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Fixed Income Analysis

# CFA二级培训项目


讲师：何　旋

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# Fixed Income整体框架




二级

- Risk
  - Yield curve & yield curve risk R35
  - Credit Risk：定量模型（一级主要是定性介绍）R38
    信用风险管理工具 R39
- Valuation：含权债券的定价（binomial tree）R36、R37

# Reading 35

## Term Structure and Interest Rates Dynamics

★★（整章都非常重要）

# Benchmark curve

## Spot curve

### Forward rate model

① Spot rate → Forward rate 计算 $[1+r(T^*+T)]^{(T^*+T)} = [1+r(T^*)]^{T^*}[1+f(T^*,T)]^{T}$

② Relationship：

- ✓ upward-sloping spot curve → upward-sloping forward curve
- ✓ When the spot curve is upward sloping, the forward curve will lie above the spot curve.

## Forward curve ★★

### Forward pricing model

$$F(T^*,T) = \frac{P(T^*+T)}{P(T^*)}$$ No-arbitrage

### Active bond portfolio management

① Yield curve movement

- ✓ If future spot rate will be lower than forward rate, forward contract value increase.

② Riding the yield curve

- ✓ upward-sloping curve: an investor will purchase bonds with maturities longer than his investment horizon.

## Par curve → Bootstrapping 计算: 已知par rate算spot rate

## YTM, spot rate and return on bond

YTM相当于是spot rate的平均数

## The swap rate curve★★ →

优点（和government bond yield比）

① reflect the credit risk of banks

② not regulated, more comparable in different countries

③ many maturities

## Spread ★★

Spread =yield $_{\text{subject bond}}$-yield $_{\text{benchmark}}$

| | 计算 | 性质（spread反映的风险） |
|---|---|---|
| Swap spread | Swap rate –Treasury yield<br>Note: Treasury bond → ***same maturity, on-the-run*** | ① Default risk<br>② 但是，有些期限swap可能liquidity更好 |
| I-spread | yield $_{\text{subject bond}}$ -Swap rate<br>Notes: ***same maturity*** | ① Credit risk<br>② Liquidity risk |
| Z-spread | 试错法<br>$P_{market}=\frac{C}{(1+S_1+Z)^1}+\frac{C}{(1+S_2+Z)^2}+\frac{C+prin}{(1+S_3+Z)^3}$ | ① Credit risk<br>② Liquidity risk<br>③ Option risk<br>④ 对含权债券和不含权债券的比较不合适 |
| TED spread | LIBOR – T-bill rate<br>Note: same maturity, 常见3个月 | Default risk in the ***banking system*** |
| LIBOR-OIS spread | LIBOR – OIS rate<br>Note: OIS rate相当于银行间隔夜借贷成本，credit risk最小 | ① high: concerns about creditworthiness<br>② Low: high liquidity |

## Term Structure of Interest Rates★★

目的：解释yield curve形状

注意辨析

| Summary | Description (性质) | 解释yield curve形状 |
|---|---|---|
| Pure (Unbiased ) Expectations Theory | ① Forward rates are solely a function of expected future spot rates<br>② Investor should earn the same return over a given investment horizon<br>③ ***risk neutrality*** | ① Upward slope: rise<br>② Downward slope: fall<br>③ Flat yield: remain |
| Local Expectation Theory | ① 类似Unbiased Expectations Theory<br>② ***only for short periods: risk-neutrality***<br>③ longer periods: risk premiums exist | |
| Liquidity preference theory | ① ***forward rates: biased estimates*** of expectation of future rates<br>② include a ***liquidity premium***<br>③ liquidity premium:larger during periods of greater economic uncertainty | Upward slope:<br>① Future rates rise<br>② rates remain constant, but the addition of the liquidity premium |
| Segmented Market Theory | Yield at each maturity is determined ***independently*** of the yields at other maturities | determined by the preferences of borrowers and lenders |
| Preferred habitat theory | ① forward rates: expected future spot rates ＋premium<br>② ***premium is not related to maturity***<br>③ Premium (incentive): 为了使投资者***shift from preferred habitats*** | |

## Modern term structure models ★

定量描述Δr

- 基本不会考计算
- 注意三个模型之间的辨析

| | | 公式 | 性质 |
|---|---|---|---|
| Equilibrium Models | Cox—Ingersoll—Ross Model (CIR) | $dr = a(b-r)dt + \sigma\sqrt{r}dz$ | ① drift term & random term<br>② Mean-reverting (b),a是reverting speed<br>③ Volatility increases with r |
| | Vasicek model | $dr = a(b-r)dt + \sigma dz$ | ① Difference from CIR: volatility不随r而上升<br>② Disadvantage: r may be negative |
| Arbitrage-free model | Ho-Lee Model | $dr_t = \theta_t dt + \sigma dz_t$ | Advantage: match current market price and current yield curve |

## Yield curve factor model ★★

概念、辨析、计算

### Yield curve factor

| | Description | 重要程度 |
|---|---|---|
| Level | Upward/Downward shift (同上同下) | 77% |
| Steepness | Long-term rate和short-term rate变动幅度不同 | 17% |
| Curvature | Long-term & short-term rate和middle-term rate变动幅度不同 | 3% |
| Managing yield curve risk: $\frac{\Delta P}{P} \cong -D_L \Delta x_L - D_S \Delta x_s - D_C \Delta r_C$ | | |

### Yield curve risk

**Managing yield curve risk:**

- Effective duration: parallel shift
- Key rate duration: non-parallel shift (***shaping risk***).

### Yield curve volatility

- Important for securities with embedded options
- ***Short-term interest rates*** are generally ***more volatile*** than are long-term rates.

# Reading 36

## The Arbitrage-free Valuation Framework

## Introduction

**Arbitrage opportunity** ➡
- **Value additivity**
  - Stripping
  - Reconstitution
- **Dominance**

了解

**Introduction of Arbitrage free valuation (spot curve)** ➡

$$P_{market}=\frac{C}{(1+S_1)^1}+\frac{C}{(1+S_2)^2}+\frac{C+prin}{(1+S_3)^3}$$

了解

不适合含权债券

## Binomial interest rate tree ★★

计算

**求二叉树**

① equal probability

② middle forward rate (f): $[1+r(T^*+T)]^{(T^*+T)}=[1+r(T^*)]^{T^*}[1+f(T^*,T)]^{T}$

③ $i_{1,H}=i_{1,L}e^{2\sigma}\quad i_{2,HL}=i_{2,LL}e^{2\sigma}\quad i_{2,HH}=i_{2,LL}e^{4\sigma}$

④ The interest rate tree should generate arbitrage-free values for the benchmark security.

**Option-free bond valuation**

**Pathwise valuation:** n period, $2^{n-1}$ paths → 求算术平均

## Monte Carlo simulation ★★

不会考计算，重点掌握性质

① Binomial tree: backward induction → 适用callable, putable

② Cash flow of MBS: path dependency

③ Advantage of MCS: 解决path dependency → 适用MBS

## 总结：bond定价方法 ★

$$P_0 = \sum_{i=i}^{N} \frac{CF_i}{(1+r)^i}$$

| | Option-free bond | Bond with embedded option |
|---|---|---|
| CF | 时间、金额确定 | 不确定(取决于将来r的变化) |
| r | Single yield (YTM)：$bond\ price = \frac{CPN_1}{(1+YTM)} + \frac{CPN_2}{(1+YTM)^2} + \cdots + \frac{CPN_N + Par}{(1+YTM)^N}$ | |
| | Arbitrage-free: no-arbitrage price$= \frac{CPN_1}{(1+S_1)} + \frac{CPN_2}{(1+S_2)^2} + \cdots\cdots + \frac{CPN_N + Par}{(1+S_N)^N}$ | |
| | Binomial tree: backward induction (考虑将来r变化) → callable, putable | |
| | Monte Carlo Simulation: 解决path dependency → MBS | |

# Reading 37

## Valuation And Analysis: Bonds With Embedded Options

★★（整章都非常重要）

## 含权债券种类

概念

| | |
|---|---|
| Simple options | Callable bond |
| | Putable bond |
| | Extendible bond (可以看成是putable bond): 比如2年可以扩展成3年，相当于3年期putable，2年可执行 |
| Complex options | Estate put： contingent put option |
| | Sinking fund bonds (sinkers) |

## Valuation of callable and putable bonds ★★

**Value** ➡

| | |
|---|---|
| 计算<br>Binomial tree | $V_{callable}$: 高过strike price的价格只能取strike price |
| | $V_{putable}$: 低于strike price的价格只能取strike price |
| 计算$V_{option}$ | $V_{callable} = V_{noncallable} - V_{call}$ |
| | $V_{putable} = V_{nonputable} + V_{put}$ |
| Volatility<br>对value的影响 | 会影响binomial tree：spread out |
| | Volatility↑ → $V_{call}$↑ → $V_{callable}$↓ |
| | Volatility↑ → $V_{put}$↑ → $V_{putable}$↑ |

**OAS** ➡

| | |
|---|---|
| 计算 | Binomial tree（试错法） |
| Volatility<br>的影响 | Callable： Volatility↑ → $OAS_{call}$↓ |
| | Putable： Volatility↑ → $OAS_{put}$↑ |
| OAS：可以对比不含权债券和含权债券 | |

## Yield curve risk

| Effect of yield curve changes | | |
|---|---|---|
| Level | r↓→ $\Delta V_{callable} < \Delta V_{straight}$ | |
| | r↑ →$\Delta V_{putable} < \Delta V_{straight}$ | |
| Shape | Call | ✓ r↓ → $V_{call\ option}$ ↑<br>✓ $V_{call\ option}$ will be lower for upward sloping yield curve |
| | Put | ✓ r↑ → $V_{put\ option}$ ↑<br>✓ $V_{put\ option}$ will be lower as an upward-sloping yield curve flattens |

| 含权债券duration | 性质 | 计算 |
|---|---|---|
| ED | ✓ Callable、putable < straight bond<br>✓ r↑ → ED putable↓<br>✓ r↓ → ED callable↓ | $ED = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$ |
| EC | ✓ Callable： r↓→ negative convexity<br>✓ Putable： r↑ → more convexity | $EC = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{BV_0 \times \Delta y^2}$ |
| One-side duration | ✓ Callable： lower one-sided down-duration than one-sided up-duration<br>✓ Putable： larger one-sided down-duration than one-sided up-duration | 不要求 |
| Key rate duration | Callable/putable： exercise date和maturity date最大<br>其他性质了解 | 不要求 |

## Capped & floored floater ★★

| | |
|---|---|
| 计算<br>(binomial tree) | Option-free floater: price = par<br>Cap： coupon>cap, 只能取cap<br>Floor： coupon<floor, 只能取floor<br>注意： 折现率还是用原来的利率 |
| 计算value of cap/floor | Value of capped floater= Value of 'straight' bond – Value of embedded cap<br>Value of floored floater= Value of 'straight' bond + Value of embedded floor |
| Ratchet bonds | capped floater： extreme protection,cap = current coupon rate |

## Convertible bond ★★

| | |
|---|---|
| 计算 | ✓ **Conversion price & market conversion price**<br>• Conversion price = ***bond issue price***/conversion ratio<br>• Market conversion price = ***market price of bond***/conversion ratio<br>✓ **Conversion value、straight value、minimum value**<br>• Conversion value = market price of stock × conversion ratio<br>• Straight value is the value of the bond if it were not convertible<br>• Minimum value = ***max {Conversion value, Straight value}***<br>✓ **Market conversion premium、premium over straight value**<br>• The market conversion premium per share = market conversion price – market price<br>• Market conversion premium ratio = market conversion premium per share/market price of stock<br>• $\text{Premium over straight value} = \left(\frac{\text{Market price of convertible bond}}{\text{straight value}}\right) - 1$ |
| 性质 | ✓ **Fixed-income equivalent(busted convertible):** Market stock price < ***Conversion price***<br>✓ **Common stock equity:** Market stock price > ***Conversion price***<br>✓ **Hybrid security** |
| **Value** | **Convertible bond value =** Straight value of bond***+ Value of the call option on the stock*** |

# Reading 38

## Credit Analysis Models

## 衡量Credit risk的指标

基本概念：PD、LGD、Recovery rate　一级的概念，二级基本不考

**Expected loss** ➡

| PV (EL): maximum amount investor pay an insurer | |
|---|---|
| **Two adjustments★** | Time value adjustment: PV(EL) < EL<br>Risk-neutral probabilities: PV(EL) > EL |
| **计算★★** | PV(risk-free) – PV (risky) |

## 利用Credit spread计算

| Time | Risk-free rate | Credit spread | Total yield | Cash flow | PV(risk-free) | PV(risky) | Difference |
|---|---|---|---|---|---|---|---|
| 0.5 | 0.11% | 0.03% | 0.14% | 30 | $29.98 | $29.98 | $0.00 |
| 1.00 | 0.16% | 0.07% | 0.23% | 30 | $29.95 | $29.93 | $0.02 |
| 1.50 | 0.21% | 0.08% | 0.29% | 30 | $29.91 | $29.87 | $0.04 |
| 2.00 | 0.22% | 0.09% | 0.31% | 30 | $29.87 | $29.81 | $0.06 |
| 2.50 | 0.27% | 0.09% | 0.36% | 30 | $29.80 | $29.73 | $0.07 |
| 3.00 | 0.31% | 0.10% | 0.41% | 1,030 | $1,020.47 | $1,017.41 | $3.06 |
| | | | | total | $1,169.97 | $1,166.73 | $3.24 |

## 四大模型 ★

### 1. Credit Scoring: 重要缺点

- Do not explicitly take into account current economic conditions. i.e. ***do not improve with the economy.***
- ***Pressure from users*** of credit scores (lenders) to prioritize stability in scores over time
- Do not take into account differing probabilities of default for ***different loans taken out by the same borrower.***

### 2. Credit Rating: 重要优缺点

- Strengths of credit ratings
  - ✓ Simple to understand
  - ✓ Reduce volatility in the debt market
- Weaknesses of credit ratings
  - ✓ ***Stability*** in credit ratings comes at the expense of a reduction in correlation with default probability
  - ✓ ***Do not adjust with the business cycle***
  - ✓ In ***issuer-pays model***, there exists the conflicts of interest.

## 3. Structure model

| | |
|---|---|
| 原理<br>(类似于option) | • Stock: long公司资产的call option<br>• Bond: short put option |
| Assumption | • Company's ***assets are traded in a frictionless market*** with return and variance .<br>• The risk-free interest rate (r) is ***constant.***<br>• The company has a ***simple balance sheet structure.*** |
| 优缺点 | Strengths of structural models<br>• Provide ***option analogy*** to understand probability of default and loss given default and can be estimated using current market prices.<br>Weaknesses of structural models<br>• ***Model assumptions*** of simple balance sheet and traded assets are not realistic.<br>• Estimation procedures do ***not consider business cycle.*** |

## 4. Reduced Form Model

| | |
|---|---|
| Assumption | • Company has a ***zero-coupon bond***, and it trades in frictionless and arbitrage-free markets.<br>• The ***risk-free interest rate (r) and the state of the economy are stochastic.*** |
| 优缺点 | Strengths of structural models<br>• Since model inputs are observable, ***historical estimation procedures*** can be used.<br>• Credit risk is a allowed to ***fluctuate with the business cycle.***<br>• Reduced form models do not require specification of the company's balance sheet structure.<br>Weaknesses of structural models<br>• The ***hazard rate estimation procedures*** may not be valid. |

## Credit analysis of ABS ★ 结论

➢ **Credit analysis of ABS is different from a corporate bond.**

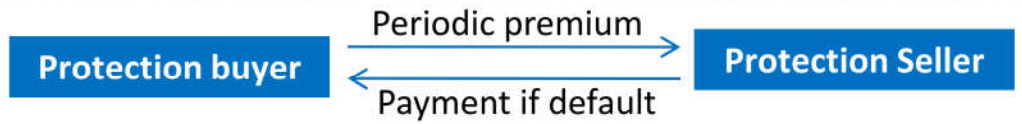
- The ***cash flow characteristics*** of an ABS differ from corporate bond.
- To value an ABS, ***either a reduced form or a structural model*** can be used.
- The credit risk metric of probability of default does not apply to an ABS; we instead use the ***probability of loss.***

# Reading 39

## Credit Default Swaps

# Basic Definitions and Concepts

**Definitions** ➡




CDS = ***insurance contract = put option*** ★★

## Important Features of CDS

- ISDA Master Agreement
- Notional principal
- CDS spread and CDS Coupon ★ → ***upfront payment***
- Credit Events ★: ***Bankruptcy, Failure to pay and Restructuring***

## Types of CDS ★★

- Single-Name CDS: one ***specific borrower***
  - ***cheapest-to-deliver obligation:*** be purchased and delivered at the lowest cost but has the same seniority
- Index CDS: ***multiple issuers***
- Tranche CDS: ***only up to pre-specified levels of losses***

**Settlement** ➡

- Physical settlement
- Cash settlement ➡ Payout amount =payout ratio × notional principal
  Payout ratio = 1- recovery rate (%)

计算★★

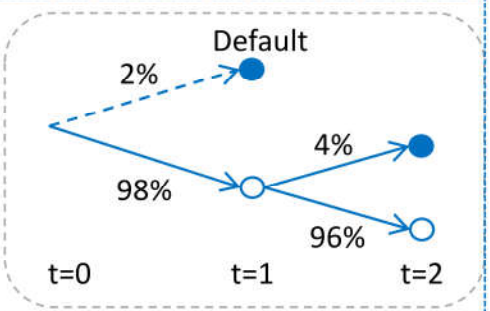
判断: Settlement Preference → 因为cash settlement要遵循***cheapest-to-deliver obligation***

# Basics of Valuation and Pricing

## 影响CDS定价因素★

Probability of default
Loss given default

➡ **Hazard rate:** The probability of default ***given*** that default has ***not already occurred***. A ***conditional probability of default***




**CDS Pricing ★★** ➡

Upfront payment (paid by protection buyer)=PV(Protection leg)- PV(Premium leg)
Upfront premium % (paid by protection buyer) ≈ (CDS spread – fixed coupon) × duration
Price of CDS (per $100 notional) ≈ $100-upfront premium (%)

**Valuation Changes in CDS ★➡**

Profit for protection buyer ≈ change in spread × duration × notional principal
Profit for protection buyer (%) ≈ change in spread (%) ×duration

**The Credit Curve ★** ➡ The credit spreads for a range of maturities

## Applications of CDS

掌握交易方法★

| Manage Credit Exposures | Valuation Differences and Basis Trading | CDO |
|---|---|---|
| **Basic application:** lender → reduce its credit exposure; CDS seller → adds credit exposure. | **Basis trade**: exploit the difference in credit spreads between bond markets and the CDS market. | **Synthetic CDO**: has ***similar credit risk exposure*** to that of a ***cash CDO*** |
| **Naked CDS:** a party with no exposure to the reference entity | **leveraged buyout**: purchase both the stock and CDS protection | |
| **Long/short trade** ★★<br>• **A curve trade:** ***curve-steepening trade*** (buying protection in a long-term CDS and selling protection in a short-term CDS) & ***curve-flattening trade*** | | |


Thank
You!