# S&SE – Software and Systems Engineering

# Assignment #3: Stochastics and Stochastic Processes

## Group 4

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## Task 1:

Variance measures how widely spread the values are likely to be and is the average of the squared difference from the mean distribution. Whereas standard deviation is the square root of the variance.

## Task 2:

### Example: Smartphone Battery Life

A smartphone's battery life exhibits a memoryless distribution because the probability of the battery discharging within a certain time frame is not influenced by its past behavior and Each unit of time has an independent probability p of the smartphone battery discharging, means that, the amount of charge consumed in a specific time interval is independent of what happened before. Whether the battery has been in use for a while or has been idle, the probability of it discharging in the next moment will be independent of the previous event.

**Possible error from the theoretical assumption could be:**

Usage Patterns:

Variations in usage patterns, such as heavy multimedia consumption or prolonged periods of inactivity, might affect the battery discharge rate and can make the battery run out at different speeds. This usage-dependent variability introduces a deviation from the constant discharge rate assumed in the memoryless model.

## Task 3:

### a) Average Lifetime of One Light:

The average lifetime (E[X]) of an exponentially distributed variable with rate $\lambda$ is given by,

$E[X]=1/\lambda$

Given, the failure rate ($\lambda$) is 0.5 per year.

So, the average lifetime of one light is.

$E[X] = 1/0.5$

E[X] = **2 years.**

### b) Average Time Until the First Light Fails:

The time until the first failure in a system of independent exponentially distributed variables is also exponentially distributed.

The rate of the "system" is the sum of the individual rates, so the rate ($\lambda_{system}$) is $4\times0.5=2$ per year.

The average time until the first light fails (E[T1]) is given by.

$E[T1] = 1/\lambda_{System}$

Therefore,

**E[T1] = 1/2 Year.**

### c) Average Time Until All Lights Have Failed:

The time until all lights fail in a system is the maximum of the individual lifetimes.

The minimum rate of the system is still 0.5 per year (the rate of the individual light with the highest failure rate).

The average time until all lights have failed E[Tall]) is given by,

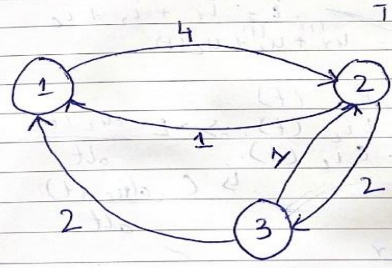
E[Tall]=1/Minimum rate

Therefore,
E[Tall]=1/0.5
E[Tall]=**2 years**.
**a)** The average lifetime of one light is **2 years.**
**b)** The average time until the first light fails is **0.5 years.**
**c)** The average time until all lights have failed is **2 years.**

## Task 4:

**a)**

$$Q = \begin{bmatrix} -4 & 4 & 0 \\ 1 & -3 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$




Transition rate diagram of the above generator matrix.

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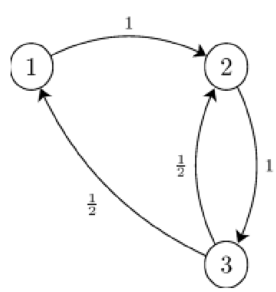
**b)**




a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ → Transition matrix from the diagram.

$$Q = \lim_{t \to 0} \frac{P(t) - I}{t}$$

Q is the generator matrix
P(t) = The transition matrix
I = Identity matrix

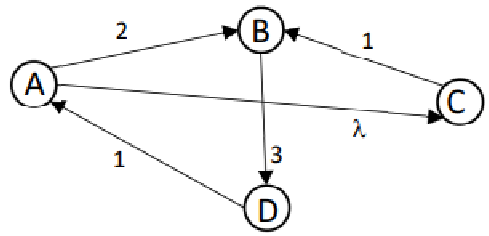
If $t \to 0$, the generator matrix will be as follows.

$$P(t) - I = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1/2 & 1/2 & -1 \end{bmatrix}$$

## Task 5:

**a)**




Task 5:

A=> Step 1: Generator Matrix

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$$Q = \begin{bmatrix} -6 & 2 & 4 & 0 \\ 0 & -3 & 0 & 3 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

Probability Vector:

$\pi = (\pi_A, \pi_B, \pi_C, \pi_D)$

System of eq.

$0 = \pi \cdot Q$

$0 = -6\pi_A + \pi_D$ — (i)

$0 = 2\pi_A - 3\pi_B + \pi_C$ — (ii)

$0 = 4\pi_A - \pi_C$ — (iii)

$0 = 3\pi_B - \pi_D$ — (iv)

$1 = \pi_A + \pi_B + \pi_C + \pi_D$ — (v)

From eq. (i)

$0 = -6\pi_A + \pi_D$

$6\pi_A = \pi_D$

$\boxed{\pi_A = \frac{1}{6}\pi_D \Rightarrow 0.1666\pi_D}$

From eq. (iv)

$0 = 3\pi_B - \pi_D$

$3\pi_B = \pi_D$

$\boxed{\pi_B = \frac{1}{3}\pi_D \Rightarrow 0.3333\pi_D}$

From eq. (iii)

$0 = 4\pi_A - \pi_C$

$\pi_C = 4\pi_A$

$\boxed{\pi_C = 4(0.1666\pi_D) \Rightarrow 0.6664\pi_D}$

Putting all values in eq. (V)

$(1 = 0.1666\pi_D + 0.3333\pi_D + 0.6664\pi_D + \pi_D)$

$1 = 2.1663\pi_D$

$\pi_D = \frac{1}{2.1663}$

$\boxed{\pi_D = 0.4616}$

$\pi_A = 0.1666\pi_D$

$\pi_A = 0.1666(0.4616)$

$\boxed{\pi_A = 0.0769}$

$\pi_B = 0.3333\pi_D$

$\pi_B = 0.3333(0.4616)$

$\boxed{\pi_B = 0.1538}$

$\pi_C = 0.6664\pi_D$

$\boxed{\pi_C = 0.6664(0.4616) \Rightarrow 0.3076}$

**b)**

B)

Frequency: (From B to D)

$= \pi \cdot T$

$= \pi_B \times 3$

$= 0.1538 \times 3$

Frequency $= 0.4614$