PANDEMIC FLU SPREAD REPORT

Jocelyn Wuici Chai & Xiyue Lin

[1]

March, 2021

[1] https://www.freepik.com/premium-vector/back-school-new-normal-lifestyle-concept-happy-kids-wearing-face-mask-social-distancing-protect-coronavirus-covid-19_8689707.htm

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ABSTRACT

The problem that we have chosen to work on in this project is an application-oriented problem, namely, the Pandemic Flu Spread (Problem #4). In our project, we studied the expected number of kids in the school that will be infected by the virus-carrier schoolboy, Tommy, due to their day-to-day interaction. Thereafter, we used simulation to predict how long this epidemic will last. We tackled this problem by using binomial models and chain binomial models. These models are central in the formulation of statistical models for estimating transmission parameters. Through our simulation, we found that the epidemic will last for approximately 63-65 days if we consider the expected number of infections below 0.5 for it to be considered as 'dying out'. However, if we instead required the expected number of infections below 0.01, then the 'epidemic' would have probably lasted until day 145-147. Note here that in our simulation, we are assuming that a kid will be immune to the virus for three days after recovery.

BACKGROUND AND DESCRIPTION OF THE PROBLEM

1. The Problem

Consider a classroom of 21 elementary school kids. 20 of the kids are healthy (and susceptible to flu) on Day 1. Tommy (the 21st kid) walks in with the flu and starts interacting with his potential victims. To keep thing simple, let's suppose that Tommy comes to school every day (whether or not he's sick) and will be infectious for 3 days. Thus, there are three chances for Tommy to infect the other kids – Days 1, 2, and 3. Suppose that the probability that he infects any individual susceptible kid on any of the three days is p = 0.02; and suppose that all kids and days are independent (which is i.i.d Bern(p) trials)
kl
If a kid gets infected by Tommy, he will then become infectious for 3 days as well, starting on the next day.

(a) What is the distribution of the number of kids that Tommy infects on Day 1?
(b) What is the expected number of kids that Tommy infects on Day 1?
(c) What is the expected number of kids that Tommy infects on Day 2 (you can count Tommy if you want)?
(d) Simulate the number of kids that are infected on Days 1, 2, … Do this many times. What are the (estimated) expected numbers of kids that are infected by Day i, i=1, 2, …? Produce a histogram detailing how long the "epidemic" will last.

2. Organization

Firstly, we start off by discussing the application areas of this problem and briefly introduce the model which we applied in our question, the $SIRS$ model, a variant based on the Reed-Frost model (SIR).

Next, we focus on the inference we made and the parameters we used for our simulation process in a more detailed fashion.

Then, we dive deeper into the discussion regarding other compartmental models used in epidemiology and other variants of SIR models such as the SIS model and the $SEIS$ model. We also elaborate on their difference and separate use cases, respectively.

Lastly, we come up with a simulation model using Python to solve the actual problem itself.

MAIN FINDINGS

1. Application Area

The main application area of this problem is the prediction of the spread of an epidemic. It is rather applicable now more than ever, given the fact that we are still affected by an ongoing pandemic. Moreover, it can also be applied to vaccination distributions as a method to end the pandemic.

2. Models and Methods Used

The binomial model is often used to estimate the transmission probability and it is the building block of the stochastic simulation models in such problems. The basic idea of the binomial model is that exposure to infection occurs in discrete contacts, which can also be discrete time units of exposure (i.e., days). Chain binomial models are dynamic models developed from simple binomial model by assuming that infection spreads from individual to individual in discrete time units, producing chains of infection governed by binomial distribution. The Reed-Frost model, which assumes that exposure to two or more infectious people at the same time are independent exposures, is a famous example of chain binomial models.
We utilized the SIRS (Susceptible – Infectious – Recovered – Susceptible) model to analyze and solve our problem. It is a slight variant of the Reed-Frost model (SIR model), which is one example of a chain binomial model.

In the Reed-Frost model, the assumption that people pass through three stages is made. They start out susceptible, denoted by S, then become infected and infectious, denoted by I, after which they recover with immunity, denoted by R. It is also called SIR models. There are other few variants of SIR models, such as SIS, $SIRS$(the one we used), $SEIR$ which will be covered later. Here, we will focus explaining SIR model. This model assumes that there is no latent period and that there are no asymptomatic infections.[2]

Here, we denoted constant N as the population size, S_t is the number of susceptible people, and I_t is the number of the infectives, and R_t is the number of immune people at time t, where $N = S_t + I_t + R_t$.

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[2] Binomial and Stochastic Transmission Models http://courses.washington.edu/b578a/readings/bookchap4.pdf

$$\Pr(I_{t+1} = i_{t+1} \mid S_t = s_t, I_t = i_t) = \binom{s_t}{i_{t+1}} \left(1 - q^{i_t}\right)^{i_{t+1}} q^{i_t(s_t - i_{t+1})}, s_t \geq i_{t+1}$$

$$where\ s_t = \frac{S_t}{N}, i_t = \frac{I_t}{N}\ , and\ r_t = \frac{R_t}{N}$$

Then, we update the number of new susceptible and recovered people.

$$S_{t+1} = S_t - I_{t+1}$$
$$R_{t+1} = R_t + I_t = \sum_{r=0}^{t} I_r$$

Since the population is constant, we have $S_t + I_t + R_t = N$ for all t. The epidemic process starts with $I_0 > 0$, and terminates at stopping time T, where:

$$T = \inf_{t \geq 0} \{t: S_t I_t = 0\}.$$

So the probability of a particular $\{i_0, i_1, i_2, \dots, i_T\}$, is given by the product of conditional binomial probabilities as

$$\begin{aligned} &\Pr\ (I_1 = i_1 \mid S_0 = s_0, I_0 = i_0) \Pr\ (I_2 = i_2 \mid S_1 = s_1, I_1 = i_1) \cdots \\ &\Pr\ (I_T = i_T \mid S_{T-1} = s_{T-1}, I_{T-1} = I_{T-1}) \\ &= \prod_{t=0}^{T-1} \binom{s_t}{i_{t+1}} \left(1 - q^{i_t}\right)^{i_{t+1}} q^{i_t(s_t - i_{t+1})} \end{aligned}$$ [3]

The prediction of the expected number of people that will get infected based on the Reed-Frost model can be simulated using a random number generator. At each time t, for each susceptible person exposed to I_t (number of infectious kids) infectives, we will generate a random number between 0 and 1. If the random number is smaller than infection probability, where we denotes as $1 - q^{I_t}$, (where $q = 1 - p$, p is the probability of Tommy infects other kids on any of the first three days, which is 0.02, and q^{I_t} denotes the probability of not being infected on a certain day t by I_t infectious kids), then the person becomes infected, else, if the probability is between the infection probability and 1, then the person will

[3] Binomial and Stochastic Transmission Models http://courses.washington.edu/b578a/readings/bookchap4.pdf

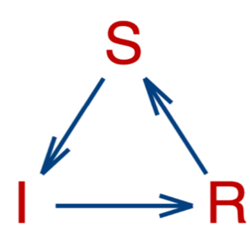
not be infected in that time interval. The actual chain depends on the series of random numbers that are generated, and therefore varies.

3. Data Collection

To make the simulation more realistic, we collected the data of the 20 most popular new-born names in 2020 from an online website called "babycenter" [4]and generously name the 20 kids[i] in Tommy's class, respectively.

4. Inference and Parameters

We assume that the infectious period for all the kids (Tommy and his 20 classmates) to be three days and the immune period for the kids to be three days as well. In other words, if a kid gets infected, he will be infectious for three days. Also, if a kid gets immune to the virus, he will only retain temporary immunity, which is also three days. After three days, they will no longer be immune to this virus. The cycle is illustrated as below (Susceptible – Infectious – Recovered – Susceptible).



5. Statistical Models Used in Epidemiology

After doing some research on the statistical models used in epidemiology settings, we have found that there are a few compartmental models used in epidemiology, where the population is assigned to compartments with labels. For example, S, I or R, which refers to Susceptible, Infectious, or Recovered, respectively. In these models, people may progress between compartments. Such models can also show how different public health interventions may affect the outcome of the epidemic. Here, we will briefly talk about a few models, SIS, $SIRS$, and $SEIR$.[5]

- SIS (Susceptible – Infectious – Susceptible) model:
    - This model is suitable for diseases that do not confer long term immunity, such as diseases caused by bacteria, i.e., common cold and influenza. Such infections do not give immunity upon recovery from infection, and individuals become susceptible again.

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[4] Top 20 most popular baby names of 2020 https://www.babycenter.com/top-baby-names

[5] Compartmental models in Epidemiology https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology#The_SEIR_model

- SIRS (Susceptible – Infectious – Recovered – Susceptible) model:
    - This model is suitable for people who have acquired immunity but lost it again to become susceptible.
- SIRD (Susceptible – Infectious – Recovered – Deceased) model:
    - This model is different from SIRS model where the individuals had deceased from the disease rather than survived and recovered.
- SEIR (Susceptible – Exposed – Infectious – Recovered) model:
    - This model allows people to pass through a latent period denoted by E. For many important infections, there is a significant incubation period which individuals have been infected but are not yet infectious themselves. During this period, the individual is in compartment E (exposed).
- SEIS (Susceptible – Exposed – Infectious – Susceptible) model:
    - This model is similar to the SEIR model above except that there will not be any immunity acquired at the end.

6. Solving the Actual Problem

(a) <u>What is the distribution of the number of kids that Tommy infects on Day 1?</u>

The distribution of the number of kids that Tommy infects on Day 1 is $I_1 \sim Binomial(20, 0.02)$

(b) <u>What is the expected number of kids that Tommy infects on Day 1?</u>

$E[I_1] = np = 20 \times 0.02 = 0.4$

∴ The expected number of kids that Tommy infects on Day 1 is 0.4.

(c) <u>What is the expected number of kids that are infected by Day 2 (you can count Tommy if you want)?</u>

$I_2 \sim Binomial(20 - 0.4, 1 - (1 - 0.02)^{1+0.4})$
$I_2 \sim Binomial(19.6, 0.0278875)$
$E[I_2] = np = 19.6 \times \ 0.0278875 = 0.5298625 \ \approx 0.53$

∴ The expected number of kids that Tommy infects on Day 2 is approximately 0.53.

(d) <u>Simulate the number of kids that are infected on Days 1, 2, …. Do this many times. What are the (estimated) expected number of kids that are infected by Day $i, i = 1, 2, \ldots$? Produce a histogram detailing how long the “epidemic” will last.</u>

Firstly, an assumption that we are making in our simulation is that once a kid has been infectious for 3 days, on the 4th day, he will enter a 3-day immunity period whereby he will not be prone to infections. After the 3 days (on the 7th day), he will again be susceptible to an infection.

Next, the number of infections on the following day, at time $t + 1$, is represented by
$I_{t+1} \sim Binomial(S_t, 1 - {q_t}^{I_t})$

where

S_t = Number of susceptible kids at time t
q_t = Probability of not getting infected at time t = $1 - 0.02$ = 0.08
I_t = Number of infectious kids at time t

To build our simulation, we started with an initial Pandas DataFrame at time = 0. This is represented by Table 1. We can see from this table that we start with Tommy being infected (*infected* = 1) and the column *infection_days_left* = 3. This column keeps track of his (and other kids') remaining days of infection. There are 21 rows in this table to represent Tommy and his 20 other classmates.

Next, if we look at Table 2, this represents the end of Day 1 results of the simulation. It seems like no kid was infected by Tommy today. To simulate a kid being infected, a Uniform(0,1) random number is generated for each kid that is susceptible to infection on that day. If the random number is less than the probability of infection $(1 - {q_t}^{I_t})$, then the kid will be infected. We can see in Table 2 that none of the random number generated is less than the probability of infection, 0.02. Note that Tommy's *infection_days_left* is now 2 as Tommy will still be infected and infectious for the next 2 days.

On day 2 (Table 3), Liam is now infected! Note that the random number generated for Liam is 0.000114 which is less than the probability of infection, 0.02. The total number of infected kids at the end of the day is also now 2 (Tommy and Liam). On day 3 (Table 4), Tommy's infection_days_left = 0, signifying that it is the last day that he will be infectious. On the other hand, Liam will be infected for the next 2 days while Jackson and Amelia were newly infected today. It is observed that the total number of infected kids is now 4. On day 4 (Table 5), Tommy is no longer infected and *immunity_countdown = 3.* This means that it is the first of 3 days that he will be immune to infections. On day 7 (Table 6), Tommy will no longer be immune and will be susceptible to infection once again (note that a random number is generated for him now since he is susceptible).

```
Table at time = 0 (Initial Table)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	3	0	0
1	Sophia	0	0	0	0
2	Liam	0	0	0	0
3	Olivia	0	0	0	0
4	Noah	0	0	0	0
5	Riley	0	0	0	0
6	Jackson	0	0	0	0
7	Emma	0	0	0	0
8	Aiden	0	0	0	0
9	Ava	0	0	0	0
10	Elijah	0	0	0	0
11	Isabella	0	0	0	0
12	Grayson	0	0	0	0
13	Aria	0	0	0	0
14	Lucas	0	0	0	0
15	Aaliyah	0	0	0	0
16	Oliver	0	0	0	0
17	Amelia	0	0	0	0
18	Cayden	0	0	0	0
19	Mia	0	0	0	0
20	Jayden	0	0	0	0

Table 1

```
----- Day 1 -----
Probability infected: 0.02
Number of infected kids at the end of the day: 1

End of Day 1 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	2	0	NaN
1	Sophia	0	0	0	0.959434
2	Liam	0	0	0	0.803961
3	Olivia	0	0	0	0.032323
4	Noah	0	0	0	0.709387
5	Riley	0	0	0	0.465001
6	Jackson	0	0	0	0.947549
7	Emma	0	0	0	0.221433
8	Aiden	0	0	0	0.267072
9	Ava	0	0	0	0.081474
10	Elijah	0	0	0	0.428619
11	Isabella	0	0	0	0.109019
12	Grayson	0	0	0	0.633787
13	Aria	0	0	0	0.802963
14	Lucas	0	0	0	0.696800
15	Aaliyah	0	0	0	0.766211
16	Oliver	0	0	0	0.342454
17	Amelia	0	0	0	0.845851
18	Cayden	0	0	0	0.428769
19	Mia	0	0	0	0.824010
20	Jayden	0	0	0	0.626496

Table 2

```
----- Day 2 -----
Probability infected: 0.02
Number of infected kids at the end of the day: 2

End of Day 2 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	1	0	NaN
1	Sophia	0	0	0	0.720324
2	Liam	1	3	0	0.000114
3	Olivia	0	0	0	0.302333

Table 3

```
----- Day 3 -----
Probability infected: 0.0396
Number of infected kids at the end of the day: 4

End of Day 3 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	0	4	NaN
1	Sophia	0	0	0	0.313424
2	Liam	1	2	0	NaN
6	Jackson	1	3	0	0.039055
17	Amelia	1	3	0	0.018288

Table 4

```
----- Day 4 -----
Probability infected: 0.0776
Number of infected kids at the end of the day: 5

End of Day 4 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	3	NaN
1	Sophia	0	0	0	0.789279
2	Liam	1	1	0	NaN
3	Olivia	0	0	0	0.447894

Table 5

```
----- Day 7 -----
Probability infected: 0.1142
Number of infected kids at the end of the day: 7

End of Day 7 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.623360
1	Sophia	1	1	0	NaN

Table 6

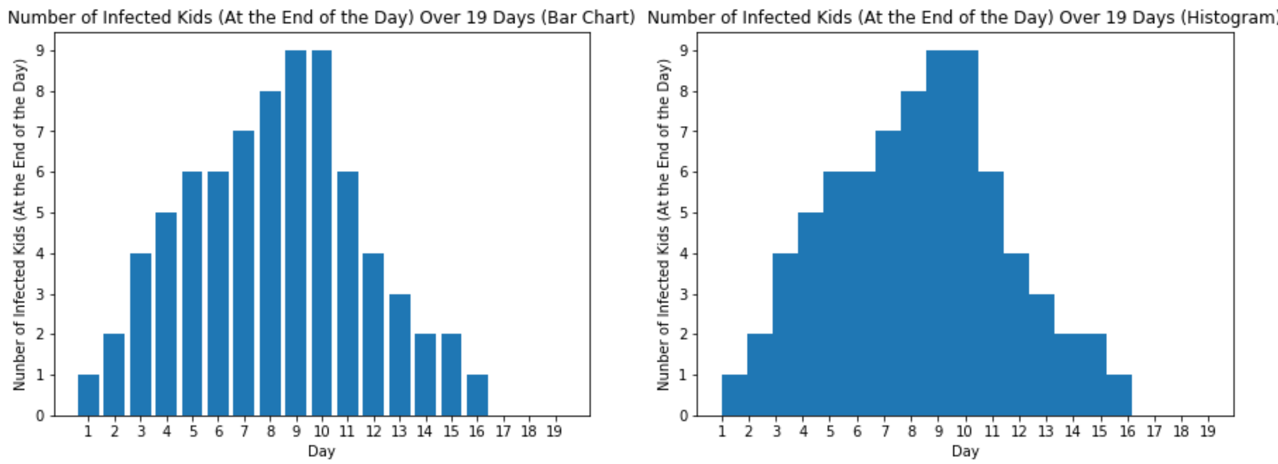


Figure 1

One run of our simulation lasts for 19 days. At the end of the run, a histogram and bar chart are presented (Figure 1). While a histogram is better at highlighting the overall shape of the distribution, we also included a bar chart as it is easier to visualize the number of infected kids on a particular day. In this run, we notice that the “epidemic” peaked on day 9 and 10 but eventually died down after day 16.

Now that we understand how one run of our simulation works, we will repeat the simulation for 10 times to find the (estimated) expected number of kids that are infected on each day. To obtain this, we take the mean of number of infected kids for each day. For example, the mean number of infected kids on day 1, 2, 3, …, 19 respectively.

```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19 (10 runs)
[1.5, 2.1, 2.9, 3.1, 3.4, 3.1, 3.1, 2.6, 2.4, 3.0, 2.9, 2.9, 3.0, 2.8, 2.6, 3.0, 3.4, 3.1, 3.1]
```

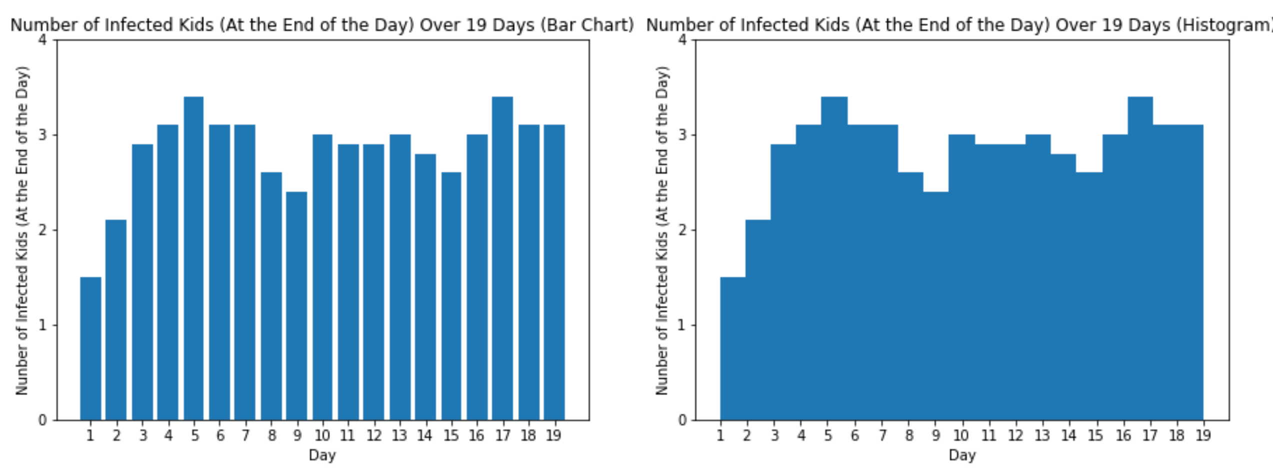


Figure 2

From our results in Figure 2, we can see that from day 3 onwards, the expected number of kids that are infected on each day fluctuates around 3. At this point, it is hard to estimate how long the “epidemic” will last from our results.

To improve our results, we will be going to do more simulations. We will do 50 simulations and 100 simulations, respectively.

```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19 (50 runs)
[1.42, 2.04, 2.8, 2.76, 3.0, 2.92, 3.0, 2.8, 2.64, 2.82, 2.66, 2.54, 2.74, 2.8, 2.82, 2.98, 3.02, 3.0, 2.92]
```

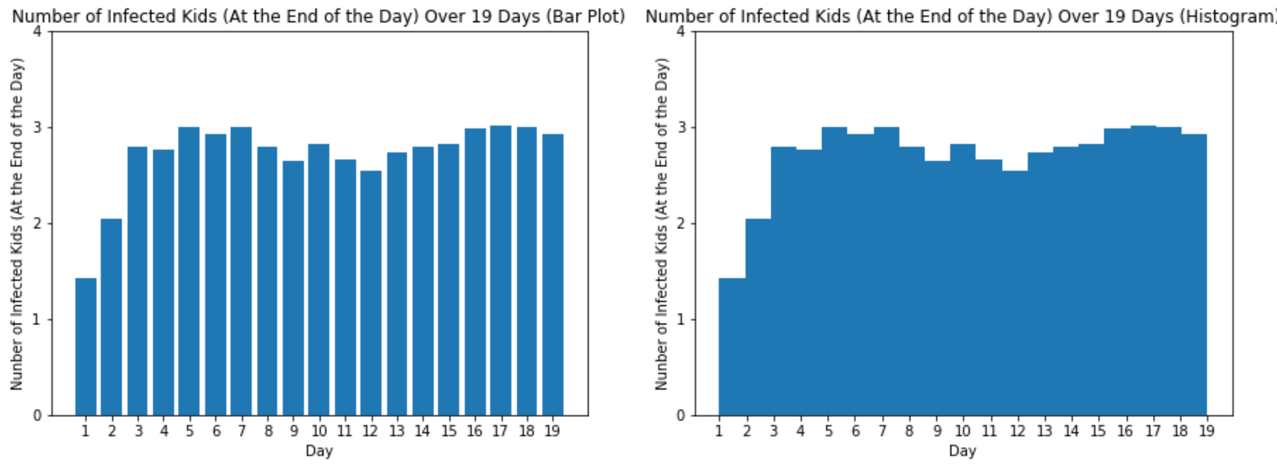


Figure 3

When we repeat the simulations for 50 times (Figure 3), the expected number of kids that are infected on each day still fluctuates around 3 after day 2, which is similar to our results when we did 10 repeated simulations. However, the range of values lies much closer to 3 this time.

```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19 (100 runs)
[1.36, 1.88, 2.67, 2.57, 3.0, 3.27, 3.43, 3.37, 3.32, 3.41, 3.21, 3.22, 3.25, 3.16, 3.1, 3.06, 2.96, 2.84, 2.76]
```

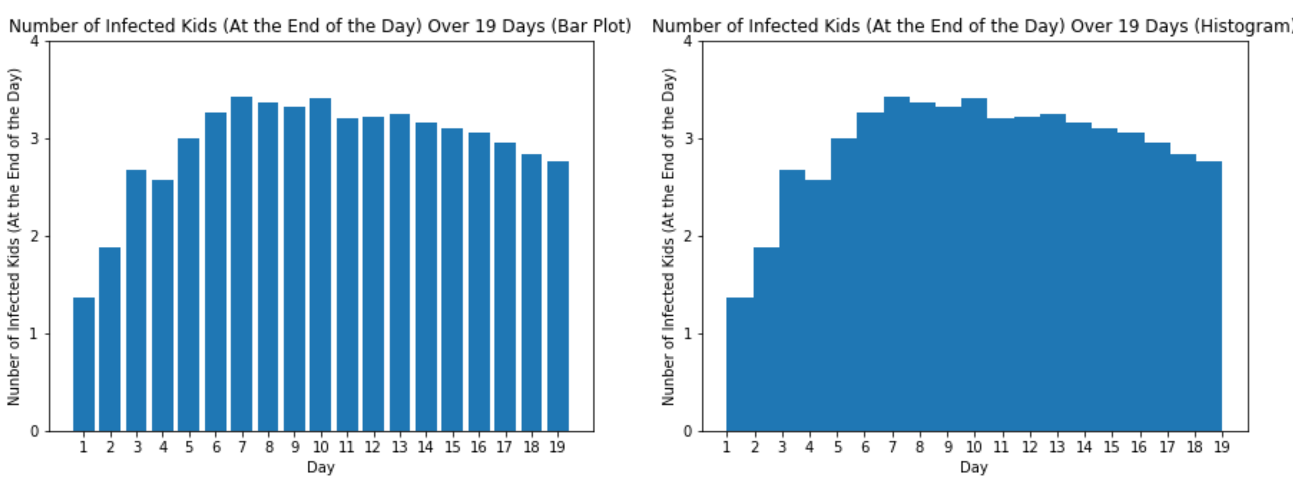
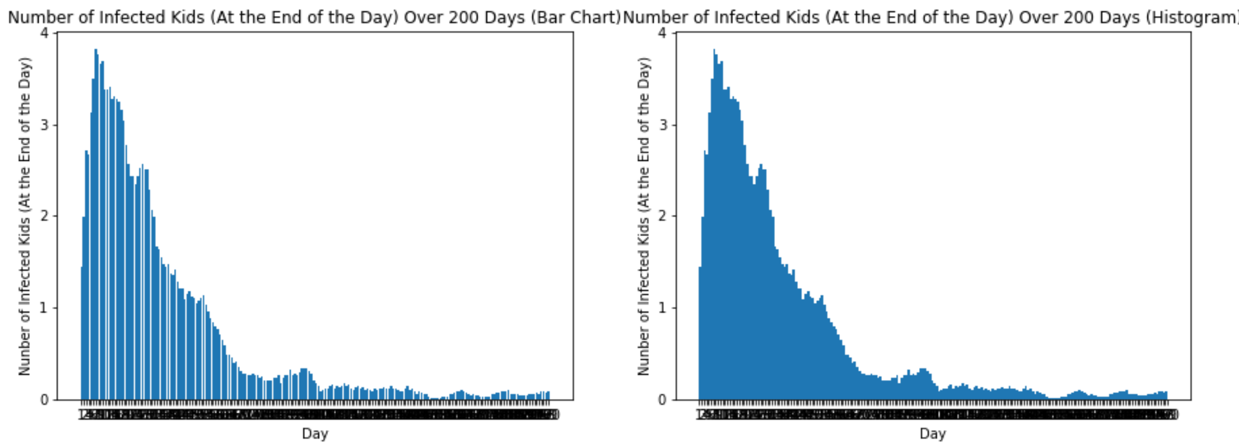


Figure 4

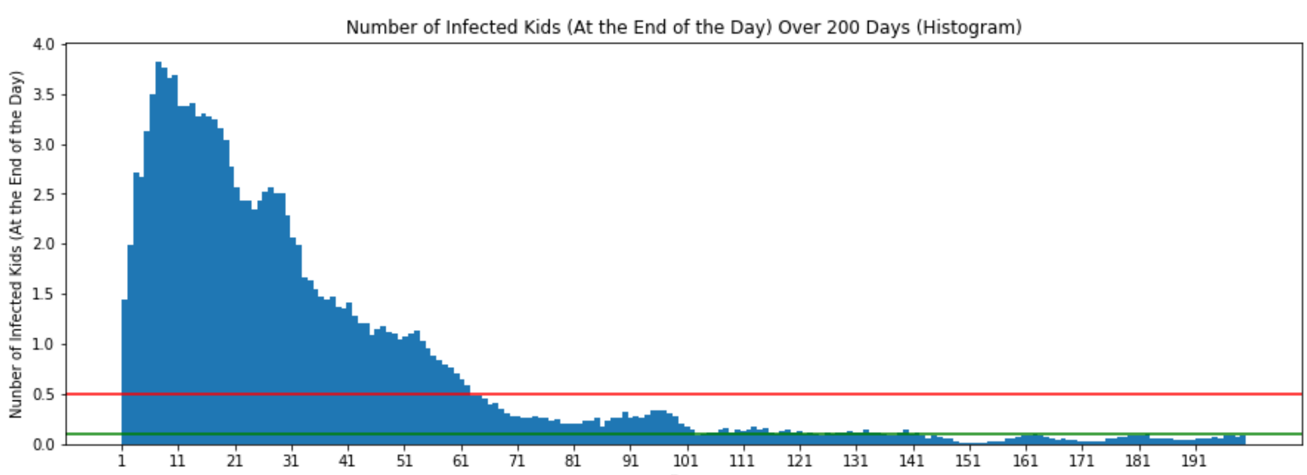
Interestingly, when we ran our simulation 100 times (Figure 4), we notice that there is an upwards trend from day 1 to day 7. It peaked around day 7 to day 10. Thereafter, there is a gentle downward slope. This stresses the importance of doing a high number of repeated simulations to get a meaningful result. Let’s keep exploring!

Seeing that the slope is going downwards, we decided to run over a longer period of time (200 days instead of 19 days) and repeat this simulation 100 times to get the (estimated) expected number of kids that are infected each day.

```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-200 (100 runs)
[1.44, 1.99, 2.71, 2.67, 3.13, 3.49, 3.82, 3.76, 3.65, 3.68, 3.37, 3.37, 3.4, 3.28, 3.31, 3.27, 3.24, 3.15, 3.03, 2.77, 2.5
7, 2.43, 2.43, 2.34, 2.43, 2.52, 2.57, 2.5, 2.51, 2.28, 2.07, 1.99, 1.66, 1.63, 1.54, 1.48, 1.44, 1.47, 1.37, 1.35, 1.41, 1.
28, 1.21, 1.2, 1.09, 1.15, 1.18, 1.12, 1.1, 1.05, 1.08, 1.11, 1.14, 1.03, 0.95, 0.88, 0.84, 0.8, 0.76, 0.71, 0.64, 0.58, 0.4
8, 0.49, 0.46, 0.4, 0.41, 0.35, 0.3, 0.28, 0.28, 0.26, 0.26, 0.28, 0.26, 0.27, 0.23, 0.25, 0.2, 0.2, 0.21, 0.21, 0.23, 0.24,
0.26, 0.18, 0.24, 0.26, 0.27, 0.32, 0.26, 0.28, 0.26, 0.29, 0.34, 0.33, 0.33, 0.3, 0.28, 0.21, 0.17, 0.14, 0.09, 0.1, 0.12,
0.12, 0.14, 0.16, 0.12, 0.15, 0.13, 0.15, 0.18, 0.15, 0.16, 0.11, 0.1, 0.13, 0.14, 0.12, 0.13, 0.1, 0.11, 0.1, 0.09, 0.11,
0.1, 0.12, 0.11, 0.13, 0.12, 0.11, 0.15, 0.11, 0.11, 0.1, 0.09, 0.08, 0.11, 0.14, 0.1, 0.12, 0.08, 0.06, 0.08, 0.07, 0.06,
0.06, 0.03, 0.01, 0.01, 0.01, 0.01, 0.01, 0.02, 0.02, 0.03, 0.05, 0.06, 0.07, 0.08, 0.08, 0.1, 0.08, 0.07, 0.06, 0.04, 0.05,
0.04, 0.04, 0.02, 0.02, 0.03, 0.02, 0.02, 0.05, 0.05, 0.07, 0.07, 0.08, 0.08, 0.08, 0.1, 0.06, 0.06, 0.05, 0.05, 0.04, 0.04,
0.04, 0.04, 0.05, 0.06, 0.06, 0.07, 0.06, 0.09, 0.09, 0.07, 0.09]
```



After simulating the epidemic for 200 days (100 runs), the final (detailed) histogram below shows that the “epidemic” will eventually die out! If we consider the expected number of infections below 0.5 for it to be considered as ‘dying out’, then the epidemic would have only lasted until day 63-65. However, if we want to consider a number closer to 0 (anything below 0.01, green line), then the ‘epidemic’ would have probably lasted until day 145-147.



Conclusion

An epidemic will eventually come to an end. It might take some time, in our case, it lasts around 147 days. However, in a real pandemic, such as Covid-19, it lasts a lot longer. And how fast it ends depends on a lot of other mitigating factors. As an epidemic progresses, reactions to the epidemic may very well change the contact rates. There are counter measures that can be adopted as well, such as social distancing, mask wearing, or lockdown. This will in turn alter the contact rates in some regard and reduce the spread of the epidemic.

[6]

It is also noted that models like SIR can also be used to model vaccination distributions, which will be a valuable and useful counter measure to end the epidemic. To do so, we can introduce an additional compartment to the SIR model, V, for vaccinated individuals. That is another interesting area to be explored!

[7]

Appendix

[6]https://image.freepik.com/free-vector/back-school-new-normal-lifestyle-concept-happy-kids-wearing-face-mask-social-distancing-protect-coronavirus-covid-19-group-children-friends-go-school-isolated_83111-885.jpg

[7]https://image.freepik.com/free-vector/kids-vaccination-benefits-promotion-illustration-with-child-health-protection-shield-symbol-doctor-with-syringe-cartoon_1284-32968.jpg

Appendix

March 10, 2021

1 Simulation Mini Project 1: Pandemic Flu Spread

Firstly, we start by creating a pandas DataFrame to store and keep track of our simulation.

[1]:
```
# Import libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Initialize variable
zero = np.zeros(21,dtype='int8')

# Prepare the initial dataframe
names =␣
 ↪['Tommy','Sophia','Liam','Olivia','Noah','Riley','Jackson','Emma','Aiden',␣
 ↪'Ava',
        ␣
 ↪'Elijah','Isabella','Grayson','Aria','Lucas','Aaliyah','Oliver','Amelia','Cayden','Mia','Ja

kids = pd.DataFrame({'names':names,'infected': zero,'infection_days_left':
                     zero,'immunity_countdown':zero,'random_no':zero,
                     "prob_infected":zero})
kids.loc[kids["names"]=="Tommy","infected"] = 1
kids.loc[kids["names"]=="Tommy","infection_days_left"] = 3
kids_starting = kids.copy()
print("Table at time = 0 (Initial Table)")
kids[["names","infected","infection_days_left","immunity_countdown","random_no"]]
```

```
Table at time = 0 (Initial Table)
```

[1]:

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	3	0	0
1	Sophia	0	0	0	0
2	Liam	0	0	0	0
3	Olivia	0	0	0	0
4	Noah	0	0	0	0
5	Riley	0	0	0	0
6	Jackson	0	0	0	0

7	Emma	0	0	0	0
8	Aiden	0	0	0	0
9	Ava	0	0	0	0
10	Elijah	0	0	0	0
11	Isabella	0	0	0	0
12	Grayson	0	0	0	0
13	Aria	0	0	0	0
14	Lucas	0	0	0	0
15	Aaliyah	0	0	0	0
16	Oliver	0	0	0	0
17	Amelia	0	0	0	0
18	Cayden	0	0	0	0
19	Mia	0	0	0	0
20	Jayden	0	0	0	0

Next, we create functions to help us with our simulation.

[2]:
```
def infected_or_not(df):
    # If kid is not already infected and not within immunity window, the kid will get infected
    # if random no is less than infection probability
    if (df["random_no"] < df["prob_infected"]) and (df["infected"] == 0) and (df["immunity_countdown"] == 0):
        return 1

    # If the kid has recovered from an infection, set infection to 0
    elif(df["immunity_countdown"] == 3):
        return 0

    # If the kid is still within the 3 day infection window, the kid will still be infected
    else:
        return df["infected"]

def days_left(df):
    # If this is the first day of infection, start countdown
    if (df["infected"] == 1) and (df["infection_days_left"] == 0):
        return 3

    # If this is not the first day of infection, reduce the no. of days left of infection by 1
    elif df["infection_days_left"] != 0:
        return df["infection_days_left"] - 1

    # If the kid is not infected, retain the number of infection days left (ie. 0)
    else:
```

```
        return df["infection_days_left"]

def immu_days_left(df):
    # If this is the last day of infection, start immunity countdown at 4
    # (ie. The kid will immune for 3 days starting the next day)
    if df["infection_days_left"] == 1:
        return 4

    # If the kid is currently within the immunity window, reduce the number of␣
 ↪days by 1
    elif df["immunity_countdown"] != 0:
        return df["immunity_countdown"] - 1

    # If the kid is not within immunity window, retain current number of␣
 ↪immunity days left (ie. 0)
    else:
        return df["immunity_countdown"]

def plot_graphs(x,y):
    # Create figure and axes
    fig = plt.figure(figsize=(15,5))
    ax1 = fig.add_subplot(1, 2, 1)
    ax2 = fig.add_subplot(1, 2, 2)

    # Plot bar plot
    ax1.bar(x, height=y)
    ax1.set_xticks(x)
    ax1.set_yticks(np.arange(0,max(y)+1,1))
    ax1.set_title("Number of Infected Kids (At the End of the Day) Over␣
 ↪"+str(len(x))+" Days (Bar Chart)")
    ax1.set_xlabel("Day")
    ax1.set_ylabel("Nunber of Infected Kids (At the End of the Day)")

    # Plot histogram
    ax2.hist(x, weights=y, bins = len(x))
    ax2.set_xticks(np.arange(1, len(x)+1, 1.0))
    ax2.set_yticks(np.arange(0,max(y)+1,1))
    ax2.set_title("Number of Infected Kids (At the End of the Day) Over␣
 ↪"+str(len(x))+" Days (Histogram)")
    ax2.set_xlabel("Day")
    ax2.set_ylabel("Nunber of Infected Kids (At the End of the Day)")
```

Then we create a function that will simulate one run of our simulation.

[3]:
```
def simulate_flu(seed, display_tables = 0, display_graphs = 1, days = 19):
```

```
    # Initialize variables
    kids = kids_starting.copy()
    kids["prob_infected"] = 0.02
    kids["random_no"] = np.random.rand(21)
    np.random.seed(seed)
    no_infected = 1
    y = []
    x = []

    # Simulate pandemic for 19 days
    for d in range(1,days+1):

        if display_tables == 1:
            print("----- Day "+str(d)+" -----")
            print("Probability infected: " + str(round(1-(1-0.
↪02)**no_infected,4)))
        x.append(d)

        # Infect other kids
        kids["infected"] = kids.apply(infected_or_not, axis = 1)

        # Adjust the infection days left (add days for new infections and␣
↪reduce days for ongoing infections)
        kids["infection_days_left"] = kids.apply(days_left, axis = 1)

        # Total number of infected kids at the end of the day
        no_infected = np.sum(kids["infected"])
        y.append(no_infected)
        if display_tables == 1:
            print("Number of infected kids at the end of the day: "␣
↪+str(no_infected))

        # If the kid is already infected or currently in immunity period, set␣
↪random number to none
        kids.
↪loc[(kids["infected"]==1)&(kids["infection_days_left"]<3),"random_no"] = None
        kids.loc[kids["immunity_countdown"]!=0,"random_no"] = None

        if display_tables == 1:
            # Print end of day results (who's infected by the end of the day)
            print("\nNote: A kid that is infected today has infected = 1,␣
↪infection days left = 3 and random number < infection probability")
            print("      A kid that is infected today will be infectious for␣
↪the next 3 days.")
            print("      After recovery, a kid will be immune to infections for␣
↪3 days.")
```

```
            print("      The random number is NaN when the kid is already␣
↪infected (1st-3rd infectious day) or within immunity period.")
            print("\nEnd of Day " + str(d) + " results (Who is infected by the␣
↪end of the day?)")
           ␣
↪display(kids[["names","infected","infection_days_left","immunity_countdown","random_no"]])

        # Update probability of infection for the next day
        kids["prob_infected"] = 1-(1-0.02)**no_infected

        # Update random number for the next day
        kids["random_no"] = np.random.rand(21)

        # Update immunity days
        kids["immunity_countdown"] = kids.apply(immu_days_left,axis = 1)

    if display_graphs == 1:
        plot_graphs(x,y)

    return y
```

Now, we simulate one run of our simulation, printing the end of day results table for each day.

[4]:
```
# Simulate one run of our simulation
_ = simulate_flu(1,1)
```

```
----- Day 1 -----
Probability infected: 0.02
Number of infected kids at the end of the day: 1

Note: A kid that is infected today has infected = 1, infection days left = 3 and
random number < infection probability
      A kid that is infected today will be infectious for the next 3 days.
      After recovery, a kid will be immune to infections for 3 days.
      The random number is NaN when the kid is already infected (1st-3rd
infectious day) or within immunity period.

End of Day 1 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	2	0	NaN
1	Sophia	0	0	0	0.643416
2	Liam	0	0	0	0.665396
3	Olivia	0	0	0	0.346453
4	Noah	0	0	0	0.945650
5	Riley	0	0	0	0.230128
6	Jackson	0	0	0	0.363021
7	Emma	0	0	0	0.597689

```
8      Aiden         0                    0                   0   0.232230
9        Ava         0                    0                   0   0.707566
10    Elijah         0                    0                   0   0.876802
11  Isabella         0                    0                   0   0.985916
12   Grayson         0                    0                   0   0.804083
13      Aria         0                    0                   0   0.052461
14     Lucas         0                    0                   0   0.327487
15   Aaliyah         0                    0                   0   0.926618
16    Oliver         0                    0                   0   0.207116
17    Amelia         0                    0                   0   0.246280
18    Cayden         0                    0                   0   0.579567
19       Mia         0                    0                   0   0.489981
20    Jayden         0                    0                   0   0.175113

----- Day 2 -----
Probability infected: 0.02
Number of infected kids at the end of the day: 2

Note: A kid that is infected today has infected = 1, infection days left = 3 and
random number < infection probability
      A kid that is infected today will be infectious for the next 3 days.
      After recovery, a kid will be immune to infections for 3 days.
      The random number is NaN when the kid is already infected (1st-3rd
infectious day) or within immunity period.

End of Day 2 results (Who is infected by the end of the day?)

       names  infected  infection_days_left  immunity_countdown  random_no
0      Tommy         1                    1                   0        NaN
1     Sophia         0                    0                   0   0.720324
2       Liam         1                    3                   0   0.000114
3     Olivia         0                    0                   0   0.302333
4       Noah         0                    0                   0   0.146756
5      Riley         0                    0                   0   0.092339
6    Jackson         0                    0                   0   0.186260
7       Emma         0                    0                   0   0.345561
8      Aiden         0                    0                   0   0.396767
9        Ava         0                    0                   0   0.538817
10    Elijah         0                    0                   0   0.419195
11  Isabella         0                    0                   0   0.685220
12   Grayson         0                    0                   0   0.204452
13      Aria         0                    0                   0   0.878117
14     Lucas         0                    0                   0   0.027388
15   Aaliyah         0                    0                   0   0.670468
16    Oliver         0                    0                   0   0.417305
17    Amelia         0                    0                   0   0.558690
18    Cayden         0                    0                   0   0.140387
19       Mia         0                    0                   0   0.198101
```

20	Jayden	0	0	0	0.800745

----- Day 3 -----
Probability infected: 0.0396
Number of infected kids at the end of the day: 4

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability

A kid that is infected today will be infectious for the next 3 days.

After recovery, a kid will be immune to infections for 3 days.

The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 3 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	1	0	4	NaN
1	Sophia	0	0	0	0.313424
2	Liam	1	2	0	NaN
3	Olivia	0	0	0	0.876389
4	Noah	0	0	0	0.894607
5	Riley	0	0	0	0.085044
6	Jackson	1	3	0	0.039055
7	Emma	0	0	0	0.169830
8	Aiden	0	0	0	0.878143
9	Ava	0	0	0	0.098347
10	Elijah	0	0	0	0.421108
11	Isabella	0	0	0	0.957890
12	Grayson	0	0	0	0.533165
13	Aria	0	0	0	0.691877
14	Lucas	0	0	0	0.315516
15	Aaliyah	0	0	0	0.686501
16	Oliver	0	0	0	0.834626
17	Amelia	1	3	0	0.018288
18	Cayden	0	0	0	0.750144
19	Mia	0	0	0	0.988861
20	Jayden	0	0	0	0.748166

----- Day 4 -----
Probability infected: 0.0776
Number of infected kids at the end of the day: 5

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability

A kid that is infected today will be infectious for the next 3 days.

After recovery, a kid will be immune to infections for 3 days.

The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 4 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	3	NaN
1	Sophia	0	0	0	0.789279
2	Liam	1	1	0	NaN
3	Olivia	0	0	0	0.447894
4	Noah	0	0	0	0.908596
5	Riley	0	0	0	0.293614
6	Jackson	1	2	0	NaN
7	Emma	0	0	0	0.130029
8	Aiden	1	3	0	0.019367
9	Ava	0	0	0	0.678836
10	Elijah	0	0	0	0.211628
11	Isabella	0	0	0	0.265547
12	Grayson	0	0	0	0.491573
13	Aria	1	3	0	0.053363
14	Lucas	0	0	0	0.574118
15	Aaliyah	0	0	0	0.146729
16	Oliver	0	0	0	0.589306
17	Amelia	1	2	0	NaN
18	Cayden	0	0	0	0.102334
19	Mia	0	0	0	0.414056
20	Jayden	0	0	0	0.694400

----- Day 5 -----
Probability infected: 0.0961
Number of infected kids at the end of the day: 6

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 5 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	2	NaN
1	Sophia	1	3	0	0.049953
2	Liam	1	0	4	NaN
3	Olivia	0	0	0	0.663795
4	Noah	0	0	0	0.514889
5	Riley	0	0	0	0.944595
6	Jackson	1	1	0	NaN

7	Emma	0	0	0	0.903402
8	Aiden	1	2	0	NaN
9	Ava	0	0	0	0.139276
10	Elijah	0	0	0	0.807391
11	Isabella	0	0	0	0.397677
12	Grayson	0	0	0	0.165354
13	Aria	1	2	0	NaN
14	Lucas	0	0	0	0.347766
15	Aaliyah	0	0	0	0.750812
16	Oliver	0	0	0	0.725998
17	Amelia	1	1	0	NaN
18	Cayden	0	0	0	0.623672
19	Mia	0	0	0	0.750942
20	Jayden	0	0	0	0.348898

----- Day 6 -----
Probability infected: 0.1142
Number of infected kids at the end of the day: 6

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability

A kid that is infected today will be infectious for the next 3 days.

After recovery, a kid will be immune to infections for 3 days.

The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 6 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	1	NaN
1	Sophia	1	2	0	NaN
2	Liam	0	0	3	NaN
3	Olivia	0	0	0	0.964840
4	Noah	0	0	0	0.663441
5	Riley	0	0	0	0.621696
6	Jackson	1	0	4	NaN
7	Emma	0	0	0	0.949489
8	Aiden	1	1	0	NaN
9	Ava	0	0	0	0.578390
10	Elijah	0	0	0	0.408137
11	Isabella	0	0	0	0.237027
12	Grayson	0	0	0	0.903380
13	Aria	1	1	0	NaN
14	Lucas	1	3	0	0.002870
15	Aaliyah	0	0	0	0.617145
16	Oliver	0	0	0	0.326645
17	Amelia	1	0	4	NaN
18	Cayden	0	0	0	0.885942

19	Mia	0	0	0	0.357270
20	Jayden	0	0	0	0.908535

```
----- Day 7 -----
Probability infected: 0.1142
Number of infected kids at the end of the day: 7

Note: A kid that is infected today has infected = 1, infection days left = 3 and
random number < infection probability
      A kid that is infected today will be infectious for the next 3 days.
      After recovery, a kid will be immune to infections for 3 days.
      The random number is NaN when the kid is already infected (1st-3rd
infectious day) or within immunity period.

End of Day 7 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.623360
1	Sophia	1	1	0	NaN
2	Liam	0	0	2	NaN
3	Olivia	0	0	0	0.690897
4	Noah	0	0	0	0.997323
5	Riley	0	0	0	0.172341
6	Jackson	0	0	3	NaN
7	Emma	0	0	0	0.932595
8	Aiden	1	0	4	NaN
9	Ava	1	3	0	0.066000
10	Elijah	0	0	0	0.755463
11	Isabella	0	0	0	0.753876
12	Grayson	0	0	0	0.923025
13	Aria	1	0	4	NaN
14	Lucas	1	2	0	NaN
15	Aaliyah	1	3	0	0.019880
16	Oliver	1	3	0	0.026211
17	Amelia	0	0	3	NaN
18	Cayden	0	0	0	0.246211
19	Mia	0	0	0	0.860028
20	Jayden	0	0	0	0.538831

```
----- Day 8 -----
Probability infected: 0.1319
Number of infected kids at the end of the day: 8

Note: A kid that is infected today has infected = 1, infection days left = 3 and
random number < infection probability
      A kid that is infected today will be infectious for the next 3 days.
      After recovery, a kid will be immune to infections for 3 days.
```

The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 8 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.552822
1	Sophia	1	0	4	NaN
2	Liam	0	0	1	NaN
3	Olivia	0	0	0	0.279184
4	Noah	0	0	0	0.585759
5	Riley	0	0	0	0.969596
6	Jackson	0	0	2	NaN
7	Emma	1	3	0	0.018647
8	Aiden	0	0	3	NaN
9	Ava	1	2	0	NaN
10	Elijah	0	0	0	0.807105
11	Isabella	0	0	0	0.387861
12	Grayson	0	0	0	0.863542
13	Aria	0	0	3	NaN
14	Lucas	1	1	0	NaN
15	Aaliyah	1	2	0	NaN
16	Oliver	1	2	0	NaN
17	Amelia	0	0	2	NaN
18	Cayden	1	3	0	0.044552
19	Mia	1	3	0	0.107494
20	Jayden	0	0	0	0.225709

----- Day 9 -----
Probability infected: 0.1492
Number of infected kids at the end of the day: 9

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 9 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.712989
1	Sophia	0	0	3	NaN
2	Liam	1	3	0	0.012556
3	Olivia	1	3	0	0.071974
4	Noah	0	0	0	0.967276
5	Riley	0	0	0	0.568100

6	Jackson	0	0	1	NaN
7	Emma	1	2	0	NaN
8	Aiden	0	0	2	NaN
9	Ava	1	1	0	NaN
10	Elijah	0	0	0	0.581359
11	Isabella	0	0	0	0.970020
12	Grayson	0	0	0	0.846829
13	Aria	0	0	2	NaN
14	Lucas	1	0	4	NaN
15	Aaliyah	1	1	0	NaN
16	Oliver	1	1	0	NaN
17	Amelia	0	0	1	NaN
18	Cayden	1	2	0	NaN
19	Mia	1	2	0	NaN
20	Jayden	0	0	0	0.486345

----- Day 10 -----
Probability infected: 0.1663
Number of infected kids at the end of the day: 9

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 10 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.606329
1	Sophia	0	0	2	NaN
2	Liam	1	2	0	NaN
3	Olivia	1	2	0	NaN
4	Noah	0	0	0	0.579745
5	Riley	0	0	0	0.380141
6	Jackson	0	0	0	0.550948
7	Emma	1	1	0	NaN
8	Aiden	0	0	1	NaN
9	Ava	1	0	4	NaN
10	Elijah	1	3	0	0.066335
11	Isabella	0	0	0	0.370084
12	Grayson	0	0	0	0.629718
13	Aria	0	0	1	NaN
14	Lucas	0	0	3	NaN
15	Aaliyah	1	0	4	NaN
16	Oliver	1	0	4	NaN
17	Amelia	0	0	0	0.804755

18	Cayden	1	1	0	NaN
19	Mia	1	1	0	NaN
20	Jayden	0	0	0	0.524670

----- Day 11 -----
Probability infected: 0.1663
Number of infected kids at the end of the day: 6

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 11 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.924808
1	Sophia	0	0	1	NaN
2	Liam	1	1	0	NaN
3	Olivia	1	1	0	NaN
4	Noah	0	0	0	0.772178
5	Riley	0	0	0	0.907816
6	Jackson	0	0	0	0.931972
7	Emma	1	0	4	NaN
8	Aiden	0	0	0	0.234362
9	Ava	0	0	3	NaN
10	Elijah	1	2	0	NaN
11	Isabella	0	0	0	0.950176
12	Grayson	0	0	0	0.556653
13	Aria	0	0	0	0.915606
14	Lucas	0	0	2	NaN
15	Aaliyah	0	0	3	NaN
16	Oliver	0	0	3	NaN
17	Amelia	0	0	0	0.604310
18	Cayden	1	0	4	NaN
19	Mia	1	0	4	NaN
20	Jayden	0	0	0	0.918733

----- Day 12 -----
Probability infected: 0.1142
Number of infected kids at the end of the day: 4

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.

After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 12 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.394876
1	Sophia	0	0	0	0.963263
2	Liam	1	0	4	NaN
3	Olivia	1	0	4	NaN
4	Noah	0	0	0	0.135079
5	Riley	0	0	0	0.505662
6	Jackson	1	3	0	0.021525
7	Emma	0	0	3	NaN
8	Aiden	0	0	0	0.827115
9	Ava	0	0	2	NaN
10	Elijah	1	1	0	NaN
11	Isabella	0	0	0	0.332064
12	Grayson	0	0	0	0.130997
13	Aria	0	0	0	0.809491
14	Lucas	0	0	1	NaN
15	Aaliyah	0	0	2	NaN
16	Oliver	0	0	2	NaN
17	Amelia	0	0	0	0.878832
18	Cayden	0	0	3	NaN
19	Mia	0	0	3	NaN
20	Jayden	0	0	0	0.459880

----- Day 13 -----
Probability infected: 0.0776
Number of infected kids at the end of the day: 3

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 13 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.546347
1	Sophia	0	0	0	0.798604
2	Liam	0	0	3	NaN
3	Olivia	0	0	3	NaN
4	Noah	0	0	0	0.599110

5	Riley	1	3	0	0.015533
6	Jackson	1	2	0	NaN
7	Emma	0	0	2	NaN
8	Aiden	0	0	0	0.807361
9	Ava	0	0	1	NaN
10	Elijah	1	0	4	NaN
11	Isabella	0	0	0	0.577857
12	Grayson	0	0	0	0.184010
13	Aria	0	0	0	0.787929
14	Lucas	0	0	0	0.612031
15	Aaliyah	0	0	1	NaN
16	Oliver	0	0	1	NaN
17	Amelia	0	0	0	0.679069
18	Cayden	0	0	2	NaN
19	Mia	0	0	2	NaN
20	Jayden	0	0	0	0.976759

```
----- Day 14 -----
Probability infected: 0.0588
Number of infected kids at the end of the day: 2

Note: A kid that is infected today has infected = 1, infection days left = 3 and
random number &lt; infection probability
      A kid that is infected today will be infectious for the next 3 days.
      After recovery, a kid will be immune to infections for 3 days.
      The random number is NaN when the kid is already infected (1st-3rd
infectious day) or within immunity period.

End of Day 14 results (Who is infected by the end of the day?)
```

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.376580
1	Sophia	0	0	0	0.973784
2	Liam	0	0	2	NaN
3	Olivia	0	0	2	NaN
4	Noah	0	0	0	0.574712
5	Riley	1	2	0	NaN
6	Jackson	1	1	0	NaN
7	Emma	0	0	1	NaN
8	Aiden	0	0	0	0.750022
9	Ava	0	0	0	0.858314
10	Elijah	0	0	3	NaN
11	Isabella	0	0	0	0.698057
12	Grayson	0	0	0	0.864479
13	Aria	0	0	0	0.322681
14	Lucas	0	0	0	0.670789
15	Aaliyah	0	0	0	0.450874
16	Oliver	0	0	0	0.382103

17	Amelia	0	0	0	0.410811
18	Cayden	0	0	1	NaN
19	Mia	0	0	1	NaN
20	Jayden	0	0	0	0.621919

----- Day 15 -----
Probability infected: 0.0396
Number of infected kids at the end of the day: 2

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 15 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.430247
1	Sophia	0	0	0	0.973802
2	Liam	0	0	1	NaN
3	Olivia	0	0	1	NaN
4	Noah	0	0	0	0.426701
5	Riley	1	1	0	NaN
6	Jackson	1	0	4	NaN
7	Emma	0	0	0	0.879998
8	Aiden	0	0	0	0.903842
9	Ava	0	0	0	0.662720
10	Elijah	0	0	2	NaN
11	Isabella	0	0	0	0.252367
12	Grayson	0	0	0	0.854898
13	Aria	0	0	0	0.527715
14	Lucas	0	0	0	0.802161
15	Aaliyah	0	0	0	0.572489
16	Oliver	0	0	0	0.733143
17	Amelia	0	0	0	0.519012
18	Cayden	0	0	0	0.770884
19	Mia	0	0	0	0.568858
20	Jayden	0	0	0	0.465710

----- Day 16 -----
Probability infected: 0.0396
Number of infected kids at the end of the day: 1

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability

A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 16 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.342689
1	Sophia	0	0	0	0.068209
2	Liam	0	0	0	0.377924
3	Olivia	0	0	0	0.079626
4	Noah	0	0	0	0.982817
5	Riley	1	0	4	NaN
6	Jackson	0	0	3	NaN
7	Emma	0	0	0	0.874962
8	Aiden	0	0	0	0.688413
9	Ava	0	0	0	0.569494
10	Elijah	0	0	1	NaN
11	Isabella	0	0	0	0.466880
12	Grayson	0	0	0	0.345172
13	Aria	0	0	0	0.225040
14	Lucas	0	0	0	0.592512
15	Aaliyah	0	0	0	0.312270
16	Oliver	0	0	0	0.916306
17	Amelia	0	0	0	0.909636
18	Cayden	0	0	0	0.257118
19	Mia	0	0	0	0.110891
20	Jayden	0	0	0	0.192963

----- Day 17 -----
Probability infected: 0.02
Number of infected kids at the end of the day: 0

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 17 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.499584
1	Sophia	0	0	0	0.728586
2	Liam	0	0	0	0.208194
3	Olivia	0	0	0	0.248034

4	Noah	0	0	0	0.851672
5	Riley	0	0	3	NaN
6	Jackson	0	0	2	NaN
7	Emma	0	0	0	0.233666
8	Aiden	0	0	0	0.101967
9	Ava	0	0	0	0.515857
10	Elijah	0	0	0	0.477141
11	Isabella	0	0	0	0.152672
12	Grayson	0	0	0	0.621806
13	Aria	0	0	0	0.544010
14	Lucas	0	0	0	0.654137
15	Aaliyah	0	0	0	0.144546
16	Oliver	0	0	0	0.751528
17	Amelia	0	0	0	0.222049
18	Cayden	0	0	0	0.519352
19	Mia	0	0	0	0.785296
20	Jayden	0	0	0	0.022330

----- Day 18 -----
Probability infected: 0.0
Number of infected kids at the end of the day: 0

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability
A kid that is infected today will be infectious for the next 3 days.
After recovery, a kid will be immune to infections for 3 days.
The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

End of Day 18 results (Who is infected by the end of the day?)

	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.324362
1	Sophia	0	0	0	0.872922
2	Liam	0	0	0	0.844710
3	Olivia	0	0	0	0.538441
4	Noah	0	0	0	0.866608
5	Riley	0	0	2	NaN
6	Jackson	0	0	1	NaN
7	Emma	0	0	0	0.854115
8	Aiden	0	0	0	0.098743
9	Ava	0	0	0	0.651304
10	Elijah	0	0	0	0.703517
11	Isabella	0	0	0	0.610241
12	Grayson	0	0	0	0.799615
13	Aria	0	0	0	0.034571
14	Lucas	0	0	0	0.770239
15	Aaliyah	0	0	0	0.731729

16	Oliver	0	0	0	0.259698
17	Amelia	0	0	0	0.257069
18	Cayden	0	0	0	0.632303
19	Mia	0	0	0	0.345297
20	Jayden	0	0	0	0.796589

----- Day 19 -----

Probability infected: 0.0

Number of infected kids at the end of the day: 0

Note: A kid that is infected today has infected = 1, infection days left = 3 and random number < infection probability

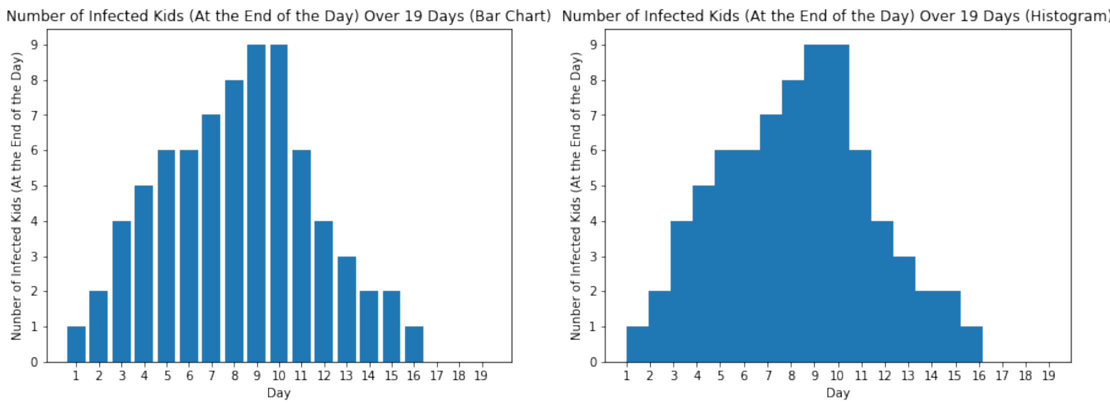
A kid that is infected today will be infectious for the next 3 days.

After recovery, a kid will be immune to infections for 3 days.

The random number is NaN when the kid is already infected (1st-3rd infectious day) or within immunity period.

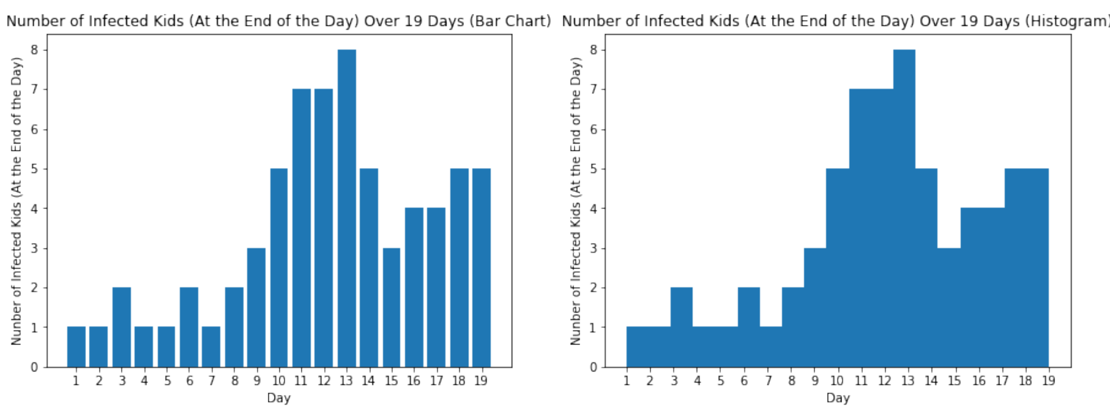
End of Day 19 results (Who is infected by the end of the day?)

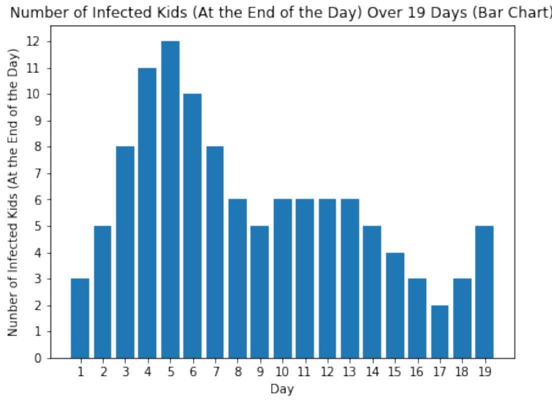
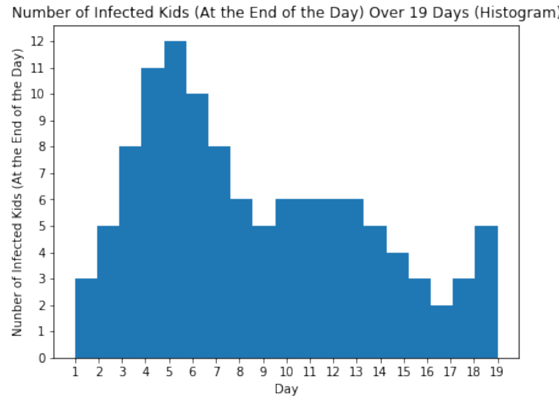
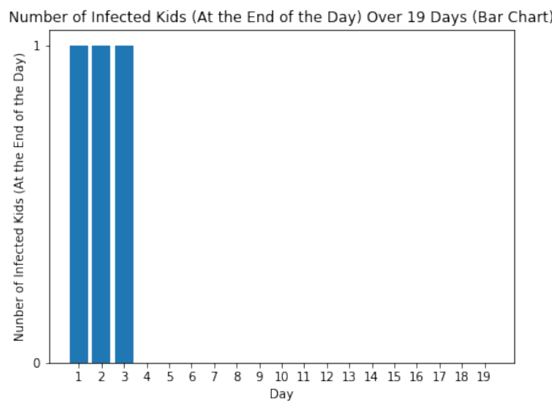
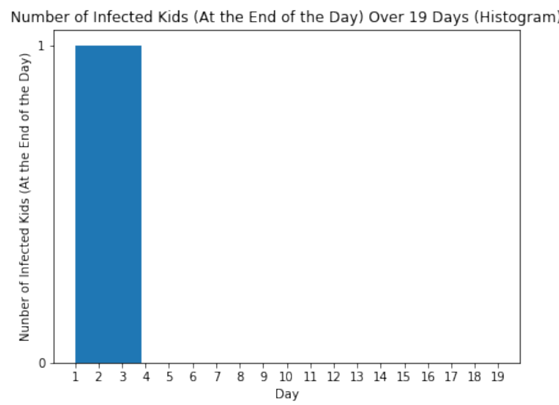
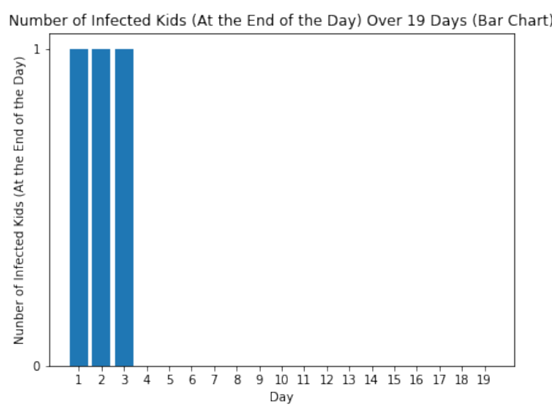
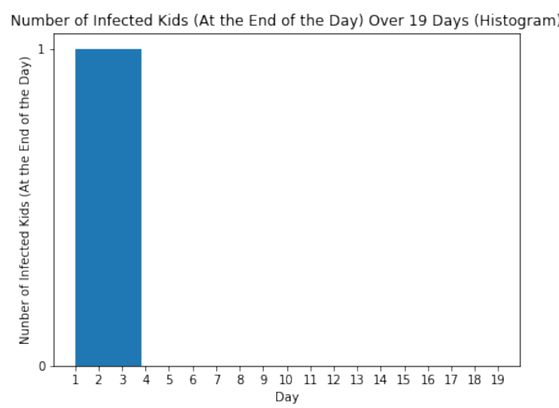
	names	infected	infection_days_left	immunity_countdown	random_no
0	Tommy	0	0	0	0.446146
1	Sophia	0	0	0	0.782749
2	Liam	0	0	0	0.990472
3	Olivia	0	0	0	0.300248
4	Noah	0	0	0	0.143006
5	Riley	0	0	1	NaN
6	Jackson	0	0	0	0.541559
7	Emma	0	0	0	0.974740
8	Aiden	0	0	0	0.636604
9	Ava	0	0	0	0.993913
10	Elijah	0	0	0	0.546071
11	Isabella	0	0	0	0.526426
12	Grayson	0	0	0	0.135428
13	Aria	0	0	0	0.355705
14	Lucas	0	0	0	0.026219
15	Aaliyah	0	0	0	0.160395
16	Oliver	0	0	0	0.745637
17	Amelia	0	0	0	0.030400
18	Cayden	0	0	0	0.366543
19	Mia	0	0	0	0.862346
20	Jayden	0	0	0	0.692678

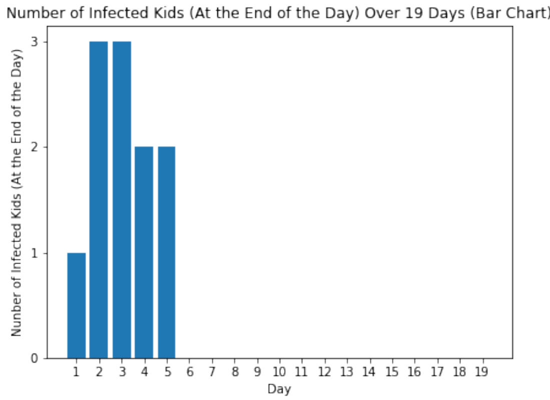


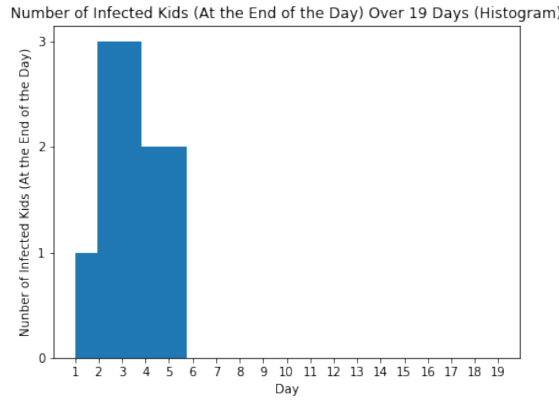
Now, we will repeat the simulation 10 times.

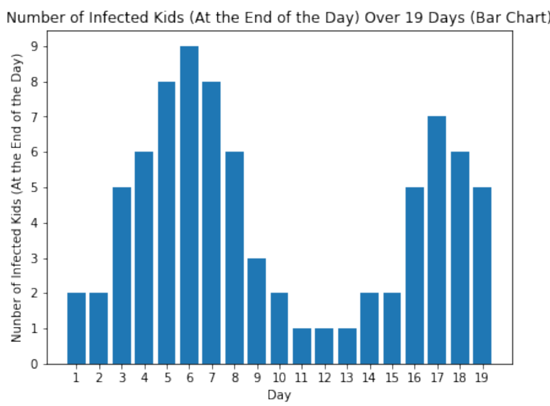
```
[5]: no_runs = 10

results = np.zeros((no_runs,19))

for k in range(0,no_runs):
    r = simulate_flu(k,0)
    results[k,:] = r
```

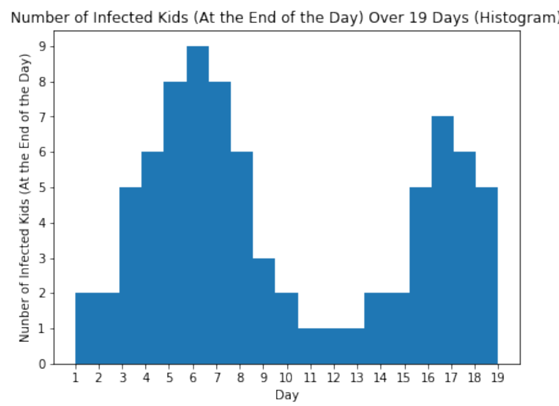


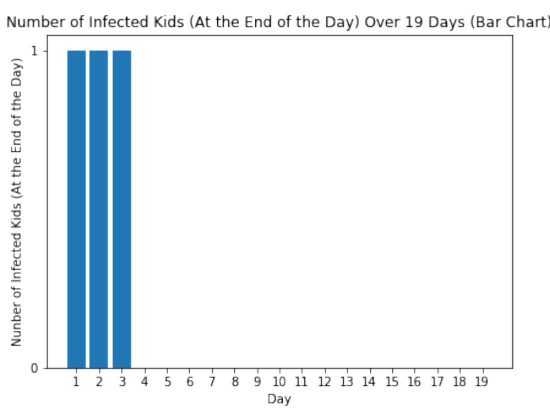

Number of Infected Kids (At the End of the Day) Over 19 Days (Bar Chart)
Nunber of Infected Kids (At the End of the Day)
Day

Number of Infected Kids (At the End of the Day) Over 19 Days (Histogram)
Nunber of Infected Kids (At the End of the Day)
Day

Number of Infected Kids (At the End of the Day) Over 19 Days (Bar Chart)
Number of Infected Kids (At the End of the Day)
Day

Number of Infected Kids (At the End of the Day) Over 19 Days (Histogram)
Number of Infected Kids (At the End of the Day)
Day

Number of Infected Kids (At the End of the Day) Over 19 Days (Bar Chart)
Nunber of Infected Kids (At the End of the Day)
Day

Number of Infected Kids (At the End of the Day) Over 19 Days (Histogram)
Nunber of Infected Kids (At the End of the Day)
Day

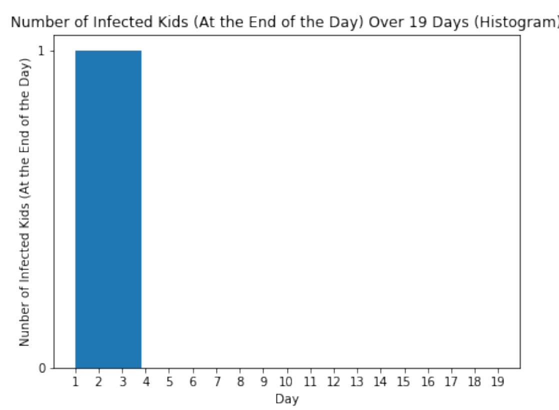

Number of Infected Kids (At the End of the Day) Over 19 Days (Bar Chart)
Nunber of Infected Kids (At the End of the Day)
Day

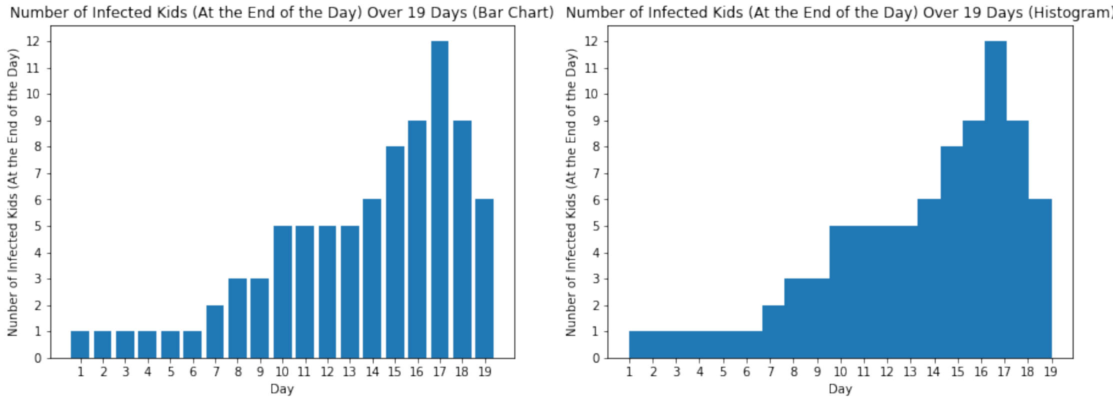

Number of Infected Kids (At the End of the Day) Over 19 Days (Histogram)
Nunber of Infected Kids (At the End of the Day)
Day

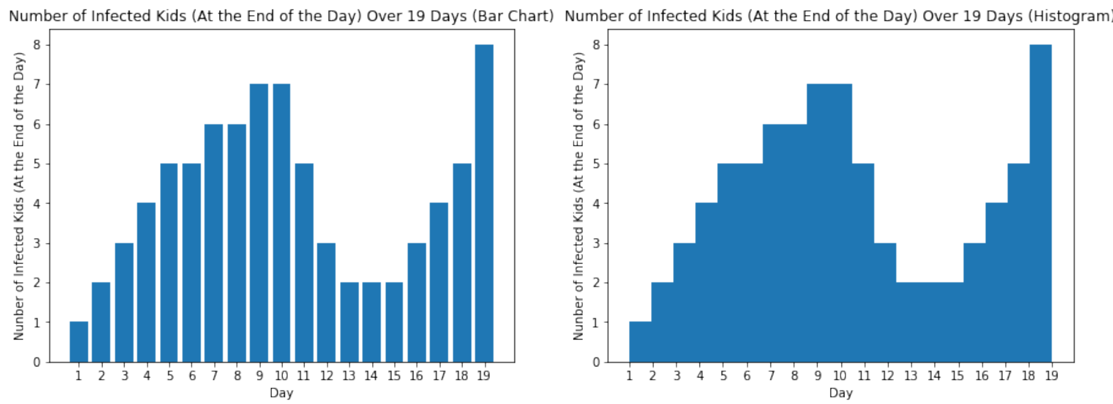

Number of Infected Kids (At the End of the Day) Over 19 Days (Bar Chart)
Nunber of Infected Kids (At the End of the Day)
Day

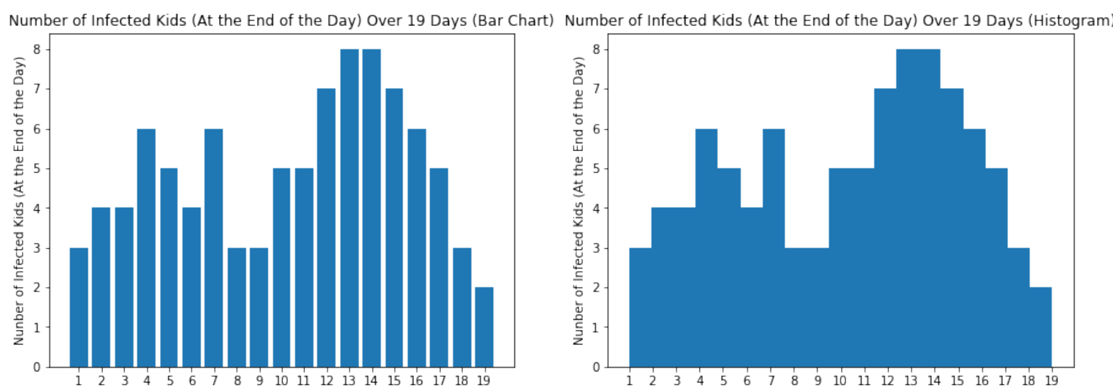

Number of Infected Kids (At the End of the Day) Over 19 Days (Histogram)
Nunber of Infected Kids (At the End of the Day)
Day


Number of Infected Kids (At the End of the Day) Over 19 Days (Bar Chart)
Nunber of Infected Kids (At the End of the Day)
Day


Number of Infected Kids (At the End of the Day) Over 19 Days (Histogram)
Nunber of Infected Kids (At the End of the Day)
Day





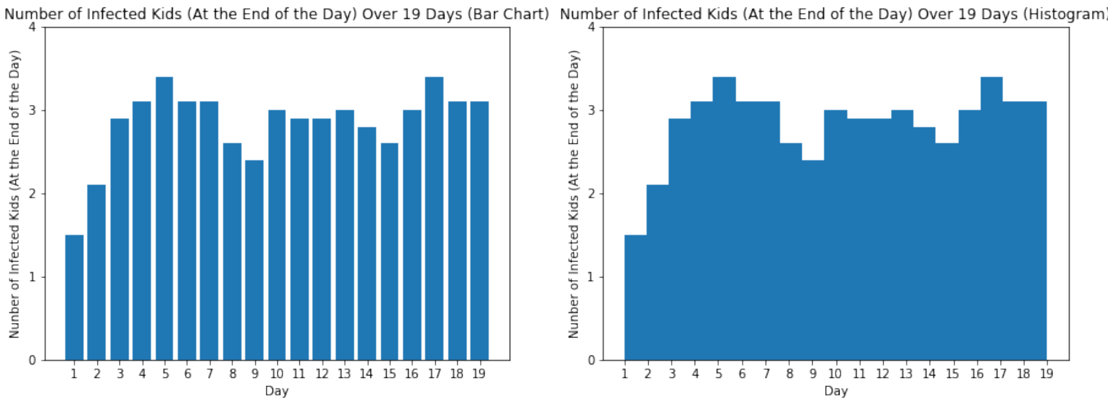


Now we take the mean of each day for the 10 runs.

[6]:
```
print("(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19␣
↪(10 runs)")
```

```
print(list(np.mean(results, axis = 0)))

plot_graphs(np.arange(1,20,1),list(np.mean(results, axis = 0)))
```

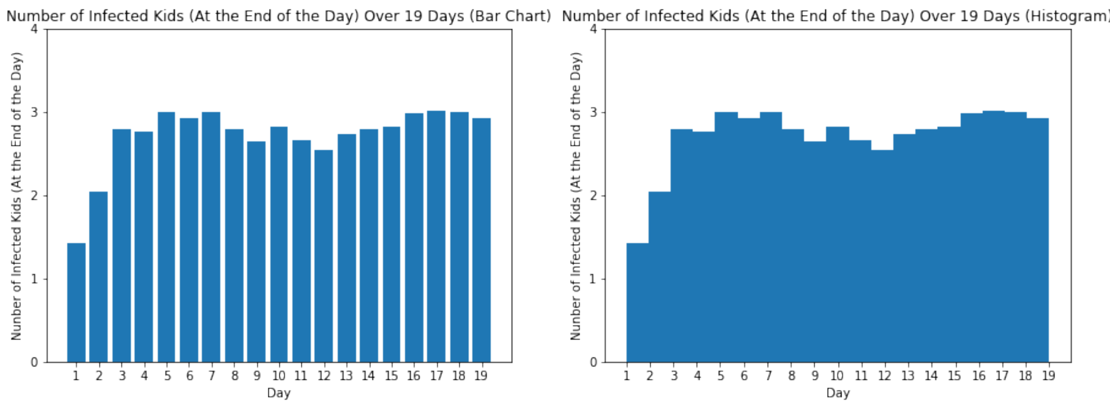
```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19 (10
runs)
[1.5, 2.1, 2.9, 3.1, 3.4, 3.1, 3.1, 2.6, 2.4, 3.0, 2.9, 2.9, 3.0, 2.8, 2.6, 3.0,
3.4, 3.1, 3.1]
```



Then, we repeat this 50 times.

[7]:
```
no_runs = 50
results = np.zeros((no_runs,19))

for k in range(0,no_runs):
    r = simulate_flu(k,0,0)
    results[k,:] = r

print("(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19␣
↪(50 runs)")
print(list(np.mean(results, axis = 0)))

plot_graphs(np.arange(1,20,1),list(np.mean(results, axis = 0)))
```

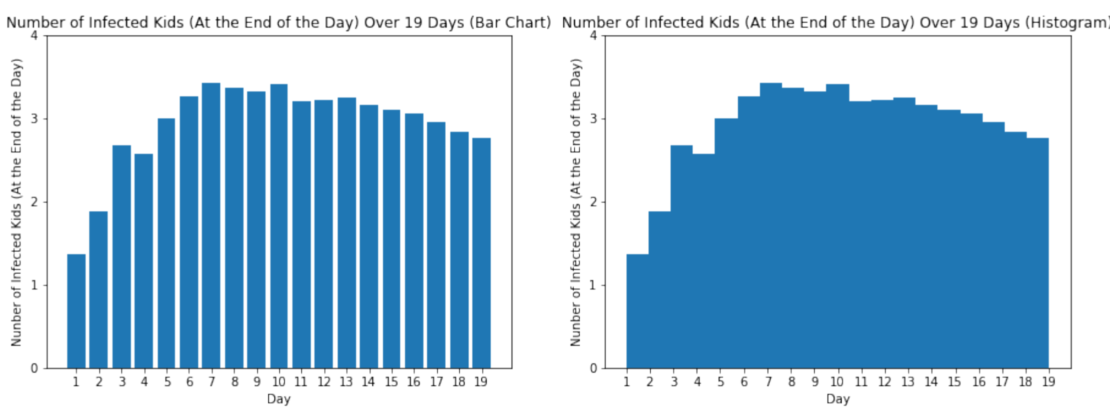
```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19 (50
runs)
[1.42, 2.04, 2.8, 2.76, 3.0, 2.92, 3.0, 2.8, 2.64, 2.82, 2.66, 2.54, 2.74, 2.8,
2.82, 2.98, 3.02, 3.0, 2.92]
```



Lastly, we run our simulation 100 times.

[8]:
```python
no_runs = 100
results = np.zeros((no_runs,19))

for k in range(0,no_runs):
    r = simulate_flu(k,0,0)
    results[k,:] = r

print("(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19␣
 ↪(100 runs)")
print(list(np.mean(results, axis = 0)))

plot_graphs(np.arange(1,20,1),list(np.mean(results, axis = 0)))
```

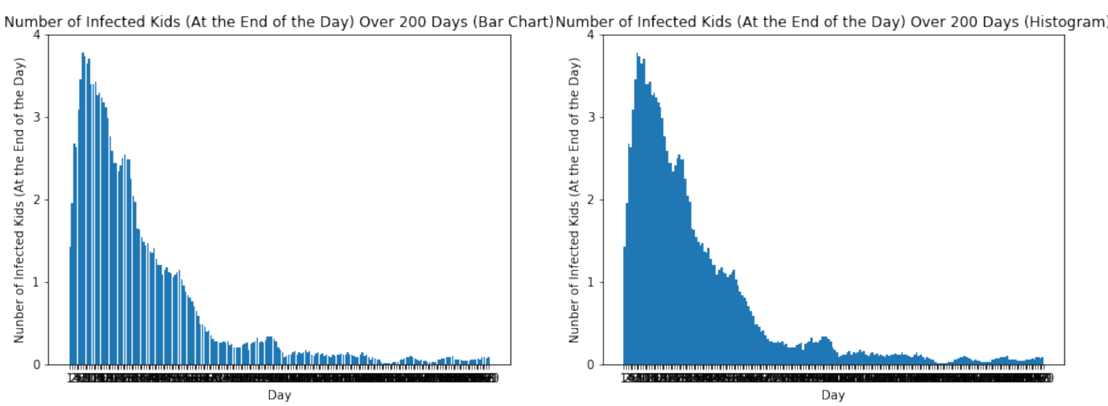
```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-19 (100
runs)
[1.36, 1.88, 2.67, 2.57, 3.0, 3.27, 3.43, 3.37, 3.32, 3.41, 3.21, 3.22, 3.25,
3.16, 3.1, 3.06, 2.96, 2.84, 2.76]
```



Noticing that there is a downwards slope after day 10, we decided to run our simulation over 200 days (instead of 19) to see if the epidemic eventually dies out.

```
[9]: d = 200
     no_runs = 100
     results = np.zeros((no_runs,d))

     for k in range(0,no_runs):
         r = simulate_flu(k,0,0,d)
         results[k,:] = r

     print("(Estimated) Expected/Mean Number of Kids that are Infected on Days␣
      ↪1-"+str(d)+" (100 runs)")
     print(list(np.mean(results, axis = 0)))

     plot_graphs(np.arange(1,d+1,1),list(np.mean(results, axis = 0)))
```

```
(Estimated) Expected/Mean Number of Kids that are Infected on Days 1-200 (100
runs)
[1.42, 1.96, 2.68, 2.63, 3.09, 3.45, 3.78, 3.73, 3.65, 3.7, 3.4, 3.4, 3.43,
3.27, 3.29, 3.23, 3.18, 3.12, 2.99, 2.76, 2.58, 2.44, 2.44, 2.33, 2.41, 2.5,
2.55, 2.49, 2.49, 2.25, 2.05, 1.97, 1.65, 1.63, 1.54, 1.48, 1.44, 1.47, 1.37,
1.35, 1.41, 1.28, 1.21, 1.2, 1.09, 1.15, 1.18, 1.12, 1.1, 1.05, 1.08, 1.11,
1.14, 1.03, 0.95, 0.88, 0.84, 0.8, 0.76, 0.71, 0.64, 0.58, 0.48, 0.49, 0.46,
0.4, 0.41, 0.35, 0.3, 0.28, 0.28, 0.26, 0.26, 0.28, 0.26, 0.27, 0.23, 0.25, 0.2,
0.2, 0.21, 0.21, 0.23, 0.24, 0.26, 0.18, 0.24, 0.26, 0.27, 0.32, 0.26, 0.28,
0.26, 0.29, 0.34, 0.33, 0.33, 0.3, 0.28, 0.21, 0.17, 0.14, 0.09, 0.1, 0.12,
0.12, 0.14, 0.16, 0.12, 0.15, 0.13, 0.15, 0.18, 0.15, 0.16, 0.11, 0.1, 0.13,
0.14, 0.12, 0.13, 0.1, 0.11, 0.1, 0.09, 0.11, 0.1, 0.12, 0.11, 0.13, 0.12, 0.11,
0.15, 0.11, 0.11, 0.1, 0.09, 0.08, 0.11, 0.14, 0.1, 0.12, 0.08, 0.06, 0.08,
0.07, 0.06, 0.06, 0.03, 0.01, 0.01, 0.01, 0.01, 0.01, 0.02, 0.02, 0.03, 0.05,
0.06, 0.07, 0.08, 0.08, 0.1, 0.08, 0.07, 0.06, 0.04, 0.05, 0.04, 0.04, 0.02,
0.02, 0.03, 0.02, 0.02, 0.05, 0.05, 0.07, 0.07, 0.08, 0.08, 0.08, 0.1, 0.06,
0.06, 0.05, 0.05, 0.04, 0.04, 0.04, 0.04, 0.05, 0.06, 0.06, 0.07, 0.06, 0.09,
0.09, 0.07, 0.09]
```



[10]:
```
# Create figure and axes
fig = plt.figure(figsize=(15,5))
ax = fig.add_subplot(1, 1, 1)

# Plot histogram
ax.hist(np.arange(1,d+1,1), weights=list(np.mean(results, axis = 0)), bins = d)
ax.axhline(y=0.10, color = 'g')
ax.axhline(y=0.5, color = 'r')
ax.set_xticks(np.arange(1, d, 10))
ax.set_title("Number of Infected Kids (At the End of the Day) Over "+str(d)+"␣
↪Days (Histogram)")
ax.set_xlabel("Day")
ax.set_ylabel("Nunber of Infected Kids (At the End of the Day)")
plt.show()
```