edges[i], &rev = edges[i ^ 1];

**int** v = e.to;

**if**(e.res() > 0 && d[u] + 1 == d[v]) {

                Weight f = pushFlow(v, T, min(e.res(), amt));

**if**(f > 0) {

                    e.flow += f, rev.flow -= f;

**return** f;

                }

            }

        }

**return** 0;

    }

    Weight maxFlow(**int** S, **int** T) {

        totalFlow = 0;

**while**( levelGraph(S, T) ) {

            p = G;

**while**(Weight f = pushFlow(S, T, numeric\_limits<Weight>::max()))

                totalFlow += f;

        }

**return** totalFlow;

    }

};

**int** main() {

**int** n, m;

**scanf**("%d%d", &n, &m);

    NetworkFlow nf(n);

**for**(**int** i=1; i<=m; ++i) {

**int** a, b, c;

**scanf**("%d%d%d", &a, &b, &c);

**if**(a == b) **continue**;

        --a; --b;

        nf.addEdge(a, b, c); // uni-directional

        nf.addEdge(a, b, c, c); // bi-directional

    }

**printf**("%lld\n", nf.maxFlow(0, n-1));

**return** 0;

}

## Bipartite Matching Using DFS Only

#include <cstdio>

#include <cstring>

#include <vector>

#include <algorithm>

**using** **namespace** std;

#define MAX\_V 1000

vector<vector<**int**> > v;

**int** backMatch[MAX\_V\*2+5];

**bool** visited[MAX\_V\*2+5];

**bool** dfs(**int** now) {

**if** ( visited[now] ) **return** **false**;

    visited[now] = **true**;

**for** ( **int** i = 0 ; i < v[now].size() ; i++ ) {

**int** next = v[now][i];

**if** ( backMatch[next] == -1 || dfs(backMatch[next]) ) {

            backMatch[next] = now;

**return** **true**;

        }

    }

**return** **false**;

}

**int** BipartiteMatching() {

**memset**(backMatch,-1,**sizeof**(backMatch));

**int** matched =0;

**for** ( **int** i = 0 ; i < v.size() ; i++ ) {

**memset**(visited,**false**,**sizeof**(visited));

**if** ( dfs(i) ) matched++;

    }

**return** matched;

}

## Bipartite Matching Using Hopcroft-Karp Algorithm

#include <cstdio>

#include <queue>

#include <vector>

#include <algorithm>

**using** **namespace** std;

#define MAX\_V 1004

**const** **int** inf = 987654321;

**int** N,M;

**int** used[MAX\_V],match[MAX\_V],d[MAX\_V];

vector<vector<**int**> > v;

queue<**int**> q;

**void** bfs() {

**for** ( **int** i = 1 ; i <= N ; i++ )

        d[i] = inf;

**for** ( **int** i = 1 ; i <= N ; i++ )

**if** ( !used[i] ) d[i] =0,q.push(i);

**while** ( !q.empty() ) {

**int** now = q.front();q.pop();

**for** ( **int** i = v[now].size() ; i-- ; ) {

**int** next = v[now][i];

**if** ( match[next] && d[match[next]] == inf )

                d[match[next]] = d[now]+1,q.push(match[next]);

        }

    }

}

**bool** dfs(**int** now) {

**for** ( **int** i = v[now].size() ; i-- ; ) {

**int** next = v[now][i];

**if** ( !match[next] || d[match[next]] == d[now]+1 && dfs(match[next]) ) {

            used[now] = **true**, match[next] = now;

**return** **true**;

        }

    }

**return** **false**;

}

**int** matching() {

**int** ret=0;

**while** ( **true** ) {

        bfs();

**int** flow=0;

**for** ( **int** i = 1 ; i<= N ; i++ )

**if** ( !used[i] && dfs(i) ) flow++;

        ret += flow;

**if** ( !flow ) **break**;

    }

**return** ret;

}

## Hungarian Method

hungarian::n = XX; // 정점 개수

hungarian::cost[i][j] = XX; // 비용 테이블

result = hungarian::hungarian(); // 최대 매칭

y = hungarian::xy[x]; // 정점 x와 연결된 정점 번호

x = hungarian::yx[y]; // 정점 y와 연결된 정점 번호

#include <cstring>

#include <queue>

#include <algorithm>

#include <limits>

**using** **namespace** std;

**namespace** hungarian

{

**typedef** **double** val\_t;

**const** **int** SIZE = 100;

**const** val\_t INF = numeric\_limits<**double**>::infinity();

    // 두 값이 같은지 비교

**inline** **bool** eq(val\_t a, val\_t b) {

**static** **const** **double** eps = 1e-9;

**return** (a - eps < b && b < a + eps);

    }

**int** n;

    val\_t cost[SIZE][SIZE];

**int** xy[SIZE], yx[SIZE];

**int** match\_num;

    val\_t lx[SIZE], ly[SIZE];

**bool** s[SIZE], t[SIZE];

**int** prev[SIZE];

    val\_t hungarian() {

**memset**(xy, -1, **sizeof**(xy));

**memset**(yx, -1, **sizeof**(yx));

**memset**(ly, 0, **sizeof**(ly));

        match\_num = 0;

**int** x, y;

**for** (x = 0 ; x < n ; x++) {

            lx[x] = cost[x][0];

**for** (y = 1 ; y < n ; y++)

                lx[x] = max(lx[x], cost[x][y]);

        }

**for** (x = 0 ; x < n ; x++)

**for** (y = 0 ; y < n ; y++)

**if** (eq(cost[x][y], lx[x] + ly[y]) && yx[y] == -1) {

                    xy[x] = y;

                    yx[y] = x;

                    match\_num++;

**break**;

                }

**while** (match\_num < n) {

**memset**(s, **false**, **sizeof**(s));

**memset**(t, **false**, **sizeof**(t));

**memset**(prev, -1, **sizeof**(prev));

            queue<**int**> q;

**for** (x = 0 ; x < n ; x++) {

**if** (xy[x] == -1) {

                    q.push(x);

                    s[x] = **true**;

**break**;

                }

            }

**bool** flg = **false**;

**while** (!q.empty() && !flg) {

                x = q.front();

                q.pop();

**for** (y = 0 ; y < n ; y++) {

**if** (eq(cost[x][y], lx[x] + ly[y])) {

                        t[y] = **true**;

**if** (yx[y] == -1) {

                            flg = **true**;

**break**;

                        }

**if** (!s[yx[y]]) {

                            s[yx[y]] = **true**;

                            q.push(yx[y]);

                            prev[yx[y]] = x;

                        }

                    }

                }

            }

**if** (flg) {

**int** t1, t2;

**while** (x != -1) {

                    t1 = prev[x];

                    t2 = xy[x];

                    xy[x] = y;

                    yx[y] = x;

                    x = t1;

                    y = t2;

                }

                match\_num++;

            }

**else** {

                val\_t alpha = INF;

**for** (x = 0 ; x < n ; x++) **if** (s[x])

**for** (y = 0 ; y < n ; y++) **if** (!t[y])

                        alpha = min(alpha, lx[x] + ly[y] - cost[x][y]);

**for** (x = 0 ; x < n ; x++) **if** (s[x]) lx[x] -= alpha;

**for** (y = 0 ; y < n ; y++) **if** (t[y]) ly[y] += alpha;

            }

        }

        val\_t ret = 0;

**for** (x = 0 ; x < n ; x++)

            ret += cost[x][xy[x]];

**return** ret;

    }

}// namespace hungarian

# Geometry

## Convex Hull (Subset of Geometry Library)

hull = convex\_hull(points); // convex hull의 꼭지점 좌표 vector

정수 좌표를 사용하고 싶다면 모든 **double**을 **int**나 **long** **long**으로 치환하라.

#include <cmath>

#include <vector>

#include <algorithm>

**using** **namespace** std;

**const** **double** eps = 1e-9;

**inline** **int** diff(**double** lhs, **double** rhs) {

**if** (lhs - eps < rhs && rhs < lhs + eps) **return** 0;

**return** (lhs < rhs) ? -1 : 1;

}

**struct** Point {

**double** x, y;

    Point() {}

    Point(**double** x\_, **double** y\_): x(x\_), y(y\_) {}

};

**inline** **int** ccw(**const** Point& a, **const** Point& b, **const** Point& c) {

**return** diff(a.x \* b.y + b.x \* c.y + c.x \* a.y

            - a.y \* b.x - b.y \* c.x - c.y \* a.x, 0);

}

**inline** **double** dist2(**const** Point &a, **const** Point &b) {

**double** dx = a.x - b.x;

**double** dy = a.y - b.y;

**return** dx \* dx + dy \* dy;

}

**struct** PointSorter {

    Point origin;

    PointSorter(**const** vector<Point>& points) {

        origin = points[0];

**for** (**int** i = 1 ; i < points.size() ; i++) {

**int** det = diff(origin.x, points[i].x);

**if** (det > 0)

                origin = points[i];

**else** **if** (det == 0 && diff(origin.y, points[i].y) > 0)

                origin = points[i];

        }

    }

**bool** operator()(**const** Point &a, **const** Point &b) {

**if** (diff(b.x, origin.x) == 0 && diff(b.y, origin.y) == 0) **return** **false**;

**if** (diff(a.x, origin.x) == 0 && diff(a.y, origin.y) == 0) **return** **true**;

**int** det = ccw(origin, a, b);

**if** (det == 0) **return** dist2(a, origin) < dist2(b, origin);

**return** det < 0;

    }

};

vector<Point> convex\_hull(vector<Point> points) {

**if** (points.size() <= 3)

**return** points;

    PointSorter cmp(points);

    sort(points.begin(), points.end(), cmp);

    vector<Point> ans;

    ans.push\_back(points[0]);

    ans.push\_back(points[1]);

**for**(**int** i = 2 ; i < points.size() ; i++) {

**while** (ans.size() > 1 &&

                ccw(ans[ans.size() - 2], ans[ans.size() - 1], points[i]) >= 0)

            ans.pop\_back();

        ans.push\_back(points[i]);

    }

**return** ans;

}

## General Geometry Library

#include <cmath>

#include <vector>

**using** **namespace** std;

**const** **double** eps = 1e-9;

**inline** **int** diff(**double** lhs, **double** rhs) {

**if** (lhs - eps < rhs && rhs < lhs + eps) **return** 0;

**return** (lhs < rhs) ? -1 : 1;

}

**inline** **bool** is\_between(**double** check, **double** a, **double** b) {

**if** (a < b)

**return** (a - eps < check && check < b + eps);

**else**

**return** (b - eps < check && check < a + eps);

}

**struct** Point {

**double** x, y;

    Point() {}

    Point(**double** x\_, **double** y\_): x(x\_), y(y\_) {}

**bool** operator==(**const** Point& rhs) **const** {

**return** diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;

    }

**const** Point operator+(**const** Point& rhs) **const** {

**return** Point(x + rhs.x, y + rhs.y);

    }

**const** Point operator-(**const** Point& rhs) **const** {

**return** Point(x - rhs.x, y - rhs.y);

    }

**const** Point operator\*(**double** t) **const** {

**return** Point(x \* t, y \* t);

    }

};

**struct** Circle {

    Point center;

**double** r;

    Circle() {}

    Circle(**const** Point& center\_, **double** r\_): center(center\_), r(r\_) {}

};

**struct** Line {

    Point pos, dir;

    Line() {}

    Line(**const** Point& pos\_, **const** Point& dir\_): pos(pos\_), dir(dir\_) {}

};

**inline** **double** inner(**const** Point& a, **const** Point& b) {

**return** a.x \* b.x + a.y \* b.y;

}

**inline** **double** outer(**const** Point& a, **const** Point& b) {

**return** a.x \* b.y - a.y \* b.x;

}

**inline** **int** ccw\_line(**const** Line& line, **const** Point& point) {

**return** diff(outer(line.dir, point - line.pos), 0);

}

**inline** **int** ccw(**const** Point& a, **const** Point& b, **const** Point& c) {

**return** diff(outer(b - a, c - a), 0);

}

**inline** **double** dist(**const** Point& a, **const** Point& b) {

**return** **sqrt**(inner(a - b, a - b));

}

**inline** **double** dist2(**const** Point &a, **const** Point &b) {

**return** inner(a - b, a - b);

}

**inline** **double** dist(**const** Line& line, **const** Point& point, **bool** segment = **false**) {

**double** c1 = inner(point - line.pos, line.dir);

**if** (segment && diff(c1, 0) <= 0) **return** dist(line.pos, point);

**double** c2 = inner(line.dir, line.dir);

**if** (segment && diff(c2, c1) <= 0) **return** dist(line.pos + line.dir, point);

**return** dist(line.pos + line.dir \* (c1 / c2), point);

}

**bool** get\_cross(**const** Line& a, **const** Line& b, Point& ret) {

**double** mdet = outer(b.dir, a.dir);

**if** (diff(mdet, 0) == 0) **return** **false**;

**double** t2 = outer(a.dir, b.pos - a.pos) / mdet;

    ret = b.pos + b.dir \* t2;

**return** **true**;

}

**bool** get\_segment\_cross(**const** Line& a, **const** Line& b, Point& ret) {

**double** mdet = outer(b.dir, a.dir);

**if** (diff(mdet, 0) == 0) **return** **false**;

**double** t1 = -outer(b.pos - a.pos, b.dir) / mdet;

**double** t2 = outer(a.dir, b.pos - a.pos) / mdet;

**if** (!is\_between(t1, 0, 1) || !is\_between(t2, 0, 1)) **return** **false**;

    ret = b.pos + b.dir \* t2;

**return** **true**;

}

**const** Point inner\_center(**const** Point &a, **const** Point &b, **const** Point &c) {

**double** wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);

**double** w = wa + wb + wc;

**return** Point(

            (wa \* a.x + wb \* b.x + wc \* c.x) / w,

            (wa \* a.y + wb \* b.y + wc \* c.y) / w);

}

**const** Point outer\_center(Point a, Point b, Point c) {

    b.x-=a.x;

    b.y-=a.y;

    c.x-=a.x;

    c.y-=a.y;

**return** Point((c.y\*(b.x\*b.x+b.y\*b.y)-b.y\*(c.x\*c.x+c.y\*c.y))/(2\*(b.x\*c.y-b.y\*c.x))+a.x,(-c.x\*(b.x\*b.x+b.y\*b.y)+b.x\*(c.x\*c.x+c.y\*c.y))/(2\*(b.x\*c.y-b.y\*c.x))+a.y);

}

vector<Point> circle\_line(**const** Circle& circle, **const** Line& line) {

    vector<Point> result;

**double** a = 2 \* inner(line.dir, line.dir);

**double** b = 2 \* (line.dir.x \* (line.pos.x - circle.center.x)

            + line.dir.y \* (line.pos.y - circle.center.y));

**double** c = inner(line.pos - circle.center, line.pos - circle.center)

        - circle.r \* circle.r;

**double** det = b \* b - 2 \* a \* c;

**int** pred = diff(det, 0);

**if** (pred == 0)

        result.push\_back(line.pos + line.dir \* (-b / a));

**else** **if** (pred > 0) {

        det = **sqrt**(det);

        result.push\_back(line.pos + line.dir \* ((-b + det) / a));

        result.push\_back(line.pos + line.dir \* ((-b - det) / a));

    }

**return** result;

}

vector<Point> circle\_circle(**const** Circle& a, **const** Circle& b) {

    vector<Point> result;

**int** pred = diff(dist(a.center, b.center), a.r + b.r);

**if** (pred > 0) **return** result;

**if** (pred == 0) {

        result.push\_back((a.center \* b.r + b.center \* a.r) \* (1 / (a.r + b.r)));

**return** result;

    }

**double** aa = a.center.x \* a.center.x + a.center.y \* a.center.y - a.r \* a.r;

**double** bb = b.center.x \* b.center.x + b.center.y \* b.center.y - b.r \* b.r;

**double** tmp = (bb - aa) / 2.0;

    Point cdiff = b.center - a.center;

**if** (diff(cdiff.x, 0) == 0) {

**if** (diff(cdiff.y, 0) == 0)

**return** result; // if (diff(a.r, b.r) == 0): same circle

**return** circle\_line(a, Line(Point(0, tmp / cdiff.y), Point(1, 0)));

    }

**return** circle\_line(a,

            Line(Point(tmp / cdiff.x, 0), Point(-cdiff.y, cdiff.x)));

}

**const** Circle circle\_from\_3pts(**const** Point& a, **const** Point& b, **const** Point& c) {

    Point ba = b - a, cb = c - b;

    Line p((a + b) \* 0.5, Point(ba.y, -ba.x));

    Line q((b + c) \* 0.5, Point(cb.y, -cb.x));

    Circle circle;

**if** (!get\_cross(p, q, circle.center))

        circle.r = -1;

**else**

        circle.r = dist(circle.center, a);

**return** circle;

}

**const** Circle circle\_from\_2pts\_rad(**const** Point& a, **const** Point& b, **double** r) {

**double** det = r \* r / dist2(a, b) - 0.25;

    Circle circle;

**if** (det < 0)

        circle.r = -1;

**else** {

**double** h = **sqrt**(det);

        // center is to the left of a->b

        circle.center = (a + b) \* 0.5 + Point(a.y - b.y, b.x - a.x) \* h;

        circle.r = r;

    }

**return** circle;

}

## Polygon Cut

// left side of a->b

vector<Point> cut\_polygon(**const** vector<Point>& polygon, Line line) {

**if** (!polygon.size()) **return** polygon;

**typedef** vector<Point>::const\_iterator piter;

    piter la, lan, fi, fip, i, j;

    la = lan = fi = fip = polygon.end();

    i = polygon.end() - 1;

**bool** lastin = diff(ccw\_line(line, polygon[polygon.size() - 1]), 0) > 0;

**for** (j = polygon.begin() ; j != polygon.end() ; j++) {

**bool** thisin = diff(ccw\_line(line, \*j), 0) > 0;

**if** (lastin && !thisin) {

            la = i;

            lan = j;

        }

**if** (!lastin && thisin) {

            fi = j;

            fip = i;

        }

        i = j;

        lastin = thisin;

    }

**if** (fi == polygon.end()) {

**if** (!lastin) **return** vector<Point>();

**return** polygon;

    }

    vector<Point> result;

**for** (i = fi ; i != lan ; i++) {

**if** (i == polygon.end()) {

            i = polygon.begin();

**if** (i == lan) **break**;

        }

        result.push\_back(\*i);

    }

    Point lc, fc;

    get\_cross(Line(\*la, \*lan - \*la), line, lc);

    get\_cross(Line(\*fip, \*fi - \*fip), line, fc);

    result.push\_back(lc);

**if** (diff(dist2(lc, fc), 0) != 0) result.push\_back(fc);

**return** result;

}

## Distance from a point to a line

#include <cmath>

#define SQ(x) ((x)\*(x))

#define dist(a, b, c, d) sqrt(SQ((a)-(c)) + SQ((b)-(d)))

// find minimum distance between a line segment(x1, y1, x2, y2) and a point (px, py)

**double** segdist(**double** x1, **double** y1, **double** x2, **double** y2, **double** px, **double** py)

{

**double** l2 = SQ(x1-x2) + SQ(y1-y2);

**if**(l2 == 0.0) **return** dist(x1,y1,px,py);

**double** t = ((px-x2) \* (x1-x2) + (py-y2) \* (y1-y2)) / l2;

**if**(t < 0) **return** dist(x2,y2,px,py);

**if**(t > 1) **return** dist(x1,y1,px,py);

**return** dist(x2 + t\*(x1-x2), y2 + t\*(y1-y2), px, py);

}

# Mathematical Stuffs

#include <cmath>

#include <climits>

#include <vector>

#include <algorithm>

**using** **namespace** std;

## Modular Power

n^k mod m을 구한다.

**long** **long** power(**long** **long** n, **long** **long** k, **long** **long** m = LLONG\_MAX) {

**long** **long** ret = 1;

**while** (k) {

**if** (k & 1) ret = (ret \* n) % m;

        n = (n \* n) % m;

        k >>= 1;

    }

**return** ret;

}

## Great Common Divisor

a와 b의 최대공약수를 구한다.

Dependencies: -

**long** **long** gcd(**long** **long** a, **long** **long** b) {

**if** (b == 0) **return** a;

**return** gcd(b, a % b);

}

## Extended GCD

ac + bd = gcd(a, b)가 되는 (c, d)를 찾는다.

Dependencies: -

pair<**long** **long**, **long** **long**> extended\_gcd(**long** **long** a, **long** **long** b) {

**if** (b == 0) **return** make\_pair(1, 0);

    pair<**long** **long**, **long** **long**> t = extended\_gcd(b, a % b);

**return** make\_pair(t.second, t.first - t.second \* (a / b));

}

## Modular Inverse

    ax = gcd(a, m) (mod m)가 되는 x를 찾는다.

Dependencies: extended\_gcd(a, b)

**long** **long** modinverse(**long** **long** a, **long** **long** m) {

**return** (extended\_gcd(a, m).first % m + m) % m;

    }

## Chinese Remainder Theorem

    x = a (mod n)가 되는 x를 찾는다.

Dependencies: gcd(a, b), modinverse(a, m)

**long** **long** chinese\_remainder(**long** **long** \*a, **long** **long** \*n, **int** size) {

**if** (size == 1) **return** \*a;

**long** **long** tmp = modinverse(n[0], n[1]);

**long** **long** tmp2 = (tmp \* (a[1] - a[0]) % n[1] + n[1]) % n[1];

**long** **long** ora = a[1];

**long** **long** tgcd = gcd(n[0], n[1]);

        a[1] = a[0] + n[0] / tgcd \* tmp2;

        n[1] \*= n[0] / tgcd;

**long** **long** ret = chinese\_remainder(a + 1, n + 1, size - 1);

        n[1] /= n[0] / tgcd;

        a[1] = ora;

**return** ret;

    }

## Binomial Calculation

nCm의 값을 구한다.

Dependencies: -

파스칼의 삼각형을 이용하거나, 미리 계산된 값을 가져오도록 이 함수를 수정하면 lucas\_theorem, catalan\_number 함수의 성능을 향상시킬 수 있다.

**long** **long** binomial(**int** n, **int** m) {

**if** (n < m || n < 0) **return** 0;

**long** **long** ans = 1, ans2 = 1;

**for** (**int** i = 0 ; i < m ; i++) {

        ans \*= n - i;

        ans2 \*= i + 1;

    }

**return** ans / ans2;

}

## Lucas Theorem

    nCm mod p의 값을 구한다.

Dependencies: binomial(n, m)

    n, m은 문자열로 주어지는 정수이다. p는 소수여야 한다.

**int** lucas\_theorem(**const** **char** \*n, **const** **char** \*m, **int** p) {

        vector<**int**> np, mp;

**int** i;

**for** (i = 0 ; n[i] ; i++) {

**if** (n[i] == '0' && np.empty()) **continue**;

            np.push\_back(n[i] - '0');

        }

**for** (i = 0 ; m[i] ; i++) {

**if** (m[i] == '0' && mp.empty()) **continue**;

            mp.push\_back(m[i] - '0');

        }

**int** ret = 1;

**int** ni = 0, mi = 0;

**while** (ni < np.size() || mi < mp.size()) {

**int** nmod = 0, mmod = 0;

**for** (i = ni ; i < np.size() ; i++) {

**if** (i + 1 < np.size())

                    np[i + 1] += (np[i] % p) \* 10;

**else**

                    nmod = np[i] % p;

                np[i] /= p;

            }

**for** (i = mi ; i < mp.size() ; i++) {

**if** (i + 1 < mp.size())

                    mp[i + 1] += (mp[i] % p) \* 10;

**else**

                    mmod = mp[i] % p;

                mp[i] /= p;

            }

**while** (ni < np.size() && np[ni] == 0) ni++;

**while** (mi < mp.size() && mp[mi] == 0) mi++;

            ret = (ret \* binomial(nmod, mmod)) % p;

        }

**return** ret;

    }

## Catalan Number

Dependencies: binomial(n, m)

**long** **long** catalan\_number(**int** n) {

**return** binomial(n \* 2, n) / (n + 1);

    }

**typedef** **long** **long** ll;

#define mod 1000000007ll

ll factorial[2222222];

ll **pow**(ll a,**int** b) {

**if** ( b == 0 ) **return** 1;

**if** ( b == 1 ) **return** a%mod;

    ll t = **pow**(a,b/2);

    t = (t\*t)%mod;

**return** (b&1)?(t\*a)%mod:t;

}

ll catalanNumber(**int** n) {

**return** ((((factorial[2\*n]\***pow**(factorial[n],mod-2))%mod)\***pow**(factorial[n+1],mod-2))%mod)%mod;

}

**int** main() {

    factorial[0] = factorial[1] = 1;

**for** ( **int** i = 2 ; i <= 2222222 ; i++ )

        factorial[i] = (factorial[i-1]\*i)%mod;

}

## Euler’s Totient Function

phi(n), n 이하의 양수 중 n과 서로 소인 것의 개수를 구한다.

Dependencies: -

// phi(n) = (p\_1 - 1) \* p\_1 ^ (k\_1 - 1) \* (p\_2 - 1) \* p\_2 ^ (k\_2-1)

**long** **long** euler\_totient2(**long** **long** n, **long** **long** ps) {

**for** (**long** **long** i = ps ; i \* i <= n ; i++) {

**if** (n % i == 0) {

**long** **long** p = 1;

**while** (n % i == 0) {

                n /= i;

                p \*= i;

            }

**return** (p - p / i) \* euler\_totient2(n, i + 1);

        }

**if** (i > 2) i++;

    }

**return** n - 1;

}

**long** **long** euler\_totient(**long** **long** n) {

**return** euler\_totient2(n, 2);

}

## Matrix Inverse

Dependencies: -

**inline** **bool** eq(**double** a, **double** b) {

**static** **const** **double** eps = 1e-9;

**return** **fabs**(a - b) < eps;

}

// returns empty vector if fails

vector<vector<**double**> > mat\_inverse(vector<vector<**double**> > matrix, **int** n) {

**int** i, j, k;

    vector<vector<**double**> > ret;

    ret.resize(n);

**for** (i = 0 ; i < n ; i++) {

        ret[i].resize(n);

**for** (j = 0 ; j < n ; j++)

            ret[i][j] = 0;

        ret[i][i] = 1;

    }

**for** (i = 0 ; i < n ; i++) {

**if** (eq(matrix[i][i],0)) {

**for** (j = i + 1 ; j < n ; j++) {

**if** (!eq(matrix[j][i], 0)) {

**for** (k = 0 ; k < n ; k++) {

                        matrix[i][k] += matrix[j][k];

                        ret[i][k] += ret[j][k];

                    }

**break**;

                }

            }

**if** (j == n) {

                ret.clear();

**return** ret;}

        }

**double** tmp = matrix[i][i];

**for** (k = 0 ; k < n ; k++) {

            matrix[i][k] /= tmp;

            ret[i][k] /= tmp;

        }

**for** (j = 0 ; j < n ; j++) {

**if** (j == i) **continue**;

            tmp = matrix[j][i];

**for** (k = 0 ; k < n ; k++) {

                matrix[j][k] -= matrix[i][k] \* tmp;

                ret[j][k] -= ret[i][k] \* tmp;

            }

        }

    }

**return** ret;

}

## Modular Matrix Inverse

Dependencies: modinverse(a, m)

    // returns empty vector if fails

    vector<vector<**long** **long**> > mat\_inverse(vector<vector<**long** **long**> > matrix, **int** n, **long** **long** mod) {

**int** i, j, k;

        vector<vector<**long** **long**> > ret;

        ret.resize(n);

**for** (i = 0 ; i < n ; i++) {

            ret[i].resize(n);

**for** (j = 0 ; j < n ; j++)

                ret[i][j] = 0;

            ret[i][i] = 1 % mod;

        }

**for** (i = 0 ; i < n ; i++) {

**if** (matrix[i][i] == 0) {

**for** (j = i + 1 ; j < n ; j++) {

**if** (matrix[j][i] != 0) {

**for** (k = 0 ; k < n ; k++) {

                            matrix[i][k] = (matrix[i][k] + matrix[j][k]) % mod;

                            ret[i][k] = (ret[i][k] + ret[j][k]) % mod;

                        }

**break**;

                    }

                }

**if** (j == n) {

                    ret.clear();

**return** ret;

                }

            }

**long** **long** tmp = modinverse(matrix[i][i], mod);

**for** (k = 0 ; k < n ; k++) {

                matrix[i][k] = (matrix[i][k] \* tmp) % mod;

                ret[i][k] = (ret[i][k] \* tmp) % mod;

            }

**for** (j = 0 ; j < n ; j++) {

**if** (j == i) **continue**;

                tmp = matrix[j][i];

**for** (k = 0 ; k < n ; k++) {

                    matrix[j][k] -= matrix[i][k] \* tmp;

                    matrix[j][k] = (matrix[j][k] % mod + mod) % mod;

                    ret[j][k] -= ret[i][k] \* tmp;

                    ret[j][k] = (ret[j][k] % mod + mod) % mod;

                }

            }

        }

**return** ret;

    }

## Matrix Determinants

Dependencies: -

**double** mat\_det(vector<vector<**double**> > matrix, **int** n) {

**int** i, j, k;

**double** ret = 1;

**for** (i = 0 ; i < n ; i++) {

**if** (eq(matrix[i][i], 0)) {

**for** (j = i + 1 ; j < n ; j++) {

**if** (!eq(matrix[j][i], 0)) {

**for** (k = 0 ; k < n ; k++)

                        matrix[i][k] += matrix[j][k];

**break**;

                }

            }

**if** (j == n)

**return** 0;

        }

**double** tmp = matrix[i][i];

**for** (k = 0 ; k < n ; k++)

            matrix[i][k] /= tmp;

        ret \*= tmp;

**for** (j = 0 ; j < n ; j++) {

**if** (j == i) **continue**;

            tmp = matrix[j][i];

**for** (k = 0 ; k < n ; k++)

                matrix[j][k] -= matrix[i][k] \* tmp;

        }

    }

**return** ret;

}

## Kirchhoff’s Theorem

    주어진 그래프에서 가능한 신장트리의 경우의 수를 구한다.

Dependencies: mat\_det(matrix, n)

**long** **long** count\_spantree(vector<**int**> graph[], **int** size) {

**int** i, j;

        vector<vector<**double**> > matrix(size - 1);

**for** (i = 0 ; i < size - 1 ; i++) {

            matrix[i].resize(size - 1);

**for** (j = 0 ; j < size - 1 ; j++)

                matrix[i][j] = 0;

**for** (j = 0 ; j < graph[i].size() ; j++) {

**if** (graph[i][j] < size - 1) {

                    matrix[i][graph[i][j]]--;

                    matrix[i][i]++;

                }

            }

        }

**return** (**long** **long**)(mat\_det(matrix, size - 1) + 0.5);

    }

## Gaussian Elimination

gaussian::run(size\_eq, size\_var, A, B, C);

A는 1차원 배열의 꼴로 주어지는 2차원 행렬이다. 배열 C의 값을 채워 넣는 루틴은 별도로 구현하라. val\_t로 **double**을 사용할 경우 **abs** 함수의 구현을 적절히 수정하라.

#include <algorithm>

**using** **namespace** std;

**long** **long** gcd(**long** **long** a, **long** **long** b)

{

**if** (b == 0)

**return** a;

**return** gcd(b, a % b);

}

**struct** rational {

**long** **long** p, q;

**void** red() {

**if** (q < 0) {

            p \*= -1;

            q \*= -1;

        }

**long** **long** t = gcd((p >= 0 ? p : -p), q);

        p /= t;

        q /= t;

    }

    rational() {}

    rational(**long** **long** p\_): p(p\_), q(1) {}

    rational(**long** **long** p\_, **long** **long** q\_): p(p\_), q(q\_) { red(); }

**bool** operator==(**const** rational& rhs) **const** {

**return** p == rhs.p && q == rhs.q;

    }

**bool** operator!=(**const** rational& rhs) **const** {

**return** p != rhs.p || q != rhs.q;

    }

**bool** operator<(**const** rational& rhs) **const** {

**return** p \* rhs.q < rhs.p \* q;

    }

**const** rational operator+(**const** rational& rhs) **const** {

**return** rational(p \* rhs.q + q \* rhs.p, q \* rhs.q);

    }

**const** rational operator-(**const** rational& rhs) **const** {

**return** rational(p \* rhs.q - q \* rhs.p, q \* rhs.q);

    }

**const** rational operator\*(**const** rational& rhs) **const** {

**return** rational(p \* rhs.p, q \* rhs.q);

    }

**const** rational operator/(**const** rational& rhs) **const** {

**return** rational(p \* rhs.q, q \* rhs.p);

    }

};

**namespace** gaussian

{

**typedef** rational val\_t;

**const** val\_t **abs**(**const** val\_t& x) {

**return** (x.p >= 0) ? x : rational(-x.p, x.q);

    }

#define GET(i, j, n) A[i \* n + j]

    // return true when solution exists, false o/w.

**bool** run(**int** size\_eq, **int** size\_var, val\_t\* A, val\_t\* B, val\_t\* C) {

**int** i = 0, j = 0, k, l;

**int** maxi;

        val\_t temp\_r;

        val\_t\* x;

        val\_t\* y;

**while** (i < size\_eq && j < size\_var) {

            maxi = i;

**for** (k = i + 1 ; k < size\_eq ; k++)

**if** (**abs**(GET(maxi, j, size\_var)) < **abs**(GET(k, j, size\_var)))

                    maxi = k;

**if** (GET(maxi, j, size\_var) != val\_t(0)) {

                x = A + i \* size\_var;

                y = A + maxi \* size\_var;

**for** (k = 0 ; k < size\_var ; k++)

                    swap(\*(x + k), \*(y + k));

                swap(B[i], B[maxi]);

                temp\_r = \*(x + j);

**for** (k = j ; k < size\_var ; k++)

                    \*(x + k) = \*(x + k) / temp\_r;

                B[i] = B[i] / temp\_r;

**for** (k = 0 ; k < size\_eq ; k++) {

**if** (k == i) **continue**;

                    temp\_r = GET(k, j, size\_var);

**for** (l = j ; l < size\_var ; l++)

                        GET(k, l, size\_var) = GET(k, l, size\_var)

                            - temp\_r \* GET(i, l, size\_var);

                    B[k] = B[k] - GET(k, j, size\_var) \* B[i];

                }

                i++;

            }

            j++;

        }

**if** (i < size\_eq)

**for** ( ; i < size\_eq ; i++)

**if** (B[i] != val\_t(0)) **return** **false**;

        // C[...] := Case by case

**return** **true**;

    }

#undef GET

} // namespace gaussian

## Simplex Algorithm

n := number of constraints

m := number of variables

matrix[0] := maximize할 식의 계수

matrix[1~n] := constraints

solution := results

solution[n] := 원하는 식의 최대값

부등식의 우변(변수 없는 쪽)이 음이 아닌 수가 되도록 정리하여 대입한다.

ex) Maximize p = -2x + 3y

Constraints: x + 3y ≤ 40

2x + 4y ≥ 10

x ≥ 0, y ≥ 0

n = 2, m = 2, matrix = [ 2 -3 1 0 0 ] , c = [ 0 ]

[ 1 3 0 1 0 ]        [ 40]

[ 2 4 0 0 -1 ]       [ 10]

**namespace** simplex

{

**const** **int** MAX\_N = 50;

**const** **int** MAX\_M = 50;

**const** **double** eps = 1e-9;

**inline** **int** diff(**double** a, **double** b) {

**if** (a - eps < b && b < a + eps) **return** 0;

**return** (a < b) ? -1 : 1;

    }

**int** n, m;

**double** matrix[MAX\_N + 1][MAX\_M + MAX\_N + 1];

**double** c[MAX\_N + 1];

**double** solution[MAX\_M + MAX\_N + 1];

**int** simplex() { // 0: found solution, 1: no feasible solution, 2: unbounded

**int** i, j;

**while** (**true**) {

**int** nonfeasible = -1;

**for** (j = 0 ; j <= n + m ; j++) {

**int** cnt = 0, pos = -1;

**for** (i = 0 ; i <= n ; i++) {

**if** (diff(matrix[i][j], 0)) {

                        cnt++;

                        pos = i;

                    }

                }

**if** (cnt != 1)

                    solution[j] = 0;

**else** {

                    solution[j] = c[pos] / matrix[pos][j];

**if** (solution[j] < 0) nonfeasible = i;

                }

            }

**int** pivotcol = -1;

**if** (nonfeasible != -1) {

**double** maxv = 0;

**for** (j = 0 ; j <= n+m ; j++) {

**if** (maxv < matrix[nonfeasible][j]) {

                        maxv = matrix[nonfeasible][j];

                        pivotcol = j;

                    }

                }

**if** (pivotcol == -1) **return** 1;

            }

**else** {

**double** minv = 0;

**for** (j = 0 ; j <= n + m ; j++) {

**if** (minv > matrix[0][j]) {

                        minv = matrix[0][j];

                        pivotcol = j;

                    }

                }

**if**(pivotcol == -1) **return** 0;

            }

**double** minv = -1;

**int** pivotrow = -1;

**for** (i = 0 ; i <= n ; i++) {

**if** (diff(matrix[i][pivotcol], 0) > 0) {

**double** test = c[i] / matrix[i][pivotcol];

**if** (test < minv || minv < 0) {

                        minv = test;

                        pivotrow = i;

                    }

                }

            }

**if** (pivotrow == -1) **return** 2;

**for** (i = 0 ; i <= n ; i++) {

**if** (i == pivotrow) **continue**;

**if** (diff(matrix[i][pivotcol], 0)) {

**double** ratio = matrix[i][pivotcol] / matrix[pivotrow][pivotcol];

**for** (j = 0 ; j <= n + m ; j++) {

**if** (j == pivotcol) {

                            matrix[i][j] = 0;

**continue**;

                        }

**else**

                            matrix[i][j] -= ratio \* matrix[pivotrow][j];

                    }

                    c[i] -= ratio \* c[pivotrow];

                }

            }

        }

    }

} // namespace simplex

# Miscellaneous

## Binary Indexed Tree

BIT::Init(size); // BIT initializing

BIT::Read(idx);  // Read

BIT::Update(idx,val); // Update

#include <vector>

**using** **namespace** std;

**namespace** BIT {

**typedef** **long** **long** ll;

**int** MAX;

    vector<ll> tree;

**void** Init(**int** size) {

        MAX=size;

        tree.resize(MAX+1);

    }

    ll Read(**int** idx) {

        ll ret=0;

**while** ( idx > 0 ) {

            ret += tree[idx];

            idx -= (idx & -idx);

        }

**return** ret;

    }

**void** Update(**int** idx,**int** val) {

**while** ( idx < MAX ) {

            tree[idx] += val;

            idx += (idx & -idx);

        }

    }

}

## Fenwick tree interval update

**const** **int** MAXN =  2222222;

**int** N;

**int** dataMul[MAXN\*2],dataAdd[MAXN\*2];

**void** internalUpdate(**int** at, **int** mul, **int** add) {

**while** (at < MAXN) {

        dataMul[at] += mul;

        dataAdd[at] += add;

        at |= (at + 1);

    }

}

**void** update(**int** left, **int** right, **int** by) {

    internalUpdate(left, by, -by \* (left - 1));

    internalUpdate(right, -by, by \* right);

}

**int** query(**int** at) {

**int** mul = 0;

**int** add = 0;

**int** start = at;

**while** (at >= 0) {

        mul += dataMul[at];

        add += dataAdd[at];

        at = (at & (at + 1)) - 1;

    }

**return** mul \* start + add;

}

## Union Find using disjoint-set

UnionFind::Init(size); // set initializing

UnionFind::Find(node); // find parent

UnionFind::MakeUnion(x,y); // union(x,y)

#include <vector>

#include <algorithm>

**using** **namespace** std;

**namespace** UnionFind{

    vector<**int**> rank;

    vector<**int**> u;

**void** Init(**int** size) {

        rank.resize(size+1,0);

        u.resize(size+1,0);

**for** ( **int** i = 0 ; i <= size ; i++ )

            u[i] = i;

    }

**int** Find(**int** now) {

**return** (u[now]==now)?now:(u[now]=Find(u[now]));

    }

**void** MakeUnion(**int** x,**int** y) {

        x = Find(x); y = Find(y);

**if** ( x == y ) **return**;

**if** ( rank[x] < rank[y] ) u[x] = y;

**else** {

            u[y] = x;

            rank[x]+=(rank[x]==rank[y]);

        }

    }

}

## KMP Algorithm

result = kmp::match(text, pattern); // 모든 matched point의 vector

#include <vector>

**using** **namespace** std;

**namespace** kmp

{

**typedef** vector<**int**> seq\_t;

**void** calculate\_pi(vector<**int**>& pi, **const** seq\_t& str) {

        pi[0] = -1;

**int** j = -1;

**for** (**int** i = 1 ; i < str.size() ; i++) {

**while** (j >= 0 && str[i] != str[j + 1]) j = pi[j];

**if** (str[i] == str[j + 1])

                pi[i] = ++j;

**else**

                pi[i] = -1;

        }

    }

    /\* returns all positions matched \*/

    vector<**int**> match(seq\_t text, seq\_t pattern) {

        vector<**int**> pi(pattern.size());

        vector<**int**> ans;

**if** (pattern.size() == 0) **return** ans;

        calculate\_pi(pi, pattern);

**int** j = -1;

**for** (**int** i = 0 ; i < text.size() ; i++) {

**while** (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];

**if** (text[i] == pattern[j + 1]) {

                j++;

**if** (j + 1 == pattern.size()) {

                    ans.push\_back(i - j);

                    j = pi[j];

                }

            }

        }

**return** ans;

    }

}

## Suffix Array O(n log^2 n) with LCP

#include <cstdio>

#include <cstring>

#include <algorithm>

**using** **namespace** std;

// L: doubling method 정렬을 위한 정보

// P[stp][i]: 길이가 1 << stp인 원래 문자열의 위치 i부터 시작하는 버켓 번호

**int** N, i, stp, cnt;

**int** A[65536];

**struct** entry {

**int** nr[2], p;

} L[65536];

**int** P[17][65536];

**int** suffix\_array[65536];

**int** lcp[65536]; // lcp(i, i + 1)

**int** cmp(**struct** entry a, **struct** entry b) {

**return** (a.nr[0] == b.nr[0]) ? (a.nr[1] < b.nr[1]) : (a.nr[0] < b.nr[0]);

}

// calclcp(x, y) = min(lcp[x], lcp[x + 1], ..., lcp[y - 1])

// binary indexed tree needed for speedup

**int** calclcp(**int** x, **int** y) { // x, y: start position in original string

**int** k, ret = 0;

**if**(x == y) **return** N - x;

**for**(k = stp - 1 ; k >= 0 && x < N && y < N ; k--)

**if**(P[k][x] == P[k][y])

            x += 1 << k, y += 1 << k, ret += 1 << k;

**return** ret;

}

**int** main(**void**) {

**int** i;

**scanf**("%d",&N);

**for**(i = 0 ; i < N ; i++) {

**scanf**("%d", &A[i]);

        P[0][i] = A[i];

    }

**for** (stp = 1, cnt = 1 ; (cnt >> 1) < N ; stp++, cnt <<= 1) {

**for** (i = 0 ; i < N ; i++) {

            L[i].nr[0] = P[stp - 1][i];

            L[i].nr[1] = (i + cnt < N) ? P[stp - 1][i + cnt] : -1;

            L[i].p = i;

        }

        sort(L, L + N, cmp);

**for** (i = 0 ; i < N ; i++) {

            P[stp][L[i].p] = (i > 0 && L[i].nr[0] == L[i - 1].nr[0]

                    && L[i].nr[1] == L[i - 1].nr[1]) ? P[stp][L[i-1].p] : i;

        }

    }

**for** (i = 0 ; i < N ; i++)

        suffix\_array[P[stp - 1][i]] = i;

**for** (i = 0 ; i + 1 < N ; i++)

        lcp[i] = calclcp(suffix\_array[i], suffix\_array[i + 1]);

**return** 0;

}

## Lowest Common Ancestor <O(n log n), O(log n)>

**void** Prepare\_LCA(**void**)

{

    // pd : distance to parent, p : parent(direct), O(nlogn)

**memset**(P, -1, **sizeof** P);

**for** (**int** i = 1; i <= N; i++) {

        D[i][0] = pd[i];

        P[i][0] = p[i];

    }

**for** (**int** j = 1; 1 << j <= N; j++) {

**for** (**int** i = 1; i <= N; i++)

**if** (P[i][j-1] != -1) {

                P[i][j] = P[P[i][j-1]][j-1];

                D[i][j] = D[P[i][j-1]][j-1] + D[i][j-1];

            }

    }

}

**int** Query\_LCA(**int** x, **int** y)

{

    // O(logn)

**int** **log**, ret = 0;

**if** (lv[x] < lv[y]) swap(x, y);

**for** (**log** = 1; 1 << **log** <= lv[x]; ++**log**); --**log**;

**for** (**int** i = **log**; i >= 0; i--) {

**if** (lv[x] - (1 << i) >= lv[y]) {

            ret += D[x][i];

            x = P[x][i];

        }

    }

**if** (x == y) **return** ret;

**for** (**int** i = **log**; i >= 0; i--) {

**if** (P[x][i] != -1 && P[x][i] != P[y][i]) {

            ret += D[x][i] + D[y][i];

            x = P[x][i]; y = P[y][i];

        }

    } **if** (p[x] != p[y]) **while** (**true**); // NOT CONNECTED

**return** ret + pd[x] + pd[y];

}

## Pick’s Theorem

On a simple polygon constructed on a grid of equal-distanced points, **for** area A,

number of interior points I, number of boundary points B, we have A=I+B/2-1.

## Combinatorial Game Theory

game sum: A xor B

game calc: minimum excluded number { Possible Games }

staircase nim: 짝수 계단에 있는 것들은 전부 소용 없음. 누구든 원래 nim 상태로 복귀시킬 수 있다.

Moore's nim\_k: k개씩 제거하는 nim. 2진수로 변환하고, k+1진수에서 xor 하듯이 carry 없 이 더한다.

misere nim: play exactly as **if** you were playing normal play nim, except **if** your winning move would lead to a position that consists of heaps of size one only. In that **case**, leave exactly one more or one fewer heaps of size one than the normal play strategy recommends.

## Combination Generator

/\*

 \* bit n개 중에 r개를 1로 바꿔준다.

 \* while은 nCr만큼 돌고 x는 모든 경우의 수 비트를 갖는다.

 \*/

**void** combination\_generator(**int** n,**int** r)

{

**int** x, s, s1, t, k;

    x=(1<<r)-1;

**while**(!(x & (1<<n))){

        s=x&-x;

        t=x+s;

        s1=t&-t;

        k=((s1/s)>>1)-1;

        x=t|k;

    }

}

## Range MinMaximum Query Using Segment Tree

**typedef** pair<**int**,**int**> ii;

**int** a[1111111];

**int** mntree[4444444];

**int** mxtree[4444444];

**void** initialize(**int** node,**int** s,**int** e) {

**if** ( s == e ) mntree[node] = mxtree[node] = a[s];

**else** {

**int** mid = (s+e)>>1;

        initialize(2\*node,s,mid);

        initialize(2\*node+1,mid+1,e);

        mxtree[node] = max(mxtree[2\*node],mxtree[2\*node+1]);

        mntree[node] = min(mntree[2\*node],mntree[2\*node+1]);

    }

}

ii query(**int** node,**int** s,**int** e,**int** i,**int** j) {

**if** ( e < i || s > j ) **return** ii(-1,-1);

**if** ( s >= i && e <= j ) **return** ii(mxtree[node],mntree[node]);

**int** mid = (s+e)>>1;

    ii p1 = query(2\*node,s,mid,i,j);

    ii p2 = query(2\*node+1,mid+1,e,i,j);

**if** ( p1 == ii(-1,-1) ) **return** p2;

**if** ( p2 == ii(-1,-1) ) **return** p1;

**return** ii(max(max(0,p1.first),max(0,p2.first)),min(max(0,p1.second),max(0,p2.second)));

}

ii update(**int** node,**int** s,**int** e,**int** idx,**int** val) {

**if** ( e < idx || idx < s ) **return** ii(mxtree[node],mntree[node]);

**if** ( s == e ) **return** ii(mxtree[node]=val,mntree[node]=val);

**int** mid = (s+e)>>1;

    ii p1 = update(2\*node,s,mid,idx,val);

    ii p2 = update(2\*node+1,mid+1,e,idx,val);

**return** ii(mxtree[node]=max(max(0,p1.first),max(0,p2.first)),

            mntree[node]=min(max(0,p1.second),max(0,p2.second)));

}

## Segment tree lazy propagation

**const** **int** MAXN =  1111111;

**int** N;

**int** tree[4\*MAXN],lazy[4\*MAXN];

**void** update(**int** left,**int** right,**int** node,**int** nodeLeft,**int** nodeRight,**int** val) {

**if** ( nodeLeft > right || nodeRight < left ) **return**;

**if** ( nodeLeft < nodeRight ) {

        lazy[2\*node] += lazy[node];

        lazy[2\*node+1] += lazy[node];

    }

    tree[node] += lazy[node];

    lazy[node] = 0;

**if** ( left <= nodeLeft && nodeRight <= right ) {

        tree[node] += val;

**if** ( nodeLeft < nodeRight ) {

            lazy[2\*node] += val;

            lazy[2\*node+1] += val;

        }

    } **else** **if** ( nodeLeft < nodeRight ) {

**int** mid = (nodeLeft+nodeRight)>>1;

        update(left,right,node\*2,nodeLeft,mid,val);

        update(left,right,node\*2+1,mid+1,nodeRight,val);

        tree[node] = max(tree[node\*2],tree[node\*2+1]);

    }

}

**int** query(**int** left,**int** right,**int** node,**int** nodeLeft,**int** nodeRight) {

**if** ( nodeLeft > right || nodeRight < left ) **return** 0;

**if** ( nodeLeft < nodeRight ) {

        lazy[2\*node] += lazy[node];

        lazy[2\*node+1] += lazy[node];

    }

    tree[node] += lazy[node];

    lazy[node] = 0;

**int** ret = 0;

**if** ( left <= nodeLeft && nodeRight <= right ) **return** tree[node];

**else** **if** ( nodeLeft < nodeRight ) {

**int** mid = (nodeLeft+nodeRight)>>1;

        ret = max(ret, query(left,right,node\*2,nodeLeft,mid));

        ret = max(ret, query(left,right,node\*2+1,mid+1,nodeRight));

        tree[node] = max(tree[node\*2],tree[node\*2+1]);

    }

**return** ret;

}

## AntiPodal Point

컨벡스 헐로 구한 점들 중 가장 먼 두 점을 구한다. (C++11)

Dependencies : convex\_hull

pair<Point,Point> AntiPodal(vector <Point>&& v)

{

int n = v.size(), ans = 0;

if (n < 3) return {v[0], v[1]};

Point p1, p2;

int p = n-1;

int q = Next(p);

while (abs(CCW(v[p], v[Next(p)], v[Next(q)])) > abs(CCW(v[p], v[Next(p)], v[q]))) {

q = Next(q);

}

int q0 = q;

while (q != 0) {

p = Next(p);

if (ans < Dist(v[p], v[q])) {

ans = Dist(v[p], v[q]);

p1 = v[p], p2 = v[q];

}// Found

while (abs(CCW(v[p], v[Next(p)], v[Next(q)])) > abs(CCW(v[p], v[Next(p)], v[q]))) {

q = Next(q);

if (v[p] != v[q0] || v[q] != v[0]) {

if (ans < Dist(v[p], v[q])) {

ans = Dist(v[p], v[q]);

p1 = v[p], p2 = v[q];

}// Found

}

else return {p1, p2};

}

if (abs(CCW(v[p], v[Next(p)], v[Next(q)])) == abs(CCW(v[p], v[Next(p)], v[q]))) {

if (v[p] != v[q0] || v[q] != v[n-1]) {

if (ans < Dist(v[p], v[Next(q)])) {

ans = Dist(v[p], v[Next(q)]);

p1 = v[p], p2 = v[Next(q)];

}// Found

}

else {

if (ans < Dist(v[Next(p)], v[q])) {

ans = Dist(v[Next(p)], v[q]);

p1 = v[Next(p)], p2 = v[q];

} // Found

}

}

}

return {p1, p2};

}

## Aho-Corasick

#include <map>

#include <queue>

#include <vector>

#include <algorithm>

#include <string>

**using** **namespace** std;

**struct** NODE {

**bool** b;

    NODE \*next[4], \*f;

    NODE(){}

};

NODE \*root;

NODE container[1111111];

**int** size;

NODE \*newNode() {

    NODE \*ret = &container[size++];

    ret->b = ret->f = 0;

**for** ( **int** i = 0 ; i < 4; i++ )

        ret->next[i] = 0;

**return** ret;

}

map<**char**,**int**> mp;

**void** createTree(vector<string>& pattern) {

    mp['A'] = 0;mp['C'] = 1;mp['G'] = 2;mp['T'] = 3;

    size = 0;

    root = newNode();

**for** ( **int** i = 0 ; i < (**int**)pattern.size() ; i++ ) {

        NODE \*now = root;

**for** ( **int** j = 0 ; j < (**int**)pattern[i].length() ; j++ ) {

**int** c = mp[pattern[i][j]];

**if** ( !now->next[c] ) now->next[c] = newNode();

            now = now->next[c];

        }

        now->b = **true**;

    }

    queue<NODE\*> q;

**for** ( **int** i = 0 ; i < 4 ; i++ )

**if** ( root->next[i] ) {

            root->next[i]->f = root;

            q.push(root->next[i]);

        }

**while** ( !q.empty() ) {

        NODE \*now = q.front();q.pop();

        NODE \*f = now->f;

**for** ( **int** i = 0 ; i < 4 ; i++ )

**if** ( now->next[i] ) {

                NODE\* &nf = now->next[i]->f;

                nf = f;

**while** ( nf != root && !nf->next[i] )

                    nf = nf->f;

**if** ( nf->next[i] ) nf = nf->next[i];

                q.push(now->next[i]);

            }

    }

}

vector<**int**> aho\_corasick(string s) {

    vector<**int**> ret;

    NODE \*now = root;

**int** ans = 0;

**for** ( **int** i = 0 ; i < (**int**)s.length() ; i++ ) {

**int** c = mp[s[i]];

**while** ( now != root && !now->next[c] ) now = now->f;

**if** ( now->next[c] ) now = now->next[c];

**if** ( now->b ) {

            ret.push\_back(i);

            now = now->f;

        }

    }

**return** ret;

}